METHODOLOGICAL NOTES

PACS number: **42.25.–p**

Can the radiance increase in a noninverted medium?

A V Shepelev

Abstract. **The possibility of the brightness and temperature of scattered radiation exceeding the attendant magnitudes for the incident radiation is predicted for the case of a noninverted medium, based on the analysis of radiation transfer equations. The irreversible nature of the process and impossibility of introducing ray approximation are identified as the necessary conditions.**

The question about the limit of the possible concentration of radiation, which is directly related to the question concerning the transformation of its brightness and temperature, has two traditional formulations. Let the radiation be incident upon an optical system and then leave it (henceforth the subscript 1 refers to the incident and the subscript 2 to the outgoing radiation). The process is assumed to be stationary with no other energy coming into the system.

1. Can the brightness of the outgoing radiation exceed the brightness of the incoming radiation ($B_2 > B_1$)?

2. Can the radiation temperature of the outgoing light flux exceed the temperature of the incoming radiation ($T_2 > T_1$)?

These formulations are in fact almost equivalent because for incoherent radiation with frequency ν the effective temperature is uniquely related to the spectral radiance [1]:

$$\frac{\mathrm{d}B}{\mathrm{d}\nu} = B_\nu = \frac{h\nu}{c\left[\exp(h\nu/kT) - 1\right]} \,. \tag{1}$$

Radiation with a Planck distribution has one and the same temperature at all frequencies. For light with radiance B and an arbitrary spectral distribution of radiance B_ν one introduces the concept of a radiation temperature which is equal to the temperature of Planck radiation with the same radiance B.

The radiation power density I is related to the radiance B as follows [2]

$$I = \int_\Omega B \,\mathrm{d}\Omega \cos\theta = \Omega B_{\mathrm{mean}} \,, \tag{2}$$

where B_{mean} is the mean value of the radiance. The solid angle Ω cannot exceed 4π (in radiation focusing, lenses and mirrors are used to enhance the solid angle). Thus, the affirmative answers to both the first and the second questions mean that the possibility of increasing the radiation power density in improving the optical system, i.e. in providing fulfillment of the condition $B_2 > B_1$, is in a sense 'unlimited'.

A V Shepelev
The Kosygin Moscow State Textile Academy
Malaya Kaluzhskaya ul. 1, 117918 Moscow, Russia
Tel. (7-095) 955-37 54
E-mail: postmast@qopt.ilz.msu.su

Received 23 June 1998, revised 11 September 1998
Uspekhi Fizicheskikh Nauk **169** (2) 213 – 215 (1999)
Translated by M V Tsaplina; edited by A Radzig

However, the answers to the first and the second questions are as a rule negative. The negative answer to the first question is typically substantiated by the Lagrange – Helmholtz law and the Abbe condition which are generalized by the Straubel theorem [3]. According to this theorem, in a nonabsorbing medium we have $n^2\,\mathrm{d}S\,\mathrm{d}\Omega = \mathrm{const}$, where n is the refractive index, and $\mathrm{d}S$ is the area of the beam cross-section. The negative answer to the second question rests on the Liouville theorem on conservation of phase volume, or on the second law of thermodynamics.

In actuality, the Straubel theorem is only valid in the framework of geometrical optics, when at all stages of an optical process the ray approximation, i.e. the feasibility of identifying the line along which the energy flux propagates, can be introduced. In particular, this theorem does not hold for isotropic scattering of radiation. Obviously, in this case Ω_2 has a fixed value equal to 4π and does not depend on Ω_1. The second law of thermodynamics does not forbid the heating of a hot body by the radiation of a colder body but only requires that the total entropy be increased. The Liouville theorem can be applied to the radiation transfer processes only if the light beam trajectory can be determined [4].

So, the laws of statistical physics and optics listed above do not forbid the possibility of increasing the brightness and the temperature of radiation. A universally known example of such an increase is optically pumped lasing [5], and there is no reason to think of this example as unique.

The brightness of incoherent radiation is proportional to the number of photons per mode [6]. This means that to increase the radiance, the process of mode population should be more efficient than its depopulation. From this it follows that no gain in the radiance is possible as long as the principle of detailed balance holds [7]. Thus, the radiance can increase only in an irreversible process, i.e. upon a build-up of the total entropy.

A convenient example of an irreversible optical process, for which the ray approximation is inapplicable, is luminescence because in this type of light scattering the direction of propagation of the secondary radiation is in no way connected with the direction of the incident radiation. The feasibility of heightening the radiance in the presence of luminescence can readily be established from a simple analysis of the radiation transfer equation.

In the simplest case, when the temperature of the medium is much lower than the radiation temperature, the transfer equation has the form [8 – 10]

$$\frac{\mathrm{d}B_2}{\mathrm{d}r} = -\alpha_2(r,s)B_2(r,s) + \rho(r,s) \,, \tag{3}$$

where α_2 is the Bouguer absorption of the medium for the luminescence radiation, ρ is the source power density, and r and s characterize the spatial coordinates and the direction, respectively. It is of importance that the source power density is proportional to the power density I_1 of the incident radiation and to the Bouguer absorption α_1 of this radiation:

$$\rho(r) = \frac{\alpha_1}{4\pi} I_1 G(r) \eta \,. \tag{4}$$

Here $G(r)$ is the geometric factor, and η is the energy efficiency of the luminescence. The power density I_1 and the radiance B_1 of the incoming light flux are related as in Eqn (2), which makes it possible with allowance for Eqns (2) – (4) to assume the radiance of the outgoing light flux to be a functional of the radiance of the incoming light (the frequencies of the incoming and outgoing radiations are different):

$$B_2(r, s) = F\big[B_1(r, s)\big] \,. \tag{5}$$

For a homogeneous isotropic medium equation (3) takes the form

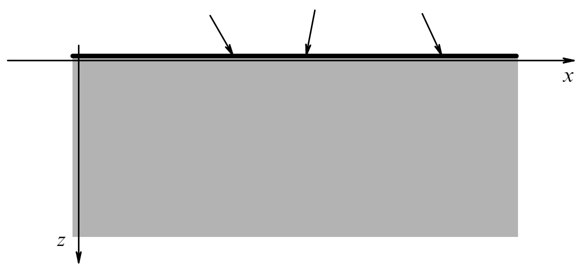
$$\frac{\mathrm{d}B_2}{\mathrm{d}x} = -\alpha_2 B_2 + \rho(x) \tag{6}$$

and its solution is [10]

$$B_2(x) = B_2(0)\exp(-\alpha_2 x) + \int_0^x \rho(x')\exp\big[-\alpha_2(x - x')\big]\,\mathrm{d}x' . \tag{7}$$

When the radiation is incident on a half-infinite medium (see the figure below) with refractive index equal to unity, the geometric factor can be estimated as

$$G(z) = \exp(-\alpha_1 z) \,.$$



For the luminescence radiation propagating along the x-axis near the boundary, expression (7) gives

$$B_2(x) = \eta \frac{\alpha_1}{\alpha_2} \frac{\Omega_1}{4\pi} B_1 \big[1 - \exp(-\alpha_2 x)\big] \,, \tag{8}$$

and for $x \gg \alpha_2^{-1}$ the outgoing radiance is equal to

$$B_2 = \eta \frac{\alpha_1}{\alpha_2} \frac{\Omega_1}{4\pi} B_1 \,. \tag{9}$$

A similar relation also holds for a luminescent optical fiber [11].

Thus, the radiance B_2 of the outgoing light flux exceeds the radiance B_1 of the incoming light flux if the condition holds:

$$\eta \frac{\alpha_1}{\alpha_2} \frac{\Omega_1}{4\pi} > 1 \,.$$

To enhance the radiance, a luminescent medium need not have an inverse population. The quantity $V = (\alpha_1/\alpha_2)\eta$ depends on the parameters of the medium only. For many materials (activated glasses and crystals, semiconductors, dye solutions) the parameter V is of the order of $10^2 - 10^3$. For example, for the activated semiconductor CdS, the quantity α_1 is equal to about 5×10^4 cm^{-1} for excitation in the region of fundamental absorption; α_2 is of the order of one reciprocal centimetre for luminescence with a wavelength of 550 nm, and the energy efficiency η of the luminescence is nearly 10% [12, 13]. For the parameter V this gives a value of the order of 5×10^3, and according to Eqn (9) the radiance of the outgoing light flus is 50 times higher than the radiance of the incident light flux provided that the latter is concentrated within a solid angle of the order of 0.1 sr. The estimates of the parameter V for glasses activated by rare-earth elements are of the same order of magnitude.

Summarizing, we should mention the following crucial points. A necessary condition for the increase of the radiation temperature and the radiance is the irreversibility of the optical process. Obviously, in the particular case of the ray approximation, the irreversibility due to the dissipation cannot, by virtue of the Straubel theorem, entail an increase of the radiance, and in the ray approximation the radiance can grow in an inverse medium only [1]. Media with inversion, in which laser generation and phenomena similar to stimulated scattering are possible, demonstrate only one of the opportunities to increase the radiance. The simple model considered above indicates the possibility of a multiple gain in radiance in a medium without inversion under luminescence which is a particular case of incoherent optical scattering.

References

1. Planck M *Vorlesungen über die Theorie des Wärmestrahlung*, 5th ed. (Leipzig: Verlag von Johan Ambrosius Barth, 1923) [Translated into English: *The Theory of Heat Radiation* (New York: Dover Publ., 1959)], [Translated into Russian (Leningrad – Moscow: Gl. Red. Obshchetekhn. Lit., 1935)]
2. Wolf E *J. Opt. Soc. Am.* **68** 6 (1978)
3. Tudorovskiĭ A I *Teoriya Opticheskikh Priborov* (The Theory of Optical Devices) (Moscow – Leningrad: Izd. AN SSSR, 1948)
4. Landau L D, Lifshitz E M *Statisticheskaya Fizika* (Statistical Physics) Part 1 (Moscow: Nauka, 1976) [Translated into English (Oxford: Pergamon Press, 1980)]
5. Sobel'man I I *Usp. Fiz. Nauk* **113** 701 (1974) [*Sov. Phys. Usp.* **17** 596 (1975)]
6. Klyshko D N *Fotony i Nelineĭnaya Optika* (Photons and Nonlinear Optics) (Moscow: Nauka, 1980)
7. Lifshitz E M, Pitaevskiĭ L P *Fizicheskaya Kinetika* (Physical Kinetics) (Moscow: Nauka, 1979) [Translated into English (Oxford: Pergamon Press, 1981)]
8. Apresyan L A, Kravtsov Yu A *Teoriya Perenosa Izlucheniya: Statisticheskie i Volnovye Aspekty* (The Theory of Radiation Transfer: Statistical and Wave Aspects) (Moscow: Nauka, 1983)
9. Chandrasekhar S *Radiative Transfer* (Oxford: Clarendon Press, 1950) [Translated into Russian (Moscow: IL, 1953)]
10. Minin I M *Teoriya Perenosa Izlucheniya v Atmosferakh Planet* (The Theory of Radiation Transfer in Planet Atmospheres) (Moscow: Nauka, 1988)
11. Shepelev A V, Shvartsburg A B, Sisakyan I N, in *Neutrons, X-Rays, and Gamma-Rays* (Proc. SPIE, Vol. 1737, Eds J M Carpenter et al.) (Bellingham, Wash.: SPIE, 1993) p. 66
12. Cuthbert J D, Thomas D G *J. Appl. Phys.* **39** 1573 (1968)
13. *Fizika i Khimiya Soedineniĭ* $A^{II}B^{IV}$ (Physics and Chemistry of II – IV Compounds) (Ed. S A) (Moscow, 1970)

[1] In wave optics, the concept of radiance is largely a matter of convention [2, 8], and the feasibility of enhancing the radiance in a medium without inversion requires special consideration in this case.