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LETTERS TO THE EDITORS

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On heat transfer (heat conduction) and the thermoelectric effect in the superconducting state

V L Ginzburg

Abstract. **As a follow-up to the author's earlier paper [1], some remarks concerning heat conduction in the superconducting state are presented and prospects for the measurement of thermoelectric coefficients discussed.**

In a recent paper [1], when discussing the electron part of thermal conductivity in the superconducting state (Section 5 of paper [1]) I was not aware of the situation. This particularly referred to allowance for the role of specimen boundaries where a superconducting current is transformed to a normal one (Fig. 1). After the Russian version of paper [1] had already been published, some points were clarified and included in a note to the English translation [1]. I would like, however, to introduce the same corrections in the Russian text in more detail allowing for some interesting newly-appeared papers [2, 3]. For the reader's convenience we shall first repeat a small part of the material presented in Section 5 of paper [1]†.

1. By complete analogy with the case of a normal-state metal, the normal current density $\mathbf{j}_n$ in a non-uniformly heated superconductor is [5 – 7]

$$\mathbf{j}_n = \sigma_n \left(\mathbf{E} - \frac{\nabla \mu_n}{e} \right) + b_n \nabla T \,, \tag{1}$$

† I am taking the opportunity of correcting the misprints noticed in the Russian text [1] (this was partly done in the English version of the paper). On page 432, line 30 from the bottom, Ref. [42] should be replaced by [45]. Formula (31) referring to the case $d \ll \delta$ should include the expression $[1 - (H_0/H_c)^2]^{3/2}$ in the right-hand side; two lines below formula (31) there should be: [see Eqn (29)]. On the same page 436, line 10 from the bottom, the word 'superheating' should be replaced by 'supercooling' and in lines 9 – 10 the brackets [see Eqn (23)] should be substituted for [see Eqn (27)]. Formula (33) includes the coefficient 0.89 from paper [61] referred to in Ref. [1]. In the recent paper [4], the ratio H_{c1}/H_{cm} was obtained to a higher accuracy, and therefore the coefficient 0.89 is replaced by $2^{-1/4} \approx 0.84$. On page 445, before formula (74) it should read $\delta_{II} = \delta_{II}(0)(1 - T/T_{c,II})^{-1/2}$. In reference [132] from paper [1], *Phys. Lett. A* **139** is substituted for *A* **138**. Reference [55] should be Boulter C J, Indeken J O *Phys. Rev. B* **54** 12407 (1996); see also J M Mishonov *J. Phys.* (France) **51** 447 (1990).

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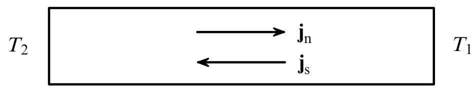

Figure 1.

where μ_n is the chemical potential referred to a single particle (excitation) with charge e, $\mathbf{E} = -\nabla\varphi$ is the electric field strength, T is the temperature, and $\sigma_n(T)$ is the electric conductivity of normal electrons (excitations). In the superconducting state, in the stationary case we have

$$\frac{\partial \Lambda \mathbf{j}_s}{\partial t} = \mathbf{E} - \frac{\nabla \mu_n}{e} = 0 \,,$$

where $\mathbf{j}_s$ is the superconducting current density; under such conditions we therefore have

$$\mathbf{j}_n = b_n(T) \nabla T \tag{2}$$

[$b_n(T)$ is certainly less than zero]‡.

In a homogeneous and isotropic sample (Fig. 1) the total current is $\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = 0$ and, therefore,

$$\mathbf{j}_s = -\mathbf{j}_n \,. \tag{3}$$

We digress here from the possibility of the appearance of a certain resultant electric charge of quasi-particles of density Q^* (the so-called charge imbalance effect). Meanwhile, this effect is of importance, in particular, for understanding the limiting transition $T \to T_c$ as $\mathbf{j}_s \to 0$ (for the corresponding references see Ref. [6] and item 7 below).

2. Even when the currents are totally compensated for [see Eqn (3)] in the superconducting state, a certain specific convective contribution to a thermal conductivity must occur owing to the presence of the current $\mathbf{j}_n$ (recall that in non-superconductors the coefficient of thermal conductivity $\varkappa$ is connected by definition with the heat flow $\mathbf{q} = -\varkappa \nabla T$ in the absence of the electric current $\mathbf{j}$; in the superconducting state the current $\mathbf{j}_n$ is present even when $\mathbf{j} = 0$). The problem of convective heat (thermal) conductivity in superconductors has already been discussed for over 50 years (see Refs [5 – 7, 10 – 13]) but remains unclear.

The total coefficient of thermal conductivity is

$$\varkappa = \varkappa_{ph} + \varkappa_e^{tot} \,, \quad \varkappa_e^{tot} = \varkappa_e + \varkappa_c \,,$$

‡ In Refs [8, 9] for the electron we see $e > 0$ [while in our formula (1) for the electron we have $e < 0$] and the designations there are somewhat different ($b = -\sigma\alpha$, where $\alpha \equiv S \equiv d\mathcal{E}/dT$ is the Seebeck coefficient or, using another terminology, the differential thermoelectric power and $\mathcal{E}$ is the thermoelectric power).

where $\varkappa_{\rm ph}$ corresponds to the thermal conductivity of phonons and $\varkappa_{\rm e}^{\rm tot}$ is the electron part of the coefficient of thermal conductivity. Next, we rather conditionally separate in $\varkappa_{\rm e}^{\rm tot}$ the electron thermal conductivity $\varkappa_{\rm e}$ under the condition that $\mathbf{j}_{\rm n} = 0$, and the part $\varkappa_{\rm c}$ of the coefficient of thermal conductivity connected with the current $\mathbf{j}_{\rm n}$. The coefficient $\varkappa$ can be measured experimentally; one can separate $\varkappa_{\rm ph}$ and $\varkappa_{\rm e}^{\rm tot}$ in various ways. However, at least in the absence of an external magnetic field, $\varkappa_{\rm e}$ and $\varkappa_{\rm c}$ are inseparable. Nevertheless, it is theoretically quite possible to pose the question of the contributions of $\varkappa_{\rm e}$ and $\varkappa_{\rm c}$, and under more involved conditions (an external magnetic field in anisotropic superconductors) the division of $\varkappa_{\rm e}^{\rm tot}$ into $\varkappa_{\rm e}$ and $\varkappa_{\rm c}$ may actually become of significance.

After the microtheory of superconductivity (BCS) was created in 1957, a sufficiently consistent estimation of $\varkappa_{\rm e}$ and $\varkappa_{\rm c}$ became possible. In Refs [11, 12], without a detailed calculation the following result is presented

$$\frac{\varkappa_{\rm c}}{\varkappa_{\rm e}} \sim \frac{k_{\rm B} T_{\rm c}}{E_{\rm F}}\,, \tag{4}$$

where $E_{\rm F}$ is Fermi energy.

The same estimate had been obtained before [10] on the basis of the two-fluid model of a superconductor and some model considerations. Finally, I obtained the same result, too [1, 7, 13]. I considered the breaking and formation of superconducting pairs at the ends of a sample (at temperatures T_2 and $T_1 < T_2$) and at the same time I felt that perhaps the heat transfer due to such a mechanism should be summed up with the heat transfer obtained by the method of the kinetic equation in the calculation of the heat flow through the sample cross-section. But this is certainly incorrect, which was noted at the end of the English translation of paper [1]. Indeed, for a sufficiently long sample (having a length $L \gg l_{\rm n}$, where $l_{\rm n}$ is the free path of normal electron-excitations), the flow through the sample cross-section calculated using the kinetic equation is equal to the heat flow due to pair transformation at the sample boundaries.

If one proceeds from the estimate (4), for conventional superconductors, when $E_{\rm F} \sim (3-10)$ eV and $T_{\rm c} \sim (1-10)$ K, one has the ratio $\varkappa_{\rm c}/\varkappa_{\rm e} \lesssim 3 \times 10^{-3}$, that is, the convective heat transfer is negligibly small. But for high-temperature superconductors (HTSC), as well as for superconductors with heavy fermions, the situation is different. So, for $E_{\rm F} \sim 0.1$ eV and $T_{\rm c} \sim 100$ K, we already have $\varkappa_{\rm c}/\varkappa_{\rm e} \sim 0.1$ according to Eqn (4). Furthermore, when the anisotropy and the unconventional pairing [14–16] are taken into account, then for the well-known reason that the Seebeck coefficient generally increases $E_{\rm F}/k_{\rm B}T$ times compared to the isotropic case. That is why it may well be that $\varkappa_{\rm c} \gtrsim \varkappa_{\rm e}$ in an HTSC.

3. From experiment it is known [17–21] that for several HTSCs the measured coefficient $\varkappa = \varkappa_{\rm ph} + \varkappa_{\rm e}^{\rm tot}$ for $T < T_{\rm c}$ has a maximum (at about $T \sim T_{\rm c}/2$; as an example see Fig. 2). What is the source of such behavior: is it due to the dependence $\varkappa_{\rm ph}(T)$ or are we dealing with the temperature run of $\varkappa_{\rm e}^{\rm tot} = \varkappa_{\rm e} + \varkappa_{\rm c}$? In principle, both are possible. Thus, the thermal conductivity of phonons may have a maximum if the leading role is played by the scattering of phonons on electrons. Indeed, the concentration of normal electrons $n_{\rm n}$ falls with T, i.e., these electrons (excitations) merely freeze out. One can hardly doubt that this is the nature of the maximum of the function $\varkappa(T)$ observed in some conventional superconductors (alloys; see Refs [22, 23] and the literature cited therein). There was an opinion (see, e.g., Refs [24, 25]) that this is also the case with HTSCs. However, as was suggested in Ref. [13], the maximum of the function $\varkappa(T)$ might be due to the appearance in a superconductor of a new channel, i.e., an electron convective heat conductivity. The crucial point is, of course, the quantity $\varkappa_{\rm c}(T)$ which should be sufficiently large. As to the temperature run of $\varkappa_{\rm c}(T)$, we shall see below (which, however, is quite obvious) that $\varkappa_{\rm c} \propto |b_{\rm n}(T)|$ and the function $b_{\rm n}(T)$, upon an unconventional pairing, has a bell-like character with a maximum at $T \sim T_{\rm c}/2$. Hence, from this point of view the hypothesis of the convective nature of the maximum of $\varkappa_{\rm e}^{\rm tot}$ at $T \sim T_{\rm c}/2$ does not meet with objections.

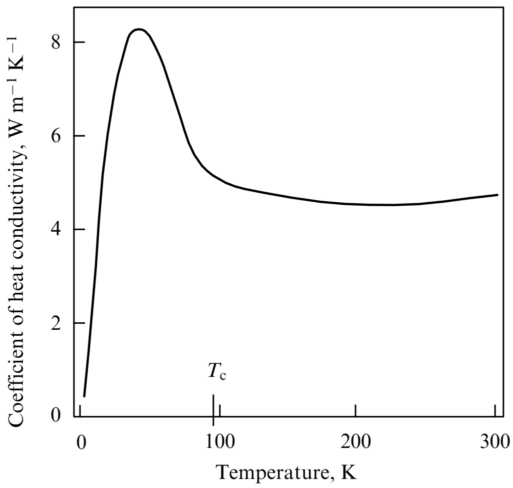

Figure 2.

4. Thus, we face two problems: firstly, to separate the phonon and the electron heat conductivities (i.e., to clarify the role of $\varkappa_{\rm ph}$ and $\varkappa_{\rm e}^{\rm tot}$) and, secondly, to distinguish between the usual electron heat conductivity $\varkappa_{\rm e}$ and the convective heat conductivity $\varkappa_{\rm c} = \varkappa_{\rm e}^{\rm tot} - \varkappa_{\rm e}$. The first problem has already been solved: the part of the thermal conductivity $\varkappa$ in an HTSC which is responsible for the maximum is mainly the electron thermal conductivity $\varkappa_{\rm e}^{\rm tot}$ [19–21, 26]. Especially convincing in this respect are the experiments [20] on the measurement of the Righi–Leduc effect (also referred to as the thermal Hall effect — the effect of the magnetic field upon thermal conductivity; see, for example, Section 27 of Ref. [9]). More precisely, the dominating role of the peaked electron thermal conductivity in an HTSC has been established only in some cases. For some HTSC materials, the characteristic peak of thermal conductivity may be associated with the phonon mechanism (see Ref. [27]), but this is apparently not typical and in any case is accessible to control. Below, the role of $\varkappa_{\rm ph}$ is assumed to be insignificant.

As for the second problem, i.e., the division of $\varkappa_{\rm e}^{\rm tot}$ into $\varkappa_{\rm e}$ and $\varkappa_{\rm c}$, it remains unsolved. In experiment, if the thermal conductivity of phonons $\varkappa_{\rm ph}$ is small or taken into account, then, as has already been mentioned, it is $\varkappa_{\rm e}^{\rm tot}$ that is measured, while the part $\varkappa_{\rm c} = \varkappa_{\rm e}^{\rm tot} - \varkappa_{\rm e}$ is, in a sense, a conditional quantity — such would be the electron thermal conductivity in the case that the current density $\mathbf{j}_{\rm n}$ were zero. Within the applicability limits of the Wiedemann–Franz law (see also below) the coefficient $\varkappa_{\rm e} \equiv \varkappa_{\rm ne}$ can be determined from the data on the electron conductivity $\sigma_n(T)$ in the superconducting state. Incidentally, such data [28] testify in some cases to a non-monotonic fall of σ_n with falling temperature (this is obviously due to the corresponding behavior of the free path $l_n(T) = v_F\tau_n(T)$). It is therefore quite possible that in such cases the indicated maximum of thermal conductivity is associated with $\varkappa_e$. The same will of course hold if we know that $\varkappa_e \gg \varkappa_c$.

5. Estimate (4) was obtained in Refs [10, 13] in an obviously unreliable way. The calculations in Refs [11, 12] seemed to be consistent, but the details were not presented. In my discussion of the problem with L P Pitaevskiĭ† it became clear that the final result (4) presented in Refs [11, 12] was due to some inexplicable error. We shall therefore present the estimate which we think is correct.

By analogy with superfluidity, the convective heat flow in an isotropic superconductor is

$$\mathbf{q}_c = -\varkappa_c \nabla T = T s_n \mathbf{v}_n \,, \tag{5}$$

where s_n is the electron entropy per unit volume (as is known, the contribution to s_n is made only by normal electrons) and $\mathbf{v}_n$ is the mean velocity of the normal component (i.e., of the same normal electrons). Next, $\mathbf{j}_n = b_n \nabla T = e n_n \mathbf{v}_n$, where n_n is the concentration of normal electrons. Hence,

$$\varkappa_c = \frac{T s_n}{e n_n} |b_n| \,. \tag{6}$$

At $T \approx T_c$, in the framework of the BCS model one can employ the expressions for a free electron gas with $T = T_c$, i.e., assume

$$s_n = \frac{1}{2}\pi^2 k_B \frac{k_B T_c}{E_F} n_n \,, \quad n_n = n = \frac{1}{2\pi^2}\left(\frac{2\pi E_F}{\hbar^2}\right)^{3/2}$$

and thus

$$\varkappa_c(T \sim T_c) \sim \frac{k_B T_c}{e}\left(\frac{k_B T_c}{E_F}\right)|b_n| \,. \tag{7}$$

According to the Wiedemann – Franz law (see Section 78 of Ref. [8]) we have

$$\sigma_n = \frac{3e^2}{\pi^2 k_B^2 T}\varkappa_e \,, \tag{8}$$

and the Seebeck coefficient (not to be confused with the entropy s_n !) is

$$S_n = \frac{|b_n|}{\sigma_n} = \frac{\pi^2 k_B^2 T}{3 e E_F} \,. \tag{9}$$

Incidentally, in such a simple model we have

$$\sigma_n = \frac{e^2 n_n \tau_n}{m} \,, \quad \varkappa_e = \frac{\pi^2 k_B^2 T n_n \tau_n}{3m} \,, \tag{9a}$$

where $\tau_n = l_n / v_F$ is the free path time, l_n is the free path length and v_F is the velocity on the Fermi surface.

Using expressions (8) and (9), we obtain from Eqn (7)

$$\frac{\varkappa_c}{\varkappa_e} \sim \left(\frac{k_B T_c}{E_F}\right)^2 . \tag{10}$$

Since the starting point in Refs [11, 12] was, in fact, expression (6), it follows that estimate (4) could be obtained instead of Eqn (10) only by mistake. In Refs [10, 13] the error was due to the assumption that an energy of the order of $\Delta \sim k_B T_c$ is released (or absorbed) upon breaking or formation of pairs with concentration n_n, while the actual pair concentration is of the order of $k_B T_c n_n / E_F$. The relative smallness of the convective heat conductivity in superconductors is connected in the end with the smallness of the electron specific heat in a metal. At the same time, for metals with a complex electron structure (in particular, for HTSCs) the ratio $\varkappa_c / \varkappa_e$ may greatly exceed estimate (10); see above and Refs [14 – 16, 53].

Of great interest is the study of thermal conductivity in the superconducting state in the presence of an external magnetic field — we mean the influence of the field on the tensor coefficient of thermal conductivity $\varkappa_{ik}(T, H)$. Measurements of the Righi – Leduc effect have already been mentioned [20]. In papers [2, 3], the influence of a transverse field $\mathbf{H}$ directed along the **c**-axis on the thermal conductivity of cuprates in the *ab*-plane was investigated. The result is as follows: in a strong field we have $\varkappa_e^{tot} \to 0$. If heat transfer is connected with the current $\mathbf{j}_n$, such a result is quite natural — clearly, a strong field $\mathbf{H}$ does not allow the current $\mathbf{j}_n \perp \mathbf{H}$ to flow, and therefore the heat flow $\mathbf{q}_c$ will be suppressed, too. But the field can, of course, also suppress the transverse heat flow $\mathbf{q}$ in the case of a conventional (i.e. non-convective) thermal conductivity. Quite obviously, the problem of thermal conductivity in the superconducting state cannot be properly analysed either with or without the field $\mathbf{H}$ until the quantities $\varkappa_{e,ik}(T, \mathbf{H})$ are calculated for s- and d-pairing. More precisely, in the absence of the field some results have already been obtained (see Refs [12, 26] and Section 98 of Ref. [8]) but with the terms $\varkappa_e$ and $\varkappa_c$ not separated‡. Since by experiment it is only the sum $\varkappa_e^{tot} = \varkappa_e + \varkappa_c$ that is measured, the clarification of the role of the current $\mathbf{j}_n$ (i.e., the role of the term $\varkappa_c$) may, of course, be thought of as uninteresting. But I repeat that in my opinion, however, the separation of the convective term $\varkappa_c$ has a physical meaning and provides an insight into the mechanism of thermal conductivity in the superconducting state, in particular, in the presence of a magnetic field [2, 3]. When dealing with the micropicture (i.e., the corresponding kinetic equation for normal excitations), the separation of the convective term does not seem to be a problem.

6. One cannot but be sorry that in spite of repeated calls [7, 29] for attention to the question of convective heat conductivity in the superconducting state, it has remained uninvestigated. It is the more strange as quite a lot of efforts (besides those referred to in paper [1] and above in the text, see also Refs [30 – 39]) have been made in studying thermal conductivity in superconductors. But still more unclear is the lack of attention to the study of thermoelectric effects in the superconducting state, while many papers are devoted to measure-

† I take the opportunity to thank L P Pitaevskiĭ for his help on this question.

‡ I suspect that in the corresponding calculations the contribution of the convective heat conductivity may have been ignored. However, as clarified in Section 98 of Ref. [8], the fact that $\mathbf{v}_s \neq 0$ only leads to higher-order corrections in $|\nabla T|$ in the calculation of the heat flow $\mathbf{q}$.

ments of thermoelectric power in the normal state of superconductors (in particular, HTSC).

Obviously, to analyse thermoelectric phenomena in the superconducting state, one should measure the coefficient $b_{\mathrm{n}}(T)$ in this state in the isotropic case and the coefficients $b_{\mathrm{n},\,ik}(T)$ in anisotropic superconductors. For this purpose, several effects can be investigated [6, 40], but I shall dwell on only two of them which I have considered myself [5–7, 29, 41]; see also the literature cited therein.

In a circuit consisting of two superconductors (Fig. 3), one should measure the magnetic field flux through the circuit:

$$\Phi = n\Phi_0 + \Phi_T\,, \quad \Phi_T = \frac{4\pi}{c}\int_{T_1}^{T_2}\left(b_{\mathrm{n,\,II}}\delta_{\mathrm{II}}^2 - b_{\mathrm{n,\,I}}\delta_{\mathrm{I}}^2\right)\mathrm{d}T\,, \quad (11)$$

where $\delta(T)$ is the magnetic field penetration depth, $\Phi_0 = hc/2e$ is the flux quantum, n is an integer; the derivation of this formula is given, for example, in Refs [6, 7]. It is desirable and, obviously, not difficult to choose as one of the superconductors in a circuit (for definiteness, superconductor I) such that for the one under consideration (superconductor II) the inequality $b_{\mathrm{n,\,II}}\delta_{\mathrm{II}}^2 \gg b_{\mathrm{n,\,I}}\delta_{\mathrm{I}}^2$ holds. Next, as distinct from 'closed' circuits, for 'open' ones (see item 7 below) an entrapped flux is typically absent (i.e., $n = 0$ in Eqn (11); the same always holds certainly for a bimetallic plate without a hole [5]). Under such conditions the quantity $\Phi_T = (4\pi/c)\int_{T_1}^{T_2} b_{\mathrm{n,\,II}}(T)\delta_{\mathrm{II}}^2(T)\,\mathrm{d}T$ is measured and the function $b_{\mathrm{n,\,II}}(T)$ can be found because the depth $\delta_{\mathrm{II}}(T)$ can be measured in quite an independent way. Formula (11) also holds for an anisotropic sample if by b_{n} and δ we understand the components $b_{\mathrm{n},\,ik}$ and δ_k that correspond to the geometry of the circuit (we mean the orientation of the crystal axes relative to the 'arms' of the circuit). The flux Φ_T for conventional superconductors was measured in a number of papers (in the last one [42] and in Refs [6, 40] earlier references are given).

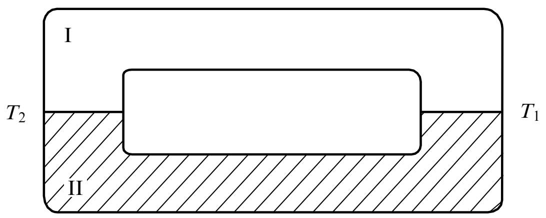

Figure 3.

In the determination of $b_{\mathrm{n},\,ik}(T)$ for an anisotropic superconductor, it seems most attractive to measure the magnetic field H_T perpendicular to the superconducting plate in which the temperature gradient ∇T makes an angle φ with the crystal axes, say, the z'-axis (Fig. 4; the role of the z'-axis can be played, for example, by the **a**-axis in HTSC cuprates). This field H_T is proportional to $|\nabla T|^2$ and to the corresponding quantities $b_{\mathrm{n},\,ik}(T)$ (see Refs [5–7, 41, 51])

$$H_T = \frac{2\pi}{c}\,\frac{\delta_0^2(\alpha_{z'}b_{z'} - \alpha_{x'}b_{x'})\sin 2\varphi}{T_{\mathrm{c}}(1 - T/T_{\mathrm{c}})^2}\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)^2, \quad (12)$$

where $\alpha_{x'}$, $\alpha_{z'}$, $b_{x'}$, and $b_{z'}$ are the principal values of the tensors α_{ik} and $b_{\mathrm{n},\,ik}$ corresponding to the crystal symmetry axes x' and z'.

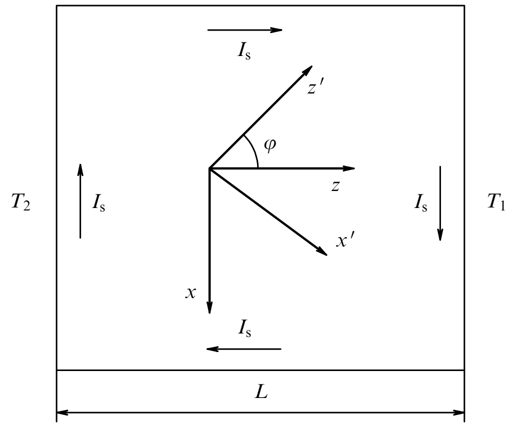

Figure 4.

The quantities $2m\alpha_k$, where $k = x', y', z'$, are involved in the familiar expression for the density of a superconducting current

$$j_{\mathrm{s},\,k} = \frac{2ie}{2m\alpha_k}\left(\Psi^*\frac{\partial\Psi}{\partial x_k} - \Psi\frac{\partial\Psi^*}{\partial x_k}\right) - \frac{2e^2}{m\alpha_k}\,A_k|\Psi|^2\,,$$

where e and m are the charge and the mass of an electron, A_k is the vector potential and the function $\Psi = \sqrt{n_{\mathrm{s}}/2}\,e^{\mathrm{i}\varphi}$ is so normalised that $n_{\mathrm{s}}/2$ is the concentration of superconducting pairs; one can also write $\Lambda_k = \Lambda\alpha_k$, $\Lambda = m/e^2n_{\mathrm{s}} = 4\pi\delta^2/c^2$, and in Eqn (12) we assumed $\delta^2 = \delta_0^2(1 - T/T_{\mathrm{c}})^{-1}$ because temperatures T close to T_{c} are being considered. As noticed in paper [43], the field H_T in Eqn (12) generally depends strongly on the coordinates because the temperature T is dependent on them. Therefore, when calculating the flux Φ_T through a plate under certain conditions one should take into account the dependence of H_T on z, and as a result the flux Φ_T is proportional to $\mathrm{d}T/\mathrm{d}z$ rather than $(\mathrm{d}T/\mathrm{d}z)^2$. But this is a minor detail.

It is amazing that there has been only a single attempt [44] to measure the field H_T in a superconducting crystal, and this attempt yielded vague results. Seemingly, now that strongly anisotropic HTSC crystals are available, measurements of the thermal field H_T must attract attention. Incidentally, it may be more convenient to measure not the field H_T, but the superconducting current flowing around the crystal [41].

7. In connection with the problem of thermal conductivity [see Eqns (6), (7)] the quantities $b_{\mathrm{n}}(T)$ and $b_{\mathrm{n},\,ik}(T)$ appreciably below $b_{\mathrm{n},\,ik}(T)$ should be measured first of all. But for the analysis of thermoelectric effects as a whole, the temperature region in the vicinity of T_{c} also seems to be of great interest. In this region, first of all for HTSCs (see the end of paper [7] and Ref. [37]) the fluctuation effects must be substantial. Secondly, since the superconducting transition is a second-order transition, it must be continuous. This is guaranteed by the fact that as $T \to T_{\mathrm{c}}$, the field penetration depth is $\delta_k \to \infty$. In the case of circuits and plates (films) the characteristic parameter is the ratio $(\delta/d)^2$, where d is the sample size (the wire diameter or the plate thickness). So, the total superconducting current in a circuit (see Fig. 3) is $I_{\mathrm{s}} \sim (\delta/d)^2 I_{\mathrm{n}}$, where I_{n} is the current in this circuit in the normal state [6]. Another aspect of the problem of the transition $T \to T_{\mathrm{c}}$ is the applicability limit of expression (2). In a non-closed sample (Fig. 1) at $T > T_{\mathrm{c}}$ we obviously have $j = j_{\mathrm{n}} = 0$, and according to Eqn (1) the equality $\mathbf{E} - \nabla\mu/e = -b\nabla T/\sigma$ holds. At $T < T_c$, far enough from the ends of a sample, relations (2) and (3) hold. But near the ends a certain charge of density Q^* (charge imbalance) is accumulated. The quantity Q^* depends on the distance from the boundaries (ends) of the sample and $\mathbf{j}_n$ depends on ∇Q^* (see Refs [40, 45–48]). This guarantees a continuous transition from the superconducting to the normal state. Measurements of the charge Q^* and the related quantities can be used [48] to determine the coefficient $b_n(T)$. Next, as $\delta(T)$ increases, the flux Φ becomes comparable with or greater than the flux quantum Φ_0 (under the conditions of [42] we still find $\Phi_T \ll \Phi_0$). As soon as the temperature approaches T_c the entrapped flux $n\Phi_0$ will possibly and, strictly speaking, inevitably increase [see Eqn (11)]. According to [49] (in which, as in Ref. [1], earlier works are cited) this is the fact accounting for the giant thermoeffect observed in Ref. [40] in circuits with a closed geometry (a cylinder, a toroid)†. Measurements of the total flux $\Phi(T)$ when $\Phi \gg \Phi_0$ have been carried out neither for the thermoeffect nor in an external magnetic field for different geometries of superconductors. Meanwhile, they seem to be very interesting and useful for the understanding of the thermodynamics and kinetics of some processes in superconductors [49, 50].

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† In the recent paper [52], as in Refs [1, 49], it was justly stressed that the giant thermoeffect in superconductors deserves great attention. The giant thermoeffect is associated in Ref. [52] with the expected strong increase of the coefficient $b_n(T)$. In paper [52] it is the coefficient $\alpha_s(T)$. Since in Ref. [52] ordinary superconductors are considered (Sn is meant), such an assumption seems to contradict the experimental facts obtained at a certain distance from T_c (see, for instance, Ref. [42]). The theoretical arguments involved in Ref. [52] are not clear to me. The possible role of convective heat transfer in HTSCs is discussed in paper [53].