

### 3. Conclusion

In conclusion we note, as one can see from Fig. 3, that the correction  $\delta R_1$  to the resistance of the normal channel caused by the proximity effect depends on the temperature  $T$  in a non-monotonic way: it is zero at  $T = 0$  (the bias voltage is zero as well), reaches a maximum at  $T \approx \varepsilon_{L_1}$  and decays to zero at higher  $T$ . Such behavior of  $\delta R_1(T)$  is related, as noted in [15], to different dependencies of two contributions to  $\delta R_1$  on the energy  $\varepsilon$ . One contribution which increases the N channel resistance is connected with a decrease of the density-of-states in the normal channel. It is described by the last term in  $M(\varepsilon)$  [see Eqn (16)]. Another contribution (anomalous) which diminishes the resistance of the normal channel is described by the first two terms in  $M(\varepsilon)$ . This contribution exactly compensates a contribution due to a change in the density-of-states of the normal channel at  $\varepsilon = 0$  and dominates at  $\varepsilon \neq 0$ . At  $T > T_c$  it leads to the Maki-Thompson contribution to the paraconductivity. Mathematically, compensation of the two contributions at  $\varepsilon = 0$  arises because at  $\varepsilon = 0$   $F^R = F^A$  and  $m_-$  in Eqn (16) becomes zero. The monotonic behavior of  $\delta R$  has been observed in an experiment [4]. It would be interesting to observe the long-range Josephson effect experimentally.

This work was supported by the Russian Fund for Fundamental Research (Project 96-02-16663a), by the Russian Grant on high  $T_c$  Superconductivity (Project 96053), and by the CRDF Project (No. RP1-165). This support is gratefully acknowledged.

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## Sign memory of the Ruderman – Kittel interaction in disordered metals and magnetic coupling in mesoscopic metal/ferromagnet layered systems

B Spivak, A Zyuzin

In the case when two paramagnetic spins are embedded in a pure non-magnetic metal at zero temperature  $T = 0$  the Ruderman – Kittel exchange interaction energy between them has the well known form [1]

$$I_{ij}(\mathbf{R}_1, \mathbf{R}_2) = I_0 \frac{\cos(2p_F|\mathbf{R}_1 - \mathbf{R}_2|)}{(p_F|\mathbf{R}_1 - \mathbf{R}_2|)^d} \delta_{ij}. \quad (1)$$

Here  $I_0$  is a factor which is proportional to the square of the exchange interaction between the localized spins and spins of conduction electrons in the metal,  $p_F$  the Fermi momentum in the metal,  $d$  is the dimensionality of the space,  $i, j$  are spin indexes and  $\mathbf{R}_1, \mathbf{R}_2$  are coordinates of the localized spins. In the case of disordered metals  $|\mathbf{R}_1 - \mathbf{R}_2| \gg l \gg p_F^{-1}$  the amplitude of the average Ruderman – Kittel exchange energy

$$\langle I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \rangle \sim \exp\left(-\frac{|\mathbf{R}_1 - \mathbf{R}_2|}{l}\right), \quad (2)$$

is exponentially small [2]. Here brackets  $\langle \rangle$  correspond to averaging over realizations of a random scattering potential (or averaging between samples) and  $l$  is the electron elastic mean free path in the metal. On the other hand, it was shown in [3–5] that the exponential decay of the average  $\langle I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \rangle$  is the consequence of randomization of the sign of  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  and that the typical amplitude of the interaction

$$\sqrt{\langle (I_{ij}(\mathbf{R}_1, \mathbf{R}_2))^2 \rangle} \sim |\mathbf{R}_1 - \mathbf{R}_2|^{-d} \quad (3)$$

decreases with distance in the same way as in the pure case. The interpretation of Eqs (2) and (3) given in [3] was that in a given sample

$$I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \sim \frac{\cos(2p_F|\mathbf{R}_1 - \mathbf{R}_2| + \delta(\mathbf{R}_1, \mathbf{R}_2))}{(p_F(|\mathbf{R}_1 - \mathbf{R}_2|)^{-d}}, \quad (4)$$

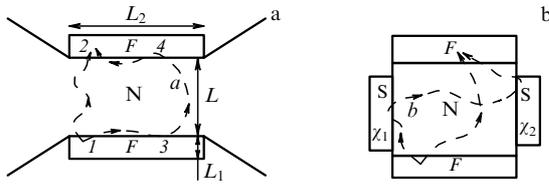
where  $\delta(\mathbf{R}_1, \mathbf{R}_2)$  has a random sign when  $|\mathbf{R}_1 - \mathbf{R}_2| \gg l$ , which means that the sign of  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  cannot be predicted.

In fact what follows from Eqs (2) and (3) is that  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  has random signs between samples. We would like to point out here that in a given sample the sign of  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  can be predicted in a sense that there are long range correlations between the signs of  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  and  $I_{k,l}(\mathbf{R}_3, \mathbf{R}_4)$  which survive even over very large distances  $|\mathbf{R}_1 - \mathbf{R}_3| \sim |\mathbf{R}_2 - \mathbf{R}_4| \sim |\mathbf{R}_2 - \mathbf{R}_3| \sim |\mathbf{R}_1 - \mathbf{R}_4| \sim R \gg l$  and for arbitrary locations of  $\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \mathbf{R}_4$ . To prove this statement we can calculate the correlation function  $(\delta I_{ij} = I_{ij} - \langle I_{ij} \rangle)$  at  $R \gg l$

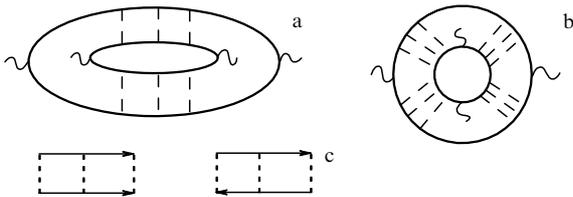
$$\langle \delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \delta I_{ij}(\mathbf{R}_3, \mathbf{R}_4) \rangle \sim R^{-4(d-1)} \quad (5)$$

with the help of the diagrams shown in Fig. 2b. We use the standard diagram technique [14] for averaging over realizations of a random potential. The solid lines in Fig. 2 correspond to electron Green functions in the Matsubara representation, dashed lines correspond to the scattering on the random potential and vertices correspond to the contact magnetic interaction. Eqn (5) shows that the above mentioned sign correlations exhibit a very slow power-law decay with the distance  $R$ . We would like to stress here that this effect exists only in disordered metal when  $R \gg l$  due to the remarkable fact that it is insensitive to the change of coordinates  $\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \mathbf{R}_4$  by a distance of order or larger than  $1/p_F$ . (In pure metal the sign correlations decay exponentially when  $R \gg 1/p_F$  due to the oscillatory nature of the Ruderman – Kittel interaction). The diagrams shown in Fig. 2b were calculated in [4, 18], though the question of the sign correlations was not discussed.

The qualitative explanation of the origin of the correlation is as follows. The mesoscopic fluctuations of the exchange energy  $\delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  result from the interference of random probability amplitudes of diffusion paths between the points  $\mathbf{R}_1$  and  $\mathbf{R}_2$ . Among these paths there are some which visit points  $\mathbf{R}_3$  and  $\mathbf{R}_4$  (An example is the line ‘a’ in Fig. 1a). On the other hand, among paths which determine the amplitude of the probability of traveling between points  $\mathbf{R}_3$  and  $\mathbf{R}_4$  there are those which again travel along the dashed lines in Fig. 1a and visit points  $\mathbf{R}_1$  and  $\mathbf{R}_2$ . This leads to the above mentioned correlation between  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  and  $I_{kl}(\mathbf{R}_3, \mathbf{R}_4)$ .



**Figure 1.** Schematic pictures of the ferromagnet-nonmagnet layered systems.



**Figure 2.** Diagrams for the correlation function  $\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle$ .

Let us now consider a system of two ferromagnetic (F) layers of sizes  $L_1, L_2, L_3$  divided by a disordered normal metal (N) layer with thickness  $L$  and  $p_F l \gg 1$  (see Fig. 1). The sign memory discussed above or the Ruderman – Kittel interaction can determine the exchange interaction energy between F layers through the metal in the case when the N layer’s thickness  $L \gg l$  is larger than the mean free path. The origin of the exchange interaction between the ferromagnetic layers is the Ruderman – Kittel type interaction: The interaction of itinerant nonmagnetic metal electrons with localized ‘f’ or ‘d’ electrons in the ferromagnets induces a spin polarization in the nonmagnetic metallic layers. This magnetization, in turn, creates the effective interaction between two localized spins in

different ferromagnetic layers with the energy

$$E(\mathbf{R}_1, \mathbf{R}_2) = I_{ij}(\mathbf{R}_1, \mathbf{R}_2) S_i^1 S_j^2, \quad (6)$$

here  $S_i^{1,2}$  are components of localized spins in the F layers and  $\mathbf{R}_{1,2}$  are their coordinates. We employ the simplest model where conduction ‘s’-electrons interact with localized ‘f’ or ‘d’ electrons in the F layers via a contact interaction with energy  $A \sum_k \delta(\mathbf{r}_k - \mathbf{R}) \mathbf{s}_k \mathbf{S}$ . Here  $\mathbf{r}_k$  and  $\mathbf{s}_k$  are coordinates and spins of conduction electrons in the metal which are labeled by the index  $k$  and  $A$  is the interaction constant. As a result we have  $I_0 = (9\pi/64)[(An)^2/E_F]$  which is of order of the ferromagnet’s critical temperature. Here  $E_F$  is the Fermi energy and  $n = p_F^3/3\pi^2$  is the concentration of electrons in the metal. Following [10–13], we will use the approximation where the total exchange interaction energy  $\bar{E}$  between the magnetic moments in the F layers is a sum of  $E(\mathbf{R}_1, \mathbf{R}_2)$  over coordinates  $\mathbf{R}_1, \mathbf{R}_2$  of the localized spins in the ferromagnetic layers.

$$\bar{E} = \sum_{\mathbf{R}_1, \mathbf{R}_2} E(\mathbf{R}_1, \mathbf{R}_2) = \sum_{\mathbf{R}_1, \mathbf{R}_2} I_{ij}(\mathbf{R}_1, \mathbf{R}_2) S_i^{(1)} S_j^{(2)} = \bar{I}_{ij} n_{1i} n_{2j}, \quad (7)$$

here  $n_{1i}$  and  $n_{2j}$  are components of unit vectors  $\mathbf{n}_1, \mathbf{n}_2$  parallel to magnetizations in the first and the second layers respectively. We will consider the case where the length  $L_2$  of the F-layers is relatively short and one can neglect fluctuations of the orientation of magnetizations along the F layers.

Both experimental and theoretical studies of this phenomenon have until now been restricted to the consideration of the infinite dimensions of both F and N layers and clean N layers  $L \gg l$ , when the value of  $\bar{I}_{ij}$  and the relative orientation of magnetizations of F layers are oscillating functions of  $L$  [7–13]. In this article we discuss the opposite case of small sample sizes and disordered N layers where the mesoscopic effects determine the exchange interaction between F layers. To find the relative angle  $\theta(\mathbf{n}_1, \mathbf{n}_2)$  between the magnetization angles in F layers one has to calculate the sign and the amplitude of the quantity  $\bar{I}_{ij}$ . In the case  $L \gg l$ ,  $\bar{I}_{ij}$  is a random sample-specific quantity, which can be characterized by its average and moments. It follows from Eqn (2) that in the case  $L \gg l$  the average exchange energy  $\langle \bar{E} \rangle$  is exponentially small and can be neglected. Therefore, in the case  $L \gg l$  the exchange energy between the ferromagnetic layers has a random sign while its characteristic value is determined by its variance  $[(\bar{I}_{ij})^2]^{1/2}$  which in turn is determined by the long range correlations between  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  and  $I_{kl}(\mathbf{R}_3, \mathbf{R}_4)$  discussed above. As a result, for example, in the case of square or cubic geometries for the N layer and at low temperatures the variance of the exchange energy has a random sign while its characteristic value is sample size independent. The diagrams which contribute to  $\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle$  in the lowest order in the parameter  $\hbar/p_F l \ll 1$  are shown in Fig. 2. The diagrams shown in Fig. 2a were considered in [13–16]. They give the main contribution to the correlation function  $\langle \delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \delta I_{kl}(\mathbf{R}_1, \mathbf{R}_2) \rangle \sim |\mathbf{R}_1 - \mathbf{R}_2|^{-d}$ . However, the contribution of these diagrams to the correlation function  $\langle \delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \delta I_{kl}(\mathbf{R}_3, \mathbf{R}_4) \rangle$  decays exponentially when  $|\mathbf{R}_1 - \mathbf{R}_3|, |\mathbf{R}_2 - \mathbf{R}_4| \gg l$ . As a result, for example, in the case  $L \sim L_2 \gg L_3 \sim L_1$  the contribution to  $\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle$  from these diagrams is of order  $I_0^2 (np_F^{-3})^4 (L_1/L)^2$ . (We assume the density of localized spins in the ferromagnets is of order  $n$ .) Though the diagrams in Fig. 2b give a much smaller contribution to  $\langle \delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \delta I_{kl}(\mathbf{R}_1, \mathbf{R}_2) \rangle$ , they describe the

long range correlations  $\langle \delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \delta I_{ij}(\mathbf{R}_3, \mathbf{R}_4) \rangle \sim R^{-4(d-1)}$  when  $|\mathbf{R}_1 - \mathbf{R}_3| \sim |\mathbf{R}_2 - \mathbf{R}_4| = R \gg l$ . As a result they give the main contribution to the correlation function of the interlayer exchange energy  $\langle \bar{I}_{ij} \bar{I}_{kl} \rangle$  at  $L \gg l$ . As a result, we have:

$$\begin{aligned} \langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle &= \frac{2}{\pi} I_0^2 E_F^2 T \sum_m \omega \int d\mathbf{R}_1 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \\ &\times [\hat{\sigma}_i \hat{P}_\omega^c(\mathbf{R}_1, \mathbf{R}_2) \hat{\sigma}_k \hat{P}_\omega^c(\mathbf{R}_2, \mathbf{R}_3) \hat{\sigma}_j \hat{P}_\omega^c(\mathbf{R}_3, \mathbf{R}_4) \hat{\sigma}_l \hat{P}_\omega^c(\mathbf{R}_4, \mathbf{R}_1) \\ &+ \hat{\sigma}_i \hat{P}_\omega^d(\mathbf{R}_1, \mathbf{R}_2) \hat{\sigma}_j \hat{P}_\omega^d(\mathbf{R}_2, \mathbf{R}_3) \hat{\sigma}_k \hat{P}_\omega^d(\mathbf{R}_3, \mathbf{R}_4) \hat{\sigma}_l \hat{P}_\omega^d(\mathbf{R}_4, \mathbf{R}_1)]. \end{aligned} \quad (8)$$

Here  $\omega = \pi(2m+1)T$  is the Matsubara frequency,  $m$  is an integer,  $T$  is the temperature and  $\hat{\sigma}_i$  are spin operators. Integration over  $\mathbf{R}_1, \mathbf{R}_3$  and  $\mathbf{R}_2, \mathbf{R}_4$  in Eqn (8) is performed over volumes of the first and the second ferromagnetic layers respectively. The results of calculation of Eqn (8) depend on the ratio between the lengths  $L, L_2, L_T = \sqrt{D/T}$ ,  $L_{so} = \sqrt{D\tau_{so}}$  and on the boundary conditions for Cooperons and Diffusons, which are shown in Fig. 2c. Here  $L_{so}, \tau_{so}$  are the spin-orbit relaxation length and time, respectively, and  $D$  is the electron diffusion coefficient in the N layer. In the case of the ‘open’ geometry of the N layer shown in Fig. 1a and  $L_T, L_{so} \gg L > l; L, L_2 \gg L_1, L_3$ , we have

$$\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle = \frac{5 \times 2^{7/2} \zeta(5/2)}{3^2 \pi^{9/2}} X \frac{I_0^2}{(p_F l)^2} (p_F L_1)^4 \delta_{ij} \delta_{kl}. \quad (9)$$

Here  $X$  is a factor, which is of order unity when  $L \sim L_2 \leq L_T$  and  $\zeta(x)$  is the zeta-function. In different limiting cases we have:

$$X = \begin{cases} \left(\frac{L_2}{L}\right)^4, & L_T > L_2 > L, \\ \frac{L_2 L_T^3}{L^4}, & L_2 > L_T > L. \end{cases} \quad (10)$$

It is interesting that in the case  $L \sim L_2 < L_T$ , Eqs (9), (10) turn out to be independent of  $L$ . In the case  $L > L_T$  the expression for  $X$  acquires an additional exponentially small factor  $\exp(-L/L_T)$ . In the case  $L_{so} > L$  the minimum of the exchange energy corresponds to a parallel or antiparallel orientation of the layer’s magnetizations ( $\theta$  equals zero or  $\pi$ ). In the opposite limit  $L_{so} \ll L$  we get the same formula as Eqn (9) but without the factor  $\delta_{ij} \delta_{kl}$ . This means that the exchange interaction between the F layers is of the Dzialoshinski-Moria type and a minimum of the exchange energy corresponds to a sample specific angle  $\theta(\mathbf{n}_1, \mathbf{n}_2)$  distributed randomly over the interval  $(0, \pi)$ . While deriving the results presented above we neglected the sensitivity of the boundary conditions for Cooperons and Diffusons shown in Fig. 2c to the change of magnetization directions in F-layers. In the case of the open sample geometry Fig. 1a this is correct, provided  $A p_F^2 L_1 / v_F \ll 1$ . To get an estimate for  $\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle$  in the opposite limit one has to substitute  $E_F$  for the factor  $A(L_1 p_F)$  in Eqn (5). For example, in the case  $L_T > L \sim L_2 > L_{so}$  we have

$$\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle \sim E_F^2 (p_F l)^{-2} \sim \frac{\hbar}{\tau}. \quad (11)$$

Here  $\tau$  is the elastic mean free path in the metal. We would like to stress again that the origin of Eqs (9)–(11) is the long range correlation of the signs of  $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$  and  $I_{kl}(\mathbf{R}_3, \mathbf{R}_4)$  which survive over distances much larger than  $l$ .

As is usual in the physics of mesoscopic metals [15, 16], the external magnetic field changes the electron interference

pattern and thereby  $\delta \bar{I}_{ij}$ , and  $\theta(\mathbf{n}_1, \mathbf{n}_2)$  turns out to be a random sample-specific oscillating function of the magnetic field  $H$ . Another way to change the relative orientations of the F-layers is demonstrated in Fig. 1b. Namely,  $\theta(\mathbf{n}_1, \mathbf{n}_2)$  is a random sample specific function of the order parameter phase difference  $(\chi_1 - \chi_2)$  in superconductors  $S_1$  and  $S_2$  shown in Fig. 1b. The reason for this is that some diffusive paths connecting points 1 and 2 in Fig. 1b can visit superconductors (line ‘b’ in Fig. 1b) and the corresponding amplitude of the probability of traveling along these paths acquires the additional phase  $(\chi_1 - \chi_2)$  [17]. Another consequence of the phase dependence of the exchange energy is that the critical Josephson current of the device shown in Fig. 1b depends on the angle  $\theta$  between the magnetizations of the F layers.

The authors would like to acknowledge useful discussions with J Bass and Y Shender. This work was supported by the Division of Material Sciences, U.S. National Science Foundation under Contract No. DMR-9625370 and the US-Israeli Binational Science Foundation grant No. 94-00243.

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## Coulomb effects in a ballistic one-channel S-S-S device

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### 1. Introduction

Coulomb effects in several different types of three-terminal devices consisting of an island connected to external leads by two weak-link contacts, and capacitatively coupled to an additional gate potential, have been extensively studied during recent years [1–3]. In the present paper we develop a theory for a system consisting of two almost ballistic one-