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Radio pulsars

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Contents

Abstract. **Recent theoretical work concerning the magnetosphere of and radio emission from pulsars is reviewed in detail. Taking into account years of little or no cooperation between theory and observation and noting, in particular, that no systematic observations are in fact being made to check theoretical predictions, the key ideas underlying the theory of the pulsar magnetosphere are formulated and new observations aimed at verifying current models are discussed.**

1. Introduction

The discovery of radio pulsars, the sources of pulsed cosmic radio emission with a characteristic period $P \sim 1$ s [1], in the late 1960s can without exaggeration be considered as one of the most important events in the astrophysics of the twentieth century. Indeed, the cosmic sources associated with neutron stars predicted as far back as the 1930s [2] had been first discovered. Such compact objects (with a mass of the order of the solar mass $M_\odot = 2 \times 10^{33}$ g, and a radius R of only 10 – 15 km) may arise due to the catastrophic compression (collapse) of normal massive stars at a late stage of their evolution or, for example, of white dwarfs whose mass exceeded the Chandrasekhar limit $M_{\mathrm{Ch}} \approx 1.4 M_\odot$ as a result of accretion. Many other cosmic sources (X-ray pulsars, X-ray novae [3, 4]) discovered afterwards have shown that neutron stars are actually some of the most numerous objects in the Galaxy. Thus, it is not surprising that A Hewish was awarded the 1974 Nobel prize in physics for the discovery of pulsars.

It is of interest that the basic physical processes responsible for the observed activity of these unusual objects had been clarified by the mid-1970s. It became immediately obvious that the exceedingly regular pulsations of the observed radio emission was due to the neutron star's rotation [5]. Some pulsars exhibited frequency stability on a scale of several years even exceeding the stability of some atomic standards, and so a new pulsar time scale is now being defined [6].

The energy source of radio pulsars is the rotational energy, and the mechanism of energy release is due to their superstrong magnetic field $B_0 \sim 10^{12}$ G [7]. The energy loss estimated by the simple magnetodipole formula

$$W_{\mathrm{md}} = -J\Omega\dot{\Omega} \approx \frac{1}{6}\,\frac{B_0^2 \Omega^4 R^6}{c^3}\,\sin^2\chi\,, \tag{1}$$

where $J \sim MR^2$ is the moment of stellar inertia, χ is the inclination of a magnetic dipole to the rotation axis, and $\Omega = 2\pi/P$ is the angular velocity of rotation, makes up $10^{31} - 10^{34}$ erg s^{-1} for the majority of pulsars. Such energy release leads to the observed deceleration rate $\mathrm{d}P/\mathrm{d}t \sim 10^{-15}$, which corresponds to a braking time $\tau_{\mathrm{D}} = P/2\dot{P} \sim 1 - 10$ million years.

Radio pulsars are thus the only cosmic objects whose evolution is determined by electrodynamic forces. Recall that the radio-frequency radiation itself makes up only $10^{-4} - 10^{-6}$ of the total energy loss. As a result, the radio luminosities of the majority of pulsars range within $10^{26} - 10^{28}$ erg s^{-1}, which is by five to seven orders of magnitude less than the solar luminosity $L_\odot \approx 3 \times 10^{33}$ erg s^{-1}. At the same time, the extremely high brightness temperature $T_{\mathrm{b}} \sim 10^{25} - 10^{28}$ K is a direct evidence in favor of the coherent mechanism of pulsar radio emission [8, 9].

More than 1200 radio pulsars had been discovered by mid-1999. Most of them present single neutron stars and only

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60 are the members of binary systems. The total number of neutron stars in our Galaxy may reach $10^9 - 10^{10}$. This fact is mostly due to the very short (on a cosmic scale) lifetime τ_D within which a neutron star can manifest itself as a radio pulsar. Hence, we are only able to register the youngest single neutron stars. Practically none of the radio pulsars radiate in other ranges of the electromagnetic spectrum. Only nine radio pulsars generate optical pulses, and seven radio pulsars produce gamma pulses. Thirty five radio pulsars emit X-rays, but this radiation is not at all always of a clearly pronounced pulsed character.

A superstrong magnetic field of a neutron star leads to a number of important consequences. First of all, the duration of synchrotron radiation ([10], Section 74)

$$\tau_s \sim \frac{1}{\omega_B}\frac{c}{\omega_B r_e} \sim 10^{-15}\ \mathrm{s} \tag{2}$$

($\omega_B = eB/m_e c$, $r_e = e^2/m_e c^2$ is the classical electron radius) appears to be much smaller than the time within which the particle escapes from the magnetosphere. Consequently, the motion of charged particles in the magnetosphere of a neutron star will be represented by the sum of their motions along the magnetic field lines and the electric drift in the transverse direction.

Further, the importance of the one-photon conversion $\gamma \to e^+ + e^-$ in a superstrong magnetic field, which takes place when a photon propagates at a sufficiently large angle to the external magnetic field [13], has been understood [11, 12]. Hence, in the dipole magnetic field of a neutron star, the necessary gamma quanta can be emitted by primary particles moving along a curved magnetic field. This means that the magnetosphere of a neutron star must be effectively filled with an electron – positron plasma which screens the longitudinal electric field, and the charge density

$$\rho_e \approx \rho_{GJ} = -\frac{\mathbf{\Omega B}}{2\pi c} \tag{3}$$

in the magnetosphere must be nonzero [14]. Such a redistribution of electric charges, as is known from the examples of the Earth and Jupiter magnetospheres, leads to the plasma's solid corotation with the star. Clearly, such a 'rotation' is impossible outside the so-called light cylinder

$$R_L = \frac{c}{\Omega}. \tag{4}$$

Hence, in the magnetosphere of a radio pulsar, two essentially different regions must be formed, namely, the regions of open and closed field lines. Particles in the field lines not intersecting the light cylinder appear to be trapped, whereas the plasma in the field lines crossing the light cylinder may go to infinity. In this case, the size of the region near the neutron star's magnetic poles, which is crossed by open field lines, viz.

$$R_0 \approx R\left(\frac{\Omega R}{c}\right)^{1/2}, \tag{5}$$

makes up only several hundred meters for normal pulsars. And on such a negligible area (on a cosmic scale) comparable in size with a stadium occur the main processes leading to the observed activity of radio pulsars.

It is of importance that the outgoing plasma also carries away electric charge (3). That is why the strong electric currents

$$I \sim I_{GJ},$$

where

$$I_{GJ} = \pi R_0^2 c \rho_{GJ}, \tag{6}$$

which are closed in the neutron star magnetosphere, must flow in the magnetosphere of a radio pulsar. A curious fact concerning this point became clear [14]: the characteristic current losses, i.e. the intensity of the energy release due to the pondermotive action of the electric currents that flow in the magnetosphere and are closed on the neutron star surface,

$$W_{cur} \sim IV \sim \frac{B_0^2 \Omega^4 R^6}{c^3}\frac{I}{I_{GJ}}, \tag{7}$$

coincide to an order of magnitude with the magnetodipole losses (1). Here

$$V \sim eEL \sim e\,B_0\,\frac{\Omega R_0}{c}\,R_0 \tag{8}$$

is the characteristic potential drop across the polar cap. By this means the analysis of the statistical characteristics of radio pulsars [15, 16] yields similar results with respect to the magnetodipole and current losses, and so the observations do not allow us to choose now between these two deceleration mechanisms.

Finally, on the basis of the picture presented above, the hollow cone model [17] was proposed, which explained perfectly well the basic geometrical properties of radio emission. As shown below, the creation of secondary particles is impossible in a rectilinear magnetic field where, first, the intensity of 'curvature' radiation is low and, second, the photons emitted by relativistic particles propagate at small angles to the magnetic field. Accordingly (see Fig. 1), in the central regions of open field lines one should expect a

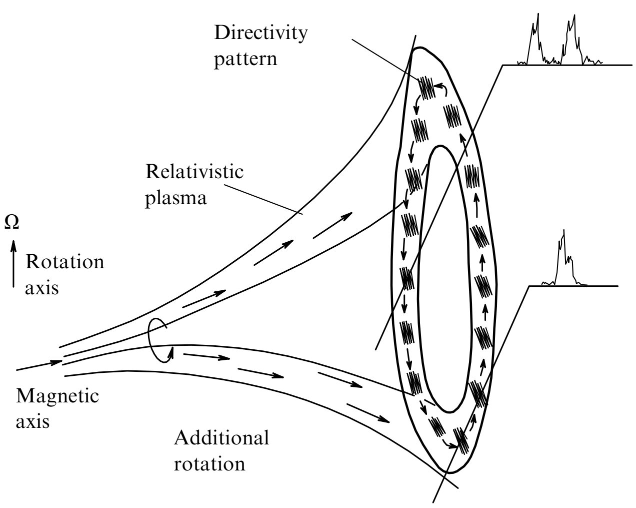

Figure 1. 'Hollow cone': the main working model of pulsar radio emission. The directivity pattern is determined by the spread of open magnetic field lines. In the central part of the pattern, the intensity of radio emission is expected to lower owing to suppressed particle creation. The shape of the mean profile depends on the orientation of the observer: for the central passing we have a double-humped profile, and for the side passing — a single-humped one. Additional plasma rotation around the magnetic axis, which is caused by the electric potential drop near the stellar surface, is observed as a subpulse drift within the confines of the mean profile.

decreased density of secondary plasma and, therefore, a lowered intensity of radio emission in the center of the directivity pattern. Digressing from the details (the mean profiles of pulsars actually have a fairly complicated structure [18 – 20]), one should expect a single-humped mean profile of pulsars for which the line of sight traverses the directivity pattern far from its center and a double-humped profile when the line of sight intersects the center of the diagram. Precisely this picture is known to be observed in reality [16, 18].

As a result, practically all the basic properties of pulsar radio emission, such as the variation of the position angle of linear polarization along the mean profile [17], the distribution of pulsars with single- and double-humped mean profiles [16], the width of the directivity pattern and even its statistical dependence on the pulsar period [16, 20], were explained. The latter fact is based on the assumption that all pulsars generate radiation at approximately the same distance r_{rad} from the neutron star. Hence, for the width of the directivity pattern we have

$$w_{\text{d}} \approx \left(\frac{\Omega r_{\text{rad}}}{c}\right)^{1/2} \approx 10\, P^{-1/2}\left(\frac{r_{\text{rad}}}{10R}\right)^{1/2} [\text{angle deg.}]\,, \quad (9)$$

i.e. $w_{\text{d}} \propto P^{-1/2}$, which agrees with observations. Moreover, some properties of radio pulsars (e.g., the drift of subpulses) indirectly confirm the existence of the region of potential drop and particle acceleration in the vicinity of the magnetic poles of a neutron star [12].

Indeed, if near the surface of a neutron star there exists a region with a longitudinal electric field, then in the open field lines located precisely above the acceleration region there appears an additional potential difference between the central and peripheral magnetic surfaces, and so the additional electric field is directed perpendicular to the magnetic axis. As a result, the additional electric drift gives rise to plasma rotation around the magnetic axis (besides the general motion around the rotation axis), which can in turn be observed as a regular displacement of radiating regions within the mean pulse (see Fig. 1). More than twenty radio pulsars with drifting subpulses are known at present [18].

Thus, the general picture of radio pulsar activity seems to have been established many years ago. At the same time, some principal questions are still far from being solved. First of all, as in the 1970s, there is no generally accepted standpoint concerning the physical nature of coherent radio emission of pulsars. In particular, it has not yet been decided whether the coherent mechanism of radio emission is of maser or antenna type. Furthermore, there is no common point of view about the structure of the neutron star magnetosphere as well [15, 16, 21]. That is why there is no generally acknowledged model of the structure of longitudinal currents circulating in the magnetosphere, which is necessary for the solution to the problem of neutron star braking, particle acceleration and energy transfer outside the light cylinder.

Nevertheless, the number of papers devoted to the key problems, namely, the theory of the radio pulsar magnetosphere and the theory of coherent radio emission has recently decreased sharply. Table 1 gives the number of publications (in percent) in the main astrophysical journals (The Astrophysical Journal, Monthly Notices of the Royal Astronomical Society, Astronomy and Astrophysics) and in Proceedings of conferences (held in Bonn, Sydney and Tokyo) devoted to radio pulsars [22 – 24]. The data of the very fruitful IAU Colloquium No. 128 (Lagow, Poland, 1990) [25] are deliberately not included in the table because the colloquium was devoted to these particular problems.

Table 1. Number of publications devoted to radio pulsars (in %).

Publications	ApJ, MN, AA(1976)	Bonn (1980)	ApJ, MN, AA(1996)	Sidney (1996)	Tokyo (1997)
Radio emission (theory)	12	9	3	1	–
Magnetosphere (theory)	20	11	3	1	4
Particle creation	1	2	14	2	9
Pulsar wind	2	4	5	10	3
Radio emission (interpretation)	9	4	12	11	5
Radio emission (observations)	18	31	16	36	11
Internal structure	7	14	6	–	18
Pulsars in binary systems, evolution	20	8	21	18	12
Connection with supernovae, proper velocities	2	6	8	2	11
High-frequency radiation	9	11	12	19	27

As is seen from the data presented in the table, the progress in the understanding of the two crucial problems of the theory of radio pulsars has actually stopped in spite of the fact that other, sometimes very refined questions such as particle creation and propagation of gamma quanta in superstrong magnetic fields [26 – 28], the effects of general relativity [29 – 33], the theories of pulsar wind [34, 36, 38, 39] and the high-frequency emission of radio pulsars [40, 41] have been under intensive investigation. Several recent important papers on the theory of radio emission [42 – 45] and the theory of the magnetosphere [46, 47] do not essentially change the general picture. There is no general view about the main theoretical questions:

What is the physical nature of coherent radio emission? and

What is the structure of electric currents flowing in the radio pulsar magnetosphere?

The theory [48 – 50] we constructed ten years ago remains, in fact, the only example of passing the whole way from a consistent model of neutron star magnetosphere, the theory of particle creation and generation of radio emission to a comparison of the quantitative predictions of the theory with observations; the comparison showed a good agreement (cf. Refs [15, 51, 52]).

The aim of the present review is a brief discussion of the principal theoretical results achieved during the past few years. Our prime concern will be the questions connected with theories of the neutron star magnetosphere and the radio emission. We therefore do not consider here such undoubtedly important problems as the internal structure of neutron stars, the evolution of radio pulsars, the mechanism of their high-frequency emission, and novel observational data. A detailed discussion of these questions can be found in surveys [53 – 57] and monographs [4, 15, 18].

2. Particle creation

2.1 Basic processes

As has already been emphasized, if we correctly understand the physical nature of the activity of single neutron stars, leading to the observed radio emission, it is due to the relativistic electron – positron plasma generated near the magnetic poles and streaming out along open magnetic field lines. To establish the parameters of an outflowing plasma, it is necessary to know the structure of the region with a longitudinal electric field, which, as we shall see, depends essentially not only on the quantum-mechanical processes of particle creation and the peculiarities of hard photon propagation through a superstrong magnetic field of a pulsar, but also on the structure of the neutron star surface.

However, before proceeding to a discussion of the details, we shall recall the main processes operating in the plasma generation region, which became clear more than two decades ago (see Fig. 2). Three years after the discovery of radio pulsars, the paper by Sturrock [11] appeared demonstrating that the magnetic field $B \sim 10^{12}$ G is quite enough for efficient electron – positron plasma generation in the magnetosphere of a neutron star.

The point is that the probability of electron – positron pair creation due to conversion of a gamma quantum with an energy $\mathcal{E}_\gamma$ moving at an angle θ to the magnetic field $\mathbf{B}$ is defined by the relation [13]

$$w = \frac{3^{3/2}}{2^{9/2}} \frac{e^3 B \cos\theta}{\hbar m_e c^3} \exp\left(-\frac{8}{3} \frac{B_{cr}}{B\cos\theta} \frac{m_e c^2}{\mathcal{E}_\gamma}\right), \tag{10}$$

where

$$B_{cr} = \frac{m_e^2 c^3}{e\hbar} = 4.4 \times 10^{13}\ \mathrm{G} \tag{11}$$

is the well-known critical field for which the distance $\hbar\omega_B$ between two neighboring Landau levels is comparable with the electron rest energy $m_e c^2$. As a result, even a sufficiently

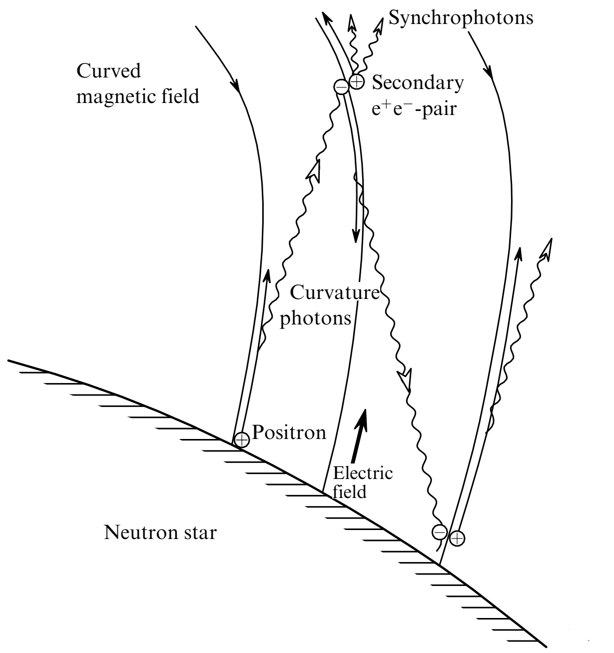

Figure 2. Structure of the region of particle acceleration and creation near the surface of a neutron star. Primary particles that have got into the region of a nonzero longitudinal electric field are accelerated along curved magnetic field lines and emit hard gamma quanta. Propagating through a curved magnetic field, these photons reach the particle creation threshold and turn to electron – positron pairs. The size H of the acceleration region is determined by the height at which effective production of a secondary plasma starts, thus screening the longitudinal electric field.

low-energy ($\mathcal{E}_\gamma \sim 10$ MeV) photon propagating across a magnetic field $B \sim 10^{12}$ G has a mean free path $l_\gamma = w/c$ much smaller than the size of a neutron star.

Photons propagating across the magnetic field are practically absent under real conditions because particles can only move along magnetic field lines and do not radiate in the transverse direction. But here the curvature of the magnetic field lines becomes important.

Primary particles of energy $m_e c^2 \gamma$ [$\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor], accelerated by the longitudinal electric field and moving along a curved magnetic field, start radiating gamma quanta with a characteristic energy

$$\mathcal{E}_\gamma \approx \frac{\hbar c}{R_c} \gamma^3 . \tag{12}$$

This 'curvature' mechanism is quite similar to synchrotron radiation: in both cases, the acceleration of charged particles is associated with the accelerated circular motion. Hence, it is only necessary to replace the Larmor radius $m_e c^2 \gamma / eB$ of the orbit by the radius R_c of curvature of the magnetic field lines. However, since the expression for R_c does not contain the Lorentz factor γ, the energy of the radiated gamma quanta increases much more rapidly with the energy of the radiating particle. In this case the energy of curvature photons may reach 10^8 MeV.

Furthermore, when propagating, hard 'curvature' gamma quanta, which are mostly emitted at small angles to the magnetic field, begin moving at increasingly large angles to the field line until the condition of pair creation is met. Since the leading part is played here by the exponential factor in Eqn (10), one can rather accurately estimate [12]

$$l_\gamma \sim R_c \frac{B_{cr}}{B} \frac{m_e c^2}{\mathcal{E}_\gamma} . \tag{13}$$

Moreover, secondary particles are produced at nonzero Landau levels and the emitted synchrophotons appear to be energetic enough for the creation of new secondary pairs, to say nothing of the fact that every primary particle emits many 'curvature' photons. As a result, a cascade type increase of the number of secondary particles occurs, which can only be stopped by screening the longitudinal electric field $E_\parallel$. Then a larger fraction of secondary particles will be produced already above the acceleration region where the longitudinal electric field is sufficiently small and so the secondary plasma is able to escape from the neutron star magnetosphere.

We shall now proceed to a discussion of the structure of plasma generation region which determines the longitudinal electric field. To make an estimate, we shall consider the one-dimensional equation for a longitudinal electric field

$$\frac{dE_\parallel}{dh} = 4\pi(\rho_e - \rho_{GJ}) , \tag{14}$$

which can be used if the gap height H is much smaller than the transverse dimension R_0 (5) of the polar cap. Unfortunately, this approximation is valid for the fastest pulsars only. Nevertheless, it contains all qualitative information on the internal gap structure.

In spite of its seeming simplicity, equation (14) involves a number of significant uncertainties. And the main uncertainty undoubtedly lies in the expression for the charge density ρ_e depending on the particle creation mechanism which is in turn determined by the longitudinal electric field.

We shall now discuss the basic properties of Eqn (14). So, for models with hindered enough ejection of particles from the neutron star surface one can put $|\rho_e| \ll |\rho_{GJ}|$ in the zeroth approximation, with the electric field on the star surface being nonzero. As a result, we have [12]

$$E_{\parallel} = E_{RS} \frac{H-h}{H}, \tag{15}$$

where

$$E_{RS} \sim 4\pi \rho_{GJ} H, \tag{16}$$

and H is the height of the region with a longitudinal electric field, which must be determined from the condition of the onset of secondary plasma production. Indeed, for $H < H_{cr}$ the longitudinal electric field is insufficient for efficient particle creation, whereas for $H > H_{cr}$ secondary plasma is conductive to rapid screening of the acceleration region. Incidentally, for a solid stellar surface this case can be realized for antiparallel directions of the magnetic axis and the axis of rotation, i.e. for $\rho_{GJ} > 0$, which means that positively charged particles should be ejected from the surface.

At the same time, if particles freely leave the neutron star surface, it is natural to assume $E_{\parallel}(0) = 0$, with the charge density ρ_e being close to ρ_{GJ}. In this case, the longitudinal electric field is only determined by the small difference between the charge density ρ_e and the critical density ρ_{GJ}. Hence, the electric field strength E_A can now be estimated to an order of magnitude as

$$E_A \sim 4\pi \rho_{GJ} \frac{H^2}{R} \sim \varepsilon_A E_{RS}, \tag{17}$$

and the additional small factor $\varepsilon_A \sim H/R \ll 1$ essentially depends on the density of the secondary electron–positron plasma.

Thus, we face an extremely interesting self-consistent problem which depends, among other things, on the secondary plasma dynamics. Since particles in a secondary pair have opposite signs, it follows that one of these particles, which was created in the region of a sufficiently strong longitudinal electric field, can in principle be stopped and accelerated in the opposite direction. As a result, such particles must themselves give birth to secondary electron–positron pairs. The presence or the absence of a noticeable backward flow of secondary particles may have a significant effect upon the internal gap structure. New papers have recently appeared that cover such processes. The section to follow is devoted to a discussion of their main features.

2.2 The surface of a neutron star

The structure of the surface layers of a neutron star is not only interesting in itself, but is also directly related to the theory of the radio pulsar magnetosphere. Indeed, the 'internal gap' structure essentially depends on the work function φ_w of particles escaping from the surface of a neutron star.

We shall recall that the model with hindered particle ejection from the surface, first considered by Ruderman and Sutherland [12], was most fruitfully and intensively developed in the 1970s. This model was based on a series of theoretical works on the structure of matter in a strong magnetic field [58–61], which predicted a high enough value for the work function of particles: $\varphi_w \approx 1-5$ keV. However, since the early 1980s, when more accurate calculations yielded a lower value for the work function, $\varphi_w \approx 0.1$ keV [62–66], models with free particle ejection have become more and more popular. The first detailed calculations of the region of particle acceleration and creation were carried out using this model by the group of J Arons [67–69].

It is of interest that the situation is still far from being clear, and not only because the accuracy of determination of the work function is insufficiently high [27]. It has turned out that even the chemical composition of the surface layers of a neutron star is unknown: they may not consist of iron atoms as was assumed in most papers. The chemical composition of the surface layers of polar caps may be changed drastically through their bombardment by energetic particles accelerated by the longitudinal electric field in the gap.

Moreover, as is now being widely discussed, the first several years after the birth of a neutron star, when its surface was unquestionably not solid, iron atoms (which are undeniably formed in the largest number as the most stable nuclei) may have 'sunk' under the action of the gravitational field [70]. It is therefore not excluded that the surface layers of neutron stars actually consist not of iron, but of much lighter atoms, those of hydrogen and helium.

Since the melting temperature estimated by the formula [53]

$$T_m \approx 3 \times 10^7 \left(\frac{Z}{26}\right)^2 \left(\frac{56}{A}\right)^{1/3} \left(\frac{\rho}{10^6 \text{ g cm}^{-3}}\right)^{1/3} [\text{K}] \tag{18}$$

depends essentially on the nuclear charge Z, at a temperature of the order of 10^6 K typical of ordinary radio pulsars, the neutron star surface must be liquid and at any rate cannot prevent free particle ejection. Furthermore, according to present views [71], at temperatures $T < 10^6$ K and magnetic fields $B_0 < 10^{13}$ G, the surface of a neutron star apparently possesses an atmosphere which also promotes free particle ejection. Incidentally, many contemporary models of thermal radio emission of pulsars [57, 72] are based on this particular picture.

2.3 Propagation of gamma quanta in a superstrong magnetic field

We shall now proceed to a discussion of the effects of high-energy photon propagation in a superstrong magnetic field near the surface of a neutron star. Clearly, this question is directly related to the mechanism of particle creation in the polar regions of radio pulsars. The quantum effects in a magnetic field close to the critical one (11) have long been known [13], but the hope for their direct observation appeared only after the discovery of radio pulsars. These processes include, for example, photon splitting $\gamma + B \rightarrow \gamma + \gamma + B$ [73, 74], a substantial change in the cross section of the one-photon $(\gamma \rightarrow e^+ + e^-)$ and two-photon $(\gamma + \gamma \rightarrow e^+ + e^-)$ pair creation, especially near the creation threshold [75], quantum synchrotron cooling due to the rapid transition of particles to a lower Landau level [76, 77], as well as propagation effects resulting from vacuum birefringence [73, 78] and the specificity of photon trajectories in the vicinity of the particle creation threshold [79–81].

Thus, in the 1970s, the possibility of a direct discovery of effects associated with the quantizing magnetic field (11) seemed to be absolutely realistic. Nevertheless, for the majority of radio pulsars these effects proved to be too weak. The point is that, for instance, the expression for the refractive index in a strong magnetic field (the formula corresponds to one of the linear polarizations):

$$n = 1 + \frac{7\alpha_{\mathrm{f}}}{90\pi}\left(\frac{B}{B_{\mathrm{cr}}}\right)^2 \tag{19}$$

includes, along with the multiplier $7/90\pi$, a fine structure constant $\alpha_{\mathrm{f}} = e^2/\hbar c \approx 1/137$, so the manifestation of significant quantum effects can be expected for fields $B > 10^{14}$ G only. For the majority of neutron stars observed as radio pulsars, one may assume to a good accuracy that gamma quanta propagate rectilinearly.

Recently, however, this question has again become topical especially in connection with the discovery of magnetars [82, 83] [i.e. sources of pulsed X-ray radiation with a period of several seconds and a magnetic field reaching $10^{15}-10^{16}$ G as estimated by formula (1)]. That is why new thorough calculations have been made to determine both the photon splitting probability [84–87] and the trajectories of hard gamma quanta near the particle creation threshold [88].

In particular, it was shown that for sufficiently high magnetic fields ($B \sim 10^{14}-10^{15}$ G) the conversion of gamma quanta through photon splitting must be essentially suppressed [89]. Consequently, the secondary plasma production must also be suppressed to a large extent. It is therefore not surprising that the majority of magnetars do not manifest themselves as radio pulsars. At the same time, no new qualitative phenomena have been found to lead to a direct observation of quantum effects in a superstrong magnetic field, and the calculations only refined the results obtained before.

2.4 Effects of general relativity

We are coming now to relativistic effects which, as distinct from the quantizing magnetic field effects discussed above, may exert a substantial influence upon particle creation near radio pulsars. It has turned out that in the model with free particle ejection from the neutron star surface (Arons type model) an important role must be played by the effects of general relativity.

Recall that on the surface of a pulsar the gravitational red shift is rather large:

$$\frac{\varphi_{\mathrm{g}}}{c^2} = \frac{2GM}{Rc^2} \approx 0.2\,, \tag{20}$$

and therefore any calculations that lay claim to an accuracy above 20% should involve relativistic effects. However, in a model with hindered particle ejection from the neutron star surface (Ruderman–Sutherland model), the allowance for such effects does not lead to significant corrections because it does not qualitatively change the structure of the electrodynamic equations.

On the other hand, in the framework of an Arons model, equation (14) contains, in addition to the small geometric factor

$$\varepsilon_{\mathrm{A}} = \left(\frac{\Omega R}{c}\right)^{1/2}, \tag{21}$$

a purely relativistic factor $\omega/\Omega \approx \varepsilon_{\mathrm{g}}$, where

$$\varepsilon_{\mathrm{g}} = \frac{\varphi_{\mathrm{g}}}{c^2}\,, \tag{22}$$

connected with dragging of inertial reference frame (the Lense–Thirring effect [90]). Here ω is the Lense–Thirring angular velocity. For the majority of radio pulsars with a period $P \sim 1$ s, the relativistic correction ε_{g} appears to be at least an order of magnitude larger than the geometric correction ε_{A}, which necessitates the allowance for the effects of general relativity.

Indeed, the longitudinal electric field in the particle acceleration and creation region originates due to the difference between the plasma charge density ρ_{e} and the Goldreich–Julian density ρ_{GJ} (3). In the general relativistic case, the Gauss equation is rewritten as [90]

$$\nabla\left(\frac{1}{\alpha}E_{\parallel}\right) = 4\pi(\rho_{\mathrm{e}} - \rho_{\mathrm{GJ}})\,, \tag{23}$$

the Goldreich–Julian density now being

$$\rho_{\mathrm{GJ}} = \frac{1}{8\pi^2}\nabla\left(\frac{\Omega - \omega}{\alpha c}\nabla\Psi\right). \tag{24}$$

Here α is the gravitational red shift, and Ψ is the magnetic flux function. With the accuracy necessary for us, we can write the quantities α and ω in the form

$$\alpha^2 = 1 - \frac{R_{\mathrm{g}}}{r}\,, \tag{25}$$

$$\omega = \Omega\,\frac{R_{\mathrm{g}}^3}{r^3} \tag{26}$$

($R_{\mathrm{g}} = 2GM/c^2$ is the gravitational radius).

In a linear approximation with respect to small quantities ε_{A} and ε_{g} we have

$$\rho_{\mathrm{GJ}} = \frac{\Omega - \omega}{2\pi c}\,\frac{B\cos\theta_{\mathrm{b}}}{\alpha}\,, \tag{27}$$

where θ_{b} is the angle between the magnetic axis and the axis of rotation. For a relativistic plasma moving at a velocity $v \approx c$, we have to the same accuracy

$$\rho_{\mathrm{e}} = C(\Psi)\,\frac{B}{\alpha}\,. \tag{28}$$

Here the quantity $C(\Psi)$ is constant along the magnetic field lines.

As we can see, the charge densities (27) and (28) change differently along the magnetic field line: the Goldreich–Julian density (27), in addition to the factor B/α, also contains the geometric factor $\cos\theta_{\mathrm{b}}$ and the gravitational factor ω. As a result, a charge-separated relativistic plasma cannot, when moving, satisfy the condition $\rho_{\mathrm{e}} = \rho_{\mathrm{GJ}}$, which gives rise to the production of a longitudinal electric field. The latter causes, in turn, the acceleration of particles, the emission of hard photons and, finally, the creation of a secondary electron–positron plasma [11]. Hence, outside the acceleration region the longitudinal electric field must already be close to zero.

So, in the Arons model, which assumes free particle ejection from the neutron star surface [68, 69], Eqn (23) should be solved with the boundary conditions

$$E_{\parallel}(h = 0) = 0\,, \tag{29}$$

which is appropriate to a zero longitudinal electric field on the pulsar surface, and

$$E_{\parallel}(h = H) = 0\,, \tag{30}$$

which corresponds to a zero longitudinal electric field on the upper boundary H of the acceleration region.

As a result, Eqn (23) can be rewritten in the form

$$\frac{\mathrm{d}E_{\parallel}}{\mathrm{d}h} = -K_{\mathrm{A}}\left(h - \frac{H}{2}\right), \tag{31}$$

where

$$K_{\mathrm{A}} = 4\pi\left[\frac{\mathrm{d}(\rho_{\mathrm{e}} - \rho_{\mathrm{GJ}})}{\mathrm{d}h}\right]_{h=H/2}. \tag{32}$$

Finally, one obtains

$$E_{\parallel} = -\frac{1}{2}\,K_{\mathrm{A}}h(H - h)\,, \tag{33}$$

and for $\mathbf{\Omega B} > 0$ [29, 30] we have

$$K_{\mathrm{A}} = \frac{3}{2}\,\frac{\Omega B_0}{cR}\left[4\,\frac{\omega}{\Omega}\cos\chi + \varepsilon_{\mathrm{A}}\cos\varphi_{\mathrm{m}}\sin\chi + O(\varepsilon_{\mathrm{g}}^2) + \ldots\right]. \tag{34}$$

The second summand in brackets in Eqn (34), which is proportional to ε_{A}, governs the geometric effect considered in Ref. [68]. Such an acceleration regime can only be realized on the half of the polar cap, $-\pi/2 < \varphi_{\mathrm{m}} < \pi/2$ ($K_{\mathrm{A}} > 0$), for which the magnetic field lines are curved in the direction of the rotation axis, and therefore $\cos\theta_{\mathrm{b}}$ increases with distance from the star surface. Such field lines were called 'preferable'. In the region $\pi/2 < \varphi_{\mathrm{m}} < 3\pi/2$ ($K_{\mathrm{A}} < 0$), where the magnetic field lines, on the contrary, tend to become perpendicular to the axis of rotation, the arising longitudinal electric field may cause a cessation rather than acceleration of the particle motion. In the framework of this model, the acceleration and generation of secondary particles will only proceed in half of the region of open field lines, and accordingly the directivity pattern of radio emission must also be semicircular [69]. This, however, contradicts the observational data [18] which agree well with the hollow cone model.

At the same time, the allowance for the effects of general relativity leads to the appearance of an additional summand proportional to ε_{g}. According to Eqn (34), for $4\omega/\Omega > \varepsilon_{\mathrm{A}}\tan\chi$ the main contribution to the quantity K_{A} is made by the gravitational correction. So, for a star with a uniform density, when

$$\frac{\omega}{\Omega} = \frac{2}{5}\,\varepsilon_{\mathrm{g}}\,, \tag{35}$$

this condition can be rewritten in the form

$$P > 10^{-3}\left(\frac{R}{10^6\ \mathrm{cm}}\right)^2\left(\frac{M}{M_{\odot}}\right)^{-2}\ [\mathrm{s}]\,. \tag{36}$$

Hence, for practically all the observed pulsars the effects of general relativity are predominant, and all the open field lines are 'preferable'.

Thus, the allowance for the effects of general relativity is actually responsible for a qualitative change in the conclusions following from the Arons model. Stationary generation becomes possible over the entire polar cap surface.

2.5 Generation of particles in the magnetosphere

We shall now briefly consider the impact of the physical processes discussed above on particle production in the vicinity of a neutron star surface. We are first of all interested in the effects of a superstrong magnetic field $B > 10^{14}$ G typical of magnetars. As has already been mentioned above, noticeable effects of a quantizing magnetic field may be expected for such strong magnetic fields only [84, 88].

It has long been understood that a strong magnetic field must suppress the generation of secondary plasma. First, for fields exceeding 10^{13} G, a secondary electron – positron pair must be created at a lower Landau level, which leads to a suppression of synchrotron radiation [91, 92]. Second, the nontriviality of permittivity of vacuum near a threshold of pair creation at the zeroth Landau levels for a transverse photon momentum close to $2m_{\mathrm{e}}c$ may lead to gamma-quantum deflection along the direction of the magnetic field with the creation of not two free particles, but their bound state — positronium [80, 81].

Third, the photon splitting $\gamma + \mathrm{B} \to \gamma + \gamma + \mathrm{B}$ leading to a decrease of the photon energy and the suppression (although not complete) of pair creation [87] becomes significant. So, it becomes clear why magnetar radio emission turns out to be suppressed. However, the magnetic fields of the majority of radio pulsars are insufficiently large for such phenomena to be registered.

At the same time, the interaction between primary particles accelerated in the gap and soft X-ray photons emitted by a heated neutron star surface may turn out to be of importance for ordinary radio pulsars (the importance of inverse Compton scattering in the region of particle production was first noted in paper [93]). The hard gamma quanta produced in the act of such an interaction appeared to have an energy sufficient to create secondary electron – positron pairs and, therefore, to affect the structure of the particle acceleration region [94, 95]. Finally, the work function of particles also has an appreciable effect upon the electric field structure. That is why the uncertainty in this question presents as before an obstacle to the construction of a consistent model for the acceleration region.

Nevertheless, new important results have recently been obtained in this branch of the theory. In particular, papers [32, 33] should be mentioned which consider both the general relativity effects and the inverse (nonresonance and resonance) Compton scattering on X-ray photons emitted by a neutron star surface.

It is of interest that in this model the acceleration region may fail to adjoin the surface of a neutron star but will rather be hanging above its magnetic poles. However, for a comprehensive analysis the kinetic effects should necessarily be taken into account exactly as was done by Gurevich and Istomin [96] for the acceleration region near a neutron star surface in a Ruderman – Sutherland model and in the recent papers [95, 97] on the 'outer' gap.

Recall that an analysis of kinetic effects is necessary, in particular, for the solution of the problem of a backward particle flux, which is in turn directly related to the structure of the plasma generation region. Indeed, in the Arons model the electric charge density on the boundaries of the acceleration region does not coincide with the Goldreich – Julian density. It is only in this case (Fig. 3) that the solution of the Poisson equation (14) allows fulfillment of the boundary condition $E_{\parallel} = 0$ not only on the neutron star surface $h = 0$, but also on the upper boundary of the acceleration region.

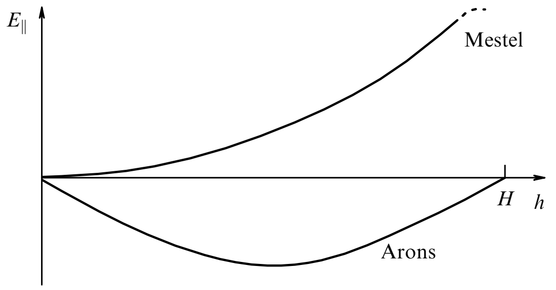

Figure 3. Longitudinal electric fields in the Arons [69] and Mestel [47] models on 'preferable' field lines for $\mathbf{\Omega B} > 0$. In the Mestel model, the plasma charge density ρ_e on the star surface is equal to the Goldreich–Julian density ρ_{GJ} (and, accordingly, $dE/dh = 0$), while in the Arons model the charge density for $h = 0$ differs from ρ_{GJ} owing to the presence of the backward particle flux. Although in both cases the electric field is equal to zero on the star surface, the directions of the electric field and, therefore, of the particle acceleration appear to be different.

As a result, on preferable field lines for $\mathbf{\Omega B} > 0$ the solution of equation (14) acquires the form of Eqn (33): $E_\|(h) \propto h(h - H)$. But as is readily seen, for the existence of such a solution a backward particle flux is necessary, which is determined in a self-consistent way from equation (14). In the Arons model it makes up

$$\frac{j_{\mathrm{back}}}{j} \approx \varepsilon_A \sim 10^{-2}\,. \tag{37}$$

If a backward particle flux is absent, equation (14) leads to a quite different solution $E_\| \propto h^2$ in which the longitudinal electric field appears to be aligned in the opposite direction. Consequently, particle acceleration becomes possible only on 'nonpreferable' magnetic field lines. It is this particular model with a naturally rather small backward particle flux that has been developed by Mestel et al. [21, 47, 98, 99] over many years. As we can see, only a consistent kinetic model can prompt a choice between the two realizations (in this connection see papers [21, 100, 101]).

2.6 Comparison with observations

Concluding, it is necessary to say a few words about the possibility of observational verification of the indicated physical effects in addition to the hollow-cone model mentioned in the Introduction. To begin, we shall consider the question of the maximum radio pulsar period $P_{\max}$. As has already been said, radio emission is associated with the secondary electron–positron plasma produced in the polar regions of a neutron star. That is why, the condition

$$H(P, B) < R_0(P) \tag{38}$$

may be thought of as the 'ignition condition' separating the active and passive parameter ranges, when the neutron star does not manifest itself as a radio pulsar.

It is well known that in Ruderman–Sutherland type models relation (38) leads to a reasonable value of the ultimate period

$$P_{\max} \approx \left(\frac{B_0}{10^{12}\ \mathrm{G}}\right)^{8/15} [\mathrm{s}] \approx 1-3\ \mathrm{s}\,. \tag{39}$$

Incidentally, this condition is usually depicted as a 'death line' in the $P-\dot{P}$ diagram. Such good agreement can undoubtedly be taken as a direct evidence of the picture discussed.

For Arons type models, the ultimate period must be much smaller [69], namely

$$P_{\max} = 0.1-0.3\ \mathrm{s} \tag{40}$$

owing to the much smaller values of the accelerating potential. Hopes for raising the ultimate period through making allowance for the effects of general relativity have not been justified [102].

However, different solutions are possible here, for example, a displacement of the magnetic dipole with respect to the pulsar center [102] or the existence near the neutron star surface of a sufficiently strong nondipole magnetic field [103] leading to a decrease of the radius R_c of curvature of magnetic field lines and, therefore, to an increase in the particle creation efficiency. Nevertheless, Arons type models encounter certain difficulties.

We shall also mention some more possibilities concerning direct verification of the existence of a plasma production region. The information on processes proceeding in a particle creation region might first of all be obtained from 'relic photons', i.e. hard gamma quanta with an energy insufficient for conversion into an electron–positron pair. The possibility of direct recording of such photons has long been discussed [12, 96] (moreover, the spectra and intensities of the anticipated radiation have been determined for many models [28, 96]), but the situation is not yet quite clear. The point is that in those rare cases where radio pulsars are at the same time sources of pulsed gamma-ray emission (as is the case with a pulsar in the Crab Nebula), their gamma-ray emission from the particle generation region cannot apparently compete with other mechanisms of gamma-ray emission, for instance, with radiation from an 'outer' gap.

It should be recalled that particles can be generated not only in the vicinity of the neutron star surface, but also in the region of the so-called outer gap located on those open magnetic field lines which have a section perpendicular to the axis of rotation and on which the Goldreich–Julian charge density (3) changes its sign [94].

Clearly, in the case of a charge-separated plasma flowing out of the magnetosphere, the condition of zero longitudinal electric field $\rho_e = \rho_{GJ}$ cannot be fulfilled. Hence, near the surface $\rho_{GJ} = 0$ one should also expect the appearance of a longitudinal electric field, accelerated particles and, therefore, the production of secondary electron–positron plasma. Since the internal gap lies near the light cylinder, where the magnetic field is by many orders of magnitude weaker than the field on the neutron star surface, the one-photon conversion cannot play the leading role in the creation of secondary particles. However, it has turned out that the two-photon conversion $\gamma + \gamma_X \to e^+ + e^-$ must be sufficiently effective, where hard gamma quanta γ are emitted as before by primary particles accelerated in the gap and γ_X corresponds to thermal X-ray photons emitted by the neutron star surface. The outer gap structure has now been rather thoroughly calculated on the basis of the analysis of kinetic effects [95, 97], although the very existence of the outer gap cannot be thought of as proved.

Finally, direct information on the potential drop must be contained in the subpulse drift velocity because it is directly related to the potential drop across the gap. For example, the recent analysis reported in Ref. [104] also seems to testify in favor of Ruderman–Sutherland type models. However, further analysis is needed here.

It should be emphasized that the general properties of the secondary electron–positron plasma flowing out of the magnetosphere appeared to be generally insensitive to the structural details of the acceleration region. For the majority of models [12, 96, 105], both the density and the energy spectrum of the outflowing plasma prove to be fairly universal. That is why one can say with confidence that plasma streaming along open magnetic field lines in a pulsar magnetosphere consists of a beam of primary particles of energy $\mathcal{E} \sim 10^7$ MeV and a density close to the Goldreich–Julian density and also of a secondary electron–positron component whose energy spectrum has to a good accuracy the following power-like form

$$N(\mathcal{E}) \propto \mathcal{E}^{-2}\,, \tag{41}$$

and the energies themselves lie within the range from $\mathcal{E}_{\min} \sim 10-100$ MeV to $\mathcal{E}_{\max} \sim 10^4$ MeV. The total secondary plasma density is 10^3-10^4 times higher than the Goldreich–Julian density.

Such a model has been considered in the overwhelming majority of papers devoted to the theory of pulsar radio emission. It is of importance that the distribution functions of electrons and positrons must be displaced relative to each other (which fact has already been pointed out by Ruderman and Sutherland [12]). Only in this case can the charge density of an outflowing plasma coincide with the Goldreich–Julian density.

3. Radio pulsar magnetosphere

3.1 Energy loss of radio pulsars

As has already been said, the problem of the structure of the neutron star magnetosphere is first of all associated with the energy loss of radio pulsars. To answer this question, it is necessary to determine the structure of the electric charges and currents flowing in a pulsar magnetosphere. It is these currents that lead, when closing at the neutron star surface, to the appearance of a braking torque and thus determine the radio pulsar braking. And it is these longitudinal electric currents that produce the toroidal component of the magnetic field, forming an electromagnetic energy flux (the Poynting vector) escaping from the magnetosphere.

At the same time, we shall see that when redistributed in the pulsar magnetosphere, electric charges are capable of screening the magnetic dipole radiation of the neutron star. Properly speaking, the question of the relative role of current and magnetodipole losses is crucial in the theory of the radio pulsar magnetosphere.

Before proceeding to a quantitative analysis of the energy loss of a neutron star, it seems timely to recall the equation describing the magnetosphere of an oblique rotator. All the quantities are assumed here to depend on the time t and the angular coordinate φ only in the combination $\varphi - \Omega t$. Such quasi-stationary equations are in a sense analogous to a transition to a rotating coordinate system. This approach is however much broader because it can also be applied outside the light cylinder, where a transition to a rotating coordinate system is impossible.

As a result, the Maxwell equation

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

can be rewritten in the form [106]

$$\nabla \times \mathbf{E} = -\nabla \times [\boldsymbol{\beta}_{\mathrm{R}} \times \mathbf{B}]\,, \tag{42}$$

where

$$\boldsymbol{\beta}_{\mathrm{R}} = \frac{1}{c}\,[\boldsymbol{\Omega} \times \mathbf{r}]\,. \tag{43}$$

Equation (42) can immediately be reduced to

$$\mathbf{E} + \boldsymbol{\beta}_{\mathrm{R}} \times \mathbf{B} = -\nabla\psi\,, \tag{44}$$

where $\psi = \Phi - (\boldsymbol{\beta}_{\mathrm{R}}\mathbf{A})$ has the meaning of the electric potential in a rotating coordinate system, which leads to particle acceleration in the region of a longitudinal electric field [see Eqn (14)]. In particular, for an ideally conducting neutron star, when

$$\mathbf{E}_{\mathrm{int}} + \boldsymbol{\beta}_{\mathrm{R}} \times \mathbf{B}_{\mathrm{int}} = 0\,, \tag{45}$$

we have $\psi = 0$.

For the case of a zero longitudinal electric field $(\mathbf{EB}) = 0$, we multiply scalarly Eqn (44) by $\mathbf{B}$ to obtain

$$(\mathbf{B}\,\nabla\psi) = 0\,. \tag{46}$$

This means that the potential ψ is constant on magnetic surfaces $\Psi = \mathrm{const}$:

$$\psi = \psi(\Psi)\,. \tag{47}$$

Hence, in the region of closed field lines we simply have

$$\psi = 0\,. \tag{48}$$

At the same time, in the area of open magnetic field lines which are separated from the neutron star by the region of longitudinal electric field (where condition (47) certainly does not hold), the potential ψ is nonzero. According to Eqn (44), the angular velocity Ω_{F}, which enters in the definitions of the electric field and the Goldreich–Julian density, will in the axisymmetric case be rewritten as

$$\Omega_{\mathrm{F}} = \Omega - 2\pi c\,\frac{\mathrm{d}\psi}{\mathrm{d}\Psi}\,, \tag{49}$$

so that

$$\mathbf{E} = -\frac{\Omega_{\mathrm{F}}}{2\pi c}\,\nabla\Psi\,. \tag{50}$$

As one can see, the angular velocity Ω_{F} is determined by a concrete mechanism of plasma generation near the surface of the neutron star and has the meaning of the angular velocity of plasma rotation. Indeed, determining the drift velocity $\mathbf{U}_{\mathrm{dr}}$ with the aid of relations (44) and (47), for an arbitrary inclination χ we have

$$\mathbf{U}_{\mathrm{dr}} = c\,\frac{\mathbf{E} \times \mathbf{B}}{B^2} = \Omega_{\mathrm{F}}[\mathbf{e}_z \times \mathbf{r}] + j\mathbf{B}\,,$$

where j is a scalar function. Consequently, the particle velocity is the sum of the velocity of corotation with the angular velocity Ω_{F} and the sliding velocity along the magnetic field. It is precisely the difference of the angular velocity Ω_{F} from Ω that leads to the subpulse drift discussed above. As a result, electric corotation currents are induced in the magnetosphere, the current density being

$$\mathbf{j}_{\rm cor} = \rho_{\rm GJ}\Omega_{\rm F}[\mathbf{e}_z \times \mathbf{r}]\,. \tag{51}$$

As regards the longitudinal currents, they are conveniently normalized to the Goldreich – Julian current density $j_{\rm GJ} = c\rho_{\rm GJ}$. The total electric current I flowing within the field tube with a magnetic flux Ψ can be written as

$$I(\Psi) = i_0 I_{\rm GJ}\,. \tag{52}$$

Here

$$I_{\rm GJ} = \frac{B_0\,\Omega^2 R^3}{2c} \tag{53}$$

is the characteristic total current flowing through the polar cap surface.

In what follows, it will be convenient for us to introduce a dimensionless accelerating potential $\beta_0 = \psi/\psi_{\max}$, where

$$\psi_{\max} = \left(\frac{\Omega R}{c}\right)^2 RB_0 \tag{54}$$

is the maximum potential drop V (8) in the acceleration region. As a result, the angular velocity $\Omega_{\rm F}$ above the acceleration region where the secondary plasma screens the longitudinal electric field, will in the axisymmetric case be determined as

$$\Omega_{\rm F} = (1-\beta_0)\Omega\,. \tag{55}$$

The appearance of electric currents in the pulsar magnetosphere is of paramount importance because longitudinal currents must lead to a neutron star braking. Clearly, the total current flowing out of the pulsar surface must be equal to zero. As a result, currents must flow along the pulsar surface, closing the longitudinal currents circulating in the magnetosphere. The pondermotive action of these currents gives rise to a deceleration of the radio pulsar.

To show this, we shall write the rate of the energy loss in the form

$$W_{\rm cur} = -\mathbf{\Omega}\mathbf{K}\,. \tag{56}$$

Here

$$\mathbf{K} = \frac{1}{c}\int\big[\mathbf{r}\times[\mathbf{j}_{\rm s}\times\mathbf{B}]\big]\,{\rm d}\mathbf{S} \tag{57}$$

is the braking torque associated with the Ampere force of the surface currents $\mathbf{j}_{\rm s}$. This torque is aligned antiparallel to the magnetic moment of the neutron star. Allowing now for definitions (52) and (56), we arrive at

$$W_{\rm cur} = k\,\frac{B_0^2\Omega^4 R^6}{c^3}\,i_0\cos\chi\,, \tag{58}$$

where the coefficient $k \approx 1$ depends on the geometry of the polar cap.

As has already been said, when expressed in symbols, the energy loss (58) coincides with the magnetodipole loss (1). However, the magnetodipole loss (1) is absent for the axisymmetric case $\chi = 0$. Furthermore, the energy loss (58) is proportional to the electric current (52) circulating in the magnetosphere. Making use of the known values of the moment of inertia and the radius of a neutron star $J \approx 10^{45}$ g cm^2 and $R \approx 10^6$ cm [108, 109], we obtain the estimate of the deceleration rate for $B_0 \sim 10^{12}$ G:

$$\dot{P} \sim 10^{-15}\,, \tag{59}$$

which is in agreement with actual observations [18].

We emphasize that the current loss $W_{\rm cur}$ (58) does not coincide, even in the force-free approximation, with the Poynting vector flux

$$W_{\rm em} = \frac{c}{4\pi}\int[\mathbf{E}\times\mathbf{B}]\,{\rm d}\mathbf{S} = \frac{1}{2\pi c}\int I(\Psi)\,\Omega_{\rm F}(\Psi)\,{\rm d}\Psi\,. \tag{60}$$

Indeed, using identity transformations, one can rewrite formula (56) as [16]

$$W_{\rm cur} = W_{\rm em} + W_{\rm part}\,, \tag{61}$$

where, according to Eqn (44), the summand

$$W_{\rm part} = \int \psi\mathbf{j}_{\rm e}\,{\rm d}\mathbf{S} = \frac{1}{2\pi c}\int I(\Psi)\big[\Omega - \Omega_{\rm F}(\Psi)\big]\,{\rm d}\Psi \tag{62}$$

corresponds to the energy acquired by primary particles in the acceleration region. Given this, we have

$$\frac{W_{\rm part}}{W_{\rm cur}} \approx \beta_0\,. \tag{63}$$

Notice that although $\beta_0 \approx 1$ for slow pulsars with $P \approx 1$ s (when a considerable portion of the total loss goes to plasma generation and acceleration), the total energy carried away by particles turns out to be lower than the energy carried away by the electromagnetic field, because only a small part of the energy acquired by primary particles is ultimately transferred to the secondary plasma. The greater part of the energy goes to the generation of sufficiently soft gamma quanta for which the neutron star magnetosphere appears to be transparent. As a result, the applicability condition of the force-free approximation proves to hold even for pulsars with $\beta_0 \approx 1$. Incidentally, it is for this reason that the relative fraction of energy radiated by slow pulsars in the gamma range must be close to unity, which is observed in reality [110].

At the same time, the loss of angular momentum will be entirely due to the electrodynamic loss (57):

$$K_{\rm cur} = \frac{1}{2\pi c}\int I(\Psi)\,{\rm d}\Psi\,, \tag{64}$$

exactly as it should be, because the angular momentum of photons, $\mathcal{L}_{\rm ph}$, emitted near the surface of a star is much smaller than $\Omega\mathcal{L}_{\rm ph}$. Owing to Eqns (60) and (62), the condition

$$W_{\rm tot} = \Omega K_{\rm tot} \tag{65}$$

(which, for a rotating neutron star, holds by definition) also appears to be identically fulfilled for the outgoing radiation. However, as we have seen, this relation cannot be obtained without the additional summand (62). An attempt to solve the energy loss problem exclusively in the framework of the force-free approximation inevitably leads to misunderstanding [111, 112].

3.2 Theory of the neutron star magnetosphere

So, the question of deceleration of the radio pulsar rotation is reduced to a determination of the longitudinal electric current circulating in the magnetosphere of a neutron star, which can in turn be only done in the framework of solving the complete problem of the magnetosphere structure. And the question of the relative role of current and magnetodipole losses can also be resolved only in the framework of the complete problem.

As is well known, since the 1970s the model of an axisymmetric force-free magnetosphere [113, 114] has been the main line of inquiry, where the plasma energy density was assumed to be substantially lower than the magnetic field energy density. In this limit, the structure of the magnetosphere is described by the equation [15, 16, 98, 115, 116]

$$-\left(1-\frac{\Omega_{\mathrm{F}}^2\varpi^2}{c^2}\right)\nabla^2\Psi+\frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi}-\frac{16\pi^2}{c^2}\,I\,\frac{\mathrm{d}I}{\mathrm{d}\Psi}+\frac{\varpi^2}{c^2}\,\Omega_{\mathrm{F}}(\nabla\Psi)^2\,\frac{\mathrm{d}\Omega_{\mathrm{F}}}{\mathrm{d}\Psi}=0\,. \qquad (66)$$

Its solution determines the induction of the poloidal magnetic field

$$\mathbf{B}_{\mathrm{p}}=\frac{1}{2\pi\varpi}\,[\nabla\Psi\times\mathbf{e}_{\varphi}]\,. \qquad (67)$$

On the contrary, in the framework of the force-free approximation, the key quantity, i.e. the longitudinal current I flowing in the magnetosphere, is a free parameter. In particular, the first exact solutions in the axisymmetric case [98, 113] [as in the case of an oblique rotator [48], where we had to solve an equation similar to Eqn (66)] were just obtained for a zero longitudinal current (and for $\Omega_{\mathrm{F}}=\mathrm{const}$). In this approximation, the force-free equation becomes linear irrespective of the inclination χ, which allows us to obtain its solution in a rather simple way.

Exact solutions to the force-free equations for a zero longitudinal current and $\Omega_{\mathrm{F}}=\Omega=\mathrm{const}$ have repeatedly been discussed in the scientific literature (see, e.g., Refs [15, 16, 21] and Fig. 4). That is why we shall only formulate the basic conclusions inferable from the solutions obtained.

(1) A solution can be constructed only within the confines of the light surface $|\mathbf{E}|=|\mathbf{B}|$ which, for a zero longitudinal current, coincides with the light cylinder $\varpi=c/\Omega$. Outside the light cylinder, the electric field becomes stronger than the magnetic one, which violates the freezing-in condition $\mathbf{E}+c^{-1}[\mathbf{v}\times\mathbf{B}]=0$. As a result, on the light cylinder surface, the particle energy formally becomes infinite. In the general case, the light surface is not coincident with the light cylinder but is always at larger distances.

(2) On the light cylinder, irrespective of the angle of axis inclination χ the magnetic field must be perpendicular to its surface [48, 117]. This mathematical result leads to a very important physical conclusion: the Poynting vector here does not have a normal component, which means that the electromagnetic energy flux through the light cylinder surface is equal to zero. Consequently, in the absence of longitudinal current the secondary plasma that fills the magnetosphere must completely screen the magnetic dipole radiation of a neutron star. Hence, the rotational energy loss of a radio pulsar can only be tied to the pondermotive action of currents flowing in the magnetosphere of a neutron star, and so formula (58) completely determines the radio pulsar deceleration rate.

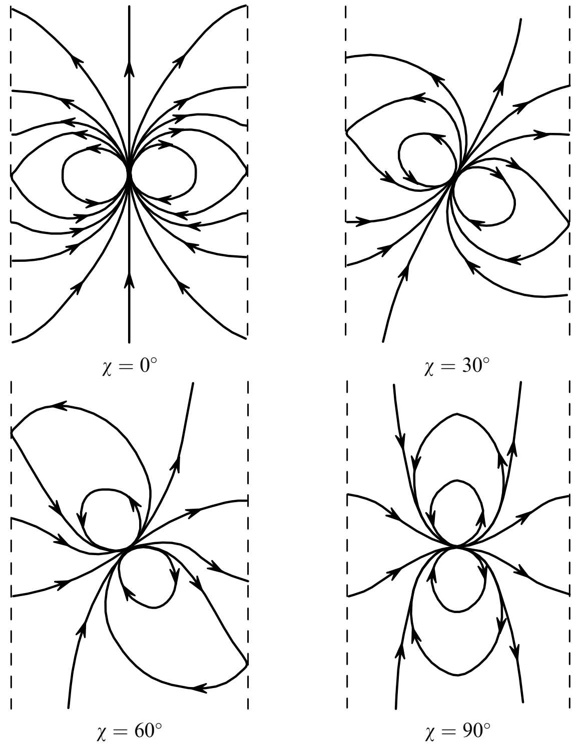

Figure 4. Structure of the neutron star magnetosphere for zero longitudinal currents and an arbitrary angle of inclination χ of the magnetic dipole axis to the axis of revolution [16, 98, 113]. The applicability region of the approach under study is restricted to the light cylinder.

(3) In the absence of longitudinal current, magnetic field lines are concentrated near the equator. In other words, the toroidal currents $\mathbf{j}=\rho_{\mathrm{GJ}}\,[\mathbf{\Omega}\times\mathbf{r}\,]$ due to the corotation of the Goldreich–Julian density ρ_{GJ} tend not to collimate but, on the contrary, spread magnetic field lines. As a result, the magnetic field along the axis of rotation falls exponentially rather than by a power law.

As concerns solutions with a nonzero longitudinal electric current, even the simplest force-free equation in this situation becomes nonlinear, which makes its analysis extremely difficult. Except for the Michel solution [114], in which the magnetic field is monopole, and some other exact solutions [48, 122] (see also Refs [46, 118, 121]), the problem still remains unsolved. Technically, this is connected with the fact that equation (66) contains a critical surface — a light cylinder, the passage through which requires expansion of the solution in terms of eigenfunctions that have no singularities on this surface.

This problem up to now has been solved only analytically, which could be done only for a certain class of functions $I(\Psi)$, that is, for the case when the current density is constant over the entire region of open magnetic field lines [i.e. when $I(\Psi)=k\Psi$] and the closure of current proceeds along the separatrix between open and closed field lines. In such a statement, equation (66) turns out to be linear not only in the region of closed, but also open magnetic field lines, and the main problem is reduced to 'matching' the solutions in these two regions (an extensive discussion of this issue can be found in Refs [16, 122]).

A singularity at the light cylinder has recently been passed numerically (using an iteration procedure) for the first time [123]. This allowed the investigation of the case of an arbitrary profile of the current $I(\Psi)$. However, the results as yet obtained in this field are preliminary, although the approach itself seems to be fairly fruitful.

Nonetheless, the analysis of the solutions already obtained suggests a number of important conclusions.

(1) When a longitudinal electric current is coincident with the Goldreich – Julian current (the Michel solution), there occurs a complete compensation of two opposite processes, namely, decollimation due to toroidal currents and collimation thanks to presence of longitudinal currents. As a result, the monopole magnetic field, which is an exact solution in the absence of particles, appears to be an exact solution of equation (66) in the presence of plasma as well. The exact magnitude of the critical current depends, of course, on the concrete geometry of the poloidal magnetic field. However, one can say with confidence that $j_{\rm cr} \approx \rho_{\rm GJ} c$.

(2) For $j_{\parallel} > j_{\rm cr}$, the light surface (which, we recall, does not coincide with the light cylinder in the general case) goes to infinity. This means that for sufficiently high longitudinal currents the solution can be extended to infinity, the magnetic surfaces being collimated in the direction of the axis of rotation [118] (precisely what is needed to explain the jets registered recently for radio pulsars [119, 120]).

(3) If there exist some physical restrictions from above on the longitudinal current ($j_{\parallel} < j_{\rm cr}$), then the magnetosphere contains a 'natural boundary', that is, a light surface. The complete problem, which also includes external regions, cannot be solved in the framework of conventional magnetic hydrodynamics if for no other reason than regions with a multistream flow inevitably occur in this case.

The latter point should be somewhat clarified. The question of a possible limitation of the longitudinal current owing to the 'interaction' between regions of closed and open field lines was first considered in Ref. [48]. The 'nonlinear Ohm's law'

$$\beta_0(i_0) = \beta_{\rm max}\left[1 - \left(1 - \frac{i_0^2}{i_{\rm max}^2}\right)^{1/2}\right] \tag{68}$$

was formulated, which relates the electric current $I = i_0 I_{\rm GJ}$ circulating in the magnetosphere to the potential drop $\psi = \beta_0 \psi_{\rm max}$ across the particle acceleration region.

If relation (68) actually holds, then in Ruderman – Sutherland type models (in which, we recall, the electric current in the plasma production region can be arbitrary) the longitudinal current i_0 is to be determined from relationship (68). In particular, for a sufficiently low potential drop $\beta_0 < 1$, the longitudinal current must also be small. This means that the light surface on which additional particle acceleration is inevitable, must be located at a finite distance from the neutron star. At the same time, the existence of a light surface makes the theory much more sophisticated: not a single reliable result on plasma behavior outside the light surface has yet been obtained.

However, one should not think that the light surface may be located at a finite distance only in the framework of the Ruderman – Sutherland model of hindered particle ejection from the neutron star surface. For the example of the force-free approximation one can see that the light surface goes to infinity only for sufficiently large longitudinal currents. As is shown in the section to follow, this conclusion also remains true in the more general magnetohydrodynamic case. That is why, with any additional restrictions from above on the longitudinal electric current, one can expect the appearance of a light surface at a finite distance from the radio pulsar.

However, in the framework of Arons type models (which assume free particle ejection from the star surface), the longitudinal electric current is fixed and, which is especially important, the density of this current is close to the Goldreich – Julian density. Hence, it is not excluded that in the actual dipole geometry of the magnetic field of a pulsar this current is insufficient for a continuous (in particular, transonic) plasma outflow to distances large compared with the radius of a light cylinder. An exact proof of this fact requires, of course, a special study.

Returning to the problem of longitudinal current, we shall emphasize once again that this issue remains quite open. The only thing to be asserted with confidence is that the longitudinal current circulating in the radio pulsar magnetosphere does not apparently exceed the critical one: $I \approx I_{\rm GJ}$. Thus, the question of the exact value of the total energy loss $W_{\rm tot}$ and the existence of a light surface on which, as will be seen below, an additional particle acceleration is possible, remains unclarified. For many applications, the estimate $I \approx I_{\rm GJ}$ appears however to be sufficient, and so relationship (7) for $I/I_{\rm GJ} = 1$ is a good approximation to the rate of the energy loss $W_{\rm tot}$.

3.3 Pulsar wind

The question of the pulsar wind has also remained open for many years. In other words, no consistent model has up to now been constructed which would simultaneously describe the energy transport from a neutron star surface to infinity and efficient particle acceleration, i.e. practically a complete conversion of the electromagnetic field energy to the energy of the outflowing plasma (there exists unambiguous evidence in favor of such a conversion; see Section 3.4).

At the same time, great attention was paid in the 1970s to the motion of relativistic particles in an intense electromagnetic wave of a rotating magnetic dipole [124, 125] and since the 1980s, when it became clear that particles must play a decisive role in the pulsar wind, the principal direction has been the magnetohydrodynamic approach [34 – 37, 39, 126 – 128]. This approach was simultaneously (and possibly firstly) discussed in connection with the problem of the formation of jets from active galactic nuclei [129 – 133] and young stellar objects [134 – 139]. The authors, in fact, considered the possibility of constructing a complete solution, i.e. the extension of the solutions obtained in the force-free approximation for internal regions of magnetosphere to the pulsar wind region.

The point is that the force-free approximation, within which the first results were obtained, encounters some difficulties. First of all, within this approximation one cannot determine the fraction of energy transported by relativistic particles. Moreover, since in the force-free approximation the electric current $I(\Psi)$ is constant in magnetic field lines, there is no hope of consistently considering the question of the current closure.

As to the magnetohydrodynamic approach, it is rather easy to describe in this framework both the energy conversion from the electromagnetic field to the particles and the whole of the magnetic field structure [140, 141]. Since in this approximation the electric current I need not be constant in magnetic field lines, the question of the closure of current can be investigated as well. Unfortunately, this does not refer to the angular velocity $\Omega_{\mathrm{F}}(\Psi)$ of plasma rotation which remains constant on magnetic surfaces.

Finally, it is exceedingly important that within the framework of the complete set of magnetohydrodynamic equations, the electric current itself circulating in the magnetosphere is not already a free parameter but should be determined from the critical conditions on singular surfaces [34, 142]. In other words, one of the main problems encountered by the theory, i.e. the construction of a current system and, as a consequence, the determination of the energy loss, can be stated mathematically rigorously.

The essence of this approach can briefly be formulated as follows. For axisymmetric and stationary flows (and also with accurate fulfilment of the freezing-in condition $\mathbf{E}+c^{-1}[\mathbf{v}\times\mathbf{B}]=0$ and the ideality condition $(\mathbf{v}\nabla s)=0$ [143]) in the general case there exist five 'integrals of motion' that are invariant on axisymmetric magnetic surfaces. These are the energy flux (Bernoulli integral)

$$E(\Psi)=\frac{\Omega_{\mathrm{F}} I}{2\pi}+\gamma\eta\mu \tag{69}$$

($\mu \approx m_{\mathrm{e}}c^2$ is the relativistic enthalpy), the z-component of angular momentum $L(\Psi)$, the angular velocity $\Omega_{\mathrm{F}}(\Psi)$, the entropy $s(\Psi)$, and the particle-to-magnetic field flux ratio $\eta(\Psi)$. As a result, in a given poloidal field all physical characteristics of the flow (including the particle energy) can be determined using sufficiently simple algebraic relations. Since the Bernoulli integral (69) now contains the contributions of both the Poynting vector and the particles, in this approach it actually becomes possible to consistently consider the process of energy conversion from the electromagnetic field to the relativistic plasma.

At the same time, the problem of finding the poloidal magnetic field itself is much more difficult. This is first of all connected with the complex structure of the stream (trans-field) equation describing equilibrium configurations. This stream equation is a mixed-type nonlinear differential equation in partial derivatives, which contains integrals of motion in the form of free functions and changes from elliptic type equation to that of hyperbolic on singular surfaces. In the case of cold plasma $s=0$, the most interesting for radio pulsars, such singular surfaces will be the Alfven surface (coincident in the force-free limit with a light cylinder) and the fast magnetosonic surface.

The latter circumstance is of particular importance: beyond the fast magnetosonic surface at large distances from a neutron star, the stream equation, as distinct from Eqn (66), becomes hyperbolic. The basic technical difficulty which obstructs a consistent analysis of all possible solutions is the existence of singular surfaces whose position should also be determined from the solution. Furthermore, as is shown in the sequel, the results obtained in the approximation of a given poloidal magnetic field may differ qualitatively from the results of a self-consistent analysis. That is why the question of the determination of particle energy can unfortunately be solved only in the framework of the complete problem.

It is of interest that the first results obtained in the magnetohydrodynamic approach were reported by F Michel as far back as 1969 [126]. First of all he introduced the key relativistic parameter — the magnetization

$$\sigma=\frac{e\Omega\Psi_{\mathrm{tot}}}{4\lambda m_{\mathrm{e}}c^3}, \tag{70}$$

which has the meaning of the electromagnetic-to-particle energy flux ratio near the star surface. Here Ψ_{tot} is the total magnetic flux in the source.

We emphasize that for simplicity Michel considered solely the case of a monopole magnetic field. One should therefore take care when determining the quantity Ψ_{tot} for real astrophysical objects. In particular, for radio pulsars we have

$$\Psi_{\mathrm{tot}}=\pi B_0 R_0^2 \approx \pi B_0 R^2\,\frac{\Omega R}{c}, \tag{71}$$

which corresponds to the magnetic flux in the region of open field lines only. As a result, one gets

$$\sigma=\frac{eB_0\Omega^2 R^3}{4\lambda m_{\mathrm{e}}c^4}. \tag{72}$$

Thus, for characteristic parameters of radio pulsars ($P\sim 1$ s, $B_0\sim 10^{12}$ G) we have $\sigma\sim 10^4-10^5$ and only for the fastest ones ($P\sim 0.1$ s, $B_0\sim 10^{13}$ G) the parameter σ reaches values of the order of 10^6-10^7. A high value of the parameter σ shows that the main contribution to the energy flux in the internal magnetosphere regions is made by the electromagnetic field flux.

It has turned out that there exists a very simple relation between the Michel magnetization parameter σ and the particle energy $\gamma m_{\mathrm{e}}c^2$ on a fast magnetosonic surface [126]:

$$\gamma\approx\sigma^{1/3}. \tag{73}$$

This means that here, too, the particle-to-electromagnetic field energy flux ratio

$$\frac{W_{\mathrm{part}}}{W_{\mathrm{em}}}\approx\sigma^{-2/3} \tag{74}$$

must be much less than unity.

Finally, the critical value of current for which the condition of a smooth passage through a fast magnetosonic surface holds (i.e. the flow is transonic) was shown to be also close to the Goldreich – Julian current:

$$j_{\mathrm{cr}}\approx\rho_{\mathrm{GJ}}c. \tag{75}$$

For longitudinal currents different from the critical one, the structure of the flow remains close to that in the force-free case. In particular, for $j_{\parallel}<j_{\mathrm{cr}}$ the light surface $|\mathbf{E}|=|\mathbf{B}|$ is at a finite distance from the neutron star.

Recall that beginning from paper [126] the generally accepted point of view has been that a fast magnetosonic surface is located at infinity. Since then the result has been reproduced many times [144 – 146]. However, this conclusion has turned out to hold only with the assumption that the poloidal magnetic field is assigned.

For an example of an exact solution it has recently been demonstrated [147] that in the self-consistent case, when the poloidal magnetic field is not given but is capable of varying through the presence of currents flowing in the magnetosphere, the fast magnetosonic surface is located at a finite distance from the star. In particular, we have [147]

$$r_{\mathrm{f}}(r,\theta)\approx\sigma^{1/3}\sin^{-1/3}\theta\,R_{\mathrm{L}}\qquad\text{for}\qquad\sigma>\gamma_{\mathrm{in}}^3, \tag{76}$$

where γ_{in} is the characteristic Lorentz factor of particles found near the neutron star surface, and in addition [39]

$$r_{\mathrm{f}}(r,\theta)\approx\left(\frac{\sigma}{\gamma_{\mathrm{in}}}\right)^{1/2}R_{\mathrm{L}}\qquad\text{for}\qquad\sigma<\gamma_{\mathrm{in}}^3. \tag{77}$$

Here it turned out that the acceleration of particles beyond the boundaries of the fast magnetosonic surface ceases almost completely [147, 148], so that estimate (74) virtually relates to the particle flux to infinity.

As a result, none of the magnetohydrodynamic theories allowed the construction of a reasonable pulsar wind model. All the attempts to find a self-consistent solution containing efficient particle acceleration failed. We recall that this conclusion refers exactly to the relativistic case; for nonrelativistic plasma flows, on the contrary, the acceleration efficiency must be high [126].

Thus there is an apparent contradiction between the necessity of efficient particle acceleration, which follows from observations, and the absence of such an acceleration in 'smooth' magnetohydrodynamic models in which the electric current is determined from the critical conditions on singular surfaces and the light surface is located at infinity. It is therefore not surprising that various models ready to overstep the limits of the 'classical' scheme are nowadays being intensively discussed.

Efficient particle acceleration may first of all be brought about through the above-mentioned property of relativistic flows: for small electric currents, the light surface $|\mathbf{E}| = |\mathbf{B}|$ is located at a finite distance from the light cylinder, and for $i_0 \ll 1$ — in its vicinity. Therefore, if the interaction of the regions of closed and open field lines actually does lead to a limitation of the longitudinal current i_0 (or some other causes exist for the current i_0 to be fixed in the plasma generation region), one should expect the appearance of a light surface and efficient particle acceleration. Incidentally, this conclusion has recently been drawn by the authors of paper [149], who also obtained a number of crucial results in the relativistic wind theory [36, 37].

The first calculations of such an acceleration were carried out long ago [48]. In the simplest cylindrical geometry of the problem, by way of solution of two-fluid hydrodynamics equations (governing the difference between the electron and positron motions) it has been shown that a considerable portion of the energy transported by the electromagnetic field within the confines of the light surface, in the thin transition layer

$$\Delta r \sim \lambda^{-1} R_{\mathrm{L}} \tag{78}$$

near the light surface is transferred to plasma particles ($\lambda \sim 10^3 - 10^5$ is the multiplicity of particle creation near the surface of a neutron star). Here, as is seen from Fig. 5, the longitudinal current circulating in the magnetosphere is closed almost completely. As a result, the high efficiency of particle acceleration finds a natural explanation.

We note, however, that the presence of a light surface brings substantial intricacy into the whole problem of the structure of the neutron star magnetosphere. In this case one can reliably describe only the internal magnetosphere regions. The questions about the further fate of accelerated particles, energy transfer through large distances, and the closure of current remain, in fact, open. To be solved, these problems require going beyond the scope of one-fluid hydrodynamics; they are unlikely to be solved at all within an analytical approach.

The calculations done in Ref. [48] only referred to cylindrical geometry in which it was impossible, for example, to consider magnetic-surface and electric-potential perturbations consistently. In particular, it remained unclear to what extent the solutions obtained were general. It is only recently that an analogous result based on solutions of equations of two-fluid hydrodynamics has also been obtained for a more realistic geometry [150] where the poloidal magnetic field is close to a monopole one. Practically all the results obtained for cylindrical geometry have been shown to hold for a more realistic two-dimensional geometry. The particle energy has been confirmed here to reach values of the order of

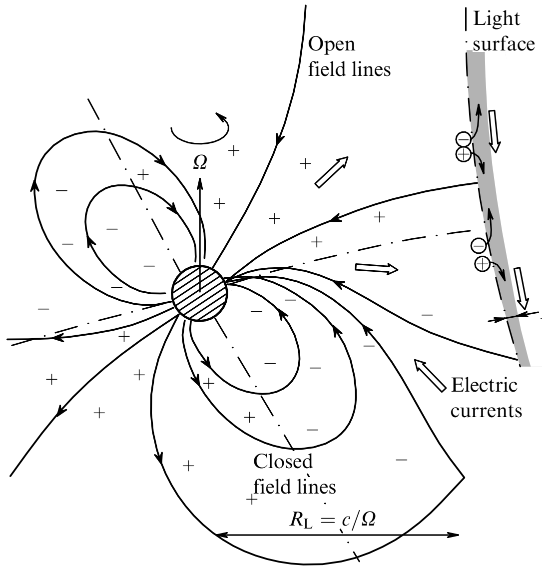

Figure 5. Particle acceleration in the model posed in Ref. [16]. If, apart from the critical condition, other physical limitations on the longitudinal current ($j_{\parallel} < j_{\mathrm{cr}}$) exist on a fast magnetosonic surface, then the magnetosphere contains a 'natural boundary', i.e. a light surface $|\mathbf{E}| = |\mathbf{B}|$, where the freezing-in condition does not hold. For this reason, electrons and positrons are accelerated in opposite directions along the electric field, inducing a strong poloidal electric current. In a thin layer $\Delta r \approx R_{\mathrm{L}}/\lambda$, the longitudinal current is almost completely closed and the particle energy flux becomes comparable with the total energy loss.

$$\begin{aligned}\mathcal{E}_{\mathrm{e}} &\sim eB_0R\,\frac{1}{\lambda}\left(\frac{\Omega R}{c}\right)^2 \\ &\sim 10^4\left(\frac{\lambda}{10^3}\right)^{-1}\frac{B_0}{10^{12}\,\mathrm{G}}\left(\frac{P}{1\,\mathrm{s}}\right)^{-2}\ [\mathrm{MeV}]\,,\end{aligned} \tag{79}$$

but not greater than 10^6 MeV. However, as in the one-dimensional case, the question concerning the construction of a solution outside the light surface remained unsettled. Nevertheless, the solution constructed may well serve as a 'seed' for further numerical computations because in the internal region simple analytical relations have been obtained for all physical parameters.

As a matter of fact, a similar model was discussed by Mestel and Shibata [47] who also assumed the existence of a dissipation domain near a light cylinder (Fig. 6). The difference is that in the latter paper only a slight variation of the longitudinal current was assumed, whereas the relative variation of the electric potential along magnetic field lines (and, therefore, a change of the angular velocity Ω_{F}) was considered to be large. The light surface was again moved towards infinity. At large distances from the neutron star, the main energy flux is associated, as before, with the Poynting vector.

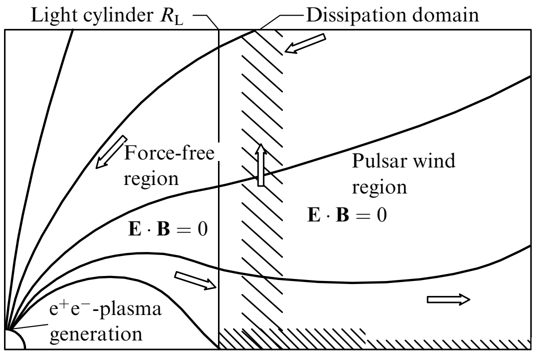

Figure 6. Particle acceleration in the model posed in Ref. [47]. Here, only a slight variation of the longitudinal current near a light surface is assumed, whereas the relative variation of the electric potential along the magnetic field lines (and, therefore, the variation of the angular velocity Ω_F) is thought of as large, although insufficient to change the particle energy radically.

Note that in this model the properties of the transition layer are only postulated. Thus, the possibility of the existence of such a layer remains open. The main property of this layer — a large variation of the angular velocity Ω_F with a relatively small variation of the longitudinal current — is in contradiction with the properties of the acceleration region in the vicinity of the light surface. As shown by the analysis of the equations of two-fluid magnetic hydrodynamics [48, 150], it is the longitudinal current rather than the electric potential that must change most rapidly in the direction perpendicular to the transition layer.

The result obtained can easily be explained. The point is that near the light surface the particle energy formally tends to infinity, i.e. the freezing-in condition is violated, which requires a passage to more accurate two-fluid equations. This physically leads to an oppositely directed acceleration of electrons and positrons along the electric field. As a consequence, a strong poloidal electric current arises, in the induction of which the total electron and positron density $n = \lambda|\rho_{GJ}|/|e|$ participates. It is the poloidal current that leads to a sharp decrease of the toroidal magnetic field, i.e. to a decrease of the Poynting vector.

As regards the electric potential, its variation in the transition layer is determined by the electric charge density which is proportional to the difference of the electron and positron densities only. Since the particle density exceeds the Goldreich–Julian density $n_{GJ} = |\rho_{GJ}|/|e|$ by many orders of magnitude, the relative change of current in the layer must appreciably exceed the change in the electric potential. This is in fact responsible for the appearance of the factor λ^{-1} in expression (78).

Finally, a new interesting mechanism based on the results of numerical calculations has recently been proposed by Bogovalov [39]. It has turned out that in the relativistic case, in spite of the stationary boundary conditions on the neutron star surface, under certain conditions (namely, when $W_{em} \gg W_{part}$) beyond a fast magnetosonic surface a nonstationary turbulent region may occur, in which the electric and magnetic fields undergo sharp random variations (Fig. 7). Incidentally, this result confirms once again the limited character of the conventional magnetohydrodynamic approach which cannot be taken to consider such effects. As a result, efficient particle acceleration can proceed in a turbulence region.

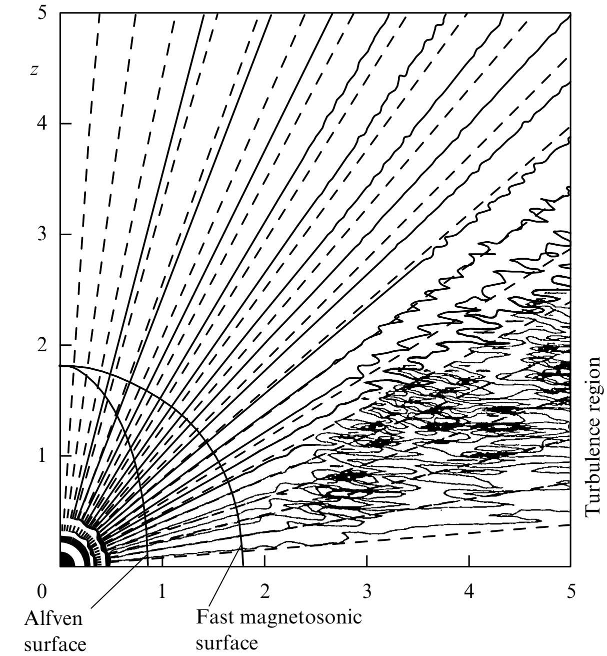

Figure 7. Particle acceleration in the model posed in Ref. [39]. It becomes possible in the turbulence region occurring outside the fast magnetosonic surface.

Thus, in spite of understanding the importance of the pulsar wind and particle acceleration problem and in spite of the large number of papers devoted to this subject, no satisfactory model exists at present. As has already been emphasized, one of the main reasons is the impossibility of formulating simple enough equations describing the behavior of relativistic plasma in the case when its energy density is comparable with that of the electromagnetic field. That is why practically nothing definite can now be said about the energy spectrum of particles escaping from the magnetosphere or about their radiation. It is only clear that already at small distances from the light cylinder particles must transfer a substantial fraction of energy compared to the total energy flux.

3.4 Analysis of observations

We shall now discuss direct observational tests that might cast light on the real structure of the pulsar magnetosphere. First of all this concerns the mechanism of neutron star braking. Recall that the above picture of radio pulsar braking is now not conventional. The point is that beginning with the paper by Pacini [7] published before the official announcement of the discovery of radio pulsars, the magnetodipole energy loss (1) was treated as that underlying the braking mechanism. According to this model, the overwhelming portion of energy must be carried away by low-frequency electromagnetic waves and only a small fraction could be connected with the pulsar wind. As has already been said, the current loss (7) hardly differs in magnitude from the magnetodipole loss, and therefore the analysis of the statistical distribution of pulsars unfortunately does not allow a choice between these two braking mechanisms [16]. In the theoretical context, the sole unambiguous indication of the absence of magnetic dipole radiation is, in our opinion, the above-mentioned result, i.e. an exact solution for the magnetosphere of an oblique rotator in the case of zero longitudinal current [48], which (the solution) does not contain an electromagnetic energy flux beyond the magnetosphere.

The only direct test offering some insight into the mechanism of radio pulsar braking centers around the determination of the so-called braking index

$$n_{\mathrm{br}} = \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2}\,, \tag{80}$$

which can only be found if the second derivative $\ddot{\Omega}$ of the angular velocity of rotation is known. However, since much time is needed to determine this quantity (not to mention the fact that in many cases the second derivative $\ddot{\Omega}$ cannot be distinguished against the background of low-frequency disturbances [151]), it has been measured for only four radio pulsars.

It has turned out that in all the four cases $n_{\mathrm{br}} < 3$ (Table 2), whereas in the framework of magnetodipole loss we must have [15]

$$n_{\mathrm{br}} = 3 + 2\cot^2\chi\,. \tag{81}$$

It is already this circumstance that can be regarded as a direct contradiction between the model of magnetodipole loss and observations. That is why numerous attempts have been made to 'correct' relation (81), for instance, at the expense of the magnetic field evolution [156, 157] or the interaction between the superfluid component in the core of a neutron star and its hard crust [158, 159]. However, the majority of such effects may lead to insignificant corrections and cannot appreciably affect the quantity (81).

Table 2. Radio pulsar braking indices.

Pulsar	P, s	$\dot{P}$, 10^{-15}	n_{br}	Ref.
B0531 + 21	0.033	421	2.51 $\pm$0.01	[152]
B0540 − 693	0.050	479	2.24 $\pm$0.04	[153]
B0833 − 45	0.089	124	1.40 $\pm$0.20	[154]
B1509 − 58	0.150	1490	2.837 $\pm$0.001	[155]

As to the mechanism of current-induced braking, one can obtain [49]

$$n_{\mathrm{br}} = 1.93 + 1.5\tan^2\chi\,, \tag{82}$$

which is in good agreement with observations. In any case, the determination of the braking index for other radio pulsars, as well as the second-order braking index $n_{\mathrm{br}}^{(2)} = \Omega^2\dddot{\Omega}/\dot{\Omega}^3$ (this index is now available for one pulsar only [18]) would appreciably clarify the nature of radio pulsar energy loss.

Next, using the explicit form of 'Ohm's law' $i_0 = i_0(\beta_0)$ (68) and the potential drop in the Ruderman – Sutherland model, one can rewrite the inequality $P \ll P_{\max}$ in the form

$$Q \ll 1\,,$$

where

$$Q = 2\left(\frac{P}{1\,\mathrm{s}}\right)^{11/10}\left(\frac{\dot{P}}{10^{-15}}\right)^{-4/10}. \tag{83}$$

The parameter $Q \approx i_0$, which can be directly obtained from observations, proved to be very convenient for determining the main characteristics of radio pulsars [20, 49, 160]. For instance, in the hollow cone model, the ratios of the internal radius r_{in} of the directivity pattern and the height H of the acceleration region to the size R_0 of the polar cap are written as

$$\frac{r_{\mathrm{in}}}{R_0} \approx Q^{7/9}\,, \tag{84}$$

$$\frac{H}{R_0} \approx Q\,. \tag{85}$$

One can thus conclude that pulsars with the parameter $Q \approx 1$ must have a narrow radiation cone and, therefore, a double-humped mean profile. Such pulsars must show irregularities of radiation, mode switching effects, etc. On the contrary, pulsars with the parameter $Q \ll 1$ have a stable single-humped mean profile. This is precisely the picture to be observed in reality [20, 160].

Finally, we recall that the determination of the evolution of the inclination χ of the magnetic dipole axis to the axis of rotation might become a direct test. Since for current losses the braking torque $\mathbf{K}$ is directed oppositely to the magnetic moment of the neutron star [16], the Euler equation leads to conservation of the projection of the angular velocity of rotation onto the axis perpendicular to the torque $\mathbf{K}$. Hence, during the evolution the quantity [16]

$$\Omega\sin\chi = \mathrm{const} \tag{86}$$

must be conserved.

Consequently, the angle χ between the axis of rotation and the magnetic dipole axis must increase upon current loss (rather than decrease as in the case of magnetic dipole radiation), and the typical time of its evolution must coincide with the characteristic time of pulsar period variation $\tau_{\mathrm{D}} = P/2\dot{P}$ [49]. Unfortunately, no method has yet been found to determine the direction of evolution of the inclination χ for individual pulsars. Statistically, the prediction of an increase of the angle χ, as is well known, does not contradict observations [49].

The latter statement should also be clarified. The point is that observations show an unconditional *decrease* in the mean angles χ of inclination with increasing pulsar period P and decreasing period derivative $\dot{P}$ [161, 162]. That is why the mean inclination of the axes formally decreases with increasing dynamic age τ_{D}. However, this does not at all mean that for each individual pulsar the angle of inclination decreases with time. Such a behavior of the mean inclination χ of axes can also be realized if for each pulsar the angles χ increase according to formula (86).

Indeed, as is shown in Fig. 8, with the given values of the pulsar period P and the magnetic field induction B, particle

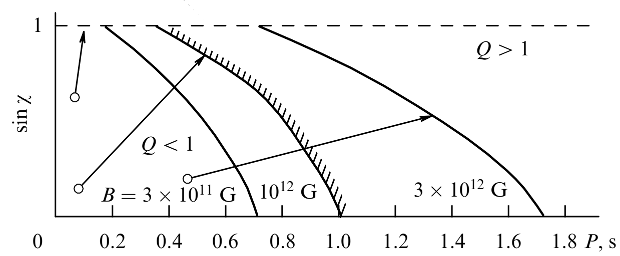

Figure 8. Extinction line of pulsars in the $P - \sin\chi$ diagram for different magnetic fields B. The arrows indicate the evolution of individual pulsars in the current loss model (86). Particle creation is inhibited when the angles χ are close to 90°. Therefore, neutron stars with the angle χ higher than and to the right of the extinction line will not behave as radio pulsars irrespective of the braking model.

creation is suppressed for angles χ close to 90° because for such angles the Goldreich–Julian charge density (3) decreases appreciably thus causing a decrease of the electric potential drop near the neutron star surface. As a result, stable generation of secondary particles becomes impossible. Hence, owing to such a dependence of pulsar extinction lines on the angle χ, the *mean* angle of inclination of axes may also decrease with increasing dynamic age, for instance, for a homogeneous pulsar distribution over the plane $P - \sin\chi$. The extensive analysis carried out in Refs [16, 49] on the basis of the kinetic equation governing the pulsar distribution confirmed that the observed pulsar distribution over the angle of inclination of axes does not contradict the hypothesis (86) for an increase of the angle χ for each individual pulsar.

However, the most convincing evidence in favor of the absence of magnetodipole loss was in our opinion the discovery of time-dependent optical radiation from companions in some close binary systems containing radio pulsars [163]. Such an optical radiation with a periodicity exactly coincident with the orbital period of the binary system is naturally associated with heating the part of the companion star that faces the radio pulsar.

It has turned out that the energy re-emitted by the companion is practically coincident with the total energy emitted by the radio pulsar into a corresponding solid angle. Clearly, this fact cannot be understood from the model of magnetic dipole radiation since the coefficient of the low-frequency wave transformation cannot be close to unity. Only if a considerable portion of the energy is connected with a relativistic particle flux, the heating of the star surface will be sufficiently effective.

Concluding, we should say that although the key physical processes proceeding in the vicinity of neutron stars were understood many years ago, the structural theory of the pulsar magnetosphere and the pulsar wind theory are still very far from being completed. In particular, to construct a quantitative model of neutron star magnetosphere, new non-trivial ideas are needed and the numerical methods should be developed especially in connection with the pulsar wind problem.

4. Radio emission

4.1 The theory of radio emission

As has already been said, there is no generally accepted outlook for the problem of the physical nature of pulsar coherent radio emission. Except the 'initial material', i.e. the parameters of the electron–positron plasma flowing along open magnetic field lines, there is in fact not a single point of general concord. For this reason it is very difficult to carry out a somewhat comprehensive analysis of the most fresh results devoted to this subject, the more so as practically no new fruitful ideas have been reported in a matter of recent years. Therefore it seems pertinent not to give here a detailed comparison of various papers but only to recall the principal directions in which the theory of pulsar radio emission has been developed.

As is well known, two basic groups of theories exist which are related with either the maser or the antenna mechanisms of coherence [164]. The maser mechanisms deal with various realizations of an inverse population that amplifies transmitted radiation, whereas the antenna mechanisms assume the existence of an effective 'bunching' (i.e. the formation of charged bunches) and the high brightness temperature is determined by a coherent summation of amplitudes of individually radiating particles.

It however became clear very soon that the 'classical' antenna mechanism encounters serious difficulties (see, e.g., Refs [164, 165] and the recent paper [166]). The point is that the observed radio emission requires a fairly long lifetime of charged bunches, while the simplest estimates show that their size rapidly exceeds (if only through insignificant velocity dispersion) the 'coherence length', i.e. the length of radiation formation.

Furthermore, the formation of charged particle bunches themselves appeared to be questionable. At the present time, all the models somehow suggest the existence of instability in the electron–positron plasma flowing along open magnetic field lines.

We shall immediately stress that the main problem is not so much to find the instability (and to determine the increment) as to analyze its nonlinear stage. Indeed, both the output power in the maser mechanism and the number of radiating particles in a bunch are determined by the instability saturation process. And the study of the nonlinear stage of instability growth is an incomparably more sophisticated problem.

As to the fundamental instability, it has turned out that both longitudinal [167–170] and Alfven waves [50, 171–173] may be unstable. The basic mechanisms under discussion may be grouped as follows.

(1) A relativistic electron–positron plasma stream flowing along curved magnetic field lines is unstable (the first papers concerned with this topic were [174, 175]). The instability also occurs in the limit of an infinitely strong magnetic field and is in fact associated with the possibility of synchrotron radiation of plasma particles [176]. We recall that this very model underlay the theory of pulsar radio emission, which was ultimately developed to concrete quantitative predictions [16, 50]. In particular, the dependences of the radio emission window width on the wave period and frequency were found, the high-frequency and low-frequency breakdowns in the spectrum were determined, and the spectral indices and the total radio luminosity were estimated. A detailed analysis of the observational data has shown good agreement between the theory and experiment.

(2) The instability is caused by the 'boundedness' of the region of open field lines along which a relativistic electron–positron plasma flows [42, 44]. In other words, the instability is due to specially chosen conditions on the boundary between open and closed field lines. The predictions of this model cannot unfortunately be now compared with other predictions because the plasma permittivity tensor was not determined in the explicit form in the framework of this approach.

(3) The instability is due to the kinetic effects induced by the nonequilibrium behavior of the distribution function of particles exhibiting a wide energy spectrum. This may first of all be the anomalous Doppler effect on cyclotron resonance [177, 178].:

$$\omega - \mathbf{kv} - \frac{s\omega_{\mathrm{B}}}{\gamma} = 0 \quad \text{for} \quad s = -1\,. \tag{87}$$

In this case, instability occurs for waves with a refractive index $n > 1$. Along with radiation, there proceeds particle excitation to higher Landau levels. As has been shown [178], condition (87) can actually be fulfilled in the radio pulsar magnetosphere, although rather far from the neutron star surface. Clearly, the analysis of such an instability requires that the finite character of the magnetic field be taken into account.

(4) The oscillation build-up is determined by a two-stream instability. Recall that the two-stream instability was already proposed as a radiation generation mechanism in paper [12], but later it became obvious that its efficiency was not high enough. The point is that, as suggested in Ref. [12], the electrons and positrons move in the same direction, and their small velocity difference is only due to the necessity that the electrodynamic condition $\rho_{\rm e} = \rho_{\rm GJ}$ be met. For this reason, the velocity difference appears to be insufficient for a rapid build-up of oscillations. However, as has recently been shown in paper [45], the two-stream instability can still play a significant role under certain conditions. When the energy distribution function of particles is wide enough for the condition $\rho_{\rm e} = \rho_{\rm GJ}$ to be fulfilled, a small fraction of the electrons (or positrons) must move in the opposite direction, i.e. towards the main plasma stream, which can lead to a rapid build-up of oscillations. A more lengthy discussion of this issue can be found in the recent review [179].

(5) Finally, the instability can be simply due to nonstationarity of particle creation in the region of their generation [16, 180]. In particular, it is not excluded that nonstationarity is capable of building-up the two-stream instability [181, 182]. Moreover, completely open is the question of the electron–positron plasma stream stability caused by the variation of the Goldreich–Julian density along the magnetic field line, which can also lead to an oscillation build up [51]. It is clear that for a successful solution of this problem, it is necessary to investigate the kinetic effects.

We shall note that in connection with the first mechanism of instability, critical remarks [183–186] were expressed concerning the validity of the permittivity tensor for an inhomogeneous plasma:

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{pmatrix} = \delta_{ij} - 2\pi \mathrm{i}\, \frac{R_{\rm c}^{2/3}}{k_\parallel^{1/3}} \int \mathrm{d}p_\varphi\, \frac{\omega_{\rm p}^2}{\omega}\, \frac{\partial f^{(0)}}{\partial p_\varphi} \begin{pmatrix} \dfrac{\mathcal{F}''(\zeta)}{(k_\parallel R_{\rm c})^{2/3}} & -\mathrm{i}\, \dfrac{\mathcal{F}'(\zeta)}{(k_\parallel R_{\rm c})^{1/3}} \\ \mathrm{i}\, \dfrac{\mathcal{F}'(\zeta)}{(k_\parallel R_{\rm c})^{1/3}} & \mathcal{F}(\zeta) \end{pmatrix} \tag{88}$$

($\varepsilon_{zz} = 1$, $\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zx} = \varepsilon_{zy} = 0$, $\mathbf{B} \parallel \mathbf{e}_y$) that underlay the theory [176]. In the previous equation

$$\mathcal{F}(\zeta) = \mathrm{Ai}(\zeta) + \mathrm{i}\,\mathrm{Gi}(\zeta) = \frac{1}{\pi} \int_0^\infty \mathrm{d}\tau\, \exp\left(\mathrm{i}\tau\zeta + \mathrm{i}\, \frac{\tau^3}{3}\right), \tag{89}$$

the prime implies a derivative with respect to the variable

$$\zeta = 2(\omega - k_\parallel v_\varphi)\, \frac{R_{\rm c}^{2/3}}{k_\parallel^{1/3} v_\varphi}, \tag{90}$$

and $R_{\rm c}$ is the radius of the magnetic field line lying in the xy-plane.

In spite of the fact that each objection was given an exhaustive explanation [187, 188], it seems pertinent to return to this question, the more so as papers [183–186] are considered, as before, as a serious objection to the possibility of maser amplification of waves in a curved infinitely strong magnetic field. We shall only discuss essentially important questions and shall not touch upon papers containing evident arithmetic errors.

To begin with, the author of Ref. [184] expressed doubt that the tensor (88) correctly describes the interaction between the wave and the plasma particles. However, this remark was the result of a misunderstanding, and the author withdrew his objections [188]. The point is that the tensor (88), as was specially emphasized in Refs [50, 176], exactly corresponds to the necessary transformation

$$\varepsilon_{ij}(\omega, \mathbf{k}, \boldsymbol{\eta} \to \mathbf{r}) = \int \mathrm{d}\boldsymbol{\xi}\, \varepsilon_{ij}(\omega, \boldsymbol{\xi}, \boldsymbol{\eta})\exp(-\mathrm{i}\mathbf{k}\boldsymbol{\xi}) \tag{91}$$

from the space tensor $\varepsilon_{ij}(\mathbf{r}, \mathbf{r}')$ entering into the constitutive relation

$$D_i(\mathbf{r}) = \int \mathrm{d}\mathbf{r}'\, \varepsilon_{ij}\left(\mathbf{r} - \mathbf{r}', \frac{\mathbf{r} + \mathbf{r}'}{2}\right) E_j(\mathbf{r}'), \tag{92}$$

where $\boldsymbol{\xi} = \mathbf{r} - \mathbf{r}'$, and $\boldsymbol{\eta} = (\mathbf{r} + \mathbf{r}')/2$ [189]. We henceforth consider the case of a stationary medium where time integration is elementary. Therefore, for simplicity we shall sometimes not write down the correspondent dependences on the time t and frequency ω.

As is well known, it is precisely the transformation (91) that allows the dielectric tensor that correctly describes the wave–particle interaction to be obtained (see, e.g., Ref. [190]). Recall that in the expansion of the total electric field $\mathbf{E}_{\rm tot}(\mathbf{r}, t)$ in terms of slowly varying amplitudes $\mathbf{E}(\mathbf{r})$, viz.

$$\mathbf{E}_{\rm tot}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \exp\left[\mathrm{i}\big(\mathbf{k}(\mathbf{r})\mathbf{r} - \omega t\big)\right], \tag{93}$$

the Maxwell equations can be represented as an infinite chain after corresponding transformations:

$$D_{ij}\big(\varepsilon_{ij}^{(0)}\big) E_i E_j^* + \nabla \mathbf{S}\big(\varepsilon_{ij}^{(1)\rm H}\big) + \frac{\omega}{8\pi}\, \varepsilon_{ij}^{(0)\rm AH} E_i E_j^* + \ldots = 0, \tag{94}$$

where

$$D_{ij}\big(\varepsilon_{ij}^{(0)}\big) = k_i k_j - k^2 \delta_{ij} + \frac{\omega^2}{c^2}\, \varepsilon_{ij}^{(0)}, \tag{95}$$

and the indices 'H' and 'AH' imply the Hermitian and anti-Hermitian parts of the tensor ε_{ij}.

As is readily seen, to the zeroth order with respect to the derivatives $\partial E/\partial r$, equation (94) corresponds to the algebraic dispersion equation $\det D_{ij} = 0$. To the first order, it has the form of the equation of energy

$$\nabla \mathbf{S}\big(\varepsilon_{ij}^{(1)\rm H}\big) + \frac{\omega}{8\pi}\, \varepsilon_{ij}^{(0)\rm AH} E_i E_j^* = 0. \tag{96}$$

Here

$$S_k\big(\varepsilon_{ij}^{(1)\rm H}\big) = -\frac{c^2}{16\pi\omega} \left(k_j \delta_{ik} + k_i \delta_{jk} - 2k_k \delta_{ij} + \frac{\omega^2}{c^2}\, \frac{\partial \varepsilon_{ij}^{(1)\rm H}}{\partial k_k} + \ldots\right) E_i^* E_j \tag{97}$$

corresponds to the Poynting vector. This procedure is however not unique, and there generally exist infinitely many different chains of tensors: $\varepsilon_{ij}^{(0)}(\omega, \mathbf{k}, \mathbf{r})$, $\varepsilon_{ij}^{(1)}(\omega, \mathbf{k}, \mathbf{r})$, etc. satisfying the expansion (94). However, the dielectric tensor $\varepsilon_{ij}^{(0)}$ entering into the algebraic dispersion equation

$$\det D_{ij}\big(\varepsilon_{ij}^{(0)}\big) = 0 \tag{98}$$

coincides with the dielectric tensor $\varepsilon_{ij}^{(1)}$ in the equation of energy (96) only if they both coincide with the tensor $\varepsilon_{ij}(\omega, \mathbf{k}, \mathbf{r})$ due to the transformation (91). And it is only in this case that the wave damping out (or excitation) found from the imaginary part of the wave vector $\mathbf{k}$ obtained in turn from the solution of the algebraic dispersion equation (98) is exactly coincident with the wave attenuation (or excitation) following from the equation of energy (96).

Another objection was based on the results of papers [183, 191] which, in particular, failed to lead to the dielectric tensor (88) and, therefore, to instability for an infinitely strong curved magnetic field. Within this approach, the magnetic field was assumed to be circular and an expansion in terms of the normal modes $\exp(\mathrm{i}\nu\varphi)$ was employed. However, as was already mentioned in Ref. [187], a separate harmonic cannot describe correctly the wave–plasma interaction because a freely propagating wave contains, in fact, an infinite number of harmonics. But a wave with a fixed ν can only be realized under special conditions, for instance, in the presence of a conducting boundary.

Indeed, the formal expansion in terms of cylindrical harmonics yields the following expressions for the electric induction $\mathbf{D}$ (see, e.g., Ref. [192]):

$$D_r(r, \varphi) = E_r(r, \varphi)\,, \tag{99}$$

$$D_\varphi(r, \varphi) = E_\varphi(r, \varphi) - \sum_{\nu=-\infty}^{\infty} E_\varphi(r, \nu)\, K(r, \nu) \exp(\mathrm{i}\nu\varphi)\,, \tag{100}$$

with

$$K(r, \nu) = \frac{4\pi e^2}{m_\mathrm{e}} \int \mathrm{d}p_\varphi\, \frac{f^{(0)}}{\gamma^3(\omega - \Omega_\mathrm{c}\nu)^2} = \frac{4\pi e^2}{\omega} \int \mathrm{d}p_\varphi\, \frac{v_\varphi}{\omega - \Omega_\mathrm{c}\nu}\, \frac{\partial f^{(0)}}{\partial p_\varphi}\,, \tag{101}$$

where $\Omega_\mathrm{c} = v_\varphi / r$. It is of importance here that the unperturbed particle distribution function $f^{(0)}$ depends on cylindrical coordinates: $f^{(0)} = f^{(0)}(r, \varphi, z, p_r, p_\varphi, p_z)$.

Relations (99) and (100) are however not algebraic constitutive relations because they contain the spectral density $\mathbf{E}(r, \nu)$ but not $\mathbf{E}(r, \varphi)$; furthermore, they do not coincide in form with the integral constitutive relation (92). Exploiting the definition

$$E_\varphi(r, \nu) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi'\, E_\varphi(r, \varphi') \exp(-\mathrm{i}\nu\varphi') \tag{102}$$

and the obvious geometric relationships

$$R_\mathrm{c} + x = r\cos\varphi\,, \qquad y = r\sin\varphi\,, \tag{103}$$

$$R_\mathrm{c} + x' = r'\cos\varphi'\,, \qquad y' = r'\sin\varphi'\,, \tag{104}$$

one can rewrite Eqns (99) and (100) in the form of the constitutive relation (92):

$$\delta D_x(\mathbf{r}) = \frac{1}{2\pi} \iint \frac{r'\,\mathrm{d}r'\,\mathrm{d}\varphi'}{r'} \sum_{\nu=-\infty}^{\infty} E_\varphi(r', \varphi')\, \delta(r - r') \times K(r, \nu) \exp\big[\mathrm{i}\nu(\varphi - \varphi')\big] \sin\varphi\,, \tag{105}$$

$$\delta D_y(\mathbf{r}) = -\frac{1}{2\pi} \iint \frac{r'\,\mathrm{d}r'\,\mathrm{d}\varphi'}{r'} \sum_{\nu=-\infty}^{\infty} E_\varphi(r', \varphi')\, \delta(r - r') \times K(r, \nu) \exp\big[\mathrm{i}\nu(\varphi - \varphi')\big] \cos\varphi\,. \tag{106}$$

Here $\delta\mathbf{D} = \mathbf{D} - \mathbf{E}$.

As a result we obtain

$$\varepsilon_{yy}(\mathbf{r}, \mathbf{r}') = 1 - \frac{1}{2\pi} \sum_{\nu=-\infty}^{\infty} \frac{1}{r'}\, \delta(r - r')\, K(r, \nu) \times \exp\big[\mathrm{i}\nu(\varphi - \varphi')\big] \cos\varphi \cos\varphi'\,, \tag{107}$$

$$\varepsilon_{yx}(\mathbf{r}, \mathbf{r}') = \frac{1}{2\pi} \sum_{\nu=-\infty}^{\infty} \frac{1}{r'}\, \delta(r - r')\, K(r, \nu) \times \exp\big[\mathrm{i}\nu(\varphi - \varphi')\big] \cos\varphi \sin\varphi'\,, \tag{108}$$

$$\varepsilon_{xy}(\mathbf{r}, \mathbf{r}') = \frac{1}{2\pi} \sum_{\nu=-\infty}^{\infty} \frac{1}{r'}\, \delta(r - r')\, K(r, \nu) \times \exp\big[\mathrm{i}\nu(\varphi - \varphi')\big] \sin\varphi \cos\varphi'\,, \tag{109}$$

$$\varepsilon_{xx}(\mathbf{r}, \mathbf{r}') = 1 - \frac{1}{2\pi} \sum_{\nu=-\infty}^{\infty} \frac{1}{r'}\, \delta(r - r')\, K(r, \nu) \times \exp\big[\mathrm{i}\nu(\varphi - \varphi')\big] \sin\varphi \sin\varphi'\,. \tag{110}$$

As is expected, the expressions for $\varepsilon_{ij}(\mathbf{r}, \mathbf{r}')$ contain an infinite sum of cylindrical harmonics. Moreover, it can be shown that the transformation (91) brings us back to the dielectric tensor (88). The objection to the validity of the tensor (88) is therefore ungrounded.

Finally, there was raised an objection to the very possibility of maser wave amplification in a curved magnetic field [186], because a theorem is known according to which the maser amplification is impossible under synchrotron radiation [193]. However, this theorem is only valid for the isotropic particle distribution functions when, roughly speaking, for each particle moving in one direction there exists a particle of the same energy but moving in the opposite direction. Under curvature radiation the particle distribution function is strongly anisotropic because at each point there exist particles moving in only one direction.

Furthermore, as shown in Refs [50, 176], the dielectric tensor (88) possesses the following important properties. First, using the Einstein coefficient method (which relates the wave damping increment and the radiation intensity of a single particle), one can show that the anti-Hermitian part of the tensor (88) actually corresponds to the intensity of curvature radiation.

Second, the optical depth τ determined using the anti-Hermitian part of the tensor (88) exactly coincides over the range $\tau \ll 1$ with the expression derived by Chugunov and Shaposhnikov [194] within quite a different approach. Hence, in our opinion the permittivity tensor (88) is correct — at least, it is the only tensor to be used in the algebraic dispersion equation (98).

As for the nonlinear stage of instability growth, the following processes have been considered in this connection.

(1) The instability growth can be restricted by the nonlinear processes that relate the amplitudes of three different normal waves capable of propagating in the magnetosphere [169, 195, 196]. As a result, the amplification of oscillations of unstable waves can be restricted by energy conversion into

other normal modes which are in turn freely able to escape from the magnetosphere of a neutron star (see Section 4.2).

(2) One of the early versions of such a nonlinear interaction was already considered in paper [197]. The model assumed the outflowing electron – positron plasma to have an intense longitudinal electromagnetic wave playing the role of a 'third body' in the interaction between radiation and relativistic particles. Resonance wave scattering is realized in these conditions provided that

$$\omega - k_{\parallel} v_{\mathrm{D}} = s(\omega' - k'_{\parallel} v_{\mathrm{D}}) \,, \tag{111}$$

where s is the number of the harmonic, and v_{D} is the drift velocity which, for three-wave processes with $s = 1$, coincides to a necessary accuracy with the initial velocity of particles.

As a result, conditions were found under which the absorption coefficient appeared to be negative [198]:

$$\omega < s\omega' \,, \qquad \frac{\partial f}{\partial p_{\mathrm{D}}} < 0 \,, \tag{112}$$

$$\omega > s\omega' \,, \qquad \frac{\partial f}{\partial p_{\mathrm{D}}} > 0 \,. \tag{113}$$

For $s = 1$, condition (112) is analogous to that characterizing the occurrence of the stimulated Raman scattering (SRS) under which the wave energy is pumped over to the Stokes line, and condition (113) is analogous to the SRS to the anti-Stokes line thanks to the inverse population of levels. That is why, if in the former case the instability is due to the energy stored in the longitudinal electromagnetic wave, then in the latter, the radiation is due to the decrease in the relativistic particle energy [165].

(3) The radiation mechanism of stabilization of cyclotron instability was investigated in paper [199]. Owing to a short duration of synchrotron radiation τ_{s} (2) near the pulsar surface, particles pass to lower Landau levels practically instantaneously. However, such a one-dimensional particle distribution function remains stable only in a strong enough magnetic field. At a large distance from a neutron star one should expect an oscillation build-up and, therefore, an increase of the pitch-angles for particles escaping from the magnetosphere. So, for any distance from the pulsar a stable value of the pitch-angle θ must exist, for which the cyclotron instability tending to increase the angle θ is exactly balanced by the reaction of the cyclotron radiation. Thus, one can estimate the intensity of generated radiation and its characteristic frequencies. Exact calculations have shown that such a mechanism may actually be responsible for the high-frequency emission of radio pulsars [200], but it can hardly be used to explain their radio emission.

(4) Finally, the modulation instability which may lead, in particular, to the formation of a Langmuir soliton network was considered as a mechanism of saturation of the instability [201 – 205]. As has been shown [42, 191], solitons are capable of stabilizing the growth of beam instability. If, in addition, solitons carried a nonzero electric charge, they themselves might serve as a source of coherent radio emission. In this case, a stable spatial structure might explain, for example, the micropulse structure of mean radio pulsar profiles.

Summarizing, we shall repeat once again that in spite of the great interest (especially in the 1970s – 1980s) in the problem of generation of coherent radio emission, the theory has been developed up to concrete quantitative predictions accessible for its direct verification only in some exceptional cases. Therefore, great additional efforts are needed, including those in the framework of the models already constructed.

4.2 Wave propagation in the magnetosphere

The construction of a consistent theory of radio emission actually encounters substantial difficulties, but the question about the formation of a directivity pattern might well have been solved many years ago. Recall that, as has already been stressed, the principal geometric properties of radio emission are perfectly well explained in the framework of the hollow cone model. But the observational material accumulated up to date and the high accuracy in the determination of mean and individual profiles require a more thorough comparison of the results of observations with the theoretical predictions. The simplest model (which, in particular, assumes a rectilinear wave propagation in the magnetosphere) is now quite insufficient for this purpose.

The point is that the frequencies of the observed radio waves (100 MHz – 10 GHz) are fairly low, and as a result the permittivity of plasma in which radio-frequency radiation propagates notably differs from unity. This means that a distinguished role in the formation of the directivity pattern of radio emission must be played by refraction. This range of questions also involves the determination of limiting polarization when radiation leaves the magnetosphere of a neutron star and possible radio wave absorption through cyclotron resonance (the latter two questions become especially topical owing to the new observations of mean pulse polarization [206]).

These questions are relatively simple because for the construction of a corresponding model it suffices to restrict our consideration to a linear interaction between electromagnetic radiation and plasma. Nevertheless, in spite of the fact that the importance of all these effects was qualitatively established long ago [50, 207 – 209], a consistent analysis of the problem appeared only in recent years [206, 210, 211].

As an example, we shall consider in more detail the question of radio wave propagation in the internal regions of a neutron star magnetosphere, which is directly related to the construction of the directivity pattern. Clearly, this fact should also be taken into account when one determines the spread of open field lines and the level of radio emission generation. So, as was first demonstrated by Barnard and Arons [207], the dispersion curves of normal waves are nontrivial for parameters typical of a radio pulsar magnetosphere.

Figure 9 shows that when a radio wave is propagating in the curved magnetic field of a neutron star, the longitudinal plasma wave *2* is transformed into a transverse wave capable of escaping from the magnetosphere as the angle θ between the wave vector and the magnetic field increases. And vice versa, wave *4*, which for small angles θ is transverse, becomes a quasi-longitudinal Alfven wave with a frequency $\omega \approx \mathbf{kv} \approx kc\cos\theta$, which is unable to propagate through large distances.

We shall consider for simplicity the permittivity tensor of plasma in an infinitely strong rectilinear magnetic field. In the hydrodynamic limit it has the form [172]

$$\varepsilon_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \left\langle \dfrac{\omega_{\mathrm{p}}^2}{\gamma^3 (\omega - \mathbf{kv})^2} \right\rangle & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{114}$$

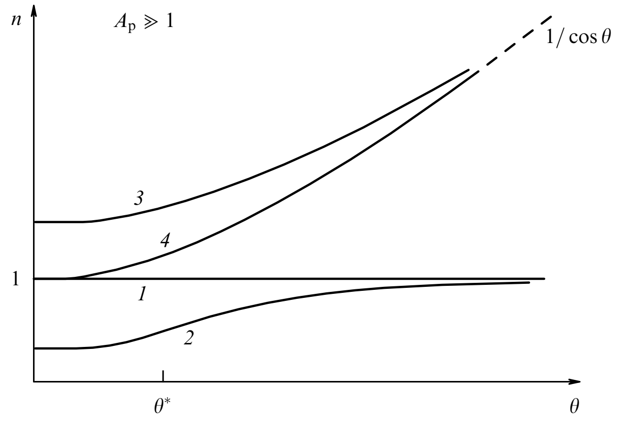

Figure 9. Dispersion curves of normal waves for parameters typical of radio pulsar magnetosphere [16]. For $\theta = 0$, *1* and *4* are two transverse waves with $n = 1$, while *2* and *3* are longitudinal plasma waves propagating in two directions relative to the plasma, but drifting together with particles. For an oblique propagation and sufficiently large angles $\theta > \theta^*$ defined by Eqn (121), the solution of the dispersion equation leads, as before, to two transverse ($n_{1,2} \approx 1$) and two quasi-longitudinal ($\omega_{3,4} \approx \mathbf{kv}$) waves. In a curved magnetic field, the longitudinal plasma wave *2* is transformed into a transverse wave (capable of escaping from the magnetosphere) with an increase of the angle θ between the wave vector and the magnetic field.

Here $\omega_\mathrm{p}^2 = 4\pi e^2 n_\mathrm{p}/m_\mathrm{e}$ is the plasma frequency, and the angular brackets designate averaging over the particle distribution function. As is well known, four normal waves can propagate in plasma in this case.

For a relativistic plasma moving at a velocity $v \approx c$ and small angles θ, we obtain for the refractive indices $n = kc/\omega$ the following particular values:

$$n_1 = 1\,, \tag{115}$$

$$n_2 \approx 1 + \frac{\theta^2}{4} - \left(\frac{\omega_\mathrm{p}^2}{\omega^2}\langle\gamma^{-3}\rangle + \frac{\theta^4}{16}\right)^{1/2}, \tag{116}$$

$$n_3 \approx 1 + \frac{\theta^2}{4} + \left(\frac{\omega_\mathrm{p}^2}{\omega^2}\langle\gamma^{-3}\rangle + \frac{\theta^4}{16}\right)^{1/2}, \tag{117}$$

$$n_4 \approx 1 + \frac{\theta^2}{2}\,. \tag{118}$$

These expressions hold at a sufficiently high plasma density:

$$A_\mathrm{p} = \frac{\omega_\mathrm{p}^2}{\omega^2}\langle\gamma\rangle \gg 1\,. \tag{119}$$

The exact expressions for the refractive indices n_j can be found, for instance, in papers [50, 212].

We shall now discuss the basic properties of these waves. When propagating along the magnetic field ($\mathbf{k} \parallel \mathbf{B}$, i.e. $\theta = 0$), *1* and *4* are two transverse waves with $n = 1$ and a wave vector $\mathbf{E}$ perpendicular to the external magnetic field, which leads to almost the cessation of wave–plasma interaction (the electric field of a wave cannot distort the trajectory of a particle moving along an infinitely strong magnetic field). Waves *2* and *3* are longitudinal plasma waves propagating in two directions relative to the plasma, but carried away together with particles (mathematically, this is a consequence of the fact that the denominator $\omega - \mathbf{kv}$ is substantially smaller than the factor $\mathbf{kv}$). Therefore, both longitudinal waves propagate in the same direction and, owing to the relativistic behavior of the particles, their refractive index also appears to be close to unity:

$$n_{2,3} = 1 \mp \left(\frac{\omega_\mathrm{p}^2}{\omega^2}\langle\gamma^{-3}\rangle\right)^{1/2}. \tag{120}$$

As a result, the frequencies of waves *2* and *3* turn out to be far from the plasma frequency. This means, in particular, that for a relativistic plasma the plasma frequency ω_p is not the distinguished frequency at which radio wave emission may be expected (this delusion can still be encountered in the scientific literature).

Next, for an oblique propagation and sufficiently large angles $\theta > \theta^*$, where

$$\theta^* \sim \left(\frac{\omega_\mathrm{p}^2}{\omega^2}\langle\gamma^{-3}\rangle\right)^{1/4}, \tag{121}$$

the solution of the dispersion equation leads, as before, to two transverse ($n_{1,2} \approx 1$) and two quasi-longitudinal ($\omega \approx \mathbf{kv}$, i.e. $n_{3,4} \approx 1/\cos\theta$) waves. As concerns the transition region, one observes here a nontrivial transformation of transverse waves into longitudinal and vice versa. Wave *1*, whose electric vector is perpendicular to the plane $\mathbf{kB}$ fails, as before, to interact with the plasma, and so we have $n_1 = 1$. Wave *1* remains transverse over the entire range of angles, whereas wave *2* is transverse at large angles θ only.

This picture refers to a rectilinear magnetic field. It has turned out, however, that for parameters typical of a radio pulsar magnetosphere, neither corrections due to finiteness and curvature of the magnetic field, nor the kinetic effects exert an appreciable effect upon the wave propagation (this does not concern wave amplification). Hence, in the zeroth approximation one may assume $\omega_j(\mathbf{k},\mathbf{r}) = ck/n_j$, where the refractive indices n_j are determined as before by relations (115)–(118).

As a result, the dispersion curves in Fig. 9 can be used for a curvilinear magnetic field as well. However, in a homogeneous external magnetic field the angle θ remains constant upon wave propagation, whereas in a curvilinear magnetic field it varies gradually according to the laws of geometrical optics. It is therefore only modes *1* and *2* that can leave the neutron star magnetosphere. As to the quasi-longitudinal Alfven waves *3* and *4*, for $\theta > \theta^*$ they propagate, as expected, along the magnetic field until, for still larger angles θ, they get into the rarefied plasma region where their propagation becomes impossible.

Thus, even if in these modes the radio emission is generated at the same altitude (and at the same angles θ with the magnetic field), the directivity patterns of these waves differ notably from each other. Indeed, as is assumed in the hollow cone model, wave *1* propagates rectilinearly. Therefore, the aperture angle of the directivity pattern is coincident with the spread of magnetic field lines in the generation region. For wave *2* and sufficiently small angles θ, the refractive index n_2 substantially differs from unity. As a result, for $\theta < \theta^*$ its trajectory curves sidewise from the magnetic axis (Fig. 10). Only for sufficiently large θ does the propagation of this wave also become rectilinear.

The curvature of the trajectory of the normal wave *2* for small angles θ must tell significantly upon the width of the mean profile. As has already been said, the integration of equations of geometrical optics for the simplest case — a

Figure 10. Trajectories of normal waves in the curved magnetic field of a radio pulsar [16]. Wave *1* whose electric vector is always perpendicular to the external magnetic field propagates rectilinearly. As to wave *2*, for $\theta < \theta^*$ defined by Eqn (121) its trajectory curves sidewise from the magnetic axis, and the directivity pattern for it is therefore significantly wider. When propagating along magnetic field lines, waves *3* and *4* fail to leave the neutron star magnetosphere.

homogeneous plasma density within the confines of open field lines — was first carried out in paper [207] and was then used in Ref. [50] to determine the directivity pattern of radio emission. In this case concrete quantitative predictions were made, for example, in respect of the dependence of the radio emission mean profile on the frequency ν. So, for wave *2* we have

$$w_{\mathrm{d}} \propto \nu^{p} \,, \tag{122}$$

where $p = -0.14$ or $p = -0.29$, whereas for wave *1* we arrive at

$$w_{\mathrm{d}} \propto \nu^{-0.5} \,. \tag{123}$$

Many pulsars are known to have this particular dependence.

In addition, it has been hypothesized that different modes observed in radio emission pulses may be related to normal modes *1* and *2*. Nevertheless, the results of these papers have practically never been used in the analysis of observations. Some papers considering wave propagation for a more realistic plasma density profile corresponding to the hollow cone model [206, 210] have appeared only in the past two or three years. The investigations have not yet been accomplished, but one may hope that a consistent theory (at least in this part of it) will be constructed in the near future.

Thus, the theory of pulsar radio emission must include not only the generation mechanism itself, but the whole complex of questions involved in the formation of the radio emission directivity pattern. These are the problems of propagation and absorption of normal waves in the magnetosphere of a rotating neutron star and those concerning the limiting polarization upon radiation ejection from the magnetosphere [213]. There is unfortunately no unanimous standpoint concerning this range of questions.

5. Radio pulsars as a cosmic laboratory

As mentioned above, the physical parameters typical of neutron stars (superstrong gravitational and magnetic fields, high-energy particles) are inaccessible to ground-based laboratories. Therefore, radio pulsars have allowed the investigation of the properties of matter under extreme conditions. Moreover, over many years radio pulsars have been successfully employed as probes of the interstellar medium. We may mention, for example, the direct measurements of the electron density and the magnetic field intensity in the Galaxy [18], as well as the transillumination of the stellar wind in binary systems [214]. We shall present here only the most demonstrative examples in which radio pulsar observations promoted an appreciable advance in the verification of theoretical predictions. It should be recalled that the large possibilities of performing nontrivial 'experiments' are due to accreting neutron stars as well [4].

5.1 Gravitational waves

One of the most expressive observational properties of radio pulsars is the possibility to experimentally verify the predictions of general relativity. Indeed, since radio pulsars incorporating into close binary systems resemble an exact watch moving in the gravitational field of a companion star, they provide unique information on the spacetime curvature.

The point is that among more than sixty 'binary' radio pulsars six cases are known where the companion is also a neutron star. In four cases the orbit appears to be rather close (i.e. it has a period not exceeding several hours), so that one can register all the post-Newtonian effects such as the periastron motion, the gravitational red shift and Shapiro's delay (the time delay in the arrival of pulses due to time deceleration upon signal propagation).

It becomes possible, in particular, because in all the six binary systems the orbital eccentricity appears to be extremely high. Since a radio pulsar moves in the variable gravitational field of its companion, the gravitational red shift — the change in the angular velocity of neutron star rotation measured by an observer at infinity — will be substantially time-dependent. As a result, although the effect itself takes a few hundredths of a second, it may be distinguished with confidence. The exceedingly narrow mean profiles of radio emission pulses (less than a millisecond for pulsar 1913 + 16), which allowed a highly accurate time fixation, were also important here.

As can be seen from Table 3, owing to their unique physical parameters, the general relativity effects in the four systems appear to be exceedingly large. For example, the angular velocity of periastron motion $\dot{\omega}$ may reach several angle degrees a year, which exceeds the analogous rate of Mercury perihelion motion by four orders of magnitude. Within the 25 years since the discovery of the binary pulsar 1913 + 16, its orbit has turned by more than 90°. Moreover, additional information connected with the general relativity effects allowed the determination of all the orbit parameters for these systems and also, which is particularly interesting, the masses of both stars in the binary system. This is the most exact determination of a star mass yet attained in astronomy.

As regards the other systems that do not enter in such close pairs, only the periastron angular velocity that has been

Table 3. Basic parameters of binary systems containing two neutron stars.

Pulsar	P, s	P_b, d	e	$\dot{\omega}$, deg./year	$\dot{P}_b$, 10^{-12}	$\frac{M}{M_\odot}$
J1141 − 6545	0.3930	0.198	0.172	5.5		1.2 1.2
J1518 + 4904	0.0409	8.634	0.249	0.011		1.56 1.05
B1534 + 12	0.0379	0.421	0.274	1.756	−0.152(3)	1.339(3) 1.339(3)
B1913 + 16	0.0590	0.323	0.617	4.227	−2.425(1)	1.4411(3) 1.3874(3)
B2127 + 11C	0.0305	0.335	0.681	4.462		1.349(40) 1.363(40)
B2303 + 46	1.0664	12.34	0.658	0.010		1.30 1.34

measured for them so far. However, since it depends, in fact, only on the total mass of the system:

$$\dot{\omega} = 3\left(\frac{P_b}{2\pi}\right)^{-5/3} (1-e^2)^{-1} \left(\frac{G}{c^3}\right)^{2/3} (M_1 + M_2)^{2/3}, \quad (124)$$

the results obtained also allowed a sufficiently accurate estimate of the neutron star masses. The masses of all the objects turned out to coincide (with an error of no more than 10%) with the Chandrasekhar limit $M_{\mathrm{Ch}} \approx 1.4 M_\odot$. This result is undoubtedly of fundamental importance for the theory of neutron star formation. We recall that from the theoretical point of view a neutron star may have a much wider mass range, namely, from $0.1 M_\odot$ to $(2-3) M_\odot$ [53].

Finally, a change in the orbital period was registered in two cases, and the origin of this change is naturally associated with the energy loss due to gravitational wave radiation. In other words, the predictions of the general theory of relativity were firstly confirmed (at least indirectly) to a higher order (c^{-5}) than the post-Newtonian corrections (c^{-2}).

Furthermore, the predictions of general relativity for pulsar 1913 + 16 proved to be valid to an accuracy of fractions of a percent:

$$\frac{\dot{P}_b^{(\mathrm{obs})}}{\dot{P}_b^{(\mathrm{th})}} = 1.0001 \pm 0.0014 \,. \quad (125)$$

As is well known, R Hulse and J Taylor were awarded the 1993 Nobel prize in physics for their achievements in this field. Indeed, the discovery of a binary system with a lifetime of 200 million years, i.e. much shorter than that of the Universe, became one of the most important events of the past decades. This means that neutron stars must merge rather frequently. The model of the merging of two neutron stars is now considered to be one of the highly probable causes of cosmological gamma bursts [215, 216]; the same process seems to be extremely promising for the discovery of gravitational waves [217].

As to the binary system containing pulsar 1534 + 12, the situation appeared to be more nontrivial [218] because for this system both parameters

$$r = \frac{G M_{\mathrm{comp}}}{c^3}\,, \quad (126)$$

$$s = \sin i\,, \quad (127)$$

determining Shapiro's delay time

$$t_{\mathrm{Sh}} = -2r \ln\Big[1 - e\cos u - s\big(\sin\omega(\cos\omega - e)(1-e^2)^{1/2}\cos\omega \sin u\big)\Big] \quad (128)$$

(u is the eccentric anomaly, e and i are the eccentricity and the angle of inclination of the orbit) have been singled out. This became possible owing to the more advantageous position of the plane of the binary system's orbit (it is seen practically on edge), which leads to a substantial heightening of general relativity effects upon the passage of a signal near the very surface of the companion star.

In consequence of this, as distinct from the first system, both parameters were verified simultaneously. And if for the parameter s the theory is in good agreement with experiment:

$$\frac{s^{(\mathrm{obs})}}{s^{(\mathrm{th})}} = 1.010 \pm 0.008\,, \quad (129)$$

for the rate of orbital period variation the theoretical predictions and experiments are far from agreement:

$$\frac{\dot{P}_b^{(\mathrm{obs})}}{\dot{P}_b^{(\mathrm{th})}} = 0.87 \pm 0.09\,. \quad (130)$$

The authors of Ref. [218] are not inclined to associate this contradiction with a possible violation of the general theory of relativity itself. The point is that the determination of the parameters of a binary system must be significantly affected by its acceleration relative to the Earth which, in turn, depends on the distance to the binary system. Since the accuracy in the determination of the distance to radio pulsars is now not high (it depends on the free electron density along the line of sight), it is not excluded that this fact leads to observed disagreement.

We shall finally recall that the possibility of applying the Einstein formula for the gravitational radiation of real objects is not obvious either. In particular, a variation of the orbital period may be due to the tidal effects caused by a finite star size (the Einstein formula is only valid for point masses). But as has been shown, a binary system consisting of neutron stars is a sufficiently 'pure' physical laboratory [219, 220], and so neutron stars may be regarded as point objects for their compactness.

5.2 Equation of state of nuclear matter

Another no less important line of inquiry is the verification of the structural theory of matter for densities exceeding nuclear ones. The problem is that over two dozen neutron star models now exist that are based on different approaches to the solution of the problem concerning the equation of state of nuclear matter [53]. They are obtained by numerical integration of hydrostatic equilibrium equations with allowance for general relativity effects (the Oppenheimer – Volkov equations). Showing a not bad agreement for ordinary nuclei, these models however differ greatly for densities and pressures typical of neutron stars.

Such ambiguity is in particular due to the fact that for densities typical of internal regions of neutron stars, within the range of internucleon forces there are a large number of particles. That is why, even for a sufficiently well-known nucleon – nucleon interaction potential the derivation of the equation of state is a nontrivial many-body problem, which leads to ambiguity in the results obtained. Many other exotic possibilities associated with various phase transitions in the central regions of a neutron star can be added as well. This may be the crystallization of a nuclear liquid [221], the transition into a quark – gluon plasma [53], pion or kaon condensation [222], the production of hyperons [53] and strange matter [223]. The majority of these possibilities were proposed as far back as the 1970s, but have not yet been directly confirmed.

As a result, the theoretical predictions for the radius and mass of a neutron star range widely. As the central density ρ_c increases, the star mass M normally grows and the radius R decreases (the star becomes more compact). For a certain density ρ_c the mass M stops growing, which typically corresponds to an extremely stable star configuration. The mass M_c of such a configuration possesses the maximum mass of the neutron star for a given equation of state.

Models of stars with a higher central density are usually unstable against collapse into a black hole and are not realized in nature. The corresponding curves of the dependence of the neutron star mass on its radius were first presented rather extensively in Ref. [109] and were then repeatedly reproduced [5, 16, 53] and became well known (Fig. 11). The softer the equation of state, the more compact the neutron star and the lower its limiting mass. So, for various soft equations of state the limiting star mass lies in the range of $(1.4-1.6)M_{\odot}$, for moderate ones in the range of $(1.6-1.8)M_{\odot}$, and for stiff ones in the range of $(1.8-3)M_{\odot}$.

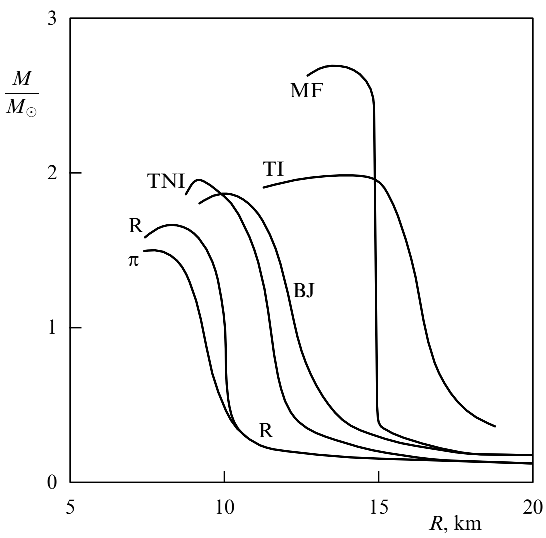

Figure 11. Diagram of the dependence of gravitational mass on the radius for different equations of state of nuclear matter [109]. Larger neutron star radii are given by stiff (TI, MF) rather than soft (R, π) equations of state. The classification corresponds to paper [224].

At the same time, the answer to the question of a true equation of state of nuclear matter can be obtained directly from observations. Clearly, there must exist only one true equation of state. Hence, all the neutron stars in the mass – radius diagram must lie along one curve. So, if at least for a single neutron star one could establish, besides the mass, also its radius to a sufficient accuracy, this would allow the equation of state of nuclear matter to be found rather reliably. Unfortunately, none of the methods [225 – 228] discussed (for many years already) up to now has yielded a reliable estimate of the neutron star radius.

We shall nonetheless enlist the basic ideas related to the equations of state of nuclear matter.

(1) Observations of thermal radiation from the surface of isolated neutron stars and their interpretation using atmosphere models and taking into account the effects of reddening and curvature of photon trajectories in the gravitational field of a pulsar allow, in principle (for sufficiently good spectral and time resolution in the X-ray range and applying radio-polarimetric measurements of pulsar angle of inclination), the determination of the mass and radius of a neutron star [229, 230]. The nearest prospects are associated here with observations made with the X-ray orbital observatory Chandra possessing unique angular and spectral resolutions.

(2) The equation of state affects the neutron star cooling regime. That is why the radius of a star can be determined from the thermal radiation of their surfaces, observed in some cases [57, 225]. The most promising in this respect are radio pulsars because a considerable part of the thermal (and nonthermal) radiation of accreting neutron stars is due to accreting substance. But even in this case the construction of a consistent model allowing an accurate enough quantitative comparison of the theory and observations encounters considerable difficulties [57, 231]. This is first of all related to the presence of a strong magnetic field which substantially affects the transport coefficients in the atmosphere, and a great uncertainty in the chemical composition of the atmosphere of a neutron star.

(3) The neutron star mass – radius relationship and, therefore, the equation of state of nuclear matter are responsible for the properties of the glitches, i.e. sharp jumps of the rotational period P observed for several pulsars [18]. This is explained by the fact that a sharp decrease of the period is in any case determined by the change in the moment of inertia of the star [53]. Furthermore, according to modern conceptions the glitches in the period are mostly due to the reconstruction of the superfluid component in the internal pulsar regions [158, 159], whose properties also depend on the equation of state. As a result, within a concrete period glitch model one can obtain restrictions on the mass and radius of a neutron star [232].

(4) One more method is connected with the study of X-ray bursters — sources of bursts on the neutron star surface, which are due to nuclear burning of accreting matter [227, 228, 233, 234]. Analyzing the change in the burster spectrum during flares (when the star surface is much brighter than the background created by the accreting matter) and employing concrete neutron star atmosphere models one can obtain restrictions on the radius R [235]. This appears to be possible because the spectrum and the profile of received radiation are governed by the parameters of surface gravitation and gravitational red shift, which depend differently on the neutron star mass – radius relationship.

(5) Finally, new information can be obtained by investigating the recently discovered kilo-Hertz quasi-periodic oscillations (QPO) in the radiation of X-ray binary systems; these oscillations are associated with the motion of accreting matter near the last stable Kepler orbit at the neutron star surface. Since the parameters of such an orbit are determined by the effects of general relativity and, therefore, depend on the mass – radius relationship, the properties of such oscillations enable the neutron star radius to be estimated [236, 237].

As we can see, the estimates of the neutron star radius appear to be model-dependent in all the cases.

This range of questions also involves the problem of the ultimate period of neutron star rotation. It is also determined from the equation of state of matter because it depends considerably on the neutron star radius. That is why millisecond pulsar periods reaching 1.59 ms can bear information on the structure of internal regions of neutron stars. However, as was shown rather long ago [238], these periods are still sufficiently large for such compact objects as neutron stars. Recent calculations [239] have confirmed once again that for the majority of the equations of state, the ultimate period of neutron star rotation is no more than 0.3 – 1 ms, which does not contradict observations.

Thus, no direct observational restrictions on the equation of state of nuclear matter have yet been obtained. At the present time, practically none of the equations of state contradict observations. One can reject with good reason only extremely soft equations of state, for example, models with a pion condensate that give a neutron star mass below $1.44M_{\odot}$, i.e. less than the mass of pulsar $1913+16$ which is the most massive star in the composition of close binary systems. Nevertheless, the approach considered seems to be exceedingly topical and it is not excluded that the success will be achieved in the near future.

6. Conclusions

Thus, radio pulsars attract, as before, not only observers who hope to discover new manifestations of their activity but also theoreticians for whom neutron stars remain in the first place a unique physical laboratory that allows the study of processes in extreme conditions [240]. It is therefore not surprising that during all the thirty years that have passed since the discovery of radio pulsars they have remained among the most 'popular' space sources. And this is in spite of the fact that other, not less interesting cosmic objects were discovered, such as X-ray pulsars and X-ray transients, sources of cosmic gamma bursts and gravitational lenses, whose many properties remain unknown in many respects.

Nonetheless, the structural theory of the pulsar magnetosphere and especially the theory of radio emission are still far from being completed. And although we now undoubtedly understand most of the key processes proceeding in a neutron star magnetosphere, a reliable quantitative description has only been given to some separate, while principal, elements. There now exists no conventional point of view on the structure of a radio pulsar magnetosphere and the origin of their radio emission.

Unfortunately, for radio pulsars there has been a clear discrepancy between the theory and observations for many years. And if, for example, a verification of the general theory of relativity plays the key role in organizing the observations of binary radio pulsars, practically no research is directed to the verification of modern theories of radio emission. As has already been mentioned, the theory has been led to concrete predictions only in exceptional cases, of course. Nevertheless, observations aimed at the establishment of the key properties of radio pulsars should still have been carried out. They might have included, for instance:

- determination of the braking index $n_{\rm br}$ for new pulsars;
- determination of the direction of evolution for the angle χ of axial inclination for individual pulsars;
- refinement of statistical dependences for the subpulse drift velocity;
- monitoring of relic photons generated in the internal gap region;
- detailed analysis of the properties of the radio emission directivity pattern on the basis of the results of latest observations.

Now, the increase of observational material is often not at all connected with theoretical works. That is why one of the reasons for writing this review was to recall the fundamental ideas underlying the theory of the radio pulsar magnetosphere and perhaps to stimulate novel observations directed to the verification of modern theoretical models.

In conclusion I would like to thank A V Gurevich and Ya N Istomin for their fruitful advice without which the survey could not have been written. The review is, in fact, largely based on our mutual monograph [16] which unfortunately is not easily available for Russian researchers. For this reason it seemed pertinent to repeat some crucial points of our radio pulsar model (most of the quantitative predictions can however be found in the original papers [49, 50, 241]). I am also grateful to I G Mitrofanov, G G Pavlov, A Yu Potekhin, Yu A Shibanov, and D G Yakovlev who read some sections of the manuscript and made some useful remarks. The work was sponsored by INTAS, grant No. 96-154.

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