We can introduce the generalized momentum $P_i = \partial L/\partial \dot{X}_i$ and quantize the problem, assuming the standard commutation relations $[P_i X_i] = i\hbar$ to be fulfilled. This yields the cyclotron frequency of the skyrmion as a whole $\hbar\omega_s = 2\gamma$ and the minimum energy $E_s = \gamma$. Thus, in experiments the skyrmion should exhibit a cyclotron resonance at a frequency 2γ. The quantity γ is determined as the exchange energy per electron, in the case of a completely filled Landau level

$$\gamma = \frac{e^2}{l_B}\sqrt{2\pi}\,.$$

The energy γ must be added to expression (5) obtained above for the total energy.

It is also interesting to find a term with the Hopf invariant in the action which, according to modern views, determines the skyrmion statistics [11]. For this purpose we should calculate the terms containing one time-dependent Ω_t^l and two space-dependent Ω^l in the expansion of the action in terms of Ω^l. Calculations up to the third order are rather cumbersome and require the consideration of numerous diagrams, the second-order diagrams also contributing, since their non-local character in time and space must be taken into account [9, 10]. We present only the final result corresponding to the 'fermion' character of vortex-skyrmions:

$$S_H = \pi H\,, \qquad H = \frac{1}{2\pi^2}\int e^{ljm}\Omega_t\Omega_j \times \Omega_l\,\mathrm{d}^2r\,\mathrm{d}t\,, \tag{7}$$

where e^{ljm} is the unit antisymmetric third-rank tensor. The integer Hopf invariant H is expressed in terms of Ω^l [12]. This result does not coincide with that obtained within the method of Landau functions projected onto the zero level [7] which is a sum of several spatial derivatives. At the same time, formula (7) has a standard form and agrees with that suggested in note [13].

In conclusion we emphasize once again that the solution of differential equations of the Hartree – Fock approximation is necessary, since it determines the discrepancy between our results and those obtained by projection onto a single Landau level, when the differential form of the kinetic energy in the Schrödinger equation is replaced by a constant energy. It is of special importance in the calculation of the thermodynamic energy of the vortex-skyrmion where the additional term $(-\hbar\omega_c/2)Q$ appears, which can lead to spontaneous appearance of vortices and a rearrangement of the ground state in a rather strong magnetic field.

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Microscopic derivation of the effective Lagrangian for skyrmions in an interacting two-dimensional electron gas at small *g*-factor

W Apel, Yu A Bychkov

1. Introduction

Electronic systems confined to two dimensions and exposed to a strong magnetic field continue to be studied intensively in both experiment and theory [1]. Due to the magnetic field, the electronic single particle energies form degenerate Landau levels and for the physical properties the electron – electron interaction is crucial. Recently, the spin degree of freedom has attracted a lot of attention. For a long time, it was accepted that the basic excitations are of particle – hole kind (spin-excitons), when the cyclotron energy is much larger than the characteristic Coulomb energy [2 – 5]. However, recent experiments performed at or near a filling factor of one, where one spin-split Landau level is completely filled, have changed this picture. The activation energy of the resistance measured under pressure [6], the spin polarization measured by magnetoabsorption spectroscopy [7], transport experiments in a tilted magnetic field [8], and measurements of the Knight shift with optically pumped NMR [9] are all taken as evidence that there are new basic excitations, the skyrmions. Theoretically, excitations of this kind have been studied before in the context of two-dimensional isotropic ferromagnets [10]. Only recently was it shown that one also has skyrmion quasiparticles in an interacting electron system in a magnetic field, provided the g-factor is smaller than a critical value [11]. The energy needed to create a skyrmion – antiskyrmion pair for $g \to 0$ is only half the energy needed to create a single spin-exciton with very large momentum. The charge of a skyrmion is the electron charge e. The number of reversed electron spins contained in a skyrmion was calculated in the Hartree – Fock (HF) approximation [12]; the value depends on the g-factor and is larger than one. Very recently, the quantum nature of the skyrmion quasiparticle, i.e. its spin, was derived [13] from a microscopic model by the generalization of a method used earlier [14] to derive the Hamiltonian part of the effective Lagrangian.

This paper is organized as follows: in the next section, we introduce, together with the model, our notation. Then, we summarize the derivation of the effective Lagrangian which was already partly described in previous works [14, 13]. The two following sections are devoted to a short discussion of the equations of motion and the energy – momentum tensor. In the last section, we derive a criterion for the applicability of the HF approximation.

2. Effective Lagrangian

We are studying interacting electrons in two dimensions moving in a strong magnetic field. The orbital states of the electrons are confined to the lowest Landau level; we use the

Landau gauge. Let $\hat{a}_p^\dagger$ ($\hat{b}_p^\dagger$) denote the creation operator of a state with single particle quantum number p and spin projection parallel (antiparallel) to the magnetic field. Then, the Hamiltonian reads

$$\hat{H} = \frac{1}{2}\sum_{\mathbf{q},\,p_1,\,p_2} \tilde{V}(q)\exp\left[\mathrm{i}q_x(p_2' - p_1)\right] \times \left[\hat{a}_{p_1}^\dagger \hat{a}_{p_2}^\dagger \hat{a}_{p_2'}\hat{a}_{p_1'} + (\hat{a}\to\hat{b}) + 2\hat{a}_{p_1}^\dagger \hat{b}_{p_2}^\dagger \hat{b}_{p_2'}\hat{a}_{p_1'}\right]. \qquad (1)$$

Here $\tilde{V}(q) = \exp(-q^2/2)V(q)$, where $V(q)$ is the electronic interaction; $p_{1,2}' = p_{1,2} \mp q_y$. The magnetic length l_H is used as the unit for lengths and $\hbar$ is set to 1. The Zeeman term can easily be added to (1); here it is omitted for the sake of brevity.

2.1 Hartree–Fock approximation

Next, we describe the HF approximation. The HF ground state is the Slater determinant of single particle states $|\Psi_{\mathrm{HF}}\rangle = \Pi_p \hat{A}_p^\dagger|0\rangle$. The creation operators for states in the lower (upper) HF band, $\hat{A}_p^\dagger$ ($\hat{B}_p^\dagger$), are determined from those of the original electronic states by

$$\hat{A}_p = \sum_{p_1}(U_{p,p_1}\hat{a}_{p_1} + V_{p,p_1}\hat{b}_{p_1}),$$
$$\hat{B}_p = \sum_{p_1}(W_{p,p_1}\hat{a}_{p_1} + X_{p,p_1}\hat{b}_{p_1}), \qquad (2)$$

where the matrices

$$\hat{U} = \exp\left(\tfrac{\mathrm{i}\hat{\psi}}{2}\right)\cos\tfrac{\hat{\theta}}{2}\exp\left(\tfrac{\mathrm{i}\hat{\phi}}{2}\right), \qquad \hat{V} = \exp\left(\tfrac{\mathrm{i}\hat{\psi}}{2}\right)\sin\tfrac{\hat{\theta}}{2}\exp\left(-\tfrac{\mathrm{i}\hat{\phi}}{2}\right),$$
$$\hat{W} = -\exp\left(-\tfrac{\mathrm{i}\hat{\psi}}{2}\right)\sin\tfrac{\hat{\theta}}{2}\exp\left(\tfrac{\mathrm{i}\hat{\phi}}{2}\right), \quad \hat{X} = \exp\left(-\tfrac{\mathrm{i}\hat{\psi}}{2}\right)\cos\tfrac{\hat{\theta}}{2}\exp\left(-\tfrac{\mathrm{i}\hat{\phi}}{2}\right) \qquad (3)$$

are parametrized by the Hermitian matrices $\hat{\psi}$, $\hat{\theta}$, and $\hat{\phi}$. We define the elements of the latter to be the matrix elements of the angular functions $\psi(\mathbf{r})$, $\theta(\mathbf{r})$, and $\phi(\mathbf{r})$ taken with the states of the lowest Landau level. Thus, any HF trial state is given by a choice of these three Euler angles.

From now on, we confine ourselves to HF states which vary slowly in space. Then, we can express the physical quantities in question by a gradient expansion. First, we consider the gradient expansion for the HF expectation values of the density $N(\mathbf{r})$ and the spin-density $\mathbf{S}(\mathbf{r})$. Starting from the standard definition (cf. Ref. [14]), we express these expectation values by $\hat{U}$ and $\hat{V}$, i.e., by $\hat{\psi}$, $\hat{\theta}$, and $\hat{\phi}$. Representing these matrices by the functions $\psi(\mathbf{r})$, $\theta(\mathbf{r})$, and $\phi(\mathbf{r})$ and making repeated use of the composition law Eqn (20) in Ref. [14], we finally find the results

$$N(\mathbf{r}) = \frac{1}{2\pi} + \delta N(\mathbf{r}) + O(\nabla^6),$$
$$\delta N(\mathbf{r}) = \frac{1}{4\pi}\,\mathbf{n}(\mathbf{r})\cdot\partial_x\mathbf{n}(\mathbf{r})\times\partial_y\mathbf{n}(\mathbf{r}) \qquad (4)$$

and

$$\mathbf{S}(\mathbf{r}) = \frac{1}{4\pi}\,\mathbf{n}(\mathbf{r}) + O(\nabla^4). \qquad (5)$$

The density and spin-density are thus determined by a unit vector $\mathbf{n}$,

$$\mathbf{n} \equiv (\sin\bar{\theta}\cos\bar{\phi}, \sin\bar{\theta}\sin\bar{\phi}, \cos\bar{\theta}). \qquad (6)$$

Here, the overbar denotes the average over the area of one flux quantum, i.e., (we restore the physical dimensions)

$$\bar{\theta}(\mathbf{r}) = \int\frac{\mathrm{d}^2h}{\pi l_H^2}\exp\left(-\frac{h^2}{l_H^2}\right)\theta(\mathbf{r}+\mathbf{h}). \qquad (7)$$

Quite naturally, the magnetic length appears here as the lower cut-off for the wavelength of spatial variations of density and spin-density. Expressing everything in terms of the averaged Euler angles simplifies the gradient expansion considerably since the next leading corrections vanish [$O(\nabla^4)$ in Eqns (4) and $O(\nabla^2)$ in Eqn (5)].

Now, we are ready to formulate the effective Lagrangian $\mathcal{L}$. We consider a time dependent HF state, parametrized by the time dependent functions ψ, θ, and ϕ. Then, $\mathcal{L}$ consists of a kinetic and a Hamiltonian part, $\mathcal{L} = \mathcal{L}_k - \mathcal{L}_{\mathrm{H}}$.

2.2 Hamiltonian part $\mathcal{L}_{\mathrm{H}}$ of the effective Lagrangian

We first discuss the Hamiltonian part

$$\mathcal{L}_{\mathrm{H}} = \langle\Psi_{\mathrm{HF}}|\hat{H}|\Psi_{\mathrm{HF}}\rangle. \qquad (8)$$

Since we want to study the interactions of the skyrmions, we need to push the gradient expansion up to fourth order. One has to insert the density and spin-density into the HF approximation of $\hat{H}$, Eqn (7) of Ref. [13]. This becomes a tedious task – even with the simplifications expressed in Eqns (4) and (5), where the terms of the next leading order are missing — since one needs the fourth order terms in the spin-density. Collecting all the terms up to fourth order in spatial derivatives, we finally arrive at a result which is surprisingly simple:

$$\mathcal{L}_{\mathrm{H}} = \frac{E(0)}{8\pi}\int\mathrm{d}^2r\left\{\frac{1}{4}\sum_{\alpha=x,y}(\partial_\alpha\mathbf{n})^2 - \mathbf{n}\cdot\partial_x\mathbf{n}\times\partial_y\mathbf{n}\right\} - \frac{3E(0)}{2^9\pi}\int\mathrm{d}^2r(\Delta\mathbf{n})^2 + \frac{1}{2}\sum_{\mathbf{q}}\tilde{V}(q)\left|\delta N(\mathbf{q})\right|^2. \qquad (9)$$

Here, $E(0)$ is found from

$$E(\mathbf{r}) = \int\frac{\mathrm{d}^2q}{(2\pi)^2}\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})\tilde{V}(q), \qquad (10)$$

and $\delta N(\mathbf{q})$ is the Fourier transform of $\delta N(\mathbf{r})$. This expression for $\mathcal{L}_{\mathrm{H}}$ is valid up to and including the fourth order of spatial derivatives.

We have omitted a term of fourth order which is a total derivative. Now, we demonstrate that this total derivative yields a zero contribution to $\mathcal{L}_{\mathrm{H}}$ for any solution of the equations of motion [with a fixed value of the charge $Q = \int\mathrm{d}^2r\,\delta N(\mathbf{r})$]. In this case, the term in question can be expressed by the density δN as

$$\int\mathrm{d}^2r\,\Delta\delta N(\mathbf{r}) \to 0. \qquad (11)$$

The total derivative does not contribute to the effective Lagrangian, because δN is non-singular. We have also checked explicitly the case of a solution with $Q = 1$, now for a finite Zeeman coupling in the Hamiltonian and found again that there is no contribution from this total derivative term.

The various parts in $\mathcal{L}_{\mathrm{H}}$ have been discussed in Refs [14, 13]. Here, we want to concentrate on the term of fourth order in spatial gradients in Eqn (9),

$$\int \mathrm{d}^2 r (\Delta \mathbf{n})^2 . \qquad (12)$$

This term leads to an additional interaction between the skyrmions appearing as solutions of the equations of motion, see below. We treat (12) in a perturbative way. We consider an unperturbed solution, which describes two skyrmions separated by a distance ρ. Then, we evaluate (12) and get the following additional skyrmion – skyrmion interaction energy:

$$V_{\mathrm{Sk-Sk}} = \mathrm{const} - \frac{E(0)}{2}\,\rho^{-2} + O(\rho^{-4}) . \qquad (13)$$

This interaction decays as the second power with the distance. Hence, it can be neglected in comparison with the Coulomb energy in the absence of screening, but it could be important for a more realistic screened Coulomb interaction.

2.3 Kinetic part $\mathcal{L}_{\mathrm{k}}$ of the effective Lagrangian

We now turn to the calculation of the kinetic part $\mathcal{L}_{\mathrm{k}}$,

$$\mathcal{L}_{\mathrm{k}} = \langle \Psi_{\mathrm{HF}} | \mathrm{i}\partial_t | \Psi_{\mathrm{HF}} \rangle . \qquad (14)$$

Starting from this expression, one has to perform the gradient expansion up to and including the second order in spatial gradients. Again, with repeated use of the composition law Eqn (20) in Ref. [14] this is a straight-forward calculation; there is just one technical detail. As already indicated in Ref. [13], one should carefully avoid changing the order of summation over the eigenstates (indices of the matrices), since these sums do not converge absolutely. Proceeding as described in Ref. [13] we find the following result:

$$\mathcal{L}_{\mathrm{k}} = \frac{1}{4\pi}\int \mathrm{d}^2 r \left[\partial_t \bar{\psi}(\mathbf{r},t) + \cos\bar{\theta}(\mathbf{r},t)\,\partial_t \bar{\phi}(\mathbf{r},t)\right]$$
$$- \frac{1}{16\pi}\int \mathrm{d}^2 r \left\{ \frac{\partial(\bar{\psi}, \cos\bar{\theta}, \bar{\phi})}{\partial(t,x,y)} - \partial_t \left[\cos\bar{\theta}\, \frac{\partial(\bar{\phi},\bar{\psi})}{\partial(x,y)} \right] \right\} . \qquad (15)$$

Note, that while the Hamiltonian part of the effective Lagrangian is determined by the unit vector $\mathbf{n}$, all the Euler angles ψ, θ, and ϕ enter the kinetic part.

This expression for $\mathcal{L}_{\mathrm{k}}$ is valid in first order time derivatives up to and including the second order spatial derivatives. In the derivation, we have omitted a term

$$\mathrm{i}\partial_t \int \mathrm{d}^2 r\, \delta N(\mathbf{r}) \to 0 . \qquad (16)$$

This does not contribute to the effective Lagrangian since it is the derivative of the topological charge Q. The term $\cos\bar{\theta}(\mathbf{r},t)\,\partial_t\bar{\phi}(\mathbf{r},t)$ in $\mathcal{L}_{\mathrm{k}}$ determines the dynamics of the field $\mathbf{n}$ (see below). The appearance of the Hopf term (first term in the second line in $\mathcal{L}_{\mathrm{k}}$) was thoroughly discussed in Ref. [13]. The prefactor of the Hopf term, $\Theta = \pi$ determines the spin of the skyrmion. Following the arguments in the work of Wilczek and Zee [15], we find that the skyrmion carries a spin of $1/2$.

3. Equations of motion

In this section, we discuss the equations of motion for the spin-density $\mathbf{n}(\mathbf{r},t)$ following from the effective Lagrangian $\mathcal{L} = \mathcal{L}_{\mathrm{k}} - \mathcal{L}_{\mathrm{H}}$ [cf. Eqns (15), (9)]:

$$\partial_t \mathbf{n} = \mathbf{n} \times \frac{1}{4}\,E(0)\left(\Delta\mathbf{n} + \frac{3}{16}\,\Delta\Delta\mathbf{n}\right)$$
$$- (\partial_x \mathbf{n}\,\partial_y - \partial_y \mathbf{n}\partial_x)\int \mathrm{d}^2 r'\, E(\mathbf{r}-\mathbf{r}')\,\delta N(\mathbf{r}',t) . \qquad (17)$$

Here, we have omitted the arguments, $\mathbf{r}$ and t, of the field $\mathbf{n}$. The dynamics of the unit vector $\mathbf{n}$ are determined by the term $\cos\bar{\theta}\,\partial_t\bar{\phi}$ in $\mathcal{L}_{\mathrm{k}}$. All the other parts of $\mathcal{L}_{\mathrm{k}}$ are either total time derivatives or of a topological character like the Hopf term and do not contribute to the equations of motion. As is well known, the value of the charge Q is conserved since this is a topological invariant describing the mapping $S_2 \to S_2$. Hence, the term proportional to Q in $\mathcal{L}_{\mathrm{H}}$ (the second term in the first line) also does not contribute to the equations of motion. Further, the solutions can be classified according to the value of Q.

3.1 Static solutions

The effective Lagrangian describes both spin-excitons and skyrmions (spin-textures). Spin-excitons are small angle fluctuations of $\mathbf{n}(\mathbf{r})$ around the state $\mathbf{n}(\mathbf{r}) = \mathrm{const}$ belonging to $Q = 0$. Skyrmions, on the other hand, are large angle solutions of the equations of motion (for a very small g-factor) with a given non-zero value of the topological charge Q. As an example, Fig. 1 shows how the spin rotates as one moves radially through the center of such a skyrmion ($Q = 1$). The skyrmion solutions have been well studied in the literature, cf. Refs [10, 16, 11].

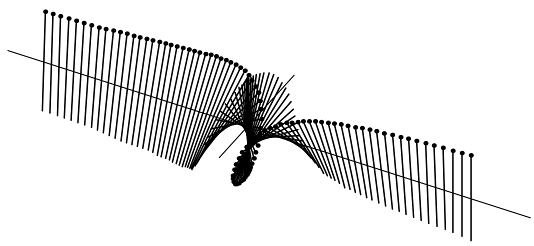

Figure 1. Spin-density $\mathbf{n}(\mathbf{r})$ for a Skyrmion state, shown as a function of $\mathbf{r} = (x, 0)$.

3.2 Electric field

In the presence of a static and homogeneous electric field $\mathbf{E}_{\mathrm{ext}}$, the energy

$$-\int \mathrm{d}^2 r\, e\mathbf{E}_{\mathrm{ext}}\mathbf{r}\,\delta N(\mathbf{r},t) \qquad (18)$$

is added to $\mathcal{L}_{\mathrm{H}}$. In the equations of motion (17), this gives an extra term on the r.h.s.:

$$\frac{l_{\mathrm{H}}^2}{\hbar}\,(eE_{\mathrm{ext}}^y\,\partial_x\mathbf{n} - eE_{\mathrm{ext}}^x\,\partial_y\mathbf{n}) . \qquad (19)$$

Here, we have restored the physical dimensions. With the ansatz $\mathbf{n}(\mathbf{r},t) = \mathbf{n}(\mathbf{r}-\mathbf{v}t)$ we see that in the presence of an external electric field, the static solution of the equations of motion (17) moves as a whole with a drift velocity given by

$$\mathbf{E}_{\mathrm{ext}} = \frac{1}{c}\,\mathbf{v}\times\mathbf{B} .$$

4. Energy – momentum tensor

In this section, we give the components of the energy–momentum tensor T which result from the effective Lagrangian. For the sake of brevity, we omit the contribution of the Coulomb term, the last term in $\mathcal{L}_{\mathrm{H}}$. Then we find for the energy density

$$T_{00} = \frac{1}{4}\,E(0) \times \left\{ \frac{1}{4} \sum_{\alpha = x,y} (\partial_\alpha \mathbf{n})^2 - \mathbf{n}\cdot\partial_x\mathbf{n}\times\partial_y\mathbf{n} - \frac{3}{64}(\Delta\mathbf{n})^2 \right\}. \quad (20)$$

Note that in our normalization,

$$\int \frac{\mathrm{d}^2 r}{2\pi}\, T_{00} = \mathcal{L}_{\mathrm{H}}\,.$$

The energy current is given by

$$T_{0\alpha} = -\frac{1}{8}\,E(0) \times \left\{ (\partial_t\mathbf{n})\cdot(\partial_\alpha\mathbf{n}) - 2\epsilon_{\alpha\beta}\mathbf{n}\cdot\partial_t\mathbf{n}\times\partial_\beta\mathbf{n} + O(\partial_t\nabla^3)\right\}. \quad (21)$$

The momentum density is

$$T_{\alpha 0} = \frac{1}{2}(\partial_\alpha\bar{\psi} + \cos\bar{\theta}\,\partial_\alpha\bar{\phi})\,. \quad (22)$$

The momentum density and the energy current are different, since the effective Lagrangian is first order in time derivatives. In the absence of Lorentz-invariance, the usual symmetry $T_{0\alpha} = T_{\alpha 0}$ is missing. The components of the momentum current are

$$T_{xx} = -\frac{1}{2}(\partial_t\bar{\psi} + \cos\bar{\theta}\,\partial_t\,\bar{\phi}) - \frac{1}{16}\,E(0)\left\{(\partial_x\mathbf{n})^2 - (\partial_y\mathbf{n})^2 + O(\nabla^4)\right\}, \quad (23)$$

(T_{yy} is given by the substitution $\partial_x \leftrightarrow \partial_y$) and

$$T_{xy} = T_{yx} = -\frac{1}{8}\,E(0)\left\{(\partial_x\mathbf{n})(\partial_y\mathbf{n}) + O(\nabla^4)\right\}. \quad (24)$$

Now, an infinitesimal spatial translation in the presence of a static and homogeneous electric field $\mathbf{E}_{\mathrm{ext}}$ yields

$$\partial_t T_{\alpha 0} + \partial_\beta T_{\alpha\beta} = -2\pi e \mathbf{E}^{\alpha}_{\mathrm{ext}}\,\delta N\,. \quad (25)$$

We collect the terms containing a time derivative and again use the ansatz $\mathbf{n}(\mathbf{r}, t) = \mathbf{n}(\mathbf{r} - \mathbf{v}t)$. Now, we find that the terms in $T_{\alpha 0}$ and $T_{\alpha\beta}$, which contain the Euler angles, combine to form the Lorentz force and we are led to the same conclusion and the same result for the drift velocity $\mathbf{v}$ as in the previous section.

5. Fluctuations around the HF approximation

Our considerations were restricted to the HF approximation and hence left open the question of in which regime the HF approximation becomes applicable. In order to come to a qualitative answer, we now calculate the size of the fluctuations of the magnetization

$$\hat{S}^z = \frac{1}{2}\sum_q (\hat{a}^\dagger_q \hat{a}_q - \hat{b}^\dagger_q \hat{b}_q) \quad (26)$$

in the HF state. Denoting the fluctuation by

$$\delta\hat{S}^z = \hat{S}^z - \langle \Psi_{\mathrm{HF}}|\hat{S}^z|\Psi_{\mathrm{HF}}\rangle\,, \quad (27)$$

by straight-forward evaluation of the expectations, using that in the HF approximation, the A-states are filled and the B-states are empty, we get

$$\langle \Psi_{\mathrm{HF}}|(\delta\hat{S}^z)^2|\Psi_{\mathrm{HF}}\rangle = \frac{1}{8\pi}\int \mathrm{d}^2 r\,\left[n^x(\mathbf{r})^2 + n^y(\mathbf{r})^2 + O(\nabla^4)\right]. \quad (28)$$

As the result, for the homogeneous HF ground state $n^z(\mathbf{r}) = 1$, the fluctuations are strictly zero; this is to be expected, because the homogeneous HF state is an eigenstate to $\hat{S}^z$. For the one-skyrmion state, on the other hand, the result is best expressed in terms of the number of reversed spins $\hat{N}_{\mathrm{rev}} = \hat{N}/2 - \hat{S}^z$. We have

$$\langle \Psi_{\mathrm{HF}}|\hat{N}_{\mathrm{rev}}|\Psi_{\mathrm{HF}}\rangle = \frac{1}{4\pi}\int \mathrm{d}^2 r\,\left[1 - n^z(\mathbf{r}) + O(\nabla^4)\right]. \quad (29)$$

Since the main contribution to the fluctuation (28) comes from large $\mathbf{r}$ in the integral where $n^z \cong 1$,

$$\langle \Psi_{\mathrm{HF}}|(\delta\hat{N}_{\mathrm{rev}})^2|\Psi_{\mathrm{HF}}\rangle \cong \langle \Psi_{\mathrm{HF}}|\hat{N}_{\mathrm{rev}}|\Psi_{\mathrm{HF}}\rangle\,. \quad (30)$$

Thus, the relative fluctuation is given by the inverse of the root of the number of reversed spins. The number of reversed spins, as the result of the competition between Coulomb and Zeeman energy, increases in proportion to $1/g^{2/3}$ [11] as the g-factor becomes small. Therefore, the relative fluctuation decreases as $g^{1/3}$ as the g-factor becomes small and this defines the limit in which the HF approximation becomes exact.

6. Conclusion

In this work, we derived the effective Lagrangian describing the low-lying excitations in a system of interacting electrons in two dimensions and a strong magnetic field at small g-factor for the case of a filling factor of one. While the effective Lagrangian determines the spin of the skyrmion quasiparticles and describes their interaction, one does not get a finite result for the skyrmion's mass at this level of the theory. Future work will show whether this is due to the projection onto the lowest Landau level, or to the HF approximation.
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Nonlinear screening, and spin and cyclotron gaps in the 2D electron gas of GaAs/AlGaAs heterojunctions

V T Dolgopolov, A A Shashkin, A V Aristov, D Schmerek, W Hansen, J P Kotthaus, M Holland

We use AlGaAs/GaAs single-heterojunction samples that contain, apart from a metallic gate on the front surface, a highly doped (4×10^{18} cm^{-3} Si) layer of thickness 200 A in the bulk of the GaAs. This layer remains well-conducting even at very low temperatures and serves as a back electrode. Whereas in standard AlGaAs/GaAs heterostructures the electrons of a 2DEG originate from a doped layer in the AlGaAs barrier, the 2DEG in our samples is created in a similar way to Si MOSFET's: the 2DEG is field-effect-induced at a positive bias V_g applied to the front gate with respect to the back contact. The bottom of the conduction band in our structure is shown in the inset to Fig. 1. A blocking barrier between the gate and the 2DEG is formed by a short-period GaAs/AlAs superlattice capped by a thin GaAs layer. A wide but shallow tunnel barrier between the back electrode and the 2DEG is created by the weak residual p-doping of the GaAs layer. The electron transfer across this tunnel barrier establishes an equilibrium between the back contact and the 2DEG.

By modulating V_g with a small ac voltage, we measure the ac current through the sample. The real part of the current depends on the tunnel resistance [1, 2] while the imaginary component is determined by the capacitance of the structure. Unlike in earlier magneto-capacitance measurements, we avoid spurious lateral transport effects using the back electrode parallel to the 2DEG. From this back electrode the electron system is charged through a tunnel barrier, regardless of the value of σ_{xx} in the 2DEG.

In our experiments [3] we studied the imaginary component of the ac current through the sample as a function of the gate voltage ($C-V$ curves). We worked in the frequency range 100 Hz to 10 kHz at magnetic fields of up to 16 T and a temperature of 25 mK. The amplitude of the ac voltage did not exceed 1 mV and corresponded to the linear regime. The majority of the measurements presented here were performed on three samples. The gate areas were 9200 μm^2 for one, and 870 μm^2 for the others.

Typical experimental dependences of the low-frequency sample capacitance upon gate voltage are presented in Fig. 2. The data were recorded at temperature ≈ 25 mK, but we checked that below 1 K all the $C(V)$ curves were temperature independent. Already for small magnetic fields the data clearly show the typical filling factor dependence of the capacitance signal: the capacitance signal oscillates with minima at integer filling factors, and an enhanced capacitance with respect to the zero field value occurs between the minima. This behaviour reflects the strong modulation of the thermodynamic density of states (DOS) by the magnetic field.

In order to convert the low-frequency $C-V$ curve $C_{\text{low}}(V_g)$ into the dependence of DOS on the gate voltage, we use the following procedure: (i) We find the high-frequency limit of the sample capacitance $C_{\text{high}}(V_g)$ (see Fig. 2). (ii) The distance x_w as a function of gate voltage is determined from the $C-V$ curve at $B = 1$ T using relation [1]

$$\frac{2eB}{h} = \frac{\kappa}{e x_w} \int_{\Delta V_{\text{LL}}} \frac{C_{\text{low}} - C_{\text{high}}}{C_{\text{high}}} \, dV_g \,. \tag{1}$$

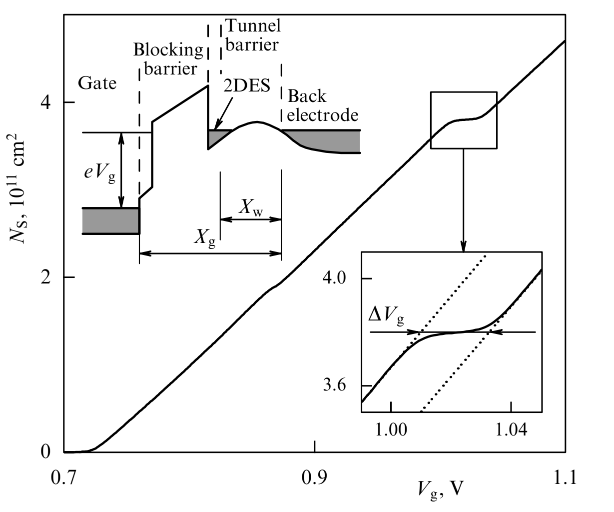

Figure 1. Electron density in the 2DES as a function of the gate voltage in a magnetic field $B = 8$ T. A blow-up of the $\nu = 2$ plateau region shows the way to determine the plateau width. A sketch of the band diagram of the device is displayed in the inset.

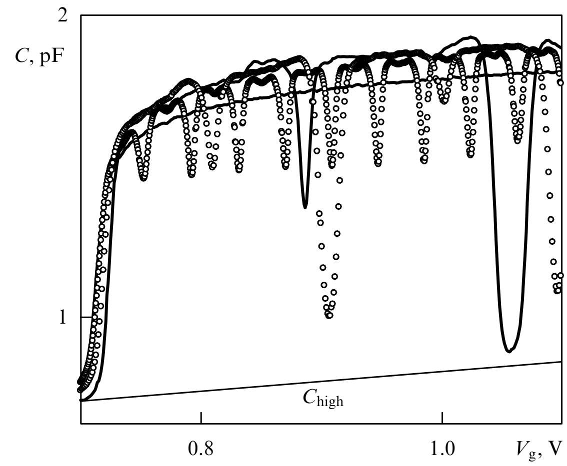

Figure 2. Experimental gate voltage dependences of the structure capacitance on the small area sample in different magnetic fields: $B = 0$ (solid line), 1 T (open circles), 5 T (dots), 9 T (bold line). The value used for $C_{\text{high}}(V_g)$ is also shown.