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Mesoscopic and strongly correlated electron systems "Chernogolovka 97"

3. Quantum chaos and resonant tunneling

The third session of the conference included the following presentations:

(1) **Larkin A I** (Minnesota University, USA; Landau Institute for Theoretical Physics, RAS, Chernogolovka, Russia) "Divergence of classical trajectories and quantum chaos";

(2) **Efetov K B** (Ruhr-Universität, Bochum, Germany) "Directed quantum chaos";

(3) **Mirlin A D** (Universität Karlsruhe, Germany) "Correlations of eigenfunctions in disordered systems";

(4) **Kravtsov V E** (ICTP, Trieste, Italy; Landau Institute for Theoretical Physics, RAS, Chernogolovka, Russia) "Level curvature distribution function beyond the random matrix theory";

(5) **Viña L** (Universidad Autónoma de Madrid, Spain), **Potemski M** (High Magnetic Field Laboratory, Grenoble, France), **Wang W I** (Columbia University, New York, USA) "Signatures of quantum chaos in the magneto-excitonic spectrum of quantum wells";

(6) **Fal'ko V I** (School of Physics & Chemistry, Lancaster University, UK) "Image of local density of states in differential conductance fluctuations in the resonance tunneling between disordered metals";

(7) **Dubrovskiĭ Yu V** (Institute of Microelectronics Technology RAS, Chernogolovka, Russia) et al. "Resonant and correlation effects in tunnel structures with sequential 2DEG in magnetic field";

(8) **König , Schoeller H, Schön G** (Institut für Theoretische Festkörperphysik, Universität Karlsruhe, Germany) "Resonant tunneling through a single-electron transistor";

(9) **Kvon Z D, Olshanetskiĭ E B, Gusev G M** (Institute of Semiconductor Physics, RAS, Siberian Branch, Novosibirsk, Russia), **Portal J C, Maude D K** (High Magnetic Field Laboratory, Grenoble, France) "Coulomb-like mesoscopic conductance fluctuations in a 2D electron gas near the filling factor $\nu = 1/2$";

(10) **Fedorov Yan V** (Universität Essen, Germany and Petersburg Nuclear Physics Institute, Gatchina, Russia), **Sommers H-J** (Universität Essen, Germany) "Resonances as eigenvalues of almost-Hermitian random matrices".

Papers 5, 6, 8 and 9 are published below. Papers 1 and 10 were published in *Phys. Rev. E* **55** R1243 (1997) and *Phys. Rev. E* **55** R4857 (1997) respectively. For papers 2 – 4 and 10 see the e-prints: 2 — cond-mat/9706055, cond-mat/9702091; 3 — cond-mat/9712153; 4 — cond-mat/9712147; 10 — cond-mat/9703152.

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Signatures of quantum chaos in the magneto-excitonic spectrum of quantum wells

L Viña, M Potemski, W I Wang

Statistical correlations in the spectra are widely regarded as the hallmark of quantum chaos. These correlations are found in a variety of complex systems in different domains, ranging from physics (nuclear, atomic, molecular and solid state), to chemistry and biology [1]. The real quantum systems to observe these phenomena need to be both complex, in the sense that their classical dynamics is chaotic, and simple, meaning that the dynamics can be analyzed and simulated numerically. The hydrogen atom in a static magnetic field is commonly studied as a paragon of a chaotic system because it fulfils both requirements [2].

Since, classically, chaotic motion takes place in situations where the Lorentz force is comparable to the Coulomb force, huge magnetic fields, of the order of 2×10^5 T, would be necessary to attain this condition for the hydrogen atom. Thus, usually, Rydberg states, with principal quantum number $n \gg 50$, where the Coulomb force of the nucleus is greatly decreased, are the object of the studies. In this case, the strength of the magnetic field necessary to balance the Coulomb and Lorentz force is only a few tesla, which can be easily obtained in the laboratory. Furthermore, to achieve the simplest systems, their dimensionality is usually reduced using a field with cylindrical symmetry around the z-axis, which gives rise to a two-dimensional (2D) Hamiltonian [3].

A quantum-mechanical equivalent to hydrogen-like atoms is obtained in solid state physics: a Wannier exciton in a semiconductor, composed of an electron in the conduction band and the hole left in the valence band after optical excitation [4]. Both particles are bound by Coulomb interaction and in a typical III – V semiconductor, such as GaAs, the dimensions of the exciton and its binding energy, 150 A and 5 meV, respectively, are of the same order of magnitude as in Rydberg states. The dimensionality of this system can be easily reduced using quantum wells (QW's) and, moreover, the fields attainable in the laboratory are enough to observe the fine structure of the spectra without the need to prepare highly excited states [5].

Some experimental work on the chaotic electron dynamics in low-dimensional semiconductor heterostructures has been performed on resonant tunneling structures in the presence of tilted magnetic fields [6 – 8]. Theoretical work on the possibility of the existence of chaos in quantum wells and dots has also appeared recently [9, 10]. We present in this work a magneto-optical study of the exciton spectrum in GaAs QW's, and investigate the statistics of the energy level distribution.

We used a p–i–n heterostructure, consisting of 5, 160 A wide, GaAs QW's sandwiched between n^+ and p^+ GaAs layers. The barriers were composed of $Ga_{0.65}Al_{0.35}As$. The study presented here was only possible due to the excellent quality of the sample (peaks FWHM ≈ 0.3 meV), which allowed the observation of the excited states of the excitons and the resolution of very closely lying states in the spectra. Highly resolved photoluminescence excitation (PLE) spectra were obtained in a polyhelix resistive magnet with fields, up to 17 T, applied in the Faraday configuration. The spectra were recorded at 2 K with circularly polarized light from a LD700 dye laser, pumped by a Kr^+-ion laser. The sample was biased to flat band conditions, to avoid further complications in the spectra which could appear due to the presence of an electric field. The use of circular polarization was also decisive to the success of the experiments to separate the spin-up and spin-down components of the magnetoexcitons.

Figure 1 depicts the low-temperature PLE spectrum at zero magnetic field, which consist of 'allowed' and 'forbidden' excitonic transitions, superimposed on a step-like background typical of 2D systems. The forbidden transitions, such as the one labeled $h_{12}(1s)$, are seen due to the presence of a residual electric field of ≈ 5 kV cm^{-1}, which could not be avoided although the sample was biased to flat band conditions. The peaks are labeled using the following notation: h (l) means a heavy (light) hole; a sub-index indicates the same confined sub-band for electrons and holes; in the case of two sub-indices the former (latter) corresponds to electrons (holes); a hydrogenic notation *nm* is used for the exciton envelope function, where *n* is the principal quantum number and *m* the angular momentum quantum number.

Due to the high quality of the sample, which is also apparent by the absence of any Stokes shift between the emission and the absorption, it was not possible to record the ground state of the system (heavy-hole exciton) in the PLE spectrum. Its energy was measured directly from the luminescence spectra. The first excited states of the heavy- and light-hole excitons, $h_1(2s)$ and $l_1(2s)$, respectively, are seen in the spectra as very sharp peaks. This is in contrast with samples of inferior quality where they appear only as shoulders. Incidentally, some of the peaks, as for example $l_1(2s)$ and $h_{13}(1s)$, have an asymmetric lineshape, typical of Fano resonances between a discrete state and the continuum of the lower lying excitonic states.

The number of states increases remarkably in the presence of an external magnetic field, as can be noticed in Fig. 2, which presents PLE spectra recorded with σ^+ (solid lines) and σ^- (dashed lines) polarized light at a field of 2.4 T. The lowest measured peak in all the spectra corresponds to $l_1(1s)$, since it was not possible to observe in PLE the $h_1(1s)$ because of the vanishing Stokes shift of the sample. The energy of $h_1(1s)$ was again obtained from PL experiments. The complexity of the spectra of this QW is clearly demonstrated in the figure. In a previous work [5], we identified the structures in the spectra in an energy range of ≈ 35 meV above $h_1(1s)$ by comparison with calculations of the magneto-excitonic spectra of a quantum well taking into account the complications of the valence band structure and using a basis of excitons including up to 4f states. The agreement between the theory and experiments was remarkably good and only some peaks of low oscillator strength observed in the experiments could not be described by the theory.

The fan diagram of the excitonic transitions obtained from σ^--PLE is shown in Fig. 3. The intensities of the peaks are colour coded (light-gray: minimum; black maximum). The excitonic nature of the transitions is appreciable in the dependence of the energies with the magnetic field: at low fields the states shift quadratically whereas the dependence is linear for higher fields. Numerous level repulsions can be observed in this diagram when it is inspected in close detail. Bending of the fan curves are already seen in this figure, as for example in the region close to 12 T and 1.58 eV. Many other interactions between states of the same symmetry occurring at lower fields ($\leqslant 5$ T) cannot be observed in this fan diagram, due to the plethora of excited states.

One of the anticrossings between states of the same symmetry for fields between 1.5 and 6 T is illustrated in

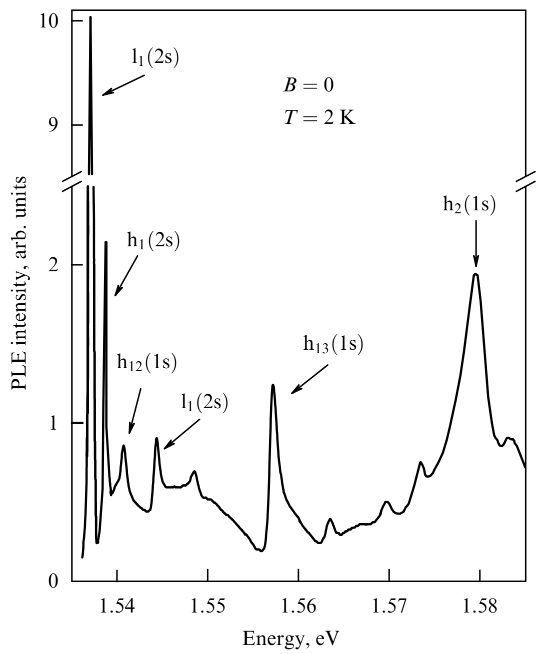

Figure 1. Zero field, low temperature, PLE spectrum of the 160 A wide GaAs QW.

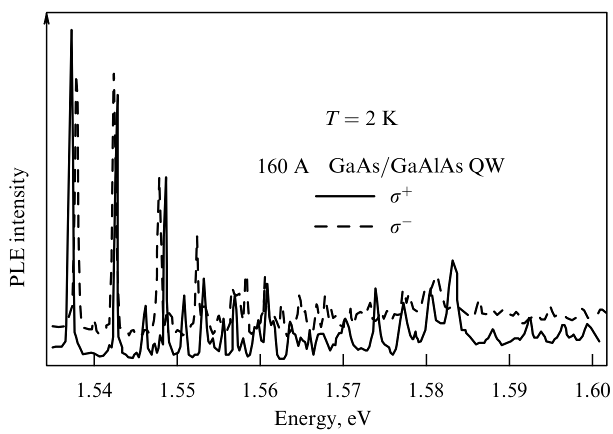

Figure 2. PLE spectra for excitation with σ^+ (solid) and σ^- (dashed) polarized light at 2.4 T. The lowest peak corresponds to the ground state of the light-hole exciton, $l_1(1s)$.

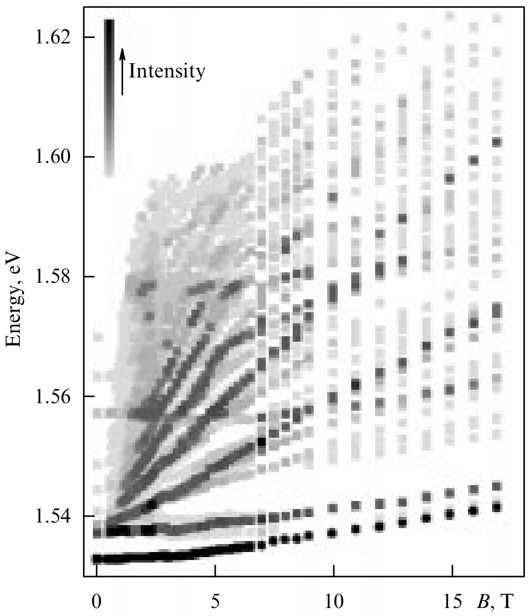

Figure 3. Energies versus magnetic field of the structures observed in the σ^--PLE spectra (squares). The dots correspond to $h_l(1s)$ measured with PL.

Fig. 4. Although the states are still labeled according to the atomic and excitonic notation, using their heavy- or light-hole character, the complicated structure of the valence band mixes their third components of angular momentum and therefore, strictly speaking, it is only possible to classify them using group theory and attending to the irreducible representations Γ_7 or Γ_8. Both states belong to the same irreducible representation, Γ_7, and therefore can interact. Increasing the field $l_l(3d\text{-})$ comes closer to $h_l(2s)$ and gains intensity. The points are shown as $\odot$ in the region of strong interaction, when they repel each other and share their oscillator strengths as demonstrated in the inset. Many couplings of this type are observed in the fan diagrams, specially in the σ^- configuration. This can lead to stochastic processes, which are known to play an important role in atomic spectra [3].

The histogram of the energy distribution of the excitonic levels is presented in Fig. 5 for a magnetic field of 2.5 T. The bars correspond to the experimental results and the symbols to the best fit with different statistical distributions. A completely random sequence of energy levels is described by a Poisson distribution, given by:

$$P(w) = \frac{1}{D}\exp\left(-\frac{w}{D}\right), \tag{1}$$

where w is the energy difference between two transitions and D is the mean local distance between the levels [11]. For this distribution small energy spacings predominate, and it presents a maximum at $w = 0$. However, if repulsions between energy levels take place, they dominate all the spectral fluctuations and the energy spacings between the levels are described by a Wigner distribution given by:

$$P(w) = \frac{\pi w}{2D^2}\exp\left(-\frac{\pi w^2}{4D^2}\right), \tag{2}$$

which assumes a linear repulsion among the levels [11]. The Wigner distribution applies only for a pure sequence of levels, which have all the same values of the quantum numbers. In the case of mixed sequences, the repulsions are moderated by the vanishing matrix elements of the interaction connecting different symmetries, the spectral distribution moves towards a random distribution: the spacing distribution becomes Poisson-like. Brody has introduced a level distribution which interpolates between the Wigner and Poisson distribu-

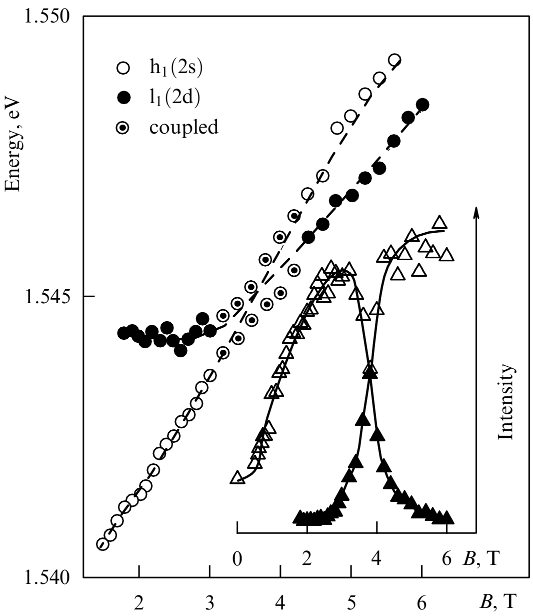

Figure 4. Energies of $h_l(2s)$ and $l_l(3d\text{-})$ versus magnetic field. The inset shows the oscillator strengths normalized to those of $l_l(1s)$.

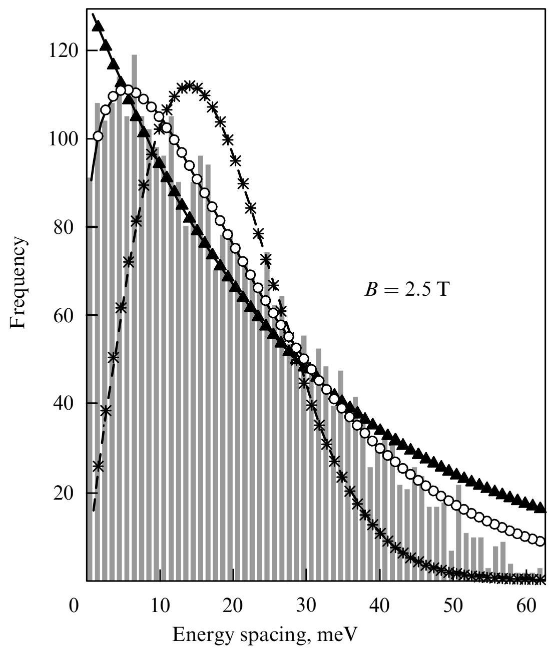

Figure 5. Histogram of energy distribution of the excitonic. Bars: experiments. Symbols: best fits to Brody (open circles), Poisson (triangles) and Wigner (stars) distributions.

tions [12]:

$$P_\beta(w) = A\left(\frac{w}{D}\right)^\beta \exp\left[-\alpha\left(\frac{w}{D}\right)^{1+\beta}\right], \qquad (3)$$

with

$$\alpha = \left[\frac{1}{D}\,\Gamma\left(\frac{2+\beta}{1+\beta}\right)\right]^{1+\beta},$$

where $\Gamma(x)$ is the Gamma function, and β is the Brody parameter. When the Brody parameter $\beta = 0$ ($\beta = 1$), the Brody distribution reduces to the Poisson (Wigner) distribution. This distribution is only heuristic and does not have a theoretical basis as a measure of underlying chaos in the system. However, in the absence of a distribution which does have a theoretical basis, it is useful since it depends only on one parameter. We obtain the best fit of our data at 2.5 T with $\beta = 0.24$ and $D = 15$ meV; at fields of ≈ 0.5 T their values are 0.05 and 10 meV, respectively. Both β and D increase with the field up to ≈ 5 T and then saturate.

In summary, we have found statistical correlations in the magnetoexcitonic spectra of GaAs QW's which can be regarded as a hallmark of quantum chaos. The separation of the energy levels obeys a Brody distribution, which interpolates between a Wigner and a Poisson distribution. The departure from a pure Wigner distribution is due to the existence of excitonic levels which belong to different irreducible representations. Those can be energetically degenerate, thus the probability of zero energy spacing grows, introducing a Poisson contribution to the distribution. Further studies, using tilted magnetic fields with respect to the growth axis of the quantum well, are being performed to investigate the effect of reducing the symmetry of the system in the energy level distribution. Additionally, the application of an external electric field can also be used to lower the symmetry.

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Image of local density of states in differential conductance fluctuations in the resonance tunneling between disordered metals

V I Fal'ko

The discussion of the local density of states (LDOS) fluctuations in disordered metals was started in the theory more than a decade ago [1, 2], and a lot is already known [3] about their statistics in the metallic regime (i.e., at $p_F l \gg 1$). Random point to point, or parametric dependences of this quantity discussed in the literature are the result of the interference of multiply scattered electron waves in a disordered metal. Recently, it was demonstrated that the amplitude of LDOS fluctuations and their statistics are mainly governed by statistical properties of the wave functions of a diffusive electron in a disordered metal [4–6], and particularly by correlations between the individual wave functions.

The direct manifestation of the density of states fluctuations in the transport experiments were discussed, at first, in the context of non-resonant tunneling between two disordered metals [7]. Later, this idea was extended [8] to the studies of resonant tunneling processes involving a resonance level in the barrier, and the contribution of LDOS fluctuations to the conductance of such a device was discussed in the linear response regime. In recent vertical transport experiments on small-area double-barrier semiconductor structures [9–13], resonant tunneling between two heavily doped semiconductors (which can be regarded as disordered metals) through a single impurity level created below the lowest quantum well sub-band by a fluctuation in the density of charged donors was observed and identified, so that more attention should now be paid to a quantitative analysis to provide a basis for a quantitative comparison with the existing experimental data. In the present paper, we report the results of such an analysis.

Under the experimental conditions of Refs [9–13], the linear response regime was hardly relevant, since, at a zero bias, the energy of a discrete impurity level, E_0 does not initially coincide with the chemical potential μ_L in the bulk electrodes coming to the resonance only after the bias voltage reaches the threshold value $V_0(E_0)$. Being essentially nonlinear, the current–voltage $I(V)$ characteristics of such a device can be divided into three typical intervals [10–14]: below the threshold, where $I \approx 0$; the threshold regime $V = V_0(E_0) \pm \Gamma/ae$, where $I(V)$ takes a step after the resonant level crosses the Fermi level μ_L in the emitter; and the interval of a plateau, $V_0(E_0) < V < V_1(E_1)$, where the current remains almost constant until the next impurity level E_1 is lowered enough to contribute to the transport. In most of the samples studied in Refs [10–13], the emitter barrier is much stronger than the collector barrier, so that in the theoretical analysis one can neglect the influence of the Coulomb blockade effect of the resonant impurity level (which plays a crucial role if the barrier configuration is perfectly symmetric [15]). If so, the width of the resonance in the conductance Γ is dominated by the electron escape from impurity to collector, $\Gamma = \Gamma_R + \Gamma_L \approx \Gamma_R$, whereas the value of the current step is mainly determined by the tunneling rate Γ_L through the thick barrier on the emitter side. In this