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Electrons in quasi-one-dimensional conductors: from high-temperature diffusion to low-temperature hopping

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1. Introduction

The past two decades have seen spectacular progress in the physics of low-dimensional disordered conductors [1, 2]. One of the directions of rapid growth is the study of electron transport in quasi-one-dimensional (1D) conductors. The experimental study of this problem is crucial for our understanding of transport mechanisms in a diversity of 1D systems: metal-film and semiconductor nanometer structures [3], heavy-doped conjugated polymers [4], carbon nanotubes [5], and many others.

It is widely believed that all electron states in low-dimensional conductors are localized [6, 7], at least in the case of weak electron – electron interaction. The extent of the electron wavefunction is characterized by the localization length ξ; for a quasi-1D conductor,

$$\xi = \frac{\pi\hbar}{e^2}\frac{W}{R_\square} = 2\pi\hbar v_{2D} D W\,, \tag{1}$$

where v_{2D} is the two-dimensional (2D) density of electron states, D is the electron diffusion constant, and W is the width of a thin-film 'wire'. In quasi-one-dimensional conductors, the largest cross-sectional dimension is smaller than ξ, and, at the same time, is much greater than the wavelength of the current carriers. In spite of localization, the conductivity of 1D conductors can be very high at room temperature. This 'metallic' conductivity is due to the strong inelastic scattering: the electron scatters to another state, localized around a different site, before it diffuses over the localization length. This is the weak localization (WL) regime. However, with decreasing temperature, a 1D conductor will inevitably become an insulator. Electron transport could proceed by hopping only in this strong localization (SL) regime.

The goal of this work is an observation of the crossover between WL and SL regimes and experimental study of electron transport on the insulating side of the crossover.

2. Crossover from weak to strong localization

The theoretical prediction of the crossover from diffusion to hopping in 1D conductors with decreasing temperature was made by Thouless [6] in 1977. However, the experimental study of this fundamental problem was delayed for 20 years. The 'gap' between the prediction and observation indicates that this is a very demanding experiment; in particular, the choice of adequate samples is important for success. Recently we observed the crossover as a function of temperature in experiments with narrow channels in the MBE-grown Si δ-doped GaAs structures [8]. The samples consisted of single sheets of Si donors with concentration $(3-5)\times10^{12}$ cm^{-2}, which were 0.1 μm below the surface of an undoped GaAs. Using the e-beam lithography and ion etching, we were able to prepare uniform conducting channels of effective width W as narrow as 0.05 μm. (Because of the side depletion, the effective width is smaller than the geometrical width by 0.15 – 0.2 μm, depending on the concentration of carriers). In order to reduce the effect of mesoscopic conductance fluctuations, we made these wires long enough (the length $L = 40-500$ μm was much greater than the localization length) and connected many wires in parallel (up to 500 wires). Parameters of several samples are listed in Table 1.

Table 1. Parameters of the samples.

Sample	1	2	3	4	5	6
W, μm	0.05	0.06	0.1	0.12	0.2	0.18
L, μm	500	500	40	500	40	500
No. of parallel 'wires'	470	470	5	470	5	470
$R_\square(T = 20\,\mathrm{K})$ kΩ	1.6	1.7	3.5	1.6	4.2	1.7
ξ, μm	0.40	0.46	0.37	1.0	0.61	1.4
Δ_ξ, K	2.1	1.5	1.1	0.35	0.34	0.17
$T_0(H = 0)$, K	2.6	1.87	1.47	0.42	0.39	0.2
$R_\xi(T = T_0)$, kΩ	20.4	21.3	28	23	24.4	24.3
H_ξ, kOe	1.0	0.74	0.56	0.17	0.17	0.083
H_ξ^{exp}, kOe	1.0	0.80	0.51	0.21	0.17	0.12
H_ξ^{exp}/T_0, kOe K^{-1}	0.37	0.43	0.35	0.50	0.44	0.59

The mean free path of electrons is small in the δ-doped layers (17 – 35 nm) because of the strong scattering of electrons by ionized impurities, and the electron motion is always diffusive at distances smaller than the wavefunction envelope of the length ξ. The relatively high concentration of carriers ensures that the number of occupied 1D sub-bands $N_{1D} = k_F W/\pi$ is large; $N_{1D} \simeq 7$ even in the narrowest sample 1. However, with respect to the quantum interference effects all the samples are one-dimensional at low temperatures $[W < \xi, L_\varphi(T)]$.

The resistance of the samples increases with decreasing temperature (Fig. 1); a slow growth of R (logarithmic above 10 K) is consistent with the theory of quantum corrections to the resistance in the WL regime [8]. However, below a certain crossover temperature, a dramatic change in the dependence $R(T)$ was observed: it becomes exponentially strong and can be fit with an activation law

$$R(T) = R_0 \exp\left(\frac{T_0}{T}\right). \tag{2}$$

The Arrehnius-type dependence (2) was observed for all the samples at $T \leqslant 0.3T_0$, where T_0 is the temperature that corresponds to the activation energy (see Fig. 1). The crossover from the one-dimensional WL dependence $R(T)$ to a stronger one occurs at $T \approx T_0$; below we identify the crossover temperature with T_0.

The proof that we observe the Thouless crossover from weak to strong localization is based on two experimental facts. Firstly, the resistance R_ξ, calculated for a wire segment of length ξ at $T = T_0$, turns out to be 24 ± 4 kΩ for different samples (see Table 1); this is consistent with the resistance $\sim h/e^2$ expected for a 1D conductor of length ξ in the vicinity of the crossover [6]. Secondly, in terms of competition between the length scales, the crossover should occur when the temperature-dependent length $L_\varphi(T)$ becomes compar-

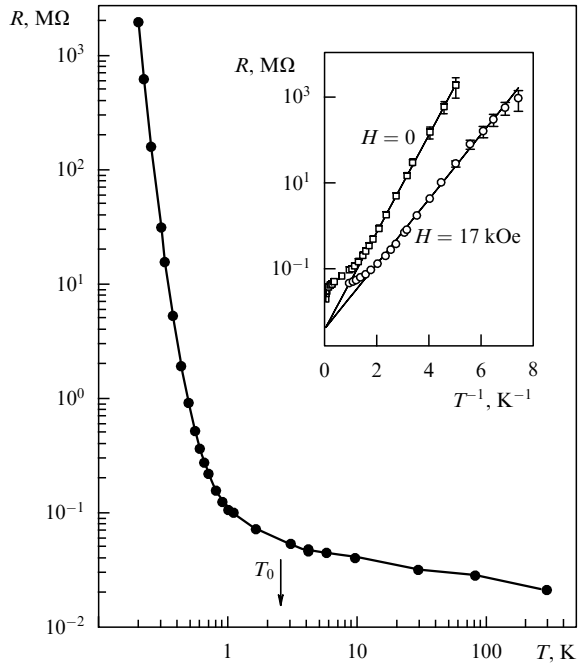

Figure 1. Temperature dependence of the resistance of sample 1 in zero magnetic field; the solid curve is a guide to the eye. The arrow indicates the temperature that corresponds to the activation energy. The insert shows R versus $1/T$ for sample 1 at $H = 0$ (□) and at $H = 17$ kOe (○). The straight lines are dependences (2) with $T_0(H = 0) = 2.6$ K and $T_0(H = 17\ \text{kOe}) = 1.73$ K.

able to the localization length [6]. The phase-breaking length can be estimated by fitting the high-temperature ($T > T_0$) magnetoresistance with the WL theory [9]. For all the samples studied, $L_\varphi(T_0)$ is approximately 1.5–3 times smaller than ξ calculated from Eqn 1 or estimated from the SL magnetoresistance (see below). Taking into account that the accuracy of estimation of the phase-breaking length is not very high in the vicinity of the crossover, and that there might be some systematic errors in calculating ξ, we conclude that our experimental data are consistent with the Thouless scenario of the WL–SL crossover in 1D conductors.

An important feature of our experiment is that we observe the crossover as a function of temperature, the electron states being exactly the same at both sides of the crossover. In this respect, our experiments differ from the measurements on gated heterostructures, where all electron parameters are changed by variation of the gate voltage [3].

3. Strong localization regime

3.1 The temperature dependence of the resistance

The activation energy $k_B T_0$ of the exponential dependence $R(T)$, observed in the SL regime (the insert in Fig. 1), is very close to the mean energy spacing of the electron states on the scale of the localization domain,

$$\Delta_\xi = (\nu_{2D}\xi W)^{-1} \tag{3}$$

The values of Δ_ξ and T_0 at $H = 0$ are listed for different samples in Table 1. The crossover has not been observed for samples wider than 0.3 μm in our temperature range ($T > 50$ mK); this is consistent with the fact that Δ_ξ should vary as W^{-2} for a fixed $R_\square$.

The Arrehnius-type temperature dependence of the resistance may be attributed to either of two models of electron transport in the SL regime: (a) hopping between neighboring electron states, which strongly overlap in space, and (b) variable-range hopping, which is substantially modified in one dimension (the so-called Kurkijarvi model) [10]. In both models, the resistance of a long 1D conductor is governed by anomalously 'resistive' (critical) hops, which cannot be by-passed in 1D. The predictions of these models differ in two important aspects. Firstly, the hopping length, L_h, is of the order of ξ in the nearest-neighbor model, whereas in the Kurkijarvi model $L_h \simeq \xi T_0/T \gg \xi$ [11, 12]. Secondly, the distance between critical hops in the nearest-neighbor model depends on the form of the distribution function of the localization length. In the Kurkijarvi model, which does not take into account fluctuations of ξ, critical hops should be exponentially rare; the distance between these hops is [11, 12]

$$L_c \simeq \frac{\xi}{2}\left(\frac{T_0}{T}\right)^{1/2} \exp\left(\frac{T_0}{T}\right). \tag{4}$$

We observed similar $R(T)$ dependences for samples with $L = 40-500$ μm; for the 'short' samples, L is smaller than the estimate of L_c from Eqn (4). The distance between critical hops can be obtained from studying the non-linearity of the current-voltage characteristics (IVCs) (see below); it turns out that L_c does not exceed 20ξ ($\sim 10-20$ μm) even at $T = 0.1T_0$. These two facts speak in favor of the nearest-neighbor model. However, we believe that these two models may merge if one takes into account both statistical fluctuations of ξ, essential in one dimension [13], and unavoidable fluctuations of the sample width.

3.2 Magnetoresistance

The magnetoresistance of the samples studied is negative over the whole temperature range. It is very anisotropic; we observed no magnetoresistance due to the H component parallel to the plane of the δ-layer. The magnetoresistance becomes exponentially large in the SL regime. One can see from Fig. 1 that the magnetoresistance is due to the field dependence of the activation energy. Indeed, T_0 is the only parameter that varies with the magnetic field: the form of $R(T)$ dependence (2) remains the same regardless of the field magnitude, and the prefactor R_0 is not affected by H.

The magnetic-field dependence of the activation energy (3) stems from the field dependence of ξ. The localization length in 1D conductors,

$$\xi = (\beta N_{1D} + 2 - \beta)l \tag{5}$$

is proportional to the symmetry index β [14]. In the absence of spin-orbit scattering, a time-reversal symmetry-breaking magnetic field induces a transition from $\beta = 1$ to $\beta = 2$, and hence a doubling of ξ for $N_{1D} \gg 1$. Doubling of the localization length results in halving the activation energy which is in accord with our data. [In fact, $N_{1D} \approx 7$ for sample 1, and T_0 in strong fields should be smaller than $T_0(H = 0)$ by a factor of 1.75.] The experiment provides the first evidence of the universal dependence of the localization length in quasi-1D conductors on the symmetry class.

It is convenient to convert the magnetoresistance into the magnetic-field dependence of T_0 (exploiting the conclusion

that the magnetoresistance stems solely from the $T_0(H)$ dependence). The $T_0(H)/T_0$ dependences measured for sample 1 at different temperatures $T \ll T_0$ are shown in Fig. 2; for all the samples, these dependences collapse onto the same universal curve. According to the theory [14], a transition from $\beta = 1$ to $\beta = 2$ should occur in magnetic fields H_ξ, where

$$H_\xi = \frac{\Phi_0}{\xi W} \tag{6}$$

is the field scale for breaking the time-reversal symmetry within the area occupied by a localized state (Φ_0 is the magnetic flux quantum). Though it is obvious that the theory predicts the correct value for the characteristic field (see Fig. 2), it would be interesting to obtain the theoretical dependence $\xi(H)$ for all magnetic fields (only limiting values of ξ are available now). Indeed, the experimental data on $R(H)$ and $R(T)$ at $H = 0$ provide us with a direct method of measuring of ξ in quasi-1D conductors. To make this method more accurate, we need the theoretical expression for the transition curve $T_0(H)$, and this is a challenge for the theory. It is worth noting, that in fields $H > H^* = \Phi_0/W^2$ one could expect a 1D – 2D crossover in the magnetoresistance of our samples. A kink at the dependence, observed in the vicinity of H^* (see Fig. 2) may be an indication of this crossover.

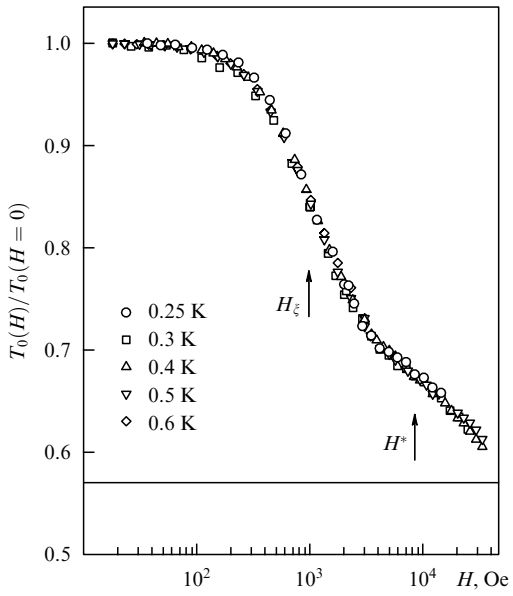

Figure 2. Magnetic field dependences of the activation energy for sample 1 at different temperatures. The arrows indicate characteristic fields $H_\xi = \Phi_0/\xi W$ and $H^* = \Phi_0/W^2$. The solid line corresponds to the high-field limit of $T_0(\beta = 2)$.

In the absence of a theoretical expression for the transition curve, we estimate H_ξ from the experimental data by fixing the level $T_0(H_\xi) = 0.85T_0(H = 0)$ (see Table 1). This choice was based on comparison of the theoretical estimate for H_ξ with experimental data for many samples. The calculated values of H_ξ are in an excellent agreement with experiment; this indicates that the shape of the transition curve is the same for all the samples. Note that the ratio of the characteristic magnetic field to the spacing between the levels at the scale ξ is a universal quantity, which depends only on the effective mass m^* of current carriers:

$$\frac{H_\xi}{\Delta_\xi} = \Phi_0 \nu_{2D} = \frac{m^* c}{e\hbar} \,. \tag{7}$$

For GaAs, where the effective mass is well known, this ratio is 0.5 kOe K^{-1}; the last row in Table 1 demonstrates that the experimental counterpart of this ratio, H_ξ^{exp}/T_0, indeed, remains the same (within $\sim 20\%$) for all the samples studied.

3.3 Non-linearity of the resistance

So far we have discussed the experimental data obtained in the linear regime (the limit of small electric fields). The study of the non-linearity of the IVCs helps to determine the distance between critical hops and to distinguish between different mechanisms of electron transport in the SL regime. The dependences of the 'resistance' $R \equiv V/I$ on the applied voltage V, measured for sample 1 at different temperatures, are shown in Fig. 3. In small electric fields, the experimental $R(V)$ curves can be fit with the dependence

$$R \equiv \frac{V}{I} = R_0 \exp\left(\frac{T_0 - eVL_c/L}{T}\right), \tag{8}$$

where L_c is the distance between critical hops, or the hopping length, L_h, if all the hops are the same. The length L_c increases with decreasing temperature; at $T \simeq 0.1T_0$ it exceeds ξ by a factor of 20. These values of L_c are insufficiently large for our data to be consistent with the Kurkijarvi model [see Eqn (4)] [10 – 12]. Hence, the most probable candidate for the electron transport mechanism in our samples in the SL regime is nearest-neighbor hopping between strongly overlapping localized electron states. We hope that future experiments will give information on the statistics of the resistance fluctuations in long 1D conductors.

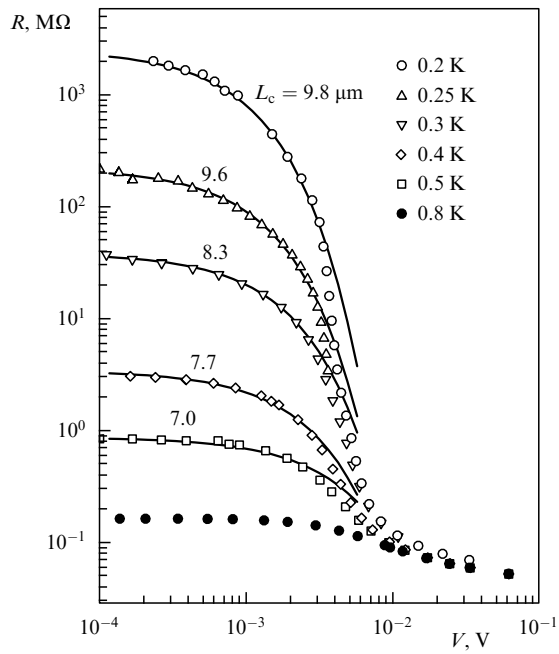

Figure 3. Resistance $R \equiv V/I$ of sample 1 versus voltage V across the sample at different temperatures. The solid curves represent the best fit of the low-voltage experimental data $R = 4.7\,[\mathrm{k\Omega}] \times \exp[(T_0 - eVL_c/L)/T]$.

4. Conclusions

This is the first experimental study of the crossover from weak to strong localization in quasi-1D conductors as a function of temperature, when one is deal with the same electron states on both sides of the crossover. In accord with the Thouless' theory, the crossover occurs when the phase-breaking length becomes comparable with the localization length. The resistance of the wire segment of length ξ is close to the quantum resistance h/e^2 at the crossover. On the insulating side of the crossover, an activation temperature dependence of the resistance is observed; the activation energy is close to the spacing between the energy levels of localized electron states within the localization domain. The exponentially strong magnetoresistance in this regime is due to the magnetic field dependence of the localization length. Our data can be considered as experimental evidence of the magnetic-field-induced doubling of the localization length in quasi-1D conductors with weak spin-orbit scattering. The study of magnetoresistance in the SL regime provides a direct measurement of the localization length in quasi-1D conductors. We estimate the distance L_c between the most resistive hops from the non-linearity of the current-voltage characteristics in the SL regime. Relatively small values of L_c contradict the Kurkijärvi model, in which the critical hops should be exponentially rare. Thus, nearest-neighbor hopping is the most probable candidate for the transport mechanism in our samples on the insulating side of the crossover.

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