

METHODOLOGICAL NOTES

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Gravitation, photons, clocks

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Contents

Abstract. **This paper is concerned with the classical phenomenon of gravitational red shift, the decrease in the measured frequency of a photon moving away from a gravitating body (e.g., the Earth). Of the two current interpretations, one is that at higher altitudes the frequency-measuring clocks (atoms or atomic nuclei) run faster, i.e. their characteristic frequencies are higher, while the photon frequency in a static gravitational field is independent of the altitude and so the photon only reddens relative to the clocks. The other approach is that the photon reddens because it loses the energy when overcoming the attraction of the gravitational field. This view, which is especially widespread in popular science literature, ascribes such notions as a 'gravitational mass' and 'potential energy' to the photon. Unfortunately, also scientific papers and serious books on the general theory of relativity often employ the second interpretation as a 'graphic' illustration of mathematically immaculate results. We show here that this approach is misleading and only serves to create confusion in a simple subject.**

1. Introduction

In the literature, two types of red shift are known: gravitational and cosmological. As a rule, they are considered independently of each other. Gravitational red shift occurs when a photon moves away from a massive body (for instance, the Earth or the Sun) that can be considered as a static object. Observable values of the red shift are usually very small. This paper is devoted to the gravitational red shift.

The cosmological red shift is the red shift of light (photons) from remote galaxies caused by their recession. This shift is often called the Hubble shift. It is large in magnitude: for the most-distant observed galaxies, $\Delta\lambda/\lambda \approx 5$, where λ is the wavelength of the radiated light. In what follows, we will not discuss cosmological red shift.

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The phenomenon of gravitational red shift was predicted by A Einstein in 1907 [1], and he discussed it in 1911 [2] before the creation of general relativity (GR). When GR was constructed by Einstein [3], gravitational red shift became one of the three classical effects of this theory (see Refs [4, 5]; as to the history of creation of GR, see Refs [6 – 8]).

Phenomenologically, without theoretical interpretation of the phenomenon, it can be described as follows: the frequency of light emitted by two identical atoms is lower for the atom 'sitting' deeper in the gravitational potential. Starting from 1960, unique experiments were carried out aimed at measuring various manifestations of the phenomenon [9 – 14]. These experiments have been discussed in excellent review papers [15 – 25], whose main purpose is to compare the experimental data with theoretical predictions, not only of GR, but of different nonstandard theories of gravitation as well. Interpretations of the gravitational red shift in the framework of the standard theory are not discussed in those reviews.

The authors of the majority of monographs on GR (see, for instance, Refs [26 – 35]) follow the interpretation given by Einstein in 1916 – 1920 (see Refs [3 – 5]), according to which the gravitational red shift is caused by the universal property of the standard clocks (atoms, atomic nuclei). In the general case of an arbitrary gravitational field and arbitrary velocities of emitting and absorbing atoms, the proper time interval between events of emission of two photons measured by the standard clock at the point of emission differs from the proper time interval between events of absorption of these photons measured by an identical standard clock at the point of absorption. The ratio of these time intervals gives the invariant description of the red shift. This formulation of the red shift was first given by H Weyl in 1923 [36].

In the case of a static gravitational potential and fixed atoms, the picture gets simplified, since there exists a distinct time (the time coordinate) upon which the metric does not depend. This time can be taken as the universal (world) time. With this choice, the energy difference of two atomic levels increases with increasing distance between an atom and the Earth, whereas the energy of a photon remains unchanged. (In what follows, we will speak about the Earth, but it could be any massive body.) Thus, the phenomenon called the red shift of a photon is actually the blue shift of an atom. As to the values of proper time at different points, they are expressed in terms of the universal time with the help of a factor that depends on the gravitational potential and, therefore, has different values at different points.

In textbooks and monographs published in recent years [37 – 40], red shift is described with the use of mathematical constructions such as orthonormalized bases (a sequence of proper frames of reference), with respect to which both the energy and the parallel transport of the 4-momentum of a photon along its world line are defined. Frequently, such a rigorous mathematical description is accompanied by non-strict verbal representations of a photon that loses its energy when it 'gets out' of the gravitational potential well. Even some classical textbooks and monographs [41 – 43] make use of a similar 'visual phraseology'. The experts on GR do not pay attention to it — for them, this is merely a tribute to the tradition of scientific popularization. However, nonexperts should be warned that the content of mathematical formulae underlying the description of the gravitational red shift drastically differs from the 'heuristic' (and incorrect) arguments, discussed above and widespread in many elementary textbooks (see, for instance, Refs [44 – 50]).

Their authors proceed from the implicit supposition that a massless photon is similar to a conventional massive non-relativistic particle, call the photon energy E divided by the speed of light squared c^2 the photon mass, and consider the 'photon potential energy' in the gravitational field. Only exceptional popular-science texts (see, e.g., Ref. [51]) do not contain this incorrect picture and emphasize that the energy and frequency of a photon do not change as it moves higher and higher.

2. Experiments

The first laboratory measurements of the gravitational red shift were performed at Harvard in 1960 by R Pound and G Rebka [9, 10] (with an accuracy of 10%). Within an accuracy of 1%, an experiment of that sort was carried out later by R Pound and J Snider [11]. Photons were moving in a tower 22.5 m high. The source and an absorber of photons (γ-rays with energy 14.4 keV) were nuclei of the isotope ^{57}Fe. To diminish possible systematic errors, observations were made both for reddening and blue-shifting of a photon. In the first case, the source was placed in a basement and the absorber, in an attic. In the second case, they changed places.

The measured frequency shift was very small, $\Delta\omega/\omega \approx 10^{-15}$. This accuracy could be achieved owing to the Mössbauer effect discovered in 1958, due to which photon lines in a crystal are extremely monochromatic. The gravitational red shift was compensated by the Doppler effect: the absorber was slowly moving in the vertical direction, thus restoring the resonance absorption of photons.

As to the interpretation of the results obtained, there is some ambiguity in the papers by Pound and colleagues. Though they mention the interpretation in terms of the clock, with reference to Einstein's papers, their paper [9] is entitled "Apparent weight of photons"; and the report made by Pound in Moscow [10] was entitled "On photon weight". From the title of the paper by Pound and Snider [11] "Effect of gravity on nuclear resonance" it can be concluded that they did not want to choose between alternative interpretations.

Unlike the original papers by Pound and colleagues, most of the reviews covering gravitational experiments [18 – 24] consider their result as a test of clock behavior in a gravitational field. Actually, the experiments themselves do not provide the choice between the two interpretations, unless GR is taken as the basis in making that choice. The reason is that they measure the relative shift of photon and nuclear frequencies, and each of the frequencies is not measured separately. The same remark also concerns the shift of the photon (radio wave) frequency with respect to the frequency of the atomic standard (a hydrogen maser) measured with a rocket that was launched to altitude 10,000 km and then fell into the ocean [12]. In this experiment, the theory of gravitational red shift was verified to an accuracy of the order 10^{-4}.

For a direct test of the dependence of the atomic-clock rate on height (without photons), experiments were carried out, in which the clock was in the air for a long time on aircraft [13, 14] (see also reviews [18 – 24]). In these experiments, the clock was brought back to the laboratory, where its reading was compared to that of an identical clock staying put on the Earth. (Besides, in experiment [14], the shift of the aircraft clock was observed from the Earth telemetrically.) It turned out that, in accordance with general relativity, the clock on-board went ahead by $\Delta T = (gh/c^2)T$, where T is the flight duration at height h, g is the gravitational acceleration, and c is the speed of light. (The accuracy of the experiment [13] that used a beam of cesium atoms was of the order of 20%. The accuracy of experiment [14] was 1.5%.)

This result was, of course, obtained when numerous background effects were taken into account. One of them was the famous 'paradox of twins': according to special theory of relativity, the moving clock after coming back to the starting point would lag behind the clock at rest. It is not difficult to derive the general formula describing the influence of both the gravitational potential ϕ and velocity u (see, for instance, Ref. [27]):

$$d\tau = dt\left[1 + \frac{2\phi}{c^2} - \frac{u^2}{c^2}\right]^{1/2}, \tag{1}$$

where τ is the proper 'physical' time of the clock, and t is the above-mentioned so-called universal world time that can be introduced in the case of a static gravitational potential and that is sometimes called laboratory time, because it is this time which is shown by a clock at rest in the laboratory, where the value of ϕ is taken zero.

In his lectures on gravitation [35], R Feynman gave a detailed explanation for the change in the clock rate because of ϕ and u. He concluded that the center of the Earth should be "a day or two younger than its surface".

Apart from the tower, rocket, and aircraft experiments, 'desk' experiments were also carried out [52, 53] with the use of the Josephson effect that compensated the gravitational shift.

As to satellite experiments, they have been repeatedly discussed in the literature (see, for instance, Refs [54, 55]), however, we do not know their results.

Besides the shift of the photon frequency, the shift of the wavelength was also measured [56] (see also Fig. 38.2 in monograph [37] that illustrates experiment [56] and reviews [15 – 25]). In this experiment, the shift of a sodium line in the solar spectrum was measured (with an accuracy of 5%) with the help of a diffraction grating. The theory of such grating experiments will be discussed below (see Section 6). The shift of the solar absorption line of potassium was measured by means of the resonance scattering of sunlight by an atomic beam [57] (with an accuracy of 6%).

3. Theory up to 1916: Einstein elevator

Since in this paper we mainly discuss the gravitational red shift in the field of the Earth, we choose the frame of reference, in which the Earth is at rest (its rotation is neglected).

As is well known, the potential is determined to within an additive constant. When a gravitational potential $\phi(r)$ is considered at a certain distance r from the center of the Earth, it is convenient to choose $\phi(\infty) = 0$. Then ϕ at any finite r is negative.

At an altitude h near the Earth surface ($h = r - R \ll R$, where R is the Earth radius), use can be made of the linear approximation:

$$\delta\phi(h) = \phi(R+h) - \phi(R) = gh\,, \tag{2}$$

where g is the standard gravitational acceleration. Note that $\delta\phi(h) > 0$ for $h > 0$. We will discuss the red shift only in the first order of the parameter gh/c^2.

The linear approximation, Eqn (2), is valid for laboratory and aircraft experiments. It is, however, obvious that it does not work for a rocket at a high altitude ($h \simeq 10^4$ km). In this case, the potential $\delta\phi(h)$ is to be replaced by the Newton potential $\phi(r)$, which is, however, unessential for the dilemma 'clocks or photons' that is the subject of the present paper.

The first papers by Einstein [1, 2] on gravitational red shift contained a lot of basic ideas that entered into numerous texts of various authors (sometimes, without a proper critical analysis). He considered the Doppler effect in a freely falling system and found the frequency of an atom (clock) to increase with increasing height (potential). A corner-stone of his reasoning was the principle of equivalence formulated by him: local equivalence between the behavior of physical systems in the gravitational potential (2) and in a properly accelerated frame of reference (an elevator). In an elevator like that, the observer cannot detect any manifestations of the gravitational field, whatever local experiments he might carry out. (Notice that experiments with unshielded electric charges are not local since the Coulomb field of such charges extends to infinity.)

Consider now, from an elevator freely falling with acceleration g, the emission and absorption of a photon. A photon with frequency ω is emitted upward by an atom at rest on the Earth surface. An identical atom that should absorb the photon is at rest at the height h. In a freely falling elevator, the gravitational field does not affect the photon, and therefore it conserves its initial frequency. Let us assume that at the moment of emission of a photon ($t = 0$) the velocity of the elevator equals zero. Then at the moment $t = h/c$, when the photon reaches the upper atom, the velocity of the latter with respect to the elevator is equal to $v = gh/c$ and directed upward: the atom 'runs away' from the photon. As a result, the photon frequency seen by the absorbing atom is diminished by the linear Doppler effect, and the photon reddens:

$$\frac{\Delta\omega}{\omega} = -\frac{v}{c} = -\frac{gh}{c^2}\,. \tag{3}$$

Consider now a different design of the experiment. Let the upper atom (an absorber) be moving in the laboratory system downward with a constant velocity $v = gh/c$. Then its velocity in the reference frame of the elevator is zero at the moment of absorption, and it can absorb the photon emitted with frequency ω resonantly, in complete agreement with experiments [9–11]. Obviously, in the reference frame connected with the freely falling elevator, it is impossible to interpret the red shift as energy loss by a photon in the process of overcoming the gravitational attraction, since there is no gravitational attraction in that elevator.

No less visual is the interpretation in the laboratory reference frame. In a static field, the frequency of a photon is conserved, whereas in the reference frame of an atom moving towards the photon, it increases due to the Doppler effect and compensates the 'blue shift' of the atom in the gravitational field.

4. General relativity: metric

Till now, we used only the special theory of relativity (constancy of the speed of light and the Doppler effect) and Newtonian gravitation in the approximation of a linear potential. As is known, a consistent relativistic description of classical gravitation is provided by general relativity with its curved space-time metric. The theory is based on the metric tensor $g_{ik}(x)$, $i,k = 0,1,2,3$ that is transformed under changes of coordinates so that the interval $\mathrm{d}s$ between two events with coordinates x^i and $x^i + \mathrm{d}x^i$

$$\mathrm{d}s^2 = g_{ik}(x)\,\mathrm{d}x^i\,\mathrm{d}x^k \tag{4}$$

remains unchanged. Setting $\mathrm{d}x^1 = \mathrm{d}x^2 = \mathrm{d}x^3 = 0$, we arrive at the relation between the interval of proper time $\mathrm{d}\tau = \mathrm{d}s/c$ and the interval of world [1] time $\mathrm{d}t = \mathrm{d}x^0/c$ for an observer at rest:

$$\mathrm{d}\tau = \sqrt{g_{00}}\;\mathrm{d}t\,. \tag{5}$$

In the static case, the integration of equation (5) gives

$$\tau = \sqrt{g_{00}}\;t\,, \tag{6}$$

where in the general case g_{00} is a function of $\mathbf{x}$, whereas g_{00} in Eqn (2) depends only on $x^3 = z = h$.

The proper time τ is measured with any standard clock. It can also be considered as the coordinate time in the so-called comoving locally inertial reference frame, i.e. in the locally inertial system that at a given moment at a given point has a zero velocity with respect to the laboratory system. (Imagine a stone thrown up from the earth at the top point of its trajectory.) If there is a set of standard clocks at different points, their proper times τ are related to the world (laboratory) time t in different ways, because g_{00} depends on $\mathbf{x}$ [see formula (6)]. This explains the aircraft experiments [13, 14].

A weak gravitational field can be described in terms of the gravitational potential ϕ, and in this case g_{00} is expressed through ϕ as follows:

$$g_{00} = 1 + \frac{2\phi}{c^2}\,. \tag{7}$$

[1] Recall that world time is sometimes called laboratory time. The first name reflects the fact that this time is the same for the whole world; the second, that it can be set using a standard clock in the laboratory [see the text after Eqn (1)]. Many authors call t coordinate time.

The physical meaning of this expression will be explained somewhat later [see formulae (8)–(10)]. According to equations (5) and (7), the clock runs slower in the laboratory which is deeper in the gravitational potential.

In analogy with Eqn (5), the rest energy of a body in the laboratory system, $E_0^{\rm lab}$, and that in the comoving locally inertial system, $E_0^{\rm loc}$, are related by the formula

$$E_0^{\rm lab} = E_0^{\rm loc}\sqrt{g_{00}} \tag{8}$$

(note that $E_0^{\rm lab}\,{\rm d}t = E_0^{\rm loc}\,{\rm d}\tau$; this relationship holds true, since the energy E is a zero component of a covariant 4-vector, whereas ${\rm d}t$ is a zero component of a contravariant 4-vector).

The rest energy of a body in the locally inertial system is the same as in the special theory of relativity (see, for instance, Refs [58, 59] and [48], p. 246, 3rd English edition):

$$E_0^{\rm loc} = mc^2\,, \tag{9}$$

whereas the rest energy in the laboratory system $E_0^{\rm lab}$ also contains the potential energy of a body in the gravitational field. Notice that in Eqn (8) this potential energy is 'hidden' in g_{00}, in conformity with the fundamental principle of general relativity: gravitation enters only through the metric. The relationship between the metric and potential, Eqn (7), can be considered as a consequence of equations (8), (9), and the relation

$$E_0^{\rm lab} = mc^2 + m\phi \tag{10}$$

that generalizes the notion of rest energy of a free particle to such in a weak gravitational field.

Now we are able to explain the red shift in the laboratory frame of reference. According to Eqn (8) or (10), the difference between energies of atomic or nuclear levels in this system, $\varepsilon_{\rm lab}$, depends on the position of an atom. The deeper the atom sits in the gravitational potential, the smaller is $\varepsilon_{\rm lab}$. For an atom-absorber that is situated at height h relative to an identical atom emitting a photon, the relative energy difference of levels equals

$$\frac{\Delta\varepsilon^{\rm lab}}{\varepsilon^{\rm lab}} = \frac{gh}{c^2}\,. \tag{11}$$

[In formula (11), like in Eqn (2), we made use of the linear approximation.] It can be said that the energy levels of an absorbing atom are slightly 'bluer' than those of an emitting atom. Equation (11) is certainly nothing else than a way of describing the difference in the rate of two atomic clocks one above the other located at height h. On the other hand, the energy (frequency) of a photon is conserved as it moves in the static gravitational potential. This can be seen, for instance, from the wave equation for the electromagnetic field in the presence of a static gravitational potential or from the equation of motion of a particle (massless or massive) in the static metric. From all the aforesaid it is clear that in the laboratory reference frame, there is no place for the interpretation according to which 'a photon loses its energy for overcoming the action of the gravitational field'.

And finally, we can discuss the experiment on the red shift by using a sequence of locally inertial reference frames that are comoving the laboratory clocks (atoms) at the moment when a photon passes through them. As we have explained above, the standard clocks in such reference frames run with the same rate, the rest energy of an atom is equal to its mass times c^2 [see Eqn (9)], and the energies of atomic levels are the same as at infinity. On the other hand, the energy of a photon in the laboratory system, $E_\gamma^{\rm lab} = \hbar\omega^{\rm lab}$, and that in a comoving locally inertial system, $E_\gamma^{\rm loc}$, are related as follows

$$E_\gamma^{\rm lab} = E_\gamma^{\rm loc}\sqrt{g_{00}}\,. \tag{12}$$

Equation (12) can be derived from equation (8) if we notice that a photon can be absorbed by a massive body and consider the increase in energy of this body. Thus, since $E_\gamma^{\rm lab}$ is conserved and $E_\gamma^{\rm loc}$ decreases with increasing height, we arrive at the following expression

$$\frac{\omega^{\rm loc}(h) - \omega^{\rm loc}(0)}{\omega^{\rm loc}(0)} = \frac{E_\gamma^{\rm loc}(h) - E_\gamma^{\rm loc}(0)}{E_\gamma^{\rm loc}(0)} = -\frac{gh}{c^2}\,, \tag{13}$$

which is just the observed red shift of a photon. However, note is to be made that the decrease in $E^{\rm loc}$ is not at all caused by the work done by the photon against the gravitational field. (The gravitational field is absent in a locally inertial system.) The energy $E_\gamma^{\rm loc}$ changes since in the given description one should pass from one locally inertial reference frame to another (from the one comoving the laboratory at the moment of emission to that at the moment of absorption).

5. Pseudoderivation and pseudointerpretation of the gravitational red shift

The simplest (and incorrect) explanation of the red shift is based on assigning the inertial gravitational mass $m_\gamma = E_\gamma/c^2$ to a photon. Owing to this mass, a photon is attracted to the Earth with the force gm_γ, as a result of which the relative change in its energy (frequency) at height h equals

$$\frac{\Delta E_\gamma}{E_\gamma} = \frac{\Delta\omega}{\omega} = -\frac{gm_\gamma h}{m_\gamma c^2} = -\frac{gh}{c^2}\,. \tag{14}$$

Notice that (up to a sign) this is exactly the formula for the blue shift of an atomic level, which is not surprising. An atom and a photon are here considered in the same way: both of them are treated nonrelativistically! This is certainly wrong for the photon. If the explanation in terms of the gravitational attraction of a photon to the Earth were correct, one should expect red-shift doubling (summation of the effects of the clock and photon) in an experiment of Pound–Rebka type.

Some readers may attempt to use Einstein's authority in order to defend the above pseudoderivation. In a paper of 1911 [2], Einstein put forward the idea that the energy is not only the source of inertia, but also the source of gravitation. He used the heuristic argument: "If there is a mass, there is an energy, and vice versa". As he realized later, this "vice versa" was not so correct as the direct statement was (a photon possesses energy, whereas its mass equals zero). Identifying the energy and mass, he calculated the energy loss of a photon moving in the vertical direction in the gravitational field of the Earth, as discussed above. Taking advantage of the same heuristic principle, he also determined the deviation of a ray of light by the Sun that was half the correct deviation. Subsequently, in the framework of GR, Einstein found this missing factor of two [3–5]. The correct formula was verified experimentally.

6. Measurement of the wavelength

In previous sections, we discussed the gravitational red shift in terms of the frequency of a photon and that of a clock.

Now, we will discuss the same effect in terms of the photon wavelength and the diffraction-grating period. Consider two gratings at different heights. The lower grating serves as a monochromator, i.e. as a source of monochromatic light. The wavelength of a photon $\lambda^{\rm lab}(z)$ corresponds to its frequency, whereas the grating period in the vertical (z) direction, $l^{\rm lab}(z)$, corresponds to the frequency of a clock.

Though the energy of a photon $E^{\rm lab}$ is conserved in a static gravitational field, its momentum $p^{\rm lab}$ is not conserved. The relation between these quantities is provided by the condition for a photon being massless. This condition in the gravitational field reads as follows

$$g^{ij}p_i p_j = 0\,, \tag{15}$$

where g^{ij}, $i, j = 0, \ldots, 3$ are contravariant components of the metric tensor, and p_j are components of the 4-momentum, $p_0 = E^{\rm lab}$, $p_3 = p^{\rm lab} = 2\pi\hbar c/\lambda^{\rm lab}(z)$ (for a photon moving along the z-axis). In our case, the metric g^{ij} can be taken in diagonal form, then, in particular, $g^{zz} = 1/g_{zz}$.

From Eqn (15) one can easily determine the change of $\lambda^{\rm lab}(z)$ with height:

$$\lambda_{\rm lab}(z) = \sqrt{\frac{g^{zz}(z)}{g^{zz}(0)}}\sqrt{\frac{g^{00}(0)}{g^{00}(z)}}\,\lambda_{\rm lab}(0)\,. \tag{16}$$

On the other hand, the grating period in the z direction, $l^{\rm lab}(z)$, changes with height, too. This is merely a familiar change of the scale in the gravitational field explained, e.g., in the book by Landau and Lifshitz (see Sect. 84 in Ref. [28]):

$$l^{\rm lab}(z) = \sqrt{-g^{zz}(z)}\;l^0\,, \tag{17}$$

where l^0 is the 'intrinsic period' of the grating in the direction z, an analog of the proper frequency of the standard clock. Thus, the grating period $l^{\rm lab}(z)$ depends on z in the following way:

$$l^{\rm lab}(z) = \sqrt{\frac{g^{zz}(z)}{g^{zz}(0)}}\;l^{\rm lab}(0)\,. \tag{18}$$

Let us now take into account that the 'grating' version of an experiment of the Pound et al. type, at the difference of heights h, would measure the double ratio $\left[\lambda(h)/l(h)\right]/\left[\lambda(0)/l(0)\right]$. The result can be represented in the form

$$\frac{\Delta\lambda^{\rm lab}}{\lambda^{\rm lab}} - \frac{\Delta l^{\rm lab}}{l^{\rm lab}} = \sqrt{\frac{g^{00}(0)}{g^{00}(h)}} = \frac{gh}{c^2}\,, \tag{19}$$

where $\Delta\lambda^{\rm lab}/\lambda^{\rm lab} = \left[\lambda^{\rm lab}(h) - \lambda^{\rm lab}(0)\right]/\lambda^{\rm lab}(0)$, $\Delta l^{\rm lab}/l^{\rm lab}$ is defined analogously. Note that this result does not depend on g^{zz}, as might be expected, since there is freedom in the choice of the scale along the z-axis, and the observables should be independent of that choice. Equation (19) is analogous to the equation that describes the experiments performed by Pound et al.:

$$\frac{\Delta\omega}{\omega} - \frac{\Delta\epsilon}{\epsilon} = \sqrt{\frac{g_{00}(0)}{g_{00}(h)}} = -\frac{gh}{c^2}\,, \tag{20}$$

where ω is the photon frequency, and $\epsilon/\hbar$ is the clock frequency [see formula (11)]. Equation (20) requires a word of explanation. In the laboratory reference frame, the first term in the left-hand side equals zero, viz.

$$\frac{\Delta\omega^{\rm lab}}{\omega^{\rm lab}} = 0\,, \tag{21}$$

as discussed in Section 3; therefore, the whole contribution comes from the second term defined by equation (11).

However, we would like to emphasize a significant distinction from the case when the photon frequency is measured. In that case, one can independently measure the difference in the rate of upper and lower clocks [$\Delta\epsilon^{\rm lab}/\epsilon^{\rm lab}$ in equation (20)], which was done in aircraft experiments; whereas in the grating case, the change of the scale [$\Delta l^{\rm lab}/l^{\rm lab}$ in Eqn (19)] cannot be measured independently. This important distinction results from the fact that the metric is time-independent but essentially depends on z.

It is to be realized that such a purely laboratory experiment cannot be performed at the present level of development of experimental physics (recall the importance of the Mössbauer effect in experiments carried out by Pound et al.). However, an experiment of that sort is feasible in measuring a sufficiently large red shift, like the shift of the sodium line from the Sun [56]. Needless to say, the initial wavelength of light in this experiment was fixed by an atom on the Sun's surface rather than by the grating.

7. Conclusions

The present paper contains little original material: for the most part, it is pedagogic. Since gravitational red shift is one of the corner-stones of general relativity, both from theoretical and experimental points of view, it is highly important that its explanation should be maximally simple but, at the same time, correct, as the explanation based on the change of the clock rate in the gravitational field. An alternative explanation in terms of the mass ascribed to the photon — and the corresponding potential energy — is wrong and produces confusion. We have demonstrated that it is incorrect and schematically discussed experiments on the red shift in the framework of the correct approach. We would like to emphasize the importance of those experiments, in which an atomic clock was lifted to a high altitude, kept there for a sufficiently long time, and then compared with a twin that never left the Earth. The clock at height was fast compared to its twin. Thus, the 'blue' shift of the clock was established as an absolute effect. Hence it follows immediately that a naive explanation of the gravitational red shift in terms of the attraction of a photon by the Earth is incorrect.

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