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5. Mesoscopic superconductivity and Coulomb blockade

In the fifth session of the conference the following presentations were made:

(1) **Petrashov** V (London) "Non-equilibrium normal-superconducting hybrid nanostructures";

(2) **Volkov A F** (Institute of Radio-engineering and Electronics RAS, Moscow, Russia), **Pavlovskiĭ V V** (Institute of Physics and Technology RAS, Moscow, Russia) "Phase-coherent phenomena in S-N-S structures";

(3) **Poirier W** (Saclay, France), **Sanquer M** (Grenoble, France) "Sub-gap conductance anomalies in GaAs-Superconductor junctions";

(4) **Andreev A F** (P L Kapitza Institute for Physical Problem RAS, Moscow, Russia) "Bose-condensation and superconductivity in mesoscopic systems: spontaneous violation of space-time symmetries";

(5) **Imry J** (Department of Condensed Matter Physics, Weizmann Institute of Science, Israel) "Inelastic scattering and dephasing in quantum-dot systems";

(6) **Spivak B** (Physics Department, University of Washington, USA; Max-Plank-Institut für Festkorperforschung Hochfeld-Magnetolabor, Grenoble, France), **Zyuzin A** (A F Ioffe Physical-Technical Institute, St. Petersburg, Russia) "Sign memory of the Ruderman – Kittel interaction in disordered metals and magnetic coupling in mesoscopic metal/ferromagnet layered systems";

(7). **Geim A** (Research Institute for Materials, University of Nijmegen, Netherlands) "Phase transitions in mesoscopic superconductors";

(8). **Averin D** (Department of Physics, Stony Brook) "Multiple Andreev reflection and electron transport in disordered S-N-S junctions";

(9). **Martin-Rodero A** (Autonomous University of Madrid, Spain) "Superconducting transport in nano-structures with several conducting channels";

(10) **Ivanov D** (L D Landau Institute for Theoretical Physics, Moscow, Russia; MIT Cambridge, USA), **Feigel'man M V** (L D Landau Institute for Theoretical Physics, Russia) "Coulomb blockade in a ballistic one-channel S-S-S device";

(11) **Matveev K** (Duke University, USA), **Larkin A I** (University of Minnesota, USA and L D Landau Institute for Theoretical Physics, Russia) "Superconductivity in ultrasmall grains";

(12) **Müller P** (Physikalisches Institut III, Universität Erlangen-Nürnberg, Germany) "Atomic scale electronics: from high T_c superconductors to self assembled supramolecules";

(13) **Soldatov E S, Khanin V V, Trifonov A S, Presnov D E, Iakovenko S A, Khomutov G V** (Department of Physics, Moscow State University, Moscow, Russia), **Gubin S P** (Institute of General and Inorganic Chemistry, Moscow Russia), **Kolesov V V** (Institute of Radio-engineering and Electronics, Moscow, Russia), **Korotkov A N** (Nuclear Physics Institute, Moscow State University, Moscow, Russia) "Room temperature molecular single-electron transistor";

(14) **Krupenin V A, Lotkhov S V** (Laboratory of Cryoelectronics, Moscow State University, Moscow, Russia), **Scherer H, Weimann Th, Zorin A B, Ahlers F -J, Niemeyer J, Wolf H** (Physikalisch-Technische Bundesanstalt, Braunschweig, Germany) "Sensing of dynamic charge states using single-electron tunneling transistors";

(15) **Kuz'min L** (Moscow State University, Moscow, Russia and Chalmers University, Sweden) "Bloch oscillations in ultrasmall Josephson junctions with a high-ohmic environment";

(16) **Shytov A V, Lee P A, Levitov L S** (Massachusetts Institute of Technology, Cambridge, USA) "Localization of quasiparticles in an NS-structure";

Papers 2, 6, 10, 13, 14, 16 are published below. The contents of a number of other presentations may be found in: № 4 — *Pis'ma Zh. Eksp. Teor. Fiz.* **63** 618 (1996) [*JETP Lett.* **63** 1018 (1996)]; **64** 618 (1996) [*JETP Lett.* **64** 664 (1996)], № 8 — cond-mat/9706087, № 9 — *Phys. Rev. Lett.* **80** 1066 (1998) (cond-mat/9711263), № 11 — cond-mat/9701041

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Phase-coherent phenomena in S-N-S structures

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1. Introduction

Progress in nanotechnology in the last few years has made it possible to produce conducting nanostructures in which new physical phenomena have been observed. Specifically, hybrid structures consisting of superconductors (S) and normal conductors (N) have been created. Metal films [1 – 5] or semiconductor layers [5 – 7] have been used as the normal conductors. The transport properties of these S/N structures have turned out to be quite unusual. Firstly, conductivity oscillations have been observed in these mesoscopic structures in a magnetic field H (i.e., in structures with dimensions less then the phase-breaking length L_φ). Oscillations of the conductivity of the N channels appeared if the structure contained superconducting or normal loops [1 – 4, 6]. Moreover, for an N channel in contact with superconductors a non-

monotonic dependence of the conductivity on the temperature T and voltage V has been observed at $T \ll T_c$ [4]. The main experimental facts have been explained in recent theoretical works. It was determined that the proximity effect plays the main role in the transport properties. For example, the conductivity of an N channel in the structure shown in Fig. 1 changes as a result of the contribution of the condensate induced by the proximity effect. Since the condensate is induced by both superconductors in a nonlocal manner, interference appears and a term $-\delta R \cos\varphi$, which depends on the phase difference φ between the superconductors, appears in the resistance of the N channel [8–10]. The phase difference increases with the magnetic field H, and this results in oscillations of the conductivity of the N channel in a magnetic fields. The non-monotonic dependence of the resistance R of an N channel on T and V has also been explained [11, 12] (see also the theoretical works in the conference proceedings in Ref. [7]). The non-monotonic dependence of the resistance $R(T, V)$ of a point contact ScN (c is a constriction) was first obtained theoretically in Ref. [13].

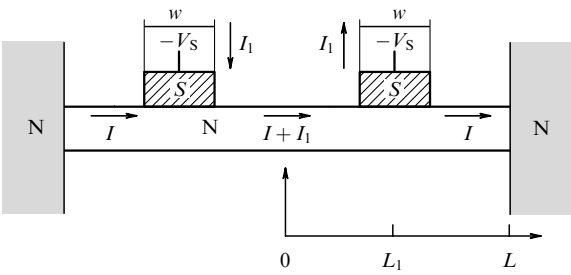

Figure 1. Schematic diagram of the system considered.

New effects have also been predicted in the theoretical works devoted to S/N structures. For example, in Refs [14, 15] it was shown that the critical Josephson current I_c in a structure of the type displayed in Fig. 1 depends on the voltage V_S between the S and N conductors, changing sign (π-contact) if V_S exceeds a certain value. In addition, it has been shown that the Josephson effect also arises in the case when current flows only through one S/N boundary. Several different configurations of S/N structures were studied in Ref. [16]. It was determined that under certain conditions the current-voltage characteristics of the S/N structures have descending segments ($\mathrm{d}I/\mathrm{d}V < 0$).

An important circumstance was noted in Ref. [17] (see also the works in Ref. [7]). It was shown that the local conductivity of an N channel changes over distances from the S/N boundary which can be much greater than the coherence length $\xi_N = \sqrt{D/2\pi T}$ (D is the diffusion coefficient). Important consequences follow from this fact. For example, phase coherence effects in the conductivity of an N channel remain even if the distance $2L_1$ between the superconductors is much greater than ξ_N. This means that the conductivity oscillations in the structure shown in Fig. 1 will also be observed in the case of a negligibly low critical current I_c. The oscillation conservation effect is due to fact that as T increases, I_c decreases exponentially ($I_c \sim \exp(-2L_1/\xi_N(T))$, and δR decreases slowly ($\delta R \sim T^{-1}$) [18].

In this work we shall consider the possibility of observing long-range, phase-coherent effects in the S/N/S structure shown in Fig. 1. We shall show in particular that Josephson-like effects may arise in this structure even when the condition

$$2L_1 \gg \xi_N(T) = \sqrt{\frac{D}{2\pi T}}, \tag{1}$$

is fulfilled, i.e., if the Josephson critical current I_c is negligibly small. However, these effects arise only in the case when a current I flows along the N channel and dissipation takes place [23].

2. Basic equations, long-range Josephson effects

As in Refs [8–18], we shall study the diffusion regime of charge transport ($l \ll \xi_N$, l is the mean free path) and we shall employ the equation for the supermatrix $\breve{G}$ whose elements are the matrix Green's functions — the retarded (advanced) functions $\hat{G}^{R(A)}$ — and the Keldysh function $\hat{G}$ [19]. These equations are supplemented by the matching conditions at the S/N boundary [20, 21].

The equation for the supermatrix $\breve{G}$, averaged over the thickness of the N film, has the form [10, 18]

$$D\partial_x(\breve{G}\partial_x\breve{G}) + \mathrm{i}\varepsilon[\breve{\sigma}_z, \breve{G}] = \varepsilon_b w \delta(x \pm L_1)[\breve{G}_S, \breve{G}]\,. \tag{2}$$

The right-hand side in Eqn (2) describes the influence of the superconductors S, where all functions $\breve{G}_S$ are assumed to be equilibrium functions, on the N channel. The coefficient ε_b is a characteristic energy which is proportional to the transmission of the S/N boundary: $\varepsilon_b = \rho D/2R_{b\,\square}d_N$, $R_{b\,\square}$ is the resistance of a unit area of the S/N boundary; ρ and d_N are the resistivity and thickness of the N film. We have neglected the interaction with phonons and depairing, making the assumption that the system is mesoscopic: $2L < \min(\sqrt{D\tau_\varepsilon}, \sqrt{D/\gamma})$, τ_ε is the energy relaxation time, and γ is the depairing rate. For simplicity, we assume that the width w of the S/N boundary is small compared to ξ_N. The elements of the supermatrix $\breve{G}$ are matrices of retarded (advanced) Green's functions $\hat{G}^{R(A)}$ and the matrix $\hat{G}$, which is related to the distribution functions f and f_0 [19].

When deriving Eqn (2), the boundary condition

$$D(\breve{G}\partial_x\breve{G}) = (\varepsilon_b d_N)[\breve{G}, \breve{G}_S]\,. \tag{3}$$

was used. Here the z-axis is normal to the plane of the S/N interface. The boundary conditions for the quasiclassical Green's functions $\breve{G}$ was derived in the general case by Zaĭtsev [20] and reduced to the simple form (3) by Kupriyanov and Lukichev [21] in the dirty case. In the case of a good S/N contact, condition (3) is reduced to the continuity of the Green's functions at the S/N interface: $\breve{G} = \breve{G}_S$. In the case of a poor contact ($\varepsilon_b \to 0$), condition (3) gives the same result for the current through the S/N interface which can be obtained using the tunneling Hamiltonian method [20]. However, for an S/N contact with an arbitrary barrier transparency condition (3) is not applicable. The point is that when deriving Eqn (3) Kupriyanov and Lukichev [21] restricted themselves to the Legendre polynomials of the zeroth and first orders in the expansion of the angle-dependent Green's function $\breve{G}$. Meanwhile, one can easily show that all the Legendre harmonics are excited near the S/N (or N/N′) interface. They decay to zero (except the Legendre polynomials of the zeroth and first order) over the mean free path away from the interface. In order to obtain a correction of the next order in ε_b to condition (3), one has to solve an integral

equation [22]. In the case of the S/N interface with an arbitrary barrier transparency, the problem of boundary conditions for the quasiclassical Green's functions becomes complicated.

In what follows we shall consider the case of a strong barrier and restrict ourselves to the lowest term in the expansion in ε_b. Then using Eqn (3) as the boundary condition, we come to Eqn (2).

The current in the system is expressed in terms of the functions f, which we must determine from Eqn (2) with the boundary condition

$$f(\pm L) = \pm F_N(\varepsilon)\,, \tag{4}$$

where $F_N(\varepsilon) = 1/2\big[\tanh(\varepsilon + eV_N)\beta - \tanh(\varepsilon - eV_N)\beta\big]$ is the equilibrium function in the N reservoirs and $\beta = (2T)^{-1}$. The solution method is the same as in Refs [15–18]. Let us multiply the elements (1,2) of Eqn (2) by $\hat{\sigma}_z$ (equation for $\hat{G}$) and calculate the trace. After one integration we obtain

$$\big[1 - m_-(x)\big]\partial_x f = \begin{cases} J + J_1 - J_S, & 0 < x < L_1\,, \\ J, & L_1 < x < L \end{cases}. \tag{5}$$

Here the function $m_-(x) = (1/8)\mathrm{Sp}\big[\hat{F}^R(x) - \hat{F}^A(x)\big]^2$ determines the correction to the conductivity of the N channel due to the proximity effect. It is the main correction in the present problem. The integration constants J and J_1 might be called partial 'currents' per unit energy. More precisely, the current I on the segment (L_1, L) is an integral over the energy

$$I = (2e\rho)^{-1} d_N \int \mathrm{d}\varepsilon J(\varepsilon)\,. \tag{6}$$

The current on the segment $(0, L_1)$ is expressed by the same formula if J is replaced by $J + J_1$. The quantity J_S is the superconducting 'current', which does not depend on x on the segments (L_1, L) and $(0, L_1)$

$$J_S = \left(\frac{1}{4}\right)\mathrm{Sp}\,\hat{\sigma}_z(\hat{F}^R\partial_x\hat{F}^R - \hat{F}^A\partial_x\hat{F}^A)\,. \tag{7}$$

The integral of J_S (7) over the energy is exponentially small if condition (1) is satisfied. As follows from Eqn (2), the constant J_1, is related to the Green's function and distribution function in the superconductor. It can be written in the form [10, 18]

$$J_1 = J_q + \tilde{J}_S, \quad J_q = \left(\frac{\rho}{d_N}\Re_b\right)\big[F_S(\varepsilon) - f(L_1)\big]\,. \tag{8}$$

Here $\Re_b = R_{b\,\square}/w\big[\nu_N\nu_S + (1/8)\mathrm{Sp}(\hat{F}^R + \hat{F}^A)(F_S^R + F_S^A)\big]^{-1}$ is the resistance of the S/N boundary per unit length in the y direction and ν_N and ν_S are the density of states in the N and S conductors. It can be shown that for $V_{N,S}$ which are small compared with T/e, the 'supercurrent' $\tilde{J}_S$, flowing through the S/N boundary equals J_S. The distribution function F_S is the equilibrium function, i.e., it is identical to the function in Eqn (4), if V_N is replaced by V_S (we measure voltages from the point 0, where the voltage is zero). Using the fact that m_- is small, we can integrate Eqn (5) and find the relation of J and J_q to F_N and F_S [see the boundary condition (4)]. We obtain the normal currents

$$\left(\frac{d_N}{\rho}\right)J = \frac{\Re_b F_N + \Re_1(F_N - F_S)}{\Re_b\Re + \Re_1\Re_2}\,,$$

$$\left(\frac{d_N}{\rho}\right)J_1 \approx J_q\left(\frac{d_N}{\rho}\right) = \frac{\Re_2 F_S + \Re_1(F_S - F_N)}{\Re_b\Re + \Re_1\Re_2}\,. \tag{9}$$

Here $\Re_b$ is determined in Eqn (8); the quantity $\Re = \Re_1 + \Re_2$, $\Re_{1,2} = R_{1,2}(1 + \langle m_-\rangle)$ can be termed the partial resistance. The spatial average $\langle m_-\rangle_{1,2}$ on the segments $(0, L_1)$ and (L_1, L) gives a decrease in the resistances on account of the proximity effect ($\langle m_-\rangle$ is negative). All resistances in Eqn (9) depend on the difference of the phases φ and on the energy; they can be represented in the form $\Re_b = R_b - \delta\Re_b\cos\varphi$ and $\Re_{1,2} = R_{1,2} - \delta\Re_{1,2}\cos\varphi$. The corrections to the resistances $\delta\Re_b$ and $\delta\Re_{1,2}$ are small in the case of a weak proximity effect. The quantities R_b and $R_{1,2}$, generally speaking, depend on the energy ε (for example, ν_S depends on ε). We assume, for simplicity, that these quantities do not depend on the energy. This is valid if it is assumed that the superconductors are gapless (the results remain qualitatively the same in the case of superconductors with a gap). Then, integrating Eqn (9) over energies, on the left-hand side we obtain the currents I and I_1 [see Eqn (6)]. Eliminating V_N from the two equations obtained, we find for V_S

$$V_S = \hbar\frac{\partial_t\varphi}{4e} = I_1\big[R_b + R_1 - (\delta R_b + \delta R_1)\cos\varphi\big] + I(R_1 - \delta R_1\cos\varphi)\,. \tag{10}$$

Here we employed the Josephson relation; R_b is the resistance of the S/N boundary, which in the case of zero-gap superconductors is approximately equal to its value in the normal state. The resistance R_1 is also approximately equal to $\rho L_1/\mathrm{d}_N$ (the φ — independent correction arising from $\langle m_-\rangle$ is small and unimportant). Integrating Eqn (10), we obtain a relation between the average voltage $\bar{V}_S$ and the fixed currents I and I_1.

$$\bar{V}_S = \sqrt{\big[(I + I_1)R_1 + I_1R_b\big]^2 - \big[(I + I_1)\delta R_1 + I_1\delta R_b\big]^2}\,. \tag{11}$$

The function $\bar{V}_S(I_1)$ is displayed in Fig. 2 for different currents I. One can see that for $I \neq 0$ this dependence is identical to the current-voltage characteristic of a standard Josephson contact. In this case the critical current is

$$I_c = I\frac{\delta R_1 R_b - \delta R_b R_1}{(R_b + R_1)^2}\,. \tag{12}$$

Therefore I_c increases in proportion to the current I. We shall show below that the correction δR_1 decreases slowly with

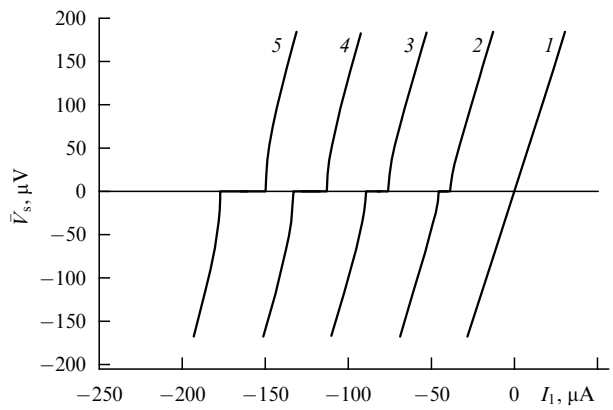

Figure 2. $\bar{V}_S$ versus the current I_1 for the following values of the current: *1* — 0; *2* — 250 μA; *3* — 500 μA; *4* — 750 μA; *5* — 1 μA. Here $\delta R_1 = 0{,}1R_1$; $R_b = 5R_1$; $R_1 = 1\ \Omega$.

increasing temperature ($\delta R_1 \sim T^{-1}$), and the correction δR_b is small if condition (1) is satisfied. Therefore, for $R_b \gg R_1$, we obtain $I_c \simeq I\delta R_1/R_b$. The maximum current I is limited by the condition that the Joule heating be small and by the condition $eV_N \simeq eIR \ll T$. In the opposite case δR_1 decreases as V_N increases. If condition (1) is not satisfied and a finite Josephson coupling exists between the superconductors, then it is easy to show that the critical current of the structure is $I_c^* = (I_c^2 + I_{cJ}^2)^{1/2}$, where I_{cJ} is the critical Josephson current. An expression for I_{cJ} can be easily obtained with the aid of Eqn (6). This expression is presented in Ref. [10]. The equilibrium phase difference φ_0 for $I_1 + IR_1/(R_b + R_1) = 0$ is $\varphi_0 = -\arcsin(I_c/I_c^*)$.

To determine δR_1 and δR_b it is necessary to find the condensate functions $\hat{F}^{R(A)}$, induced by the proximity effect. An equation for $\hat{F}^{R(A)}$ follows from Eqn (2) and is linear in the case that $\hat{F}^{R(A)}$ is small [10, 15, 18]. For $|x| < L_1$ the solution of this equation has the form

$$\hat{F}^{R(A)}(x) = F_S^{R(A)}\left[\mathrm{i}\hat{\sigma}_y \cos\left(\frac{\varphi}{2}\right) P_y \cosh(kx) + \mathrm{i}\hat{\sigma}_x \sin\left(\frac{\varphi}{2}\right) P_x \sinh(kx)\right]^{R(A)} . \tag{13}$$

Here $F_S^{R(A)}$ is the amplitude of the condensate functions in the superconductors. In the zero-gap case $F_S^{R(A)} = \pm\Delta/(\varepsilon \pm \mathrm{i}\gamma_S)$, where γ_S is the frequency of spin-flip collisions with impurities. The functions $P_{x,y}$ are:

$$P_x = \frac{b \sinh\theta_2}{(\sinh\theta + b\sinh\theta_1 \sinh\theta_2)} ,$$

$$P_y = \frac{b \sinh\theta_2}{(\cosh\theta + b\cosh\theta_1 \sinh\theta_2)} ,$$

$$b = \frac{\rho w}{(R_{b\,\square} d_N)} k , \qquad k^{R(A)} = \sqrt{\mp\frac{2\mathrm{i}\varepsilon}{D}} ,$$

$$\theta = \theta_1 + \theta_2 , \qquad \theta_{1,2} \equiv \theta'_{1,2} + i\theta''_{1,2} = kL_{1,2} . \tag{14}$$

Once the functions $\hat{F}^{R(A)}$ are known, the interference correction δR_1 to the resistance can be calculated:

$$\delta R_1 = -R_1 \int_0^\infty \mathrm{d}\varepsilon\beta \cdot \cosh^{-2}(\varepsilon\beta) \left\langle m_-(x,\varphi) - m_-\left(x, \frac{\pi}{2}\right)\right\rangle_1 . \tag{15}$$

With the aid of the expressions for $\langle m_- \rangle_1$ [see Eqn (5)] and for $\hat{F}^{R(A)}$ [see Eqn (13)] we find

$$\frac{\delta R_1}{R_1} = \int_0^\infty \mathrm{d}\varepsilon\beta \cdot \cosh^{-2}(\varepsilon\beta) M(\varepsilon) , \tag{16}$$

where

$$M(\varepsilon) = \left(\frac{1}{8}\right)\left\{|F_S|^2\left[|P_y|^2\left[\frac{\sinh(2\theta'_1)}{2\theta'_1} + \frac{\sin(2\theta''_1)}{2\theta''_1}\right] - |P_x|^2\left[\frac{\sinh(2\theta'_1)}{2\theta'_1} - \frac{\sin(2\theta''_1)}{2\theta''_1}\right] + \mathrm{Re}F_S^2\left[P_y^2\left(\frac{\sinh(2\theta_1)}{2\theta_1} + 1\right) - P_x^2\left(\frac{\sinh(2\theta_1)}{2\theta_1} - 1\right)\right]\right\} .$$

The temperature dependence of δR_1 is displayed in Fig. 3. One can see that for $T > \varepsilon_{L_1} = D/(2L_1)^2$ the quantity δR_1 decreases as T^{-1} with increasing temperature. As noted in Refs [15, 18], the slow decrease of $\delta R_1(T)$ is due to the so-called anomalous term $F_R F_A$ in $\langle m_- \rangle_1$. The special role of this term, which is non-analytic both in the upper and lower planes of ε, was noted in Ref. [24].

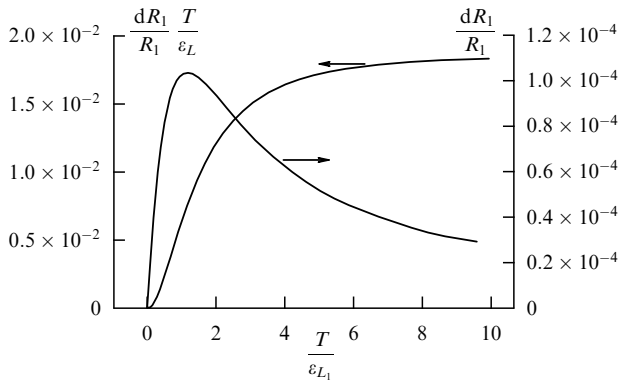

Figure 3. Interference correction δR_1 to the resistance as a function of temperature in the case $L_1 = 0.5L$, $R/R_b = 0.4$, $\gamma/\varepsilon_L = 100$, $\Delta\varepsilon_L = 30$.

The Josephson current I_S is determined by the integral of J_S (7) over all energies, i.e., the integral of products of either advanced or retarded Green's functions. It can be calculated by closing the integration contour in the upper (lower) half plane of ε and switching to summation over the Matsubara frequencies $\omega_n = \pi T(2n+1)$. For such energies the functions $F^{R(A)}$ decay exponentially over distances $k^{-1}(\omega_n) \leqslant \xi_n(T)$ away from the S/N boundary. Therefore the current I_S will be exponentially small ($I_S \sim \exp(-2L_1/\xi_N(T))$. The function $I_S(T)$ for the structure shown in Fig. 1 is presented in Ref. [18]. Similar arguments are also applicable to the calculation of δR_b, since for $T < \gamma_S$ the functions F_S^R and F_S^A can be assumed to be equal and independent of the energy. At the same time, the function $F^R F^A$, appearing in the expression for δR_1, decreases over a small (compared with T) energy $\varepsilon_{L_1} = D/(2L_1)^2$ and makes a nonzero contribution. For such energies the characteristic decay length of $F^{R(A)}(x)$ is of the order L_1, i.e., of the order of the distance between the superconductors.

In order to observe long-range Josephson effects, the critical current I_c must exceed the fluctuation current $Te/\hbar$: $I_c \gg Te/\hbar$. On the other hand, the ordinary Josephson effect is negligible if the condition $\varepsilon_{L_1} \ll T$ is fulfilled. Combining these inequalities, we obtain the condition

$$\frac{TR_b R_1}{\delta R_b R_Q} \ll \varepsilon_{L_1} \ll T , \tag{17}$$

which should be satisfied for observation of the effects under consideration. Here $R_Q = \hbar/e^2 \approx 3\ k\Omega$, and we took into account that a maximal value of I is determined by the relation $eIR \leqslant \varepsilon_{L_1}$. Otherwise δR_1 decreases with increasing I. The first inequality of (17) means that the zeroth Shapiro step on the $I_1(V_S)$ curve is absent at $I = 0$. If the second inequality of (17) is not fulfilled, then the critical current is not zero at $I = 0$ (the ordinary Josephson effect). In this case the effective critical current I_c^* should first increase with increasing I and then decrease if I exceeds ε_{L_1}/eR.

3. Conclusion

In conclusion we note, as one can see from Fig. 3, that the correction δR_1 to the resistance of the normal channel caused by the proximity effect depends on the temperature T in a non-monotonic way: it is zero at $T = 0$ (the bias voltage is zero as well), reaches a maximum at $T \approx \varepsilon_{L_1}$ and decays to zero at higher T. Such behavior of $\delta R_1(T)$ is related, as noted in [15], to different dependencies of two contributions to δR_1 on the energy ε. One contribution which increases the N channel resistance is connected with a decrease of the density-of-states in the normal channel. It is described by the last term in $M(\varepsilon)$ [see Eqn (16)]. Another contribution (anomalous) which diminishes the resistance of the normal channel is described by the first two terms in $M(\varepsilon)$. This contribution exactly compensates a contribution due to a change in the density-of-states of the normal channel at $\varepsilon = 0$ and dominates at $\varepsilon \neq 0$. At $T > T_c$ it leads to the Maki-Thompson contribution to the paraconductivity. Mathematically, compensation of the two contributions at $\varepsilon = 0$ arises because at $\varepsilon = 0$ $F^R = F^A$ and m_- in Eqn (16) becomes zero. The monotonic behavior of δR has been observed in an experiment [4]. It would be interesting to observe the long-range Josephson effect experimentally.

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Sign memory of the Ruderman – Kittel interaction in disordered metals and magnetic coupling in mesoscopic metal/ferromagnet layered systems

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In the case when two paramagnetic spins are embedded in a pure non-magnetic metal at zero temperature $T = 0$ the Ruderman – Kittel exchange interaction energy between them has the well known form [1]

$$I_{ij}(\mathbf{R}_1, \mathbf{R}_2) = I_0 \frac{\cos(2p_F|\mathbf{R}_1 - \mathbf{R}_2|)}{(p_F|\mathbf{R}_1 - \mathbf{R}_2|)^d} \delta_{ij} . \tag{1}$$

Here I_0 is a factor which is proportional to the square of the exchange interaction between the localized spins and spins of conduction electrons in the metal, p_F the Fermi momentum in the metal, d is the dimensionality of the space, i,j are spin indexes and $\mathbf{R}_1$, $\mathbf{R}_2$ are coordinates of the localized spins. In the case of disordered metals $|\mathbf{R}_1 - \mathbf{R}_2| \gg l \gg p_F^{-1}$ the amplitude of the average Ruderman – Kittel exchange energy

$$\langle I_{ij}(\mathbf{R}_1, \mathbf{R}_2)\rangle \sim \exp\left(-\frac{|\mathbf{R}_1 - \mathbf{R}_2|}{l}\right), \tag{2}$$

is exponentially small [2]. Here brackets $\langle\ \rangle$ correspond to averaging over realizations of a random scattering potential (or averaging between samples) and l is the electron elastic mean free path in the metal. On the other hand, it was shown in [3 – 5] that the exponential decay of the average $\langle I_{ij}(\mathbf{R}_1, \mathbf{R}_2)\rangle$ is the consequence of randomization of the sign of $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$ and that the typical amplitude of the interaction

$$\sqrt{\left\langle \left(I_{ij}(\mathbf{R}_1, \mathbf{R}_2)\right)^2 \right\rangle} \sim |\mathbf{R}_1 - \mathbf{R}_2|^{-d} \tag{3}$$

decreases with distance in the same way as in the pure case. The interpretation of Eqs (2) and (3) given in [3] was that in a given sample

$$I_{ij}(\mathbf{R}_1, \mathbf{R}_2) \sim \frac{\cos\left(2p_F|\mathbf{R}_1 - \mathbf{R}_2| + \delta(\mathbf{R}_1, \mathbf{R}_2)\right)}{(p_F(|\mathbf{R}_1 - \mathbf{R}_2|)^{-d}} , \tag{4}$$

where $\delta(\mathbf{R}_1, \mathbf{R}_2)$ has a random sign when $|\mathbf{R}_1 - \mathbf{R}_2| \gg l$, which means that the sign of $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$ cannot be predicted.

In fact what follows from Eqs (2) and (3) is that $I_{ij}(\mathbf{R_1}, \mathbf{R_2})$ has random signs between samples. We would like to point out here that in a given sample the sign of $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$ can be predicted in a sense that there are long range correlations between the signs of $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$ and $I_{k,l}(\mathbf{R}_3, \mathbf{R}_4)$ which survive even over very large distances $|\mathbf{R}_1 - \mathbf{R}_3| \sim |\mathbf{R}_2 - \mathbf{R}_4| \sim |\mathbf{R}_2 - \mathbf{R}_3| \sim |\mathbf{R}_1 - \mathbf{R}_4| \sim R \gg l$ and for arbitrary locations of $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R_3}$, $\mathbf{R}_4$. To prove this statement we can calculate the correlation function ($\delta I_{ij} = I_{ij} - \langle I_{ij}\rangle$) at $R \gg l$

$$\langle \delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2)\delta I_{ij}(\mathbf{R}_3, \mathbf{R}_4)\rangle \sim R^{-4(d-1)} \tag{5}$$