Physics – Uspekhi **42** (7) 733 – 734 (1999)

LETTERS TO THE EDITORS PACS numbers: **32.90.+a**, **42.65.+k**

On the history of developing the theory of tunneling ionization in atoms and ions

V S Popov

In the issue of *Physics – Uspekhi* published in May 1998 there is a review by N B Delone and V P Kraĭnov [1]. I believe myself to have the right, and even to be obliged to advance some remarks concerning the review.

1. The principal formulas presented in Sections 4.1 and 4.2 of the review [1] for the photoelectron energy and angular distributions when tunneling ionization is created in the atomic systems, were obtained in Refs [2, 3] long before papers [4, 5] were published. In addition, the problem concerning atomic level ionization by laser radiation field was solved in Refs [2, 3] not only in the adiabatic domain $\gamma \ll 1$ (low frequency, large intensity of the electric field), as was done in Refs [1, 4, 5] but for arbitrary values of the Keldysh parameter γ. For example, the momentum distribution of electrons in the case of linearly polarized radiation has the form [1]

$$w(\mathbf{p}) = w(0)\exp\left\{-\frac{1}{\omega}\left[c_1(\gamma)p_\parallel^2 + c_2(\gamma)p_\perp^2\right]\right\},$$
$$\hbar = m = e = 1\,, \qquad (1)$$

where $c_1 = \operatorname{arsinh}\gamma - \gamma(1+\gamma^2)^{-1/2}$, $c_2 = \operatorname{arsinh}\gamma$, $\gamma = \omega\sqrt{2E_\mathrm{i}}/F$, and $\mathbf{p} = (p_\parallel, p_\perp)$ is the momentum of an electron after leaving the effective potential barrier. For $\gamma \ll 1$, we have $c_1 = 1/3\gamma^3 + \ldots$, $c_2 = \gamma - 1/6\gamma^3 + \ldots$, and from Eqn (1) it immediately follows that

$$w(\mathbf{p}) = w(0)\exp\left\{-\left[\frac{\omega^2(2E_\mathrm{i})^{3/2}}{3F^3}p_\parallel^2 + \frac{(2E_\mathrm{i})^{1/2}}{F}p_\perp^2\right]\right\}, \qquad (2)$$

with this formula completely coinciding with Eqn (10) in Ref. [1]. The ensuing relationships (12) – (17) are derived from this equation, which determine the width of energy and angular distributions of photoelectrons, etc.

Similarly, the results of Section 4.2 for circularly polarized low-frequency radiation can easily be obtained from more general formulas [2] which can be found in Refs [2, 3] and are valid for arbitrary values of γ.

[1] See Eqn (53) in Ref. [3]. The notation is the same as in the review [1].
[2] The details of calculations performed by independent methods are discussed in Refs [6] and [7], respectively.

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Received 28 April 1999
Uspekhi Fizicheskikh Nauk **169** (7) 819 – 820 (1999)
Translated by V V Lobzin; edited by A Radzig

2. When considering the Coulomb correction (see Section 5.1 which is based on the paper [8]), the authors restricted themselves to the case of a stationary field, for which this correction has been known for a long time [9]. We point out, however, that in Ref. [10] the correction was computed not only for $\gamma \ll 1$, but for a much wider range of the parameter γ. Moreover, the method used in Refs [1, 8] for calculations does not actually differ from the method that was suggested earlier in Ref. [10] (taking into account the Coulomb interaction in accordance with the perturbation theory in the WKB approximation).

It is rather surprising that the relevant references are absent in Ref. [1], the more so as Refs [2, 3, 10] are well known to the authors of the review [1] and in due time were discussed in detail with N B Delone.

3. In Ref. [1], an error creeps into the discussion of Eqn (6) that was obtained in Ref. [3]. This formula is valid not for the states of a hydrogen atom (it is well known that the electron angular momentum l in the hydrogen atom is not conserved when an external electric field F is applied) but for lm-levels of an arbitrary atom without the 'accidental' degeneracy characteristic of the Coulomb interaction. Therefore, in order to apply the formula for an arbitrary atom, there is no need to generalize it with the quantum defect method (contrary to the statement in Refs [1, 11]). Then in Refs [1, 11] the expression for the asymptotic coefficient of the atomic wave function at infinity is used, the formula having been suggested by D R Hartree as early as 1927 (Eqn (7.6) in Ref. [12], see also Refs [13, 14]). Using the Stirling formula to substitute factorials leads to Eqns (7) – (9) in Ref. [1] that the authors of the latter review refer to as 'ADK formulas'.

Of course, it is difficult to object to one or another of the names, however, it is worth noting that the original contribution of the authors of the 'ADK theory' (Ammosov, Delone, and Kraĭnov) is limited to the use of the Stirling formula in order to transform the coefficients that were obtained in Refs [3, 12] and are independent of F. Thus, for example, in the case of a circularly polarized field the probability of ionization of the s-level of neutral atom is given by

$$w(F) = N\kappa^2\epsilon^{1-2n^*}\exp\left[-\frac{2}{3\epsilon}\left(1 - \frac{1}{15}\gamma^2\right)\right], \quad \gamma \ll 1\,, \qquad (3)$$

where $\kappa = \sqrt{2E_\mathrm{i}}$, E_i is the ionization potential, $\epsilon = F/\kappa^3$ is the reduced electric field strength, $n^* = 1/\kappa$ is the effective principal quantum number [12 – 14], and N is a dimensionless numerical coefficient. The 'ADK formula' can be obtained from the above equation by substitution

$$N = \left[\frac{2^{2n^*-1}}{\Gamma(n^*+1)}\right]^2 \to N_\mathrm{ADK} = \frac{1}{8\pi n^*}\left(\frac{4\mathrm{e}}{n^*}\right)^{2n^*} \qquad (4)$$

(the form of the functional dependence of w on F remains the same as in Refs [2, 3], n^* being of the order of unity [3]). Using the Stirling approximation (4) in this case is not justified — it leads to a systematic overestimation of the absolute values of the ionization probability. In particular, for a hydrogen atom (ground state, $n^* = \kappa = 1$) it is suggested to replace the coefficient $N = 4$ in the well-known formula [9] $w(F) = NF^{-1}\exp(-2/3F)$, which is asymptotically accurate in the limiting case of weak fields ($F \ll 1$), by $N_{\mathrm{ADK}} = 2\mathrm{e}^2/\pi \approx 4.70$, which cannot be considered as a good approximation (an error of 18% is introduced; for helium atom it amounts to 25%).

Physics – Uspekhi is a review journal at the level of the highest world standards. It imposes on the authors a responsibility for reliability and objectivity in the published materials. The bibliography in a review should be quite complete and, at least, should not deform the history of the problem considered, especially to the benefit of the authors. Unfortunately, the review by Delone and Kraĭnov does not meet these obvious requirements.

It is worth mentioning the peculiar mode that the authors used in citing Refs [2, 3, 10]. If the papers are received mention anywhere, the citations are connected not with the principal content of the papers, but with some problems of minor importance, and as a rule, the reader is referred to works by Delone and Kraĭnov for details. It is easy to find such examples in Refs [1, 5, 8, 15], in the monograph [16] and elsewhere.

I would like to hope that the remarks advanced will be useful to the readers of *Physics – Uspekhi*.

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[3] Thus, for s-electrons in neutral atoms the value of n^* ranges from 0.744 for He to 1.87 for Cs, whereas for singly charged positive ions $n^* = 2/\kappa$ varies from 0.848 for Li^+ up to 2.22 for Sr^+.

P.S. Comment on the letter by V S Popov entitled "On the history of developing the theory of tunneling ionization in atoms and ions"

We are grateful to V S Popov for his letter. We agree with some of the remarks concerning the matter of priority. However, we believe that in our review the physical essence of the problem is outlined completely.

NB Delone, V P Kraĭnov