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Wave phase conjugation of ultrasonic beams

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Contents

Abstract. **The current state of the acoustic wave phase conjugation (PC) problem is reviewed. The generation of phase conjugate ultrasonic waves is discussed with emphasis on the parametric method using electromagnetic pumping in solids. The giant-amplification supercritical parametric PC mode is considered in detail. Ultrasonic PC with a gain in excess of 80 dB is demonstrated for a soft magnetic ceramics-based amplifier. The high quality of supercritical parametric PC sound is confirmed by acoustic-optical visualization. Acoustic PC effects, such as anomalous sound reflection at the PC mirror and the autofocusing and self-targeting of phase conjugate sound beams incident on a scatterer in a liquid, are shown. Recent experimental results demonstrating the potential of PC for applications are presented.**

1. Introduction

The problem of the ultrasonic phase conjugation (PC) has long attracted attention because of the unusual physical properties of phase conjugate wave beams and the unique possibilities offered by the PC technique in physical research, nondestructive testing, technology, and medicine.

Systematic studies of applications of physical PC methods to the acoustics of liquids were initiated in papers [1 – 3]. The considerable progress in experiments on acoustic PC during the last decade has made these studies into a field of physical acoustics in its own right. The aim of this review is to reflect the modern state and trends in the development of the physics and techniques of phase conjugate ultrasonic beams.

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Phase conjugation means the transformation of a wave field resulting in the reversal of propagation of the waves conserving the initial spatial distribution of amplitudes and phases [4]. Unlike the usual specular reflection corresponding to the inversion of one of the spatial coordinates, PC represents the time inversion transformation. The fundamental possibility of its realization is provided by the invariance of the wave-field equations in a transparent medium with respect to time inversion. The time inversion transformation corresponds to the so-called phase conjugation of the spectral components of the field: $\widetilde{U}_\omega(\mathbf{r}) = U^*_\omega(\mathbf{r})$ or $\tilde{\varphi}(\mathbf{r}) = -\varphi(\mathbf{r})$ for phases. In turn, the phase conjugation corresponds to a change in the sign of the wave vectors in the spatial spectrum of the field. If

$$U_\omega(\mathbf{r}) = \sum_{\mathbf{k}} A_{\mathbf{k}}(\omega) \exp(\mathrm{i}\mathbf{kr}) , \qquad (1.1)$$

$$\widetilde{U}_\omega(\mathbf{r}) = \sum_{\mathbf{k}} B_{\mathbf{k}}(\omega) \exp(\mathrm{i}\mathbf{kr}) , \qquad (1.2)$$

then

$$B_{\mathbf{k}}(\omega) = A^*_{-\mathbf{k}}(\omega) . \qquad (1.3)$$

The PC transformation in a linear and stationary medium can involve a spatially uniform change in the wave-field intensity and a constant time delay. Depending on the experimental method of the PC effect production, it is referred to either as time inversion or as wave phase conjugation.

Despite the generality of the concept of phase conjugation for waves of different nature, PC in acoustics is characterized by silent features related to the wave properties of acoustic media, interactions involved in the PC process, the space-time structure of the acoustic beams being conjugated, and finally, to practical problems that can be solved using the PC phenomenon.

Almost all known manifestations of the acoustic PC are of interest for applications. The most important among them are the compensation for phase distortions in an inhomogeneous, acoustically transparent medium, including the compensation for acoustic losses in elastic scattering, autofocusing or 'self-targeting' of ultrasonic beams on scatterers, and the lensless formation of acoustic images. Figures 1 and 2 illustrate these effects.

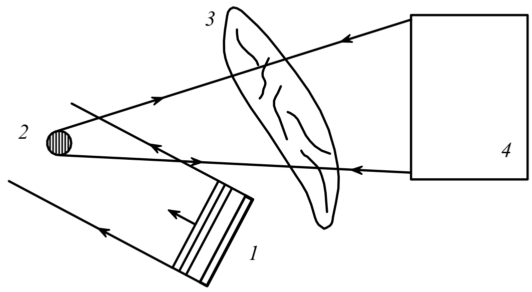

Figure 1. Schematic of autofocusing or 'self-targeting' of ultrasonic PC beams: (*1*) source of ultrasonic waves; (*2*) object; (*3*) phase-nonuniform medium; (*4*) PC mirror.

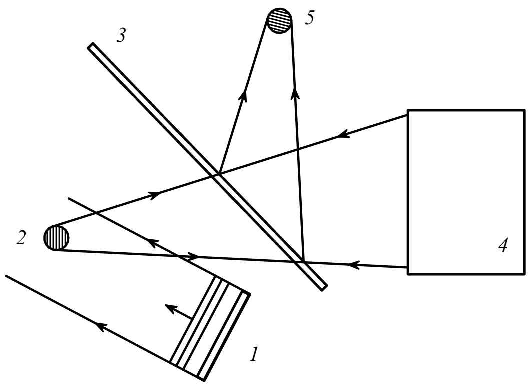

Figure 2. Schematic of lensless formation of acoustic images using PC effect: (*1*) source of ultrasonic waves; (*2*) object; (*3*) acoustic semitransparent mirror; (*4*) PC mirror; (*5*) real image of the object.

A key problem of practical applications of acoustic PC is a reasonable choice of the physical principle of generation of the conjugate wave. In the next section, the basic methods known for solving the ultrasonic PC problem in liquids are briefly analyzed.

2. Principles of generation of phase conjugate ultrasonic waves

The phase conjugation of sound waves at relatively low frequencies can be achieved using multichannel transmitting-receiving antenna arrays. Electronic channel control of the time delay of the received signal allows one to simulate on the array the amplitude-phase distribution corresponding to the phase conjugate wave. Recently, this approach was technically implemented [5, 6]. Modern microprocessors and the technology of matrix piezoelectric transducers permit the realization of PC schemes with hundreds of array elements and an operating frequency of about 5 MHz [7]. Among their advantages are the absence of fundamental restrictions on the shape of the pulse signals being processed and the possibility of deliberate correction of the synthesized amplitude-phase distribution. However, such electronic systems are still extremely complicated to control, cumbersome, and expensive.

Multichannel parametric systems provide some simplification of control and the possibility of operation in real time [8, 9]. The principle of operation is analogous to the wave phase conjugation from a reflecting surface oscillating at double frequency [10 – 12]. The amplitudes of the phase conjugate acoustic waves generated upon such reflection are usually small because of the absence of accumulating parametric effects. A quasi-harmonic signal from a separate receiving transducer in the electronic parametric system is electronically mixed with the double-frequency pump, resulting in the generation of the phase conjugate signal in each channel. In this case, the spatial phase distribution over the array reflects that in the incident wave. The input and output signals are decoupled by means of two closely spaced piezoelectric transducers placed in each channel, operating as a receiver and a transmitter, respectively. This principle has been used in a one-dimensional 300-kHz PC mirror consisting of 20 elements [8]. However, devices of this type have not gained wide acceptance because of the drastic complexity of construction of the parametric array with increasing operating frequency and the passage to two-dimensional arrays.

Alternative physical principles of wave phase conjugation in acoustics were analyzed in papers [1 – 3, 13, 14]. Attention was devoted to processes in which the generation of phase conjugate sound waves was accompanied by amplification. Similarly to nonlinear optics [15, 16], mechanisms of phase conjugation based on the intrinsic nonlinearity of an acoustic medium were considered. It is known that four-wave mixing of holographic and parametric types can produce PC in a nonlinear medium. In the case of the holographic mechanism, information on the amplitude-phase distribution in the signal wave is recorded during its interaction with the pump wave of the same frequency. The recording occurs as the spatially inhomogeneous quasi-static perturbation of the medium. The conjugate wave is generated during reading the dynamic hologram by the second pump wave propagating toward the recording wave. In the parametric mechanism, the counter-propagating pump waves produce a spatially uniform modulation of parameters of the medium at double frequency. The conjugate wave is generated during parametric interaction of the variable perturbation of the medium with the signal wave. Both holographic and parametric mechanisms principally allow one to generate a phase conjugate wave which is amplified relative to the incident wave.

However, as was noted in Refs [3, 17], a practical implementation of wave phase conjugation on the hydrodynamic nonlinearity in common liquids is hindered by the silent feature related to the absence of ultrasound dispersion, which is typical of nonlinear acoustics as a whole [18]. When the pump-wave intensity is sufficient for producing noticeable PC effect, the processes of energy transfer 'upward' over the spectrum firstly develop, resulting in the generation of sawtooth waves. To introduce dispersion and enhance nonlinearity, it was suggested to use liquids containing air bubbles [19]. In a system of such type, the nondegenerate three-wave generation of the phase conjugate wave was experimentally examined with an efficiency of about 1% [20]. The use of temperature nonlinearity was considered, which can only be applied for PC in special cases of highly viscous liquids [21]. In papers [3, 22], the holographic mechanism of wave phase conjugation in liquids containing suspended particles or gas bubbles capable of ordering under the action of acoustic pondermotive forces was discussed. PC of this type was experimentally realized with an efficiency of about 1% in Ref. [22]. In papers [23–25], the holographic mechanism of PC based on the nonlinearity of the radiative pressure of sound on the liquid–gas interface was suggested and experimentally realized. In this case, the PC efficiency in water was about 5% and was limited by side nonlinear effects of surface distortion caused by self-focusing of the sound. By changing the experimental conditions, the PC efficiency was increased almost to unity, which made it possible to observe active suppression of the sound field with the help of the wave phase conjugation considered earlier in Ref. [26].

A cardinal change in the approach to the problem of the sound wave phase conjugation was associated with abandoning the acoustic pump for modulation of the parameters of a medium. The outlook for the development of parametric PC amplifiers using uniform modulation of the sound velocity by alternating fields of a nonacoustic nature was discussed in Ref. [1]. However, the search for appropriate mechanisms in real liquids has not led to the desired practical result. Studies of the methods of ultrasonic wave phase conjugation produced in solid-state electro- and magneto-acoustic active media proved to be significantly more efficient.

3. Parametric wave phase conjugation of sound in solids

The first observations of the generation of phase conjugate sound waves in a solid were reported about thirty years ago [27]. Later, the effects of acoustic PC in crystals were often discussed, mainly in phonon-echo studies. Comprehensive references on this subject can be found in reviews and monographs [28–33]. The statement of the problem of the development of acoustic parametric PC amplifiers stimulated a search for mechanisms of dynamic control of acoustic parameters of solids, which would provide the required efficiency of ultrasonic PC.

The simplest way of obtaining acoustic PC in a solid is by modulation of the sound velocity by an alternating electromagnetic field. The difference between the velocities of acoustic and electromagnetic waves by five orders of magnitude allows one to produce an electromagnetic pump which is almost uniformly distributed in the active zone of the medium and covers a great number of the sound wavelengths. The generation of a backward wave can be interpreted as a result of the decay of a photon with frequency ω_p and the wave vector $\mathbf{k} \sim 0$ into two phonons with opposite wave vectors $\mathbf{k}_2 = -\mathbf{k}_1$ and frequencies $\omega_1 = \omega_2 = \omega_p/2$. A diagram illustrating the laws of conservation of energy and momentum in this process is shown in Fig. 3 [28, 33].

The ratio K of stationary amplitudes of the phase conjugate and signal plane waves propagating in an acoustically transparent parametrically active layer of thickness L has the simple form [2, 15, 16]

$$K = \tan\left(\frac{\Delta v}{v} kL\right), \tag{3.1}$$

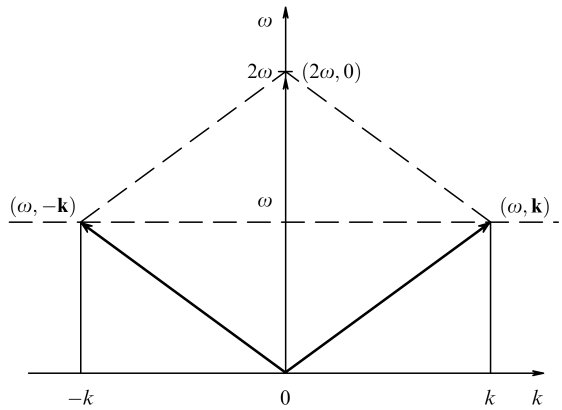

Figure 3. Vector diagram of the laws of conservation of energy and momentum in the parametric interaction of phonons with the electromagnetic pump field.

where $k = |\mathbf{k}|$, and $\Delta v/v$ is the depth of sound velocity modulation.

The depth of modulation is determined by the pump-field strength and the efficiency of its coupling with the crystal lattice of a solid. In piezoelectric crystals, the main modulation mechanism arises from a nonlinear piezoelectric effect [34, 35]. The electro-acoustic nonlinearity is most pronounced in ferroelectrics [28, 29, 36–38]. In this case, the depth of modulation of the sound velocity by an alternating electric field achieved in experiments does not usually exceed fractions of a percent [34, 39, 40]. The coefficients of the nonlinear piezoelectric effect can appreciably increase near the ferroelectric phase transitions because of the interaction of sound with a soft critical mode [28, 41, 42].

The use of the interaction of sound with soft modes of collective excitations of different physical nature substantially extends the possibilities of applications of solids for acoustic PC. Thus, in magnetics, the interaction of an alternating magnetic field with the crystal lattice involves spin excitations. In this case, the acoustic waves exist in the form of coupled magnetoelastic waves. The achievable depth of modulation of the sound velocity by the magnetic field is determined by the value of magnetostriction and dynamic features of the spin subsystem. It can amount to several or even several tens of percent. The coupling is mostly pronounced in the case of intersection of the spectra of sound and spin waves (magnetoacoustic resonance) [43–45]. Outside the resonance, the coupling can also be substantially enhanced near spin flip-flop transitions, when one of the magnon branches of the spectrum becomes a critical soft mode [46–49]. In this case, acoustic excitations (quasi-phonons) are substantially mixed with the spin component, resulting in a high sensitivity of acoustic parameters to variations in the magnetic field and in a drastic change of all acoustic properties of the matter [49, 50]. In antiferromagnetics, the relativistic magnon-phonon coupling is additionally enhanced due to the exchange interaction [51–53]. The strong influence of the magnetic field on the sound velocity is observed in magnetostriction ferrites [54], a number of rare-earth compounds with giant magnetostriction [55, 56], and in some amorphous alloys [57].

Another example of the interaction of sound with collective excitations in a solid is the phonon-plasmon interaction in semiconductors resulting in wave phase conjugation upon modulation of parameters of the semiconductor plasma by an alternating electric field or by a modulated optical pump [58 – 67].

At a given depth of modulation of the sound velocity, the parametric PC efficiency increases with increasing ratio of the length of the interaction region to the acoustic wavelength, and for fixed dimension of the active zone, it increases with increasing frequency. In Ref. [27], parametric phase conjugation of a travelling acoustic wave was observed at hypersonic frequencies ($f = 8.7$ and 4.59 GHz) under conditions of magnetoacoustic resonance in an yttrium – iron garnet single crystal. In this case, a considerable amplification, more than 55 dB, of the backward wave was detected. In piezoelectric crystals, generation of the backward wave was also studied in detail in a lithium niobate single crystal over the hypersonic range ($f = 1.43$ GHz [34], $f = 0.55$ GHz [68]). The amplification of the backward wave amounted to 67 dB [34]. In subsequent studies, generation of backward waves was also observed in other piezoelectric crystals and piezosemiconductors [40, 64, 69]. The amplification of the surface phase conjugate wave was observed in a layered piezoelectric crystal – semiconductor structure [70].

Frequency lowering to the ultrasonic range ($f < 50$ MHz), which is of interest for PC applications, is accompanied by a substantial decrease in the conversion efficiency. In recent years, the possibility of using PZT ceramics for parametric PC of bulk ultrasonic waves has been extensively studied [39, 71, 72]. By choosing a polarizing field and external mechanical strains, the efficiency of PC transformation in piezoceramics at a frequency of 10 MHz was increased to 30% [72].

Substantial parametric amplification of the bulk phase conjugate ultrasonic wave amounting to 35 dB at a frequency of 30 MHz was achieved in an antiferromagnetic hematite single crystal [73]. Earlier, parametric excitation of standing sound waves by an alternating magnetic field [74] and nondegenerate generation of the backward travelling ultrasonic wave in the acoustic pump field [75] were observed in this crystal. In the latter case, the generation was caused by the anomalous strong three-wave interaction of coupled magnetoelastic excitations [76].

The extremely high efficiency of the parametric PC of a travelling ultrasonic wave was found experimentally in a magnetostriction ceramics based on nickel ferrite [77]. The amplification coefficient of the backward wave at a frequency of 30 MHz exceeded 80 dB, its estimated intensity being hundreds of W/cm^2. The possibility of generation of highly intense phase conjugate ultrasonic waves by means of polycrystalline materials is of special interest because modern ceramic technology allows one to manufacture active elements for PC devices of any size and shape, which may be required for specific applications.

In all experimental verifications of substantial amplification of the backward travelling wave in solids [34, 70, 73, 77, 78], the pump levels exceeded the threshold of the absolute parametric instability of phonons (or quasi-phonons) specified by the condition $(\Delta v/v)kL = \pi/2$. For this reason, it is the supercritical PC mode that attracts the closest attention at present. In the next section, its main features are considered which determine the amplification dynamics and the quality of reproduction of the amplitude-phase distribution of the signal wave field.

4. Supercritical giant amplification mode of parametric ultrasonic PC in magnetic ceramics

The parametric interaction of ultrasonic beams in a solid is described, within the framework of an approximate theory of diffraction, by the system of equations [79]

$$\left[\frac{\partial}{\partial t} + (\mathbf{v}_{\lambda,\mathbf{k}}\nabla)\right] A_{\lambda,\mathbf{k}} - \frac{\mathrm{i}}{2}\,\frac{\partial^2\omega_{\lambda,\mathbf{k}}}{\partial k_i \partial k_j}\,\frac{\partial^2 A_{\lambda,\mathbf{k}}}{\partial \mathbf{r}_i \partial \mathbf{r}_j} = \mathrm{i}h_{\mathrm{p}}(\mathbf{r},t)\sum_{\nu}\Psi_{\lambda,\mathbf{k}}(\mathbf{k})A^*_{\nu,-\mathbf{k}}\exp\left[\mathrm{i}(\omega_{\mathrm{p}} - \omega_{\lambda,\mathbf{k}} - \omega_{\nu,\mathbf{k}})t\right], \tag{4.1}$$

where $\mathbf{A}_{\lambda,\mathbf{k}}$ is the complex amplitude of the acoustic mode (λ) with wave vector $\mathbf{k}$, $\omega_{\lambda,\mathbf{k}}$ and $\mathbf{v}_{\lambda,\mathbf{k}}$ are the eigenfrequency and the group velocity of the mode, respectively, $h_{\mathrm{p}}(\mathbf{r},t)$ is the slow amplitude of the parametric pump field with frequency ω_{p}, $\Psi_{\lambda,\nu}(\mathbf{k})$ is the interaction amplitude:

$$\Psi_{\lambda,\nu}(\mathbf{k}) = -\left(\mathbf{n}\frac{\partial}{\partial\mathbf{H}}\right)\hat{c}(\mathbf{H})\,\frac{\hat{a}(\mathbf{k})\hat{a}^{\nu}(\mathbf{k})}{2\rho\sqrt{\omega_{\lambda,\mathbf{k}}\omega_{\nu,\mathbf{k}}}}, \tag{4.2}$$

where $\mathbf{n}$ is the unit vector in the direction of the pump field, $\mathbf{H}$ is the strength of the external magnetic (electric) field, $\hat{c}(\mathbf{H})$ is the matrix of effective elastic moduli, ρ is the material density, $\hat{a}(\mathbf{k}) = (e_{\lambda_i}k_j + e_{\lambda_j}k_i)/2$, and $\mathbf{e}_{\lambda,\mathbf{k}}$ is the polarization vector. In the case of resonance ($\omega_{\mathrm{p}} = 2\omega_{\lambda,\mathbf{k}}$, $\nu = \lambda$) between the counter beams propagating in a flat active layer ($0 < z < L$), the transverse Fourier components $A_{\mathbf{q}}$ and $B_{\mathbf{q}}$ of the acoustic field are coupled by the same system of one-dimensional parametric equations as the amplitudes of collinear plane waves [34, 80]:

$$\begin{aligned}\frac{\partial A_{\mathbf{q}}}{\partial t} + \mathbf{v}\frac{\partial A_{\mathbf{q}}}{\partial z} - h(t)B^*_{\mathbf{q}} &= 0\,,\\ \frac{\partial B_{\mathbf{q}}}{\partial t} - \mathbf{v}\frac{\partial B_{\mathbf{q}}}{\partial z} - h(t)A^*_{\mathbf{q}} &= 0\,,\end{aligned} \tag{4.3}$$

where

$$A_q = \exp\left(\mathrm{i}\frac{\alpha q^2}{v}\,z\right)\int \mathrm{d}\mathbf{r}_\perp A_{\lambda,\mathbf{r}}\exp(-\mathrm{i}\mathbf{q}\mathbf{r}_\perp)\,,$$

$$B^*_q = \mathrm{i}\exp\left(\mathrm{i}\frac{\alpha q^2}{\mathbf{v}}\,z\right)\int \mathrm{d}\mathbf{r}_\perp A^*_{\lambda,-\mathbf{k}}\exp(-\mathrm{i}\mathbf{q}\mathbf{r}_\perp)\,.$$

Here, $\alpha = (1/2)\partial^2\omega_k/\partial q^2$, $\mathbf{r}$ is the coordinate in the beam cross section, $v = (\mathbf{v}_{\lambda,\mathbf{k}})_z$, and $h(t) = h_{\mathrm{p}}(t)\Psi_{\lambda,\lambda}(\mathbf{k})$. System (4.3) is usually supplemented with the boundary conditions $A_{\mathbf{q}}(z=0) = A^0_{\mathbf{q}}(t)$, $B_{\mathbf{q}}(z=L) = 0$, where $A^0_{\mathbf{q}}(t)$ is the signal-wave amplitude at the active layer entrance. Problem (4.3) was analyzed in detail for the case of a constant-amplitude pump in Ref. [80]. The amplification coefficient $K = |B_q(z=0)/A^0_q|$ defined as the ratio of amplitudes of the phase conjugate and incident waves at the cross section $z = 0$ has the form (3.1) for unmodulated waves ($\partial/\partial t = 0$). The amplitude restriction is provided by the balance of the energy entering the active zone from the pump source per unit time and the energy removed from the active zone by the forward amplified and phase conjugate waves. The independence of the parameters of system (4.3) from the transverse wave vector $\mathbf{q}$ provides the reproduction of the amplitude-phase distribution of the signal wave in the phase conjugate field. The account for the higher diffraction corrections in system (4.3) in Ref. [81] showed that specific distortions of the amplitude-phase distribution are small over the parameter $\pi v \tau / 2L(ka)^2 \ll 1$, where τ is the pump duration, and a is the incident-beam aperture.

For sufficiently high depths of modulation of the sound velocity, when $h > h_c = \pi v / 2L$, losses by energy removal cannot compensate for the pump, and the system becomes absolutely unstable. The generation of the backward wave becomes nonstationary over the instability threshold. The amplitude restriction appears either because of the finite pump duration or nonlinear processes developing with increasing intensity of the interacting waves. The quasi-linear and nonlinear stages of the supercritical amplification are distinctly observed in experiments [70, 79, 82, 83].

Figure 4 shows oscillograms obtained in the first experimental studies of the supercritical ultrasonic PC in magnetic ceramics [77]. The results were obtained for a sample of size $3.5 \times 7.0 \times 30.0$ mm in the standard geometry of the parametric generation of the backward plane wave. The shear ultrasonic wave in the form of a pulse of duration 3–50 μs with a frequency of 28 MHz was used as a signal wave. The wave was emitted by a piezoelectric transducer in the direction parallel to the long edge of the sample. A constant magnetizing field **H** was applied in the same direction. The burst of the parallel parametric pump field **h**||**H** at the doubled sound frequency was produced by means of an inductance coil of length $L = 5$ mm wound round the middle part of the sample. At the optimum sample magnetization ($H \cong 250$ Oe) and sufficiently high pump voltage $V_p = 2$ kV, a backward wave having all the features typical of the supercritical PC mode was detected at the transducer (see Fig. 4). Directly after the backward wave pulse is emitted during the propagation of the signal wave over the active zone, an anomalous echo signal is formed whose amplitude sharply increases with increasing the pump duration (see Figs 4b, c).

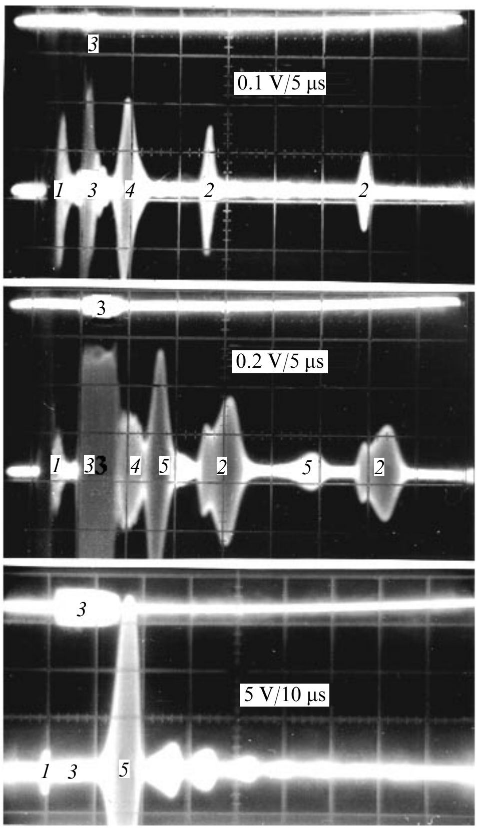

Figure 4. Oscillograms of signals from a piezoelectric transducer (the lower trace) and at the output of the pump source (the upper trace): (*1*) input pulse; (*2*) reflections from sample ends; (*3*) pump pulse; (*4*) pulse at the initial PC stage; (*5*) PC pulse in the giant amplification mode [77].

Such a development of the process of backward wave amplification is qualitatively analogous at the initial stage to the supercritical amplification at hypersonic frequencies observed earlier in lithium niobate [34]. It can be described by the nonstationary linear theory [78, 80], according to which the solution of system (4.3) in the case of the delta-like input signal and times that do not exceed the time of the double passage of the wave over the active zone ($t < T = 2L/v$) has the form

$$B^*(z=0,t) = -\frac{A_0}{t} I_1(ht)\,. \tag{4.4}$$

In the time interval $T < t < 2T$, the echo response is described by the relation

$$B^*(z=0,t) = -\frac{A_0 h}{2}\left[\frac{2}{ht} I_1(ht) - I_0\left(h\sqrt{t^2-T^2}\right) + 2\frac{t-T}{t+T} I_2\left(h\sqrt{t^2-T^2}\right) - \left(\frac{t-T}{t+T}\right)^2 I_4\left(h\sqrt{t^2-T^2}\right)\right], \tag{4.5}$$

where $I_n(ht)$ are modified Bessel functions. For $h > h_c$, solution (4.5) describes the exponentially increasing amplitude of the backward wave. Despite the fact that the input signal has time to leave the active zone by the moment $t > T$, the phase synchronization of the forward and backward waves is also retained at the stage of formation of the giant echo signal. According to experimental data (see Fig. 5) [79], later (for $t > 3T$) the quasi-linear regime passes to saturation.

Among the nonlinear mechanisms of the amplitude restriction, most important, as follows from analysis [79], is

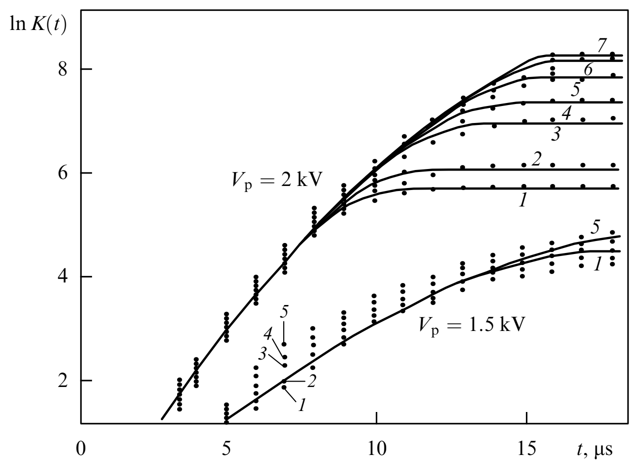

Figure 5. Dependence of the amplification coefficient of the parametric PC amplifier on the pump pulse duration t for voltages over the pump coil of $V_p = 2$ and 1.5 kV and for different voltages U_i over the emitting transducer: (*1*) 15; (*2*) 10; (*3*) 3.6; (*4*) 2.2; (*5*) 1.0; (*6*) 0.3, and (*7*) 0.1 V. Experimental data [79] and calculations [84] are shown by points and solid lines, respectively.

so-called pump depletion. When the amplitudes of the waves being amplified become quite large, the waves begin to exert the reverse effect on the pump field by decreasing its strength. This effect is described by the expression [79, 84]

$$h^{NL}(t) = h + \mathrm{i}\left(\frac{\beta}{V}\right)\int_V \mathrm{d}\mathbf{r} A_{\mathbf{k}}(\mathbf{r}, t) A_{-\mathbf{k}}(\mathbf{r}, t)\,, \qquad (4.6)$$

where β is the coefficient of coupling between the active medium of the volume V and the pump source. Accounting for the pump nonlinearity in (4.6) leaves the parameters of system (4.3) independent of the transverse wave vector and, as a consequence, does not result in the distortion of the transverse distribution of the phase conjugate wave field. For a plane wave of the form of a short rectangular pulse with duration T_0 and the amplitude A_0, the time dependence of the amplification coefficient for $t > T_0$, taking into account the pump depletion, has the form [84]

$$K(t) = \frac{\chi}{\sqrt{\exp\left[2\Gamma(T_0 - t)\right] + A_0^2\chi^2\beta/\Gamma}}\,, \qquad (4.7)$$

where Γ is the increment of the parametric instability, and $\chi \cong h_c \exp(\Gamma T_0 - 1)/\Gamma$. The saturation level is determined by the relation $K(t \to \infty) = (\Gamma/\beta A_0^2)^{1/2}$. Expression (4.7) adequately describes the saturation effect in magnetic ceramics in the case of sufficiently high supercriticality and not too weak input signals (see experimental curves *1* and *2* in Fig. 5; $V_\mathrm{p} = 2$ kV). Note that an analogous time dependence of the amplification was observed in studies on the parametric interaction of surface acoustic waves in piezosemiconductors [70].

In the case of comparatively weak input signals, the parametric amplification of thermal phonons proceeding simultaneously with amplification of the valid signal begins to play a substantial role. The amplified thermal noise not only introduces additional restrictions on the achievable intensity of the phase conjugate wave, but also leads to amplitude-phase distortions in the supercritical PC mode. For this reason, taking noise into account becomes essential in the problem of supercritical PC quality. As was shown within the framework of the small supercriticality approximation in Ref. [84], the problem of type (4.3) with the pump in the form (4.6) can be transformed to a coupled system of nonlinear equations for the phase conjugate wave amplitude $G(t) = B(z = 0, t)$ and the intensity n_f of resonance noise modes at the output from the active zone:

$$\frac{\partial G}{\partial t} - \Gamma G = h_c A_0(t) - \beta\left(|G|^2 + \sum_f n_f\right)G\,,$$

$$\frac{\partial n_f}{\partial t} - 2\Gamma_f n_f = \frac{2n_f^0}{\tau_f} - 2\beta\left(|G|^2 + \sum_f n_f\right)n_f\,, \qquad (4.8)$$

where Γ and Γ_f are increments of the signal and noise, respectively, $A_0(t)$ is the amplitude of the incident signal wave, τ_f and n_f^0 are the relaxation time and the initial thermal noise level in the active zone. System (4.8) describes the mode competition under the conditions of pump depletion. The results of numerical simulation of the amplification with the help of system (4.8) are compared with experimental data in Fig. 5 [84]. Both experimental observations [77] and the simulation show that thermal noise is only observed for sufficiently long pump durations (Fig. 6).

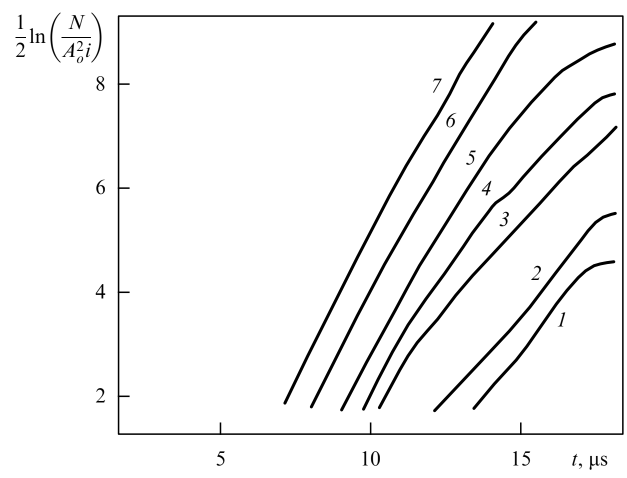

Figure 6. Time dependence of the normalized noise intensity for the voltage over the pump coil $V_\mathrm{p} = 2$ kV and for different voltages U_i over the emitting transducer: (*1*) 15; (*2*) 10; (*3*) 3.6; (*4*) 2.2; (*5*) 1.0; (*6*) 0.3, and (*7*) 0.1 V; $N = \sum_f n_f$ [84].

Competition of the modes in the nonstationary regime can suppress the noise amplification if the input signal intensity is sufficiently high, even in the case when the instability threshold for noise modes is lower than that for the signal mode. In this case, the phase conjugate wave amplitude has time to reach the saturation level before the noise level becomes noticeable. For example, for curves *1* and *2* in Fig. 5 ($V_\mathrm{p} = 2$ kV) and Fig. 6, the signal-to-noise ratio at $t = 10$ µs exceeds 40 dB. Therefore, the dominant fraction of the energy of the parametrically amplified backward sound wave in the supercritical PC mode may be concentrated in the wave that is phase conjugate relative to the signal wave.

As the pump duration becomes longer, reflections of the amplified waves from the sample boundaries can become significant. The return of the reflected waves to the active zone can result both in the increase and decrease of the backward wave amplitude, depending on the phase shift. The latter case was probably realized in the experiment considered. According to the data presented in Fig. 5 ($V_\mathrm{p} = 1.5$ kV), the amplification coefficient upon pumping near the threshold is almost independent of the input signal amplitude. At the same time, the amplification law noticeably differs from exponential at times exceeding the time of the reflected wave return to the active zone ($t > 7.5$ µs). In this case, the account of reflections in system (4.8) allows one to adequately describe the amplification dynamics [84].

The simple model of the parametric interaction of paraxial beams in a plane-parallel active layer considered above reflects only the basic properties of the supercritical ultrasonic PC mode. In practice, the wave phase conjugation of beams with a wide angular aperture is also of interest. In this case, such factors as the angular dependence of the projection of the group velocity onto the normal to a layer, the internal reflection of the waves from side surfaces of the active element, the refraction of the input beam and transformation of the modes on the interface between the active and passive media, and the anisotropy of the parametric interaction and competition between coherent modes in the saturation regime become substantial. The effect of these factors on the formation of PC of beams was partially considered in papers [14, 85, 86]. As applied to specific experimental conditions, it is discussed in Section 6 of this review.

5. Acoustic-optical visualization of phase conjugate ultrasonic beams in solids

For a long time, the possibility of using parametric acoustic effects in solids for wave phase conjugation was only confirmed by indirect experimental facts such as the time inversion in parametric trains of echo pulses (see, for example, Fig. 7 taken from Ref. [89]) and partial compensation for losses due to elastic scattering in measurements of the relaxation time of phonons by the method of parametric echo (see Refs [30, 68, 87 – 89]). In paper [90], the echo method was especially modified for observing the PC of sound beams with a complex spatial distribution of amplitudes and phases. For this purpose, a transducer of complex form was used as an emitter and receiver, which played a role analogous to that of a phase plate in optical PC experiments [4].

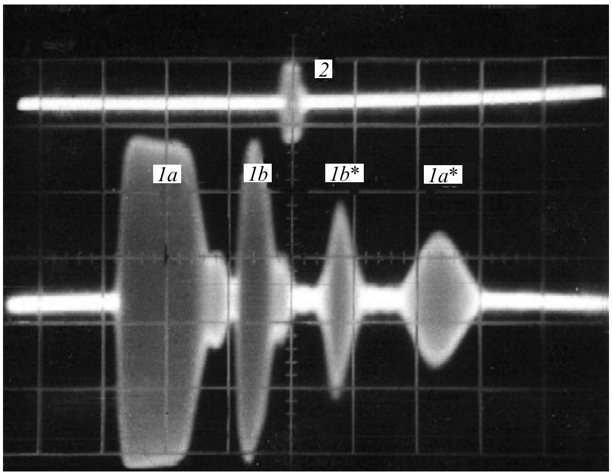

Figure 7. Oscillogram of the time inversion of a two-pulse train in a single-crystal hematite: (*1*) the initial train consisting of long (*1a*) and short (*1b*) pulses with frequency ω; (*2*) a parametric pump pulse with frequency 2ω; (*1**) phase conjugate train of pulses with frequency ω and time-inverted positions of the long (*1*a*) and short (*1*b*) pulses [89].

Lithium niobate was used as an active medium. The PC effect was confirmed by the fact that echo pulses were detected by the transducer only upon switching the parametric pump, whereas normal reflections from the system boundaries did not contribute to the detected signal because of dephasing.

The ultrasonic PC under conditions of giant supercritical amplification in ferrite was demonstrated in Ref. [78] by the example of anomalous reflection of the plane wave. In the experiment, two transducers were used which separately emitted acoustic pulses with frequency 28 MHz at angles of 45° and 135° to the entrance surface of the active element. In complete accordance with the PC principle, a giant echo pulse was detected only by the transducer emitting the seed signal.

The most informative method for studying ultrasonic PC is the acoustic-optical visualization of acoustic beams. The phase conjugate ultrasonic waves were visualized for the first time in paper [91]. The aim of the experiment was to study the possibility of autofocusing of sound beams with a broad angular spectrum in the supercritical parametric PC mode. The schematic of the experiment is shown in Fig. 8. A sample of fused quartz was used as an acoustic-optical medium of wave propagation. An ultrasonic beam emitted by a plane piezoelectric transducer at a frequency of 25 MHz was reflected from the sample's cylindrical surface and focused in the vicinity of the point *F*. The beam diverging from the focus was incident on the entrance surface of the parametrically active element. The latter was a parallelepiped of size $3.5 \times 7.0 \times 30.0$ mm made of polycrystalline nickel ferrite. After switching on the pump pulse, a giant echo pulse was emitted back from the ferrite to quartz. Oscillograms of the electric pulses at the piezoelectric emitter and a pump coil shown in Fig. 9 clearly demonstrate the giant amplification and characteristic chip on the pump pulse caused by the effect of the amplified waves on the pump discussed above.

The acoustic fields of the incident and phase conjugate waves were visualized in the stroboscopic regime using the second harmonic of a pulsed neodymium laser ($\lambda = 0.53$ μm) synchronized with a pulse signal generator. The variable

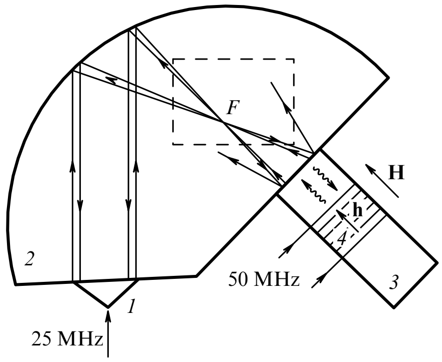

Figure 8. Schematic of the experiment on the visualization of ultrasonic fields with a broad spatial spectrum upon PC in ferrite: (*1*) ultrasonic transducer; (*2*) fused quartz acoustic line; (*3*) ferrite sample; (*4*) parametric pump coil; (*F*) region of focusing [91].

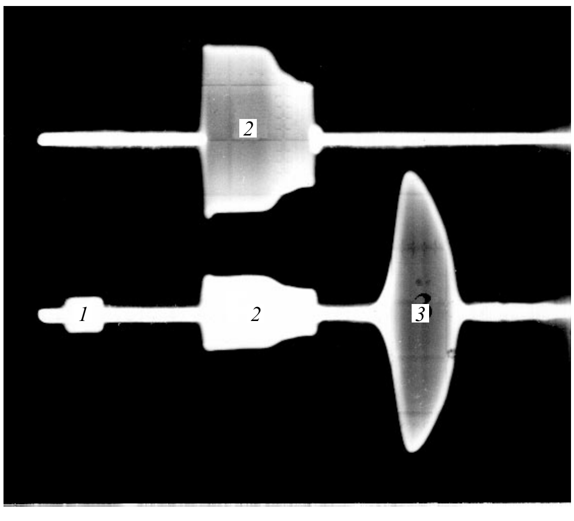

Figure 9. Oscillograms of signals from a piezoelectric transducer (the lower trace) and a pump coil (the upper trace): (*1*) input pulse; (*2*) parametric pump pulse; (*3*) PC pulse [92].

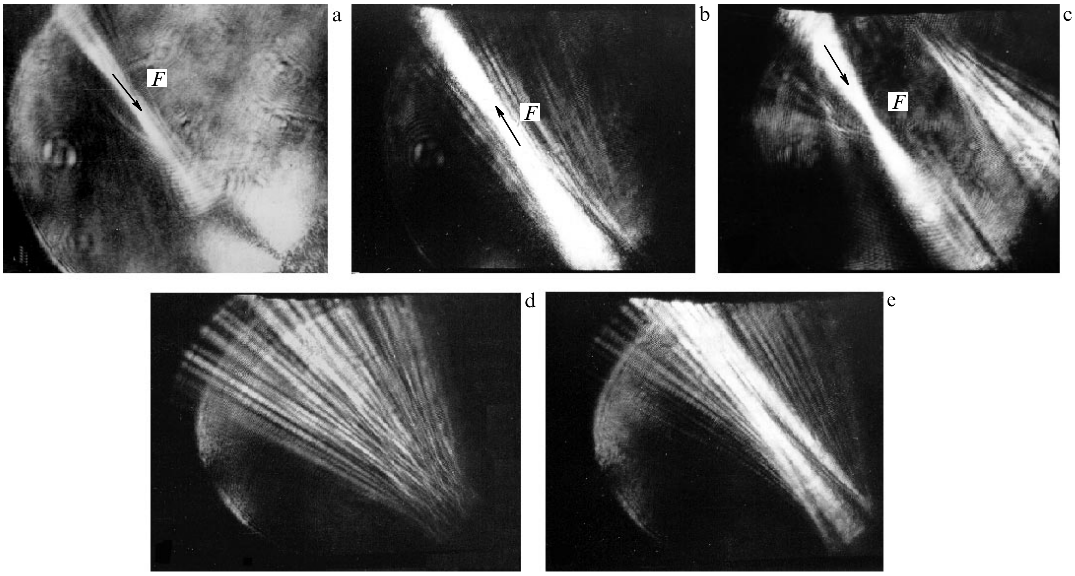

Figure 10. Acoustic beams of (a) the incident and (b, c) phase conjugate ultrasonic waves in the region F of focusing in fused quartz (c corresponds to the phase conjugate ultrasonic beam after a second passage through region F) [91]. Fluxes of parametric phonons generated by the electromagnetic pump from the thermal noise level in ferrite (d) without and (e) with a weak signal wave [79].

delay of the light and signal acoustic pulses made it possible to detect various stages of the propagation of acoustic waves in the medium region being analyzed, shown by the dashed line in Fig. 8. The linearly polarized light diffracted from the photoelastic perturbation of the medium. The diffracted beam passed through a crossed analyzer and was directed to a screen where the image of the acoustic beams was formed.

In complete accordance with the PC autofocusing principle, the beam emitted from the parametric active medium was focused in the same focal plane as the incident signal wave (Fig. 10a – c). In this case, the intensity of the phase conjugate beam was substantially greater than that of the incident beam. The high PC quality is demonstrated in Fig. 10c, where the second passage of the phase conjugate beam through the focus is shown after reflection from surfaces of the quartz acoustic line. These results represent the first direct experimental confirmation of the principal possibility and the outlook of using supercritical parametric amplification mode for ultrasonic PC.

In the case of a quite long pump duration ($t > 20$ μs) and in the absence of the incident beam, the emission of waves parametrically amplified from the thermal noise level was observed (Fig. 10d) [77].

Thus, fluxes of parametric phonons generated by the electromagnetic pump in a solid were visualized for the first time. These fluxes are characterized by fluctuations of the intensity and the direction of emission. The switching on of the signal pulse resulted in a noticeable suppression of fluctuations caused by mode competition (Fig. 10e).

6. PC and autofocusing of ultrasonic beams in a liquid – solid-state parametrically active medium system

In most practical applications of ultrasonic wave phase conjugation, a liquid is either used as the medium in which the waves are propagating or as an immersion layer between the PC amplifier and an object. For this reason, the study of the emission of phase conjugate waves from a solid active medium into a liquid is of special interest. The first experiments on the visualization of parametrically phase conjugate beams emitted into a liquid were performed in papers [92 – 95]. In Refs [94, 95], anomalous reflection was observed for a plane-parallel beam with a frequency of 56 MHz obliquely incident from a liquid onto the surface of a piezoelectric parametrically active element. The high efficiency of the magnetoacoustic PC amplifier used in Refs [92, 93] made it possible to decrease the operating frequency to 6 MHz, resulting in the opportunity to visualize the wave fronts of phase conjugate cylindrical and plane waves for the available duration of the illuminating pulse. Note that visualization of the wave fronts allows one to study the phase distribution in phase conjugate beams. In certain cases, this distribution contains basic information on specific effects caused by PC. Such an example is noted at the end of Section 8 of this review.

The schematic of the experiment on wave front visualization for PC autofocusing of cylindrical waves in water is shown in Fig. 11.

The ultrasonic pulses were focused by a cylindrical mirror and in the form of diverging beams were incident on the entrance surface of a magnetoacoustic PC amplifier. A ferrite rod 28 mm in diameter and 100 mm in length was used as a parametrically active element of the amplifier. Stroboscopic visualization of the sound was performed by the Schlieren method using 15-ns laser pulses. The distributions of the wave fronts of the incident and phase conjugate beams detected in the focal region are presented in Figs 12a and 12b, respectively. The reproduction of the cylindrical wave fronts upon autofocusing was distinctly observed within a wide angular aperture $\Delta\varphi = 30°$.

The angular dependence of the supercritical PC efficiency was especially studied in experiments on anomalous reflection of a plane wave from the surface of the PC amplifier [92, 93]. The schematic of the experiment is shown in Fig. 13. A receiver-emitter piezoelectric transducer 18 mm in diameter

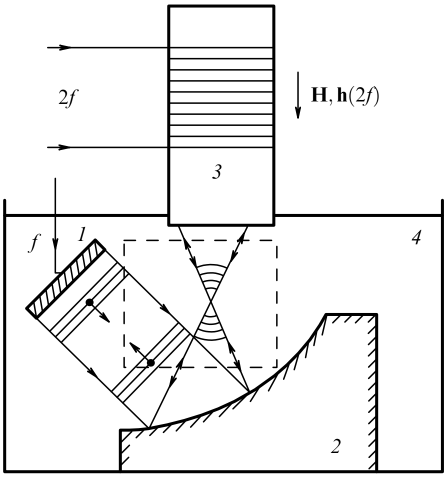

Figure 11. Scheme of the experiment on the visualization of wave fronts for the PC autofocusing of cylindrical waves in water: (*1*) ultrasonic piezoelectric transducer; (*2*) cylindrical acoustic mirror; (*3*) ferrite PC amplifier; (*4*) reservoir with water (the visualization region is shown by the dashed line) [93].

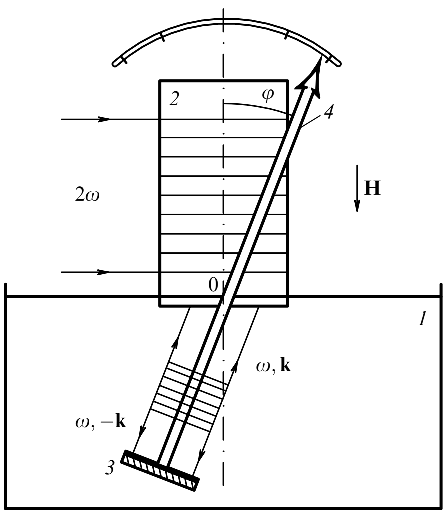

Figure 13. Schematic of the experiment on the anomalous reflection of a plane ultrasonic wave from the surface of a PC amplifier: (*1*) reservoir with water; (*2*) parametric ferrite PC amplifier; (*3*) receiver-emitter ultrasonic piezoelectric transducer; (*4*) turning lever with angular position meter [92].

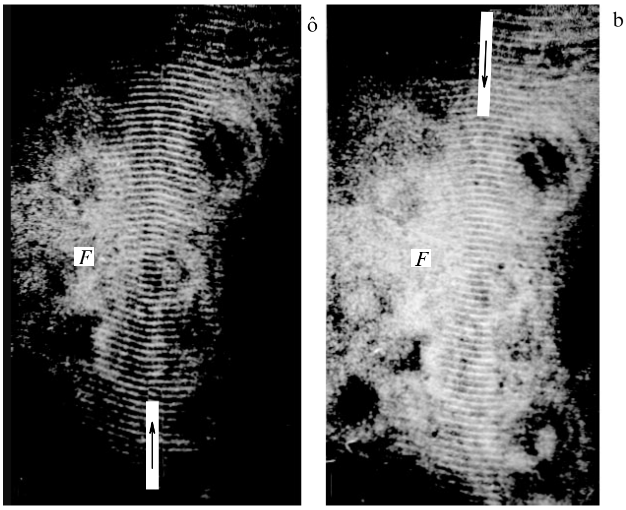

Figure 12. Images of the wave fronts of (a) the incident and (b) phase conjugate ultrasonic beams detected in the region of focusing *F*. The arrows show the direction of propagation of the waves [93].

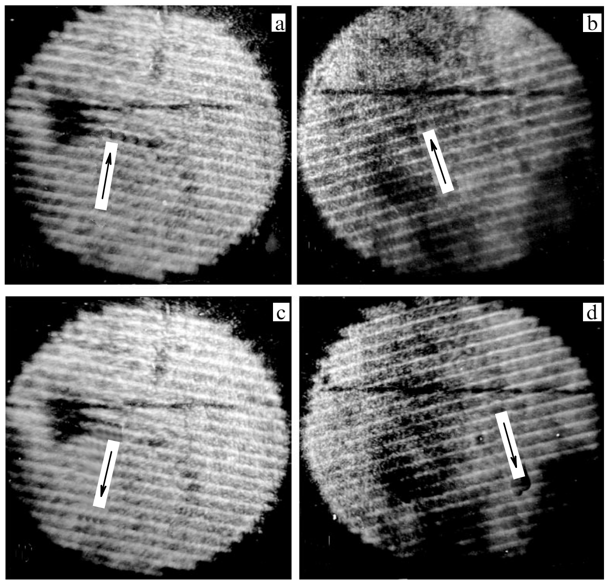

Figure 14. Images of the wave fronts of plane ultrasonic waves incident at angles of (a) $+15°$ and (b) $-15°$, and of (c, d) corresponding phase conjugate waves in water. The dark horizontal bar is the reference line [92].

was placed on a movable holder immersed in a liquid, which provided a continuous variation of the angle of incidence of the signal wave onto the entrance surface of the active element. The effect of anomalous reflection is illustrated by images of wave fronts of the incident and phase conjugate waves for angles of incidence $\varphi = \pm 15°$, shown in Fig. 14.

The results of quantitative measurements of the angular characteristics of the PC amplifiers of diameter 28 mm and lengths 150 and 100 mm are presented in Fig. 15. One can see that the optimum direction of the incident beam corresponding to maximum efficiency of PC transformation does not coincide with the normal to the entrance surface of the active element and substantially depends on the relation between its geometrical sizes. On the other hand, it is almost independent of the magnetizing field and the pump amplitude and duration. For a length of 100 mm, the optimum angle of incidence exceeds the critical angle of total internal reflection ($\varphi = 12.6°$) for the longitudinal wave. The latter circumstance suggests a substantial role of amplification for the shear waves in the active medium, which have a higher sensitivity of the sound velocity to the magnetic field, as compared to longitudinal waves (Fig. 16) [78].

The nonuniformity of the angular characteristic is reflected in the PC quality. In Ref. [93], an attempt was made to use PC compensation of phase distortions to flatten it. For this purpose, a phase plate made of solol was placed directly in front of the entrance surface of the amplifier. Despite the achieved effect of flattening of the angular characteristic, losses due to reflection and sound propagation introduced by the phase plate proved to be high. In Ref. [96], the phase-plate effect was achieved by grooving the

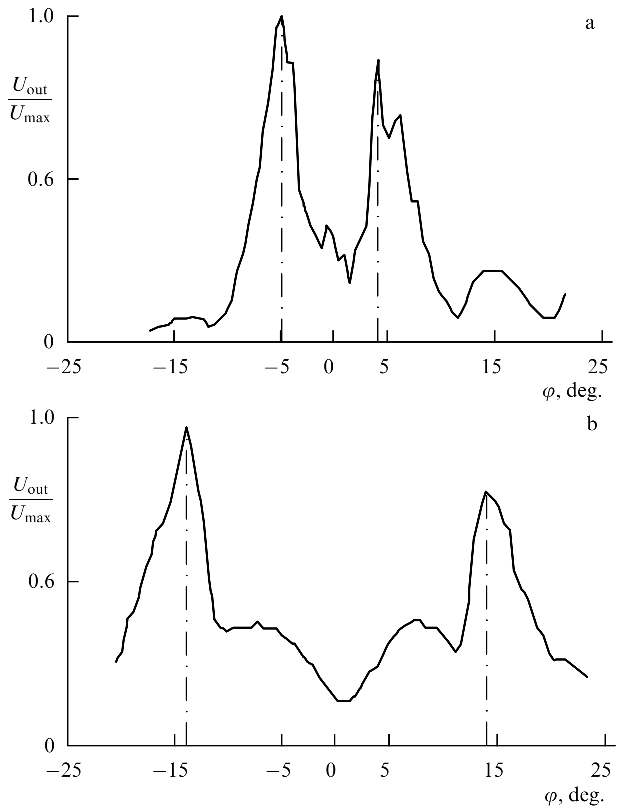

Figure 15. Angular characteristics of the efficiency of PC transformation for amplifiers 25 mm in diameter and of length (a) 150 mm and (b) 100 mm (experiment) [93].

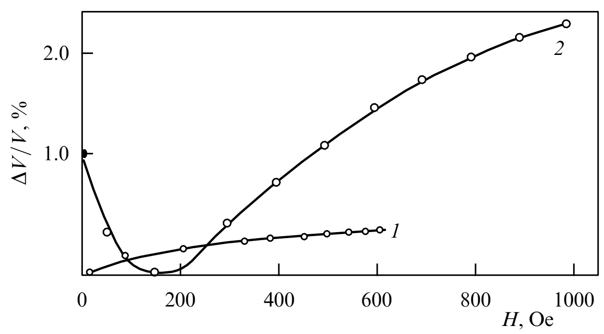

Figure 16. Dependence of the relative change in the sound velocity on the magnetic field strength for (*1*) longitudinal and (*2*) shear ultrasonic waves in ferrite [78].

entrance surface of the most active element of the amplifier. This allowed a flattening of the angular characteristic in the angular range of $\pm 15°$ by retaining the maximum amplification. The angular characteristic of the amplifier with the grooved entrance surface is shown in Fig. 17.

7. Self-targeting of parametrically phase conjugate ultrasonic beams to regular and random scatterers in a liquid

One of the most spectacular manifestations of the PC effect is the self-targeting of the phase conjugate wave beams onto objects that partially scatter the initial wave in the direction of the PC device. In this case, almost all intensity of the phase

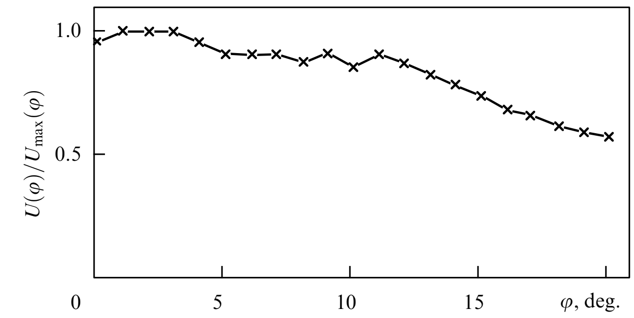

Figure 17. Normalized angular dependence of the amplitude of the phase conjugate wave in water for a ferrite sample with a grooved entrance surface [96].

conjugate wave can be concentrated on the scatterers. Together with the possibility of giant amplification of the phase conjugated beams, autofocusing permits the required spatially selective action of ultrasound on the scattering objects. The same self-targeting effect forms the basis for the method of construction of a real acoustic image of an object using PC. On the other hand, the self-targeting efficiency of the phase conjugate wave characterizes the quality of the PC amplifier. The autofocusing on the object of a pulse inverted in time by the electronic technique was studied in Ref. [97]. The self-targeting effects of parametric PC beams were first realized in papers [98 – 100]. In paper [98], a glass hemisphere 10 mm in diameter immersed in water was used as a scattering object. Figure 18 shows the schematic of the experiment. An object and a piezoelectric transducer 18 mm in diameter were fixed on a common movable holder which allowed their position relative to the PC amplifier to be changed at a time. This provided constant illumination of the object with movement. Figure 19 shows visualized beams of the incident and phase conjugate waves for two characteristic positions of the object. Although the intensity of the waves scattered by the object was too low for their visualization, the phase conjugate beams automatically focused onto the object were

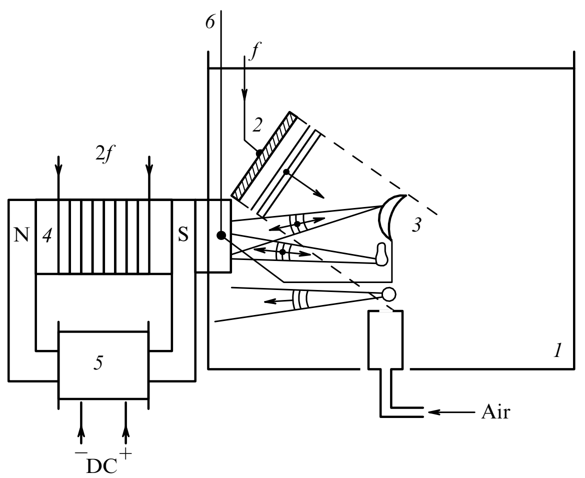

Figure 18. Scheme of experiments on the PC autofocusing of ultrasonic beams onto various objects: (*1*) reservoir with water; (*2*) ultrasonic piezoelectric transducer irradiating the objects; (*3*) objects (a glass hemisphere or a flow of emerging air bubbles); (*4*) ferrite active element with a pump coil; (*5*) electromagnet; (*6*) movable holder [98, 99].

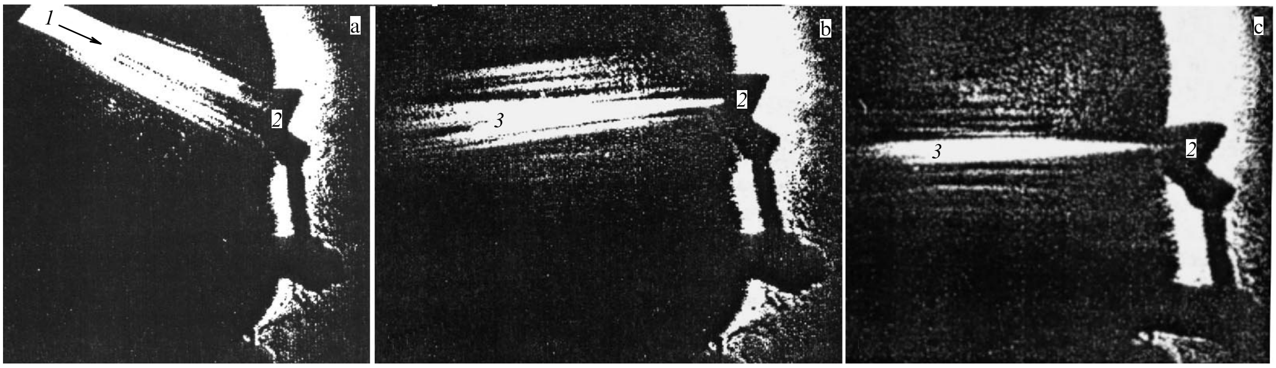

Figure 19. Stroboscopic images of (a) the ultrasonic beam (*1*) irradiating an object (*2*) in the form of a glass hemisphere, and of (b, c) the self-targeted ultrasonic beam (*3*) for two positions of the object [98].

distinctly detected in the experiment. The continuous variation of the object's position was accompanied by a corresponding re-targeting of the phase conjugate wave (cf. Figs 19b and 19c).

The real-time phase conjugation allows one to realize the follow-up regime of self-targeting of ultrasonic beams on the moving objects if their position has no time to change noticeably during the time of propagation of the forward and backward waves. In this case, the position and movement of several objects can be both regular and random. The features of the sound self-targeting to moving objects were theoretically considered in Ref. [101].

In the experimental paper [99], air bubbles floating up in water were used as random moving scatterers. The modification of the experimental scheme for this case is presented in Fig. 18. Typical results of the stroboscopic visualization of phase conjugate beams are presented in Fig. 20. As follows from the experiment, the supercritical PC mode can also provide efficient self-targeting of powerful ultrasonic beams in complex systems containing random and moving scatterers (see Fig. 20).

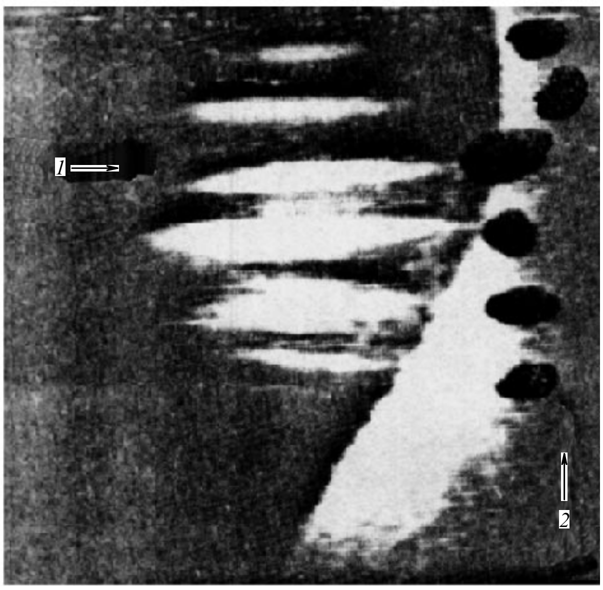

Figure 20. Stroboscopic image of ultrasonic beams (*1*) self-targeting due to PC onto a flow of air bubbles (*2*) [99].

Thus, all the recent experimental data demonstrate the possibility of widespread use of supercritical parametric ultrasonic PC in physical studies of phase conjugate ultrasonic fields and their technical applications.

8. Applications of phase conjugate ultrasonic beams

The development of ultrasonic PC methods is closely connected with applied studies demonstrating the possibilities of practical use of phase conjugated waves in modern ultrasonic technology. One of the applications of the wave phase conjugation is acoustic microscopy. In papers [102, 103], a substantial improvement in the quality of the object image was experimentally demonstrated in a scanning microscope containing a parametric piezoceramic PC mirror, when the object was surrounded by a medium with phase-distorting inhomogeneities. Note that the use of the supercritical parametric PC amplification allows one to additionally control the contrast and brightness of acoustic images. The salient features of the image formation in an acoustic microscope in the quasi-linear supercritical mode are considered in paper [104].

In Ref. [97], the application of the PC method for the nondestructive testing of titanium alloys was demonstrated. As a PC system in a B-type defectoscope, a two-dimensional electronically controlled matrix of 121 piezoelectric transducers with frequency 5 MHz was used. The use of PC permitted compensation for the phase distortions introduced by the complex surface of an object and detection of the accumulation of the α phase of titanium against a noise background caused by scattering from the granular structure of the medium.

One of the applications of acoustic PC in medicine is ultrasonic hyperthermia. In Ref. [105], autofocusing of phase conjugate waves onto the model object through bony tissue was experimentally studied in connection with hyperthermia of a brain tumor. Another promising medical application of ultrasonic PC beams is lithotrity. The self-targeting of powerful acoustic pulses to an object to be destroyed not only simplifies the system focusing but also principally allows one to continue crushing fragments of the object without any additional adjustment.

It should be noted here that autofocusing of sound pulses with the simultaneous sharp increase in intensity to the level sufficient for the destruction of solid objects is accompanied by substantial nonlinear distortions of the profile of the phase conjugate shock wave. The absence of similar distortions in a weak signal wave makes autofocusing with amplification different from PC in its own sense. For this reason, not only the generation processes but also processes of propagation of the intense phase conjugate beams in a passive nonlinear medium require special studies. The formation of nonlinear distortions is distinctly observed by the methods of acoustic-optical diffraction upon the supercritical parametric phase conjugation of powerful ultrasonic pulses in water [106]. The diffraction pattern exhibits a distinct asymmetry and shift of the maximum to higher diffraction orders, which is typical of saw-tooth waves. In its turn, the numerical simulation of the propagation of amplified phase conjugate beams in a nonlinear medium demonstrates the reproduction of the beam cross section at the location of the source of initial radiation [106].

The unusual effects of incomplete phase conjugation of sound in moving media are of some applied interest. In Ref. [107], the accumulating distortion of the wave front in a rotating cylinder placed between two PC mirrors was experimentally studied. It was also shown that the analogous phenomenon appearing upon scattering of phase conjugate waves with a broad aperture from a rotating perturbation of the medium in the form of a cylindrical vortex results in the distortion of the train of dislocation type plane wave fronts.

9. Conclusions

The elaboration of highly efficient methods of acoustic wave phase conjugation defined a new stage in the development of the physics and techniques of ultrasonic PC beams. At this stage, the main attention is already being paid not to the methods of generation of phase conjugate sound beams but to the physical properties of real phase conjugate beams, including studies of highly nonlinear phenomena observed upon PC and the interaction of powerful PC beams with matter. Of independent interest are new additional experimental possibilities for studying the dynamics of phase conjugate pairs of elementary excitations in real active media. It should be expected that the recent extensive studies of acoustic PC for applications in ultrasonic technology and medicine will result, in the near future, in the development of specialized ultrasonic PC systems for specific applications. In this case, efficient parametric PC mirrors that have a simple construction may enjoy the widest application. The important practical problem is the development of various special magnetic materials with specific magnetoacoustic properties for parametrically active elements.

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