in the figure. The 2DES remains completely polarized up to $\nu = 2/3$ (accounting for the concentration dispersion). Then electrons with spins inverted with respect to the field arise in the system, and the polarization decreases. For a further increase in ν the dependence $S_z/S_z^{\max}$ has a minimum at $\nu \sim 0.8$. Note that all the experimental curves in Fig. 5 virtually correspond to the universal dependence $S_z/S_z^{\max}$ on ν (accurate to the dispersion of the filling factor $\sigma/(eH_n/hc)$, which is different for the curves measured at various fields H_n). The universal behavior of the spin polarization in the vicinity of $\nu = 2/3$ is demonstrated in the inset of the Fig. 5 by the comparison between the experimental dependencies of ΔC on n_s and the data calculated for the case when the system is completely polarized and $dS_z/dn_s = -1/2$ in the absence of the concentration dispersion at $\nu \leqslant 2/3$, while at $\nu > 2/3$ all the electrons involved in the system have spins inverted with respect to the field, i.e. $dS_z/dn_s = 1/2$. The calculated curves in the inset are obtained by the chemical potential jump changed by the tilted magnetic field and averaged over the Gaussian distribution with the dispersion earlier determined, $\sigma = 4.2 \times 10^9$ cm^{-2}. The dependence of $S_z/S_z^{\max}$ on ν, corresponding to the values of dS_z/dn_s used in the calculations, is shown by the dashed line in Fig. 5 for the system with zero dispersion. Note that the jump of the derivative dS_z/dn_s at $\nu = 2/3$ corresponds to the completely spin polarized state of the 2DES at this filling factor and the quasi-hole (or quasi-electron) excitations with spin coinciding (reverted) with the magnetic field, as was predicted in Ref. [15] by numerical calculations for systems of few particles.

The universal dependence $S_z/S_z^{\max}$ on ν, presented in Fig. 5, demonstrates that the assumption of the independence of the 2DES spin and the Zeeman energy, which was made to derive (4), is fulfilled under the conditions of our experiment. We show that this assumption actually results in the universal dependence $S_z/S_z^{\max}$ on ν and *vice versa*. If the Coulomb energy at fixed filling factors and relative polarization is proportional to $e^2 n_s^{3/2}$ [1], then the spin-dependent part E_S of the total energy of the 2DES ground state can be written as

$$E_S = e^2 n_s^{3/2} \phi\left(\nu, \frac{S_z}{n_s}\right) - g\mu_B H S_z .$$

Here ϕ is a function of the variables ν and S_z/n_s. The equilibrium value of S_z should be found by the condition $\partial E_S/\partial S_z = 0$. It is easily seen that the universal dependence of $S_z(\nu)/n_s$ on ν results from the solution of the equation only when the Zeeman effect can be neglected and the spin value can be found from the equation $\partial\phi/\partial S_z = 0$. Additional experimental evidence in favor of the discussed assumption is the observation of the linear dependence of ΔC on $(H - H_n)$ predicted by Eqn (4) for large changes of the total field H. The corresponding data are shown in the inset of Fig. 4. The normalized values of $\Delta C/(H - H_n)$ actually correspond to the universal dependence within experimental error.

Thus, by capacitance spectroscopy, we have measured quasi-particle charge for the FQHE and the 2DES spin polarization. We have discussed the key assumptions and restrictions of the method.

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Non-singular vortex-skyrmions in a two-dimensional electron system

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The problem of the states of a thermodynamical system of two-dimensional interacting electrons in a strong magnetic field is still not solved completely. Various qualitative and phenomenological results have been obtained and extensive experimental material has been accumulated.

Recently considerable attention has been focussed on the description of states at the Landau level with of filling factor of 1. This Landau level in a strong magnetic field can be considered in the Hartree–Fock approximation with the Slater determinant corresponding to a complete filling of the level. In this case the negative exchange energy causes ferromagnetic ordering of spins.

In a ferromagnet special macroscopic excitations may form corresponding to a slow rotation of the electron spin in space. This produces a topologically nontrivial mapping of a two-dimensional plane with long-range ferromagnetic ordering onto the sphere of average spin directions [1, 2]. Similar states were suggested for the two-dimensional electron gas in a strong magnetic field [3, 4].

Leaving aside phenomenological and numerical data, we will concentrate on the results obtained by a gradient expansion of the rotation matrix [5–7]. These papers use the approximation of wave functions projected onto a single Landau level.

Let us point out the main problems of this approach. The authors of Ref. [5–7] use an electron spin rotation matrix depending on two Euler angles. However in this case, one of the Euler angles must be written as

$$\alpha = m\varphi + \tilde{\alpha}(\varphi) , \qquad (1)$$

where $\tilde{\alpha}$ is a regular periodic function of the polar angle φ in a certain coordinate system, m coinciding with the mapping

rank. But the rotation matrix must have a point singularity and, in general, may be not single valued. The Fourier expansion of the rotation matrix and its derivatives also has a singularity, which is not taken into account in the calculations performed under the assumption of all derivatives being small. A non-singular smooth rotation matrix depending on two Euler angles has a zero mapping rank and cannot describe a skyrmion. The replacement of the rotation matrix $U(\mathbf{r})$ by the matrix $\tilde{U}$ as performed in Refs [6, 7], acting on the functions of a single Landau level, also presents serious difficulties. In particular, it causes the projected matrix $\tilde{U}(\mathbf{r})$ to lose its unitary character $U^+(\mathbf{r})U(\mathbf{r}) = 1$.

All these difficulties bring about a desire to seek a way to solve the problem, which would directly use only rotation matrices smoothly and slowly changing in space, which would be free of the indicated drawbacks and, at the same time, would enable one to calculate the energy and other physical quantities in the main (first and zero) order with respect to the external magnetic field. With this object in view we will consider the complete differential Schrödinger equation not restricting ourselves to the projection onto a single Landau level.

Let us consider the complete rotation matrix parametrized by three Euler angles

$$U(\mathbf{r}) = U_z\big(\gamma(\mathbf{r})\big)\,U_y\big(\beta(\mathbf{r})\big)\,U_z\big(\alpha(\mathbf{r})\big)\,,$$

$$U_z(\alpha) = \cos\frac{\alpha}{2} + \mathrm{i}\sin\frac{\alpha}{2}\,\sigma_z\,,$$

$$U_y(\beta) = \cos\frac{\beta}{2} + \mathrm{i}\sin\frac{\beta}{2}\,\sigma_y\,,$$

where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices. Far away from the core at finite electron g-factor the average spin must be directed along the magnetic field. Therefore the angle β, measured from the direction of the magnetic field must rapidly (exponentially) vanish as $r \to \infty$. It is assumed that the matrix $U(\mathbf{r})$ has no singularities for any $\mathbf{r}$, which means the absence of singularities in the matrices

$$A_k = \mathrm{i}U^+\,\frac{\partial U}{\partial x_k} = \Omega_k^l(\mathbf{r})\sigma_l\,.$$

Expressions for Ω_k^l can easily be obtained by direct differentiation of $U(\mathbf{r})$:

$$\Omega_k^z = \frac{1}{2}\left(\partial_k\alpha + \cos\beta\,\partial_k\gamma\right),$$

$$\Omega_k^x = \frac{1}{2}\left(\sin\beta\cos\alpha\,\partial_k\gamma - \sin\alpha\,\partial_k\beta\right),$$

$$\Omega_k^y = \frac{1}{2}\left(\cos\alpha\,\partial_k\beta + \sin\beta\sin\alpha\,\partial_k\gamma\right).$$

The unit vector of the average spin direction $\mathbf{n} = (\cos\beta, \sin\beta\cos\alpha, \sin\beta\sin\alpha)$ is obtained from the z-direction by rotation through an angle α around the z-axis and then, through an angle β around the y-axis. In the case of a nonzero mapping rank the angle $\alpha(\mathbf{r})$ has a singularity in space and Ω^l has an irremovable singularity at two Euler angles $(\alpha, \gamma = 0, \beta)$. However, this singularity can be eliminated and $\Omega_k^l(\mathbf{r})$ may be non-singular, if the singular point $\gamma(\mathbf{r})$ coincides with the singularity in $\alpha(\mathbf{r})$ and occurs at the point where $\cos\beta = -1$. Thus, the matrix U must include all the three Euler angles and the angle α must have a vortex singularity with an integer quantum number, since the rotation matrix is single valued. Therefore it would be more correct to speak of non-singular vortices whose core is determined by a skyrmion, by analogy with ^{3}He [8], but as distinct from ^{3}He, the vortex numbers are not necessarily odd. The integral

$$\frac{1}{2\pi}\int \mathrm{rot}\,\Omega^z\,\mathrm{d}^2r = Q$$

is a topological invariant and expressed directly through changes in the phase of α in bypassing the contour of large radius. The smooth rotation matrix $U(\mathbf{r})$ enables one to perform a unitary transformation of electron spinors $\psi = U(\mathbf{r})\chi(\mathbf{r})$ to new spinors $\chi(\mathbf{r})$, which can be used in addition to the initial ones χ.

Though this transformation can be performed for spinors — operators of secondary quantization in the general case, for simplicity we will consider the transformation of the simplest Hartree – Fock equation with a local exchange interaction

$$\mathrm{i}\,\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(-\mathrm{i}\boldsymbol{\nabla} - \mathbf{A}_0\right)^2 - \gamma\mathbf{n}\cdot\boldsymbol{\sigma}\psi\,,$$

where γ is the exchange constant, and $\mathbf{n}(\mathbf{r})$ slowly changes in space. We use the system of units where

$$\hbar = B_0 = l_B^2 = \frac{c\hbar}{eB} = 1\,.$$

Differentiating the transformation formula and multiplying by U^+, we get

$$-\mathrm{i}\,\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(-\mathrm{i}\boldsymbol{\nabla} - \mathbf{A}_0\right)^2\chi - \frac{\mathrm{i}}{2m}\,U^+\boldsymbol{\nabla}U(-\mathrm{i}\boldsymbol{\nabla} - \mathbf{A}_0)\chi - \frac{1}{2m}\,U^+\boldsymbol{\nabla}^2U\chi - \gamma U^+\mathbf{n}\cdot\boldsymbol{\sigma}U\chi\,.$$

Adding to this expression the quantity

$$-\frac{1}{m}\left(U^+(\boldsymbol{\nabla}U)U^+\boldsymbol{\nabla}U + (\boldsymbol{\nabla}U^+)\boldsymbol{\nabla}U\right) = 0\,,$$

which is zero due to the unitarity condition ($U^+U = 1$, $\boldsymbol{\nabla}U^+U + U^+\boldsymbol{\nabla}U = 0$) and choosing the rotation matrix such that $U^+\mathbf{n}\cdot\boldsymbol{\sigma}U = \sigma_z$, we can write the equation for χ in the form

$$\mathrm{i}\,\frac{\partial\chi}{\partial t} = \frac{1}{2m}\left(-\mathrm{i}\boldsymbol{\nabla} - \mathbf{A}_0 + \Omega^l\sigma_l\right)^2\chi - \gamma\sigma_z\chi + \Omega_t^l\sigma_l\chi\,. \qquad (2)$$

This equation is a transformation of the initial one and completely equivalent to it. However, as distinct from the initial equation, its main part (at $\Omega^l = 0$) corresponds to the Schrödinger equation in a uniform magnetic field with a uniform exchange term. Small terms containing the Zeeman energy and the Coulomb interaction can be added to the expression obtained, but they are irrelevant for us as yet.

We see that the equation for χ now involves the spin-dependent vector-potential and additional potential energy expressed through Ω_k^l which is assumed to be small. This enables us to use the corresponding perturbation theory. In this case the main undisturbed part of the equation has the

Green's function

$$G_0(t-t',\mathbf{r},\mathbf{r}') = \int \frac{\mathrm{d}\omega}{2\pi}\frac{\mathrm{d}p}{2\pi}\, g_s(\omega)\exp\bigl[\mathrm{i}\omega(t'-t)\bigr] \times \exp\bigl[\mathrm{i}p(y-y')\bigr]\Phi_{\mathrm{sp}}(x)\Phi_{\mathrm{sp}}(x')\,, \quad (3)$$

where $\Phi_{\mathrm{sp}}(x)$ are oscillator functions in the magnetic field at the chosen Landau gauge. The matrices $g_s(\omega)$ correspond to a complete filling of the zero Landau level (we restrict ourselves to this case) for the spin upward states

$$g_0 = \frac{1+\sigma_z}{2}\frac{1}{\omega+\gamma-\mathrm{i}\delta} + \frac{1-\sigma_z}{2}\frac{1}{\omega-\gamma+\mathrm{i}\delta}\,, \qquad \delta\to+0\,. \quad (4)$$

The states with $s>0$ are all empty:

$$g_s = \frac{1}{\omega - s\omega_{\mathrm{c}} + \gamma\sigma_z + \mathrm{i}\delta}\,. \quad (5)$$

In these formulae we have introduced the chemical potential $\mu = \hbar\omega_{\mathrm{c}}/2$, corresponding to the filling of the lowest states. The calculation of various physical quantities using the perturbation theory is standard and uses the diagram expansion of the total Green's function G and the formula $S = \mathrm{i}\,\mathrm{Tr}\ln G$ for the action. The details of the calculation are published in Refs [9–11]. The result for the energy and electron density in the lowest order of the perturbation theory has a simple physical meaning, i.e. only rot Ω^z, adding to the external magnetic field $B_{\mathrm{eff}} = B_0 - \mathrm{rot}\,\Omega^z(\mathbf{r})$ is significant. Other quantities Ω^x, Ω^y contribute only in high order terms. The density ρ corresponds to the complete filling of the Landau level in the local effective field and represents the local density of states

$$\rho = \frac{1}{2\pi}\,B_{\mathrm{eff}}$$

and the change in the number of electrons by $\delta N = -Q$. In this case the total additional thermodynamic energy of all the electrons in the main order with no regard to changes in the potential energy has the form

$$\delta\langle H-\mu N\rangle = -\frac{\hbar\omega_{\mathrm{c}}}{2}\,Q\,.$$

This fact suggests that one should take corrections to the kinetic energy into account and solve the corresponding differential equations rather than take it to be constant as is usually done in the approximation of functions projected onto the zero Landau level. In this case the zero Landau level is actually filled but in the effective magnetic field varying in space. Note that in the absence of a skyrmion at $Q=0$ the net correction to the kinetic energy is lacking. Corrections to the density enable one to find the corresponding terms in the potential and exchange energy [11], so that the overall change in the thermodynamic energy on formation of a vortex-skyrmion has the form

$$F = \delta\langle H-\mu N\rangle = -\frac{\hbar\omega_{\mathrm{c}}}{2}\,Q + \frac{3e^2}{2l_B}\sqrt{\frac{\pi}{2}}\,Q + \frac{e^2}{2}\int\frac{\mathrm{rot}\,\Omega_z(\mathbf{r})\cdot\mathrm{rot}\,\Omega_z(\mathbf{r}')}{(2\pi)^2|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^2r\,\mathrm{d}^2r' + \int\left[\frac{J}{2}\left(\frac{\partial n_i}{\partial r_k}\right)^2 + g\mathbf{B}\cdot\mathbf{n}\,\frac{1}{2\pi(l_B)^2}\right]\mathrm{d}^2r\,. \quad (6)$$

This expression contains the change in the exchange energy caused by the changed effective magnetic field, Coulomb energy, the energy associated with heterogeneous spin direction, and the Zeeman energy (the last term) where

$$J = \frac{1}{16\sqrt{2\pi}}\,\frac{e^2}{l_B}\,.$$

The overall energy corresponds to the Hartree–Fock electron energy in the field of the non-singular rotation matrix $U(\mathbf{r})$. There are some corrections to this expression associated with zero oscillations of the rotation matrix itself. To calculate these, one should treat collective oscillations. We will restrict ourselves to the consideration of the most significant correction related to the motion of a charged vortex-skyrmion as a whole in the external magnetic field.

For this purpose we should find the skyrmion effective mass. If the skyrmion moves as a whole, the rotation matrix is the function $U(\mathbf{r}-\mathbf{X})$, where $\mathbf{X}(t)$ is the position of the skyrmion center. This yields an additional small term $\Omega^l_l\sigma_l = -\dot{\mathbf{X}}\cdot\boldsymbol{\Omega}^l\sigma_l = \mathbf{H}_1$ in the Hartree–Fock equations (2). In the isotropic case the corresponding linear in $\dot{\mathbf{X}}$ term of the skyrmion integrated action becomes zero and the expansion of the action starts with the second order term of the perturbation theory

$$\delta S = \frac{\mathrm{i}}{2}\,\mathrm{Tr}\,(H_1G_0H_1G_0)\,.$$

Taking into account only low order terms with derivatives, we present this expression, with the help of Eqn (3), in the form

$$\delta S = \frac{\mathrm{i}}{2}\,\mathrm{Tr}\int(\Omega^l\sigma_l\dot{\mathbf{X}})g_0(\omega)(\Omega^{l'}\sigma_{l'}\dot{\mathbf{X}})g_0(\omega)\exp(\mathrm{i}\omega\delta)\,\frac{\mathrm{d}\omega}{2\pi}\,\frac{\mathrm{d}^2r}{2\pi}\,\mathrm{d}t\,.$$

Contributions are made by terms with poles on opposite sides of the real axis ω. The calculation of Tr accounting for the isotropy yields

$$\delta S = \int\frac{(\dot{\mathbf{X}})^2}{2\gamma}\sum_{l\neq z}\frac{(\Omega^l)^2}{2}\,\frac{\mathrm{d}^2r}{2\pi}\,\mathrm{d}t\,.$$

Using formulae (1), we can rewrite this expression in the form

$$\delta S = \int\mathrm{d}t\,\frac{(\dot{\mathbf{X}})^2}{8\gamma}\,\frac{1}{4\pi}\int\left(\frac{\partial n_i}{\partial r_k}\right)^2\mathrm{d}^2r\,.$$

It is known [2] that the skyrmion ground state corresponds to the value

$$\frac{1}{2}\int\left(\frac{\partial n_i}{\partial r_k}\right)^2\mathrm{d}^2r = 4\pi|Q|\,,$$

which gives

$$\delta S = \int\frac{m\dot{X}^2}{2}\,\mathrm{d}t\,, \qquad m = \frac{|Q|}{2\gamma}\,.$$

Besides the kinetic energy, the Lagrangian contains a term with the vector-potential of the external field $(e/c)Q\dot{\mathbf{X}}\cdot\mathbf{A}_0$ which can also be derived by differentiation of the phase of the wave function due to transferring the charge Q. Thus, the Lagrangian for the motion of the skyrmion as a whole has the form

$$L = \frac{m\dot{X}^2}{2} + \frac{e}{c}\,Q\dot{\mathbf{X}}\cdot\mathbf{A}_0\,.$$

We can introduce the generalized momentum $P_i = \partial L/\partial \dot{X}_i$ and quantize the problem, assuming the standard commutation relations $[P_i X_i] = \mathrm{i}\hbar$ to be fulfilled. This yields the cyclotron frequency of the skyrmion as a whole $\hbar\omega_s = 2\gamma$ and the minimum energy $E_s = \gamma$. Thus, in experiments the skyrmion should exhibit a cyclotron resonance at a frequency 2γ. The quantity γ is determined as the exchange energy per electron, in the case of a completely filled Landau level

$$\gamma = \frac{e^2}{l_B}\sqrt{2\pi}\,.$$

The energy γ must be added to expression (5) obtained above for the total energy.

It is also interesting to find a term with the Hopf invariant in the action which, according to modern views, determines the skyrmion statistics [11]. For this purpose we should calculate the terms containing one time-dependent Ω_t^l and two space-dependent Ω^l in the expansion of the action in terms of Ω^l. Calculations up to the third order are rather cumbersome and require the consideration of numerous diagrams, the second-order diagrams also contributing, since their non-local character in time and space must be taken into account [9, 10]. We present only the final result corresponding to the 'fermion' character of vortex-skyrmions:

$$S_H = \pi H\,, \qquad H = \frac{1}{2\pi^2}\int e^{ljm}\Omega_t\Omega_j \times \Omega_l\,\mathrm{d}^2 r\,\mathrm{d}t\,, \tag{7}$$

where e^{ljm} is the unit antisymmetric third-rank tensor. The integer Hopf invariant H is expressed in terms of Ω^l [12]. This result does not coincide with that obtained within the method of Landau functions projected onto the zero level [7] which is a sum of several spatial derivatives. At the same time, formula (7) has a standard form and agrees with that suggested in note [13].

In conclusion we emphasize once again that the solution of differential equations of the Hartree – Fock approximation is necessary, since it determines the discrepancy between our results and those obtained by projection onto a single Landau level, when the differential form of the kinetic energy in the Schrödinger equation is replaced by a constant energy. It is of special importance in the calculation of the thermodynamic energy of the vortex-skyrmion where the additional term $(-\hbar\omega_c/2)Q$ appears, which can lead to spontaneous appearance of vortices and a rearrangement of the ground state in a rather strong magnetic field.

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Microscopic derivation of the effective Lagrangian for skyrmions in an interacting two-dimensional electron gas at small *g*-factor

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1. Introduction

Electronic systems confined to two dimensions and exposed to a strong magnetic field continue to be studied intensively in both experiment and theory [1]. Due to the magnetic field, the electronic single particle energies form degenerate Landau levels and for the physical properties the electron – electron interaction is crucial. Recently, the spin degree of freedom has attracted a lot of attention. For a long time, it was accepted that the basic excitations are of particle – hole kind (spin-excitons), when the cyclotron energy is much larger than the characteristic Coulomb energy [2 – 5]. However, recent experiments performed at or near a filling factor of one, where one spin-split Landau level is completely filled, have changed this picture. The activation energy of the resistance measured under pressure [6], the spin polarization measured by magnetoabsorption spectroscopy [7], transport experiments in a tilted magnetic field [8], and measurements of the Knight shift with optically pumped NMR [9] are all taken as evidence that there are new basic excitations, the skyrmions. Theoretically, excitations of this kind have been studied before in the context of two-dimensional isotropic ferromagnets [10]. Only recently was it shown that one also has skyrmion quasiparticles in an interacting electron system in a magnetic field, provided the *g*-factor is smaller than a critical value [11]. The energy needed to create a skyrmion – antiskyrmion pair for $g \to 0$ is only half the energy needed to create a single spin-exciton with very large momentum. The charge of a skyrmion is the electron charge e. The number of reversed electron spins contained in a skyrmion was calculated in the Hartree – Fock (HF) approximation [12]; the value depends on the *g*-factor and is larger than one. Very recently, the quantum nature of the skyrmion quasiparticle, i.e. its spin, was derived [13] from a microscopic model by the generalization of a method used earlier [14] to derive the Hamiltonian part of the effective Lagrangian.

This paper is organized as follows: in the next section, we introduce, together with the model, our notation. Then, we summarize the derivation of the effective Lagrangian which was already partly described in previous works [14, 13]. The two following sections are devoted to a short discussion of the equations of motion and the energy – momentum tensor. In the last section, we derive a criterion for the applicability of the HF approximation.

2. Effective Lagrangian

We are studying interacting electrons in two dimensions moving in a strong magnetic field. The orbital states of the electrons are confined to the lowest Landau level; we use the