an empirical procedure to determine the spin gap. Here the thermodynamic density of states $D^*(\mu)$ at $\nu \approx 1$ is compared to that at $\nu \approx 2$ in different magnetic fields. If the fields are chosen so that the gaps are equal, the dependences $D^*(\mu)$ can be expected to coincide. An example is given in the inset of Fig. 5. As can be seen from Fig. 6, the data points obtained with the help of all three methods are close to a straight line corresponding to a constant Landé factor $g \approx 5.2$. Within experimental uncertainty this behaviour cannot be described by a square-root dependence as indicated by the dashed line in Fig. 6.

The observed linear dependence of the spin gap on the magnetic field is very similar to that found in activation energy studies [12]. According to Ref. [12], the activation energy for $\nu = 1$ changes approximately linearly with magnetic field over the range 1.2 to 8 T. The corresponding g factor is about 7, which is appreciably larger than the value observed here. The difference is likely to be due to particularities of the activation energy method because at $\nu = 2$ the measured gap also exceeds the cyclotron splitting $\hbar\omega_c$ by 40%. Since, in theory, the gap values obtained by the activation technique may, because of disorder, only be smaller than gaps in the spectrum, the actual origin of the discrepancy remains to be seen. We note that optical investigations yield values of the spin gap at a filling factor $\nu = 1$ and of the cyclotron gap at $\nu = 2, 4$ [13], which are consistent with our data.

A simple estimate of the Coulomb exchange energy $e^2/\kappa l$ (where l is the magnetic length) gives values that are about an order of magnitude larger than the experimentally determined spin gaps. Two physical mechanisms may lead to decreasing exchange energy: the nonzero thickness of the 2DES and the disorder broadening of quantum levels. With the 2DES thickness and the level width as adjustable parameters, Smith et al. [14] succeed in describing the magnetic field dependence of the spin gap found in Ref. [12]. In our case, knowing the density of states we easily found the width and overlap of quantum levels [3]. The behaviour of the density of states $D(E)$ at $\nu = 1$ for the lowest magnetic field used is shown in the inset to Fig. 6. One can see that the corrections to the exchange energy due to level overlap do not exceed 1% at $B \geqslant 5$ T. As far as the finite thickness of the 2DES is concerned, it gives rise to a considerable decrease in the exchange energy at high magnetic fields while in the low-field limit its effect is negligible. Hence, this mechanism alone fails to provide an increase of the power of the theoretical square-root dependence $\Delta_s(B)$. Obviously, the approach [14] does not explain our experimental data at strong magnetic fields.

According to a recent model [15, 16], skyrmion-caused modification of the excitation spectrum at odd integer fillings results in a stronger change of Δ_s with magnetic field in the region of competition between the Zeeman and Coulomb energies. Measurements of the activation energy in tilted magnetic fields [17] indicate that the change of the spin gap attributed to skyrmion effects is smaller than 10% if $g\mu_B B/(e^2/\kappa l) \geqslant 0.015$. Using this condition we estimate that in our experiment the skyrmion effects can be neglected at $B \geqslant 5$ T. In our opinion, the theory's failure to explain the obtained experimental data is caused by the fact that the many-particle phenomena should be very sensitive to correlations of a disorder potential which is present in real systems. Thus, more theoretical work is needed taking into account disorder effects.

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Effective action and Green's function for a compressible QH edge state

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The edge of a quantum Hall (QH) system plays a central role in charge transport, because the edge states carry Hall current [1]. Also, for odd-denominator Landau level filling factors ν that correspond to incompressible quantized Hall states, the excitations on the edge form a strongly interacting one-dimensional system, which has drawn a lot of interest [2, 3]. The theoretical picture of the QH edge is based on chiral Luttinger liquid (χLL) models, involving either one or several chiral modes which may travel in the same or in opposite directions.

Another important part of the QH theory is the fermion-Chern–Simons approach, which can describe compressible QH states at even-denominator fractions such as $\nu = 1/2$, as well as the incompressible states [4, 5]. In this approach, the fractional QH effect is mapped onto the integer QH problem for new quasiparticles, composite fermions [6], which interact with a statistical Chern–Simons gauge field such that each fermion carries with it an even number p of quanta of the Chern-Simons magnetic flux. The structure of the edge can then be obtained from Landau levels for composite fermions in the average residual magnetic field [7].

Below we present a theory [12] of tunneling into the QH edge based on the composite–fermion picture. We find that under certain conditions the $I-V$ curve is described by a power-law $I \propto V^{\alpha}$. The tunneling exponent α depends only on the conductivity and interaction in the bulk, and is insensitive to the detailed structure of the edge. In this case the main effect results from the relaxation of electromagnetic disturbance caused by a tunneling electron, in which we include charge and current densities as well as the Chern–Simons field. The characteristic times and spatial scales involved in the dynamics are very large, which makes it possible to express the leading behavior in terms of measurable electromagnetic response functions, the longitudinal and Hall conductivities. Tunneling exponents can then be found as a function of the filling factor.

Effective action. The effect of interaction on the tunneling rate arises mainly due to relaxation of collective electrodynamical modes. To describe it, we employ the semiclassical effective action theory introduced elsewhere [15].

Below we list different parts of the action for our system. In imaginary time, the action for the composite fermion charge density $n(r,t)$ and current density $j_\alpha(r,t)$ reads:

$$S_n = \frac{1}{2}\sum_\omega \int \mathrm{d}^2 r \left[\frac{\rho^{(0)}_{\alpha\beta}}{|\omega|}\, j_{\alpha,-\omega}(r) j_{\beta,\omega}(r) + g n_{-\omega}(r) n_\omega(r) \right], \quad (1)$$

where ω is the Matsubara frequency. Here $\rho^{(0)}_{\alpha\beta}$ is the resistivity tensor, and $g = U + \kappa_0^{-1}$, the sum of the short–range interaction U and of the inverse compressibility (density of states) of the non-interacting system.

The coupling to the statistical gauge field is described by the Chern – Simons action:

$$S_{\mathrm{CS}} = \mathrm{i}\int \mathrm{d}t \int \mathrm{d}^2 r \left(n a_0 + \mathbf{j}\cdot\mathbf{a} + \frac{1}{4\pi p}\,\varepsilon^{\mu\nu\lambda} a_\mu\, \partial_\nu a_\lambda \right). \quad (2)$$

Note that because of charge continuity the current and charge densities are not independent. By solving the continuity equation, $\dot\rho + \nabla j = 0$, one writes n and j in terms of the displacement field:

$$n = -\nabla \mathbf{w}\,, \qquad \mathbf{j} = \dot{\mathbf{w}}\,. \quad (3)$$

The charge injected at the edge can be described by the source term localized on the boundary: $\dot w_y(y=0) = J(x,t)$, where $J = e\delta(x)\left[\delta(t+\tau) - \delta(t-\tau)\right]$ describes adding a composite fermion at the time $-\tau$ and removing it at the time τ. The corresponding part of the action is constructed by using a Lagrange multiplier:

$$S_\phi = \mathrm{i}\int \mathrm{d}x \int \mathrm{d}t\, \phi(x,t)(\dot w_y - J)\,. \quad (4)$$

Naively, the action is $S = S_n + S_{\mathrm{CS}} + S_\phi$. However, in this form the action is not gauge invariant because of charge non-conservation in the source term S_ϕ. To restore gauge invariance we add the term

$$S_{\mathrm{flux}} = \mathrm{i}\int \mathrm{d}x \int \mathrm{d}t\, a_0 \tilde J(x,t)\,, \quad (5)$$

where

$$\tilde J(x,t) = \int_{-\infty}^{t} J(x,t) = e\delta(x)\left[\theta(t+\tau) - \theta(t-\tau)\right].$$

Finally, the total action is $S_{\mathrm{total}} = S_n + S_{\mathrm{CS}} + S_\phi + S_{\mathrm{flux}}$.

The physical meaning of the term S_{flux} is the following. One notes that adding one electron to the system is equivalent to adding one composite fermion and p negative flux quanta. Thus term (4) describing the adding of one composite fermion must be supplied by a corresponding flux source term. One assumes that the time it takes the injected electron to bind p statistical gauge field quanta is very short on the overall relaxation time scale, and thus the process is described by $-p$ quanta left at the entrance point to be picked later, when the electron is removed. The term in the action describing this process is

$$\mathrm{i}\int_{-\tau}^{\tau} a_0(0,t)\,\mathrm{d}t\,,$$

which is the same as (5).

The dynamical equations are obtained by taking the variation of the action with respect to all variables, and eliminating the Lagrange multiplier ϕ. The resulting equations have the standard form:

$$\hat\rho^{(0)}\mathbf{j} = \mathbf{E}_{\mathrm{CS}} - \boldsymbol{\nabla}(gn)\,,$$

$$\frac{1}{2\pi p}\, E^\alpha_{\mathrm{CS}} = \varepsilon^{\alpha\beta} j^\beta\,,$$

$$\frac{1}{2\pi p}\, B_{\mathrm{CS}} = n + \tilde J\,, \quad (6)$$

where $\mathbf{E}_{\mathrm{CS}} = \boldsymbol{\nabla} a_0 + \dot{\mathbf{a}}$ and $B_{\mathrm{CS}} = \boldsymbol{\nabla}\times\mathbf{a}$. It is trivial to check that eliminating the Chern – Simons field leads to Ohm's law with a corrected resistivity tensor:

$$\hat\rho\mathbf{j} = -\boldsymbol{\nabla}(gn)\,, \quad (7)$$

where

$$\hat\rho = \hat\rho^{(0)} + \frac{ph}{e^2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (8)$$

is the measured resistivity tensor.

Integrating out variables in the bulk. To understand better the theory described by the action S_{total}, let us derive an effective one dimensional problem by integrating out the dynamics in the bulk, and keeping only the variables on the edge. The dynamic equations (7) in the halfplane $y > 0$ read:

$$\mathrm{i}\omega\rho_{\mu\nu} w_\nu = g\,\partial_\mu(\partial_\lambda w_\lambda)\,. \quad (9)$$

To express $\mathbf{w}(t,x,y)$ through the bulk values $\mathbf{w}(t,x,0)$ we go to the Fourier transform in x,

$$\mathbf{w}_\omega(x,y) = \int \mathbf{w}_{\omega,k}(y)\exp(\mathrm{i}kx)\,\frac{\mathrm{d}k}{2\pi}\,, \quad (10)$$

and solve (9) for w_x:

$$w_x = \mathrm{i}\,\frac{\rho_{xy}\omega - gk\,\partial_y}{\rho_{xx}|\omega| + gk^2}\, w_y\,. \quad (11)$$

The equation for $w_y(y)$ is $\left[q^2 - \partial_y^2\right] w_y = 0$, where $q^2 = k^2 + |\omega|/(g\sigma_{xx})$. In terms of the boundary value, $w_y(y) = w_y(0)\exp(-qy)$.

After substituting it into the action one gets

$$S = \int \frac{\mathrm{d}\omega\,\mathrm{d}k}{(2\pi)^2}\left\{ \frac{1}{2}\,\frac{|\omega| u_{-\omega} u_\omega}{(\sigma_{xx} q + \mathrm{i}\sigma_{xy} k\,\mathrm{sign}\,\omega)} - \phi_{-\omega}\left[\mathrm{i}\omega u + J(\omega)\right] \right\}, \quad (12)$$

where $u = w_y$ and $J(\omega) = 2\mathrm{i}e\sin(\omega\tau)$.

Finally, by integrating out u, we get

$$S = \frac{1}{2}\int \frac{\mathrm{d}\omega\,\mathrm{d}k}{(2\pi)^2}\left(\sigma_{xx} q|\omega| + \mathrm{i}\sigma_{xy} k\omega\right)\phi_{-\omega,-k}\phi_{\omega,k} + \mathrm{i}\int J(x,t)\phi(x,t)\,\mathrm{d}x\,\mathrm{d}t\,. \qquad (13)$$

This effective action represents a generalization of the standard Luttinger liquid theory of the edge mode to the compressible problem with finite σ_{xx}. Because of the relation between q and ω, the dissipative term in the action (13) is non-local in the time representation. In the incompressible case, $\sigma_{xx} = 0$, we recover the standard χLL action:

$$S = \frac{\mathrm{i}\nu}{4\pi}\int \partial_x\phi\,\partial_t\phi\,\mathrm{d}x\,\mathrm{d}t + \mathrm{i}\int J(x,t)\phi\,\mathrm{d}x\,\mathrm{d}t\,. \qquad (14)$$

In the above derivation we ignored the effects of boundary compressibility. Taken into account, these effects lead to an additional term of the form

$$\int (\partial_t\phi)^2\,\mathrm{d}x\,\mathrm{d}t\,,$$

which does not affect the long-time dynamics, and drops from the final answer.

Let us remark that we derived the action (13) with a source term. Thus, the electron creation operator can be written as $\psi^+(x,t) = \exp\left[\mathrm{i}\phi(x,t)\right]$, which coincides with the standard one-dimensional expression.

Instanton action. To find the tunneling exponent, we need to evaluate the Green's function $\left\langle \psi(0,t)\psi^+(0,0)\right\rangle$ of an electron. From the above relation of $\psi(x,t)$ to the bosonic variable $\phi(x,t)$, the Green's function can be written as

$$\left\langle \exp\left[\mathrm{i}\int J(x,t)\phi(x,t)\,\mathrm{d}x\,\mathrm{d}t\right]\right\rangle .$$

Evaluating the average with the action (13) is now straightforward. One gets $G(t) = \exp(-S)$, where

$$S = \frac{1}{2}\sum_{k,\omega}\frac{\left|J(\omega)\right|^2}{|\omega|(\sigma_{xx} q + \mathrm{i}\sigma_{xy} k\,\mathrm{sign}\,\omega)}\,. \qquad (15)$$

The integration over k gives a log-divergent answer which we cut at k_{max}:

$$S = \int \frac{\mathrm{d}\omega}{|\omega|}\left|J(\omega)\right|^2\left[\frac{\rho_{xx}}{8\pi^2}\ln\frac{g k_{\mathrm{max}}^2}{|\omega|} + \frac{1}{4\pi^2}\rho_{xy}\theta_{\mathrm{H}}\right]. \qquad (16)$$

Note that this expression does not vanish even in the absence of the Chern–Simons field and the interaction ($p = 0$, $U = 0$), which indicates that part of the answer represents the contribution of non-interacting composite fermions and must be subtracted off. The physical origin of the ultraviolet divergence at k_{max} is that for free fermions the relaxation is fast and involves large momenta $k \sim k_{\mathrm{F}}$. On the other hand, the contribution resulting from the interaction does not diverge at large momenta. To single it out, we subtract from Eqn (16) the same expression with $p = 0$ and $U = 0$. Integrating the difference over ω, we get $S - S_0 = \alpha\ln(t/t_0)$, where α is the tunneling exponent discussed below [see Eqn (17)], and t_0 is the smallest of the relaxation times, of the order of the scattering time.

Tunneling exponent. In evaluating α, we assume that the number of flux quanta carried by composite fermions is $p = 2$ for $1/3 < \nu < 1$ and $p = 4$ for $1/5 < \nu < 1/3$. Also, for simplicity, we assume that the composite fermions have 'bare' conductivities $\rho_{xx}^{(0)}$ and $\rho_{xy}^{(0)}$ which are constants that may depend, for instance, on the density, but which are independent of temperature. The measured resistivities are then $\rho_{xy} = \rho_{xy}^{(0)} + ph/e^2$ and $\rho_{xx} = \rho_{xx}^{(0)}$. The theory predicts the power-law $I \propto V^{\alpha}$, with

$$\alpha = 1 + \frac{2e^2}{\pi h}\left[\theta_{\mathrm{H}}\rho_{xy} - \theta_{\mathrm{H}}^{(0)}\rho_{xy}^{(0)}\right] + \frac{e^2\rho_{xx}}{\pi h}\ln\frac{g\kappa_0\sigma_{xx}}{\sigma_{xx}^{(0)}}\,, \qquad (17)$$

where $\theta_{\mathrm{H}} = \tan^{-1}(\rho_{xx}/\rho_{xy})$ is the Hall angle, $\theta_{\mathrm{H}}^{(0)}$ is the corresponding bare Hall angle. Other notations in Eqn (17) are as given above: $g = U + \kappa_0^{-1}$, where U is the short-range interaction constant, and $\kappa_0 = m^*/2\pi\hbar^2$ is the bare composite–fermion compressibility, determined by the effective mass m^*, which we treat as a constant.

In Figure 1, the exponent α is plotted as function of ρ_{xy} which is proportional to the electron density. The tunneling exponent (17) is a continuous and monotonically increasing function of ρ_{xy}. It is interesting, that in the limit $\rho_{xx} = 0$ the exponent α has cusp-like singularities at $\nu = 1/2$ and $\nu = 1/4$, which correspond to $\rho_{xy} = 2, 4$ in Fig. 1. To understand this, consider the vicinity of $\nu = 1/2$, where the QH state can be described as a Fermi liquid of composite fermions carrying two flux quanta each, and exposed to a 'residual' magnetic field $\delta B = 2 - \nu^{-1}$. At $\nu < 1/2$ the residual field direction coincides with the total field, and all edge modes propagate in the same direction [7]. On the other hand, at $\nu > 1/2$, the structure of the edge is qualitatively different, consisting of modes going in opposite directions. This effect makes $\nu = 1/2$ a singular density.

Of course, scattering by disorder will smear the singularity. However, it is interesting that the change in the tunneling exponent (17) resulting from finite ρ_{xx} can be either positive or negative, depending on the value of $U\kappa_0$ (see Fig. 1).

The main difference of our results from those of the χLL theory [2, 3] is that we get a *continual* tunneling exponent, whereas the χLL theory provides results for a discrete set of

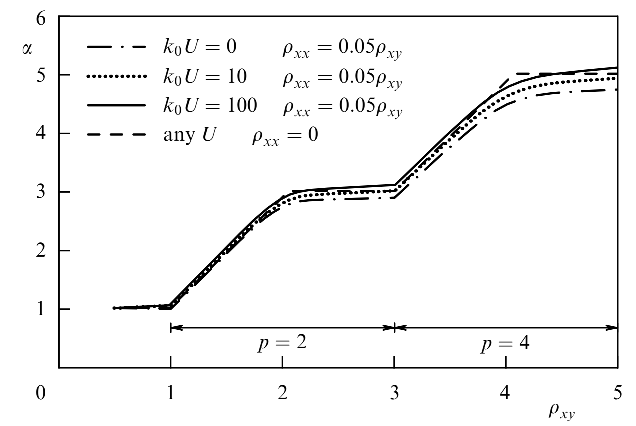

Figure 1. Tunneling exponent (17) shown as a function of ρ_{xy} with the composite-fermion flux $p = 2$ for $1 \leqslant \rho_{xy} \leqslant 3$ and $p = 4$ for $3 \leqslant \rho_{xy} \leqslant 5$. A constant Hall angle is assumed. For $\rho_{xx} = 0.05\rho_{xy}$ the exponent is plotted for three values of the model short-range interaction U. At $\rho_{xx} = 0$ the exponent is universal (no U dependence), but at finite ρ_{xx} it can be either bigger or smaller than the universal result, depending on the interaction strength $\kappa_0 U$.

incompressible densities. Moreover, the χLL theory is constructed in a different way for each incompressible filling, and provides a picture supposedly valid for densities around those fillings. For each such rational density it predicts a plateau in both the resistance and tunneling exponent. At the same time, for densities deviating sufficiently strongly from those 'good densities' the χLL theory does not give any prediction for the tunneling density of states or for the $I(V)$ curve.

However, in the domain of overlap, our results agree with the predictions of the χLL theory. Below we review the χLL theory results. In this theory, the edge of an incompressible quantized Hall state is described as a single- or multi-channel χLL. For Jain filling fractions $\nu = n/(np+1)$ with positive integer n and even p, there are n edge modes, all traveling in the same direction. In this case, Wen [2] found universally $\alpha = p+1$. By contrast, for states such as the Jain fractions with negative n, where the edges have modes going in opposite directions, α may depend on the form of the interaction. Nevertheless, Kane, Fisher and Polchinski [3] found in this case that if there is sufficient scattering between the channels, the system scales to a universal limit, with $\alpha = p + 1 - 2/|n|$.

Our result (17), taken at large Hall angle ($\theta_{\mathrm{H}} = \theta_{\mathrm{H}}^{(0)} = \pi/2$), agrees with the χLL theories discussed above. At large Hall angles and for the Jain fractions $\nu = n/(pn \pm 1)$ we use the following values of the Hall and Ohmic resistance:

$$\rho_{xy} = \left(p + \frac{1}{n}\right)\frac{h}{e^2}\,, \qquad \rho_{xy}^{(0)} = \frac{h}{ne^2}\,, \qquad \rho_{xx} = \rho_{xx}^{(0)} = 0\,. \quad (18)$$

By substituting this into Eqn (17), one gets $\alpha = 1 + |p + 1/n| - 1/|n|$, which agrees with the universal tunneling exponents found by Wen and by Kane et al.

Comparison with experiment. Recently, the physics of the edge has been probed experimentally by a tunneling conductivity measurement performed on cleft edge overgrowth systems [8–11]. In this case the 2D electron system has a sharp edge, and the confining potential is very smooth, with the residual roughness of atomic scale. With the advantage of the high quality of the system, one can explore tunneling into both incompressible and compressible QH states [9]. It is found that the tunneling conductivity is non-Ohmic at all densities $\nu < 1$. The $I - V$ curve is described by a power law $I \propto V^{\alpha}$, for $V > 2\pi k_{\mathrm{B}} T/e$, where the exponent α is a continuously increasing function of $1/\nu$ [10, 11].

The experimentally measured α is shown in Fig. 2 (courtesy Grayson and Chang). Over a large range of densities α has a striking linear dependence on $1/\nu$.

Since the density profile near the edge is sharp, our results should be applicable to this system. The experimentally measured Hall angle is large: $\rho_{xy}/\rho_{xx} \simeq 30$. Therefore, in the comparison with the theory one can ignore the effect of finite ρ_{xx}. The theoretical result for large Hall angles is shown in Fig. 2 in a broken line. As a function of the density $1/\nu$, both the experimental and the theoretical exponent come out to be a smooth and monotonically growing function. Apart from this general similarity, the details of the experimental and theoretical exponents' functional dependence are completely different. There are two main distinctions:

(i) the difference in the overall average slope α vs. $1/\nu$;

(ii) the plateau with $\alpha = 3$ in the theoretical curve which is absent in the experimental curve.

The first difference is probably explained (at least partially) by the electrostatic effect of density enhancement near the edge [10, 11]. Numerical simulations [10, 11] show that the density near the edge is about 12–16% larger than the density within the system. When the values of ν are rescaled accordingly, the average slope of the experimental curve, as well as the knee around $\nu = 1$, shifts closer to the theoretical values.

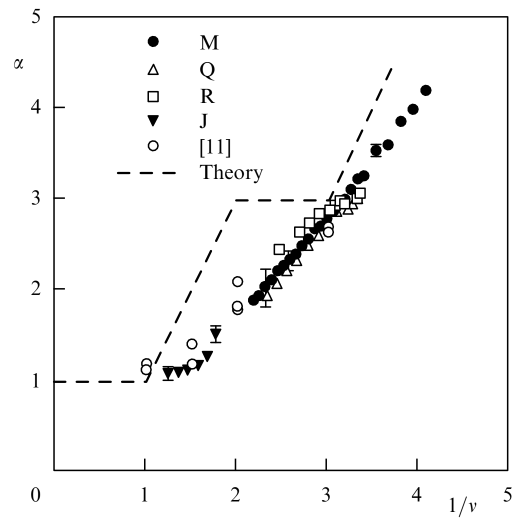

Figure 2. Experimentally measured values of the tunneling exponent α are shown (courtesy Chang and Grayson [11]). Different symbols represent different groups of samples. Theoretical curve for a large Hall angle (broken line) is shown for comparison.

The second difference is obviously more vital for the physical picture of what is going on at the QH edge. At present, there is no understanding of why there is no plateau in the measured curve. Obviously, to clarify this issue it would be extremely useful to have more data on tunneling exponents in other QH systems.

One theoretical possibility to mention, which could correct the theoretical exponent, is related to orthogonality catastrophe effects in the composite Fermi liquid. The theory presented above only accounts for the effects of shakeup of electromagnetic modes arising from small k and small ω. There may exist, however, some corrections to the spectral density of composite fermions arising from large k, i.e., from small distances. Some evidence that this might be the case is seen in the divergences found in the perturbation theory of interacting composite fermions [13, 14].

Also, having in mind the disagreement with experiment, it is of interest to think of our calculation from the point of view of its robustness to changes in the notion of a quasiparticle. Over the past years, a number of different quasiparticles in the QH problem have been proposed, such as fractional charges of several kinds and, more recently, composite fermions. It would be interesting to try to anticipate what would happen if it turns out that the quasiparticles near the edge are different from those in the bulk.

Looking from this angle, it is clear that some part of the calculation which has to do with the effective action for tunneling, is entirely robust. What may be subject to change is the expression for the electron creation operator in terms of the quasiparticle creation operator, which is used for evaluating the tunneling density of states. Under such a change, the spectral density will always remain a power law, but the exponent may change.

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Recording zero-point current and voltage fluctuations

G B Lesovik

In this paper we consider various methods for measuring current fluctuations. Our aim is to reveal a quantity which can be measured in treating fluctuations and transfer statistics on the whole. The answer is known for an average current where the quantity sought for is the averaged current operator $\langle I\rangle = \mathrm{Tr}\,\{\rho I\}$ in view of the validity of the ergodic hypothesis. The situation with current–current correlators is far less clear, since the operators should be arranged in time (in general, current operators at different times are not commutative).

In fact, this problem is reduced to the measurement of vacuum current fluctuations, and similar to the one of recording photons in optics and the measurement of vacuum electromagnetic fluctuations, although there is a significant difference.

Considerable recent attention has been focussed on the measurement of zero-point current and voltage oscillations including the very possibility of such measurements. Current fluctuations have been studied both theoretically and experimentally [1–5]. In Refs [6, 7] measurements were performed at frequencies at which zero-point oscillations can arise at practically attainable temperatures ($\hbar\omega > k_{\mathrm{B}}T$).

On the other hand, paper [8] attracted considerable interest to the possible breaking of the phase of conducting electrons by vacuum fluctuations of an electric field, which can significantly modify the localization behaviour at zero temperature.

To begin, let us consider the spectral density of fluctuations.

1. Measurement of spectral density of noise with a resonance circuit. Current fluctuations of finite frequency are usually measured by one of two main methods. In the first, the current is measured as a time-dependent function $I(t)$, for example, with a normal ammeter, and then the spectral density $S(\omega)$ is calculated numerically using a Fourier transformation.

The classical equation of motion for an ammeter coincides with the equation for an oscillator with friction and external force $\propto I(t)$

$$\ddot{\phi} = -\Omega^2\phi - \gamma\dot{\phi} + \lambda I(t)\,. \tag{1}$$

Making the Fourier transformation, we express the angle-angle correlator as:

$$\langle\phi_\omega\phi_{-\omega}\rangle = \frac{\lambda^2 I_\omega I_{-\omega}}{(\Omega^2-\omega^2)^2+\omega^2\gamma^2}\,. \tag{2}$$

To eliminate proper oscillations, it is usually assumed that $\gamma \gg \Omega_1 = (\Omega^2-\gamma^2/4)^{1/2}$.

The method is appropriate for recording ultra-low frequency noise, for instance, flicker noise, but, for various reasons, it cannot be used at high frequencies. For example, as in the case of voltage measurements with a discrete voltmeter there is a 'dead' time during which the device cannot record changes in current (below we consider the measurement of a time-dependent current–current correlator with an ammeter).

In recording high frequencies it is more suitable to use a resonance circuit (RC) as a detector coupled by inductance with the investigated conductor so that the RC is not affected by dc.

In this case the detector can still be described by Eqn (1), but now the external force is proportional to the derivative of the measured current $\lambda\dot{I}(t)$, and the circuit quality should be high, so $\gamma \ll \Omega$.

Then the detector response is a changed charge at the capacitor, $\phi \to Q$,

$$Q^2 = \int \mathrm{d}\omega\,\frac{\lambda^2\omega^2 I_\omega I_\omega}{(\Omega^2-\omega^2)^2+\omega^2\gamma^2}\,. \tag{3}$$

We have considered the same system in quantum-mechanical terms [1], assuming the circuit to have a certain temperature T_{LC}. Treating the RC as an oscillator with infinitely small damping η, we have found the correction to squared charge fluctuations, which is of second order with respect to the inductance coupling constant. The result can be formulated as follows: the measurable response of the considered detector at the resonance frequency Ω is expressed via current correlators as:

$$S_{\mathrm{meas}} = K\Big\{S_+(\Omega) + N_\Omega\big[S_+(\Omega) - S_-(\Omega)\big]\Big\}\,, \tag{4}$$

where we introduce the notations

$$S_+(\Omega) = \int \mathrm{d}t\,\big\langle I(0)I(t)\big\rangle \exp(\mathrm{i}\Omega t)\,,$$

$$S_-(\Omega) = \int \mathrm{d}t\,\big\langle I(t)I(0)\big\rangle \exp(\mathrm{i}\Omega t)\,.$$