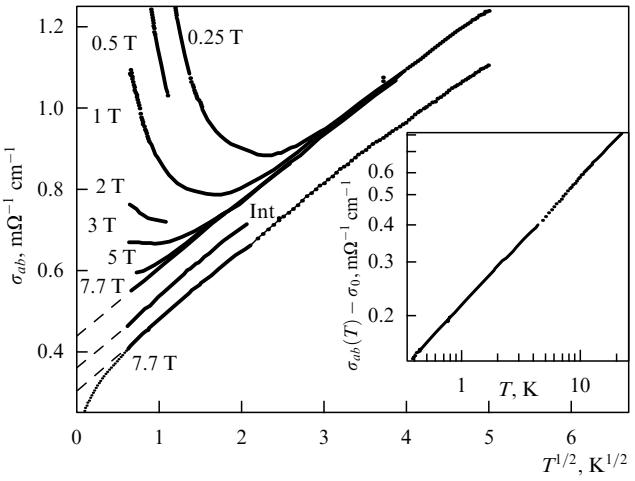

Figure 4. Plots of the conductivity $\sigma_{ab} = 1/\rho_{ab}$ versus $T^{1/2}$. The dotted line depicts, for comparison, the best logarithmic fit, $\rho_{ab} = A + B \log T$, for the 'quenched' state. Inset: temperature dependence of the conductivity on a double-logarithmic scale.

state. Such an analysis performed in [13] for ρ_c has shown that the conductivity continues to follow the power law with $\sigma_0 \to 0$, while the alternative logarithmic representation becomes inappropriate.

Therefore, a description of the normal state in $YBa_2Cu_3O_{6+x}$ as that of a 3D system in the vicinity of the metal – insulator transition seems to be preferable, and the conductivity in close vicinity of the MIT may be described by a scaling temperature dependence. The normal state underlying superconductivity is suggested to be metallic, while the M – I transition is located on the phase diagram at a distance from the SC region. Further studies are obviously highly desirable, and the fact that the normal state appeared to be just the same on both sides of the SC – NSC phase boundary is worth special notice, since it provides a possibility to deal with the problem not complicated by superconductivity.

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Quantum fluctuations and dissipation in thin superconducting wires

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1. Introduction

It is well known that fluctuations wash out the long-range order in low dimensional superconductors [1]. Does this result mean that the resistance of such superconductors always remains finite (or even infinite), or can it drop to zero under certain conditions? A lot is known about the behavior of two-dimensional (2D) superconducting films where the physics is essentially determined by the Kosterlitz – Thouless – Berezinskii (KTB) phase transition [2]. In quasi-1D superconducting wires below the (mean field) critical temperature T_c a nonzero resistivity can be caused by thermally activated phase slips (TAPS) [3]. This effect is of practical importance at temperatures close to T_c where the theoretical predictions have been verified experimentally [4]. However, as the temperature is lowered the number of TAPS decreases exponentially and no measurable resistance is predicted by the theory [3] for T not very close to T_c. Nevertheless, the experiments by Giordano [5] clearly demonstrate a notable resistivity of ultra-thin superconducting wires far below T_c. More recently strong deviations from the TAPS prediction in thin (quasi-)1D wires have been also demonstrated in other experiments [6].

The natural explanation of these observations is in terms of quantum fluctuations which generate quantum phase slips (QPS) in 1D superconducting wires. However, first estimates for the QPS tunneling rate derived from the time-dependent Ginzburg – Landau based theories [7, 8] turned out to be far too small to explain the experimental findings [5] (see [9] for more details).

More recently, the present authors [9] developed a microscopic theory describing the QPS phenomenon and demonstrated that in sufficiently thin wires QPS effects are well within the measurable range and may lead to a nonzero wire resistivity even at $T = 0$. Moreover, the existence of a new superconductor-to-metal (insulator) phase transition as a function of the wire thickness was pointed out in [9].

In the present paper we extend our theory [9] in several important aspects, in particular providing a more detailed discussion of the QPS action in various limits and paying attention to dissipative effects outside the QPS core. We also discuss a possible explanation of the recently observed negative magnetoresistance [10] within the framework of our QPS scenario.

2. The model

Our calculation is based on the effective action approach for a BCS superconductor [11]. The starting point is the partition function Z expressed as an imaginary time path-integral over the electronic fields ψ and the gauge fields $V, \mathbf{A}$, with Euclidean action

$$S = \int d^3\mathbf{r} \int_0^\beta d\tau \left\{ \bar{\psi}_\sigma \left[\partial_\tau - ieV + \xi \left(\nabla - \frac{ie\mathbf{A}}{c} \right) \right] \psi_\sigma - g\bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow + ieVn_i + \frac{\mathbf{E}^2 + \mathbf{B}^2}{8\pi} \right\}.$$

Here $\beta = 1/T$, $\xi(\nabla) \equiv -\nabla^2/2m - \mu$, en_i denotes the background charge density of the ions, and $\hbar = k_B = 1$. A Hubbard – Stratonovich transformation introduces the energy gap Δ as an order parameter and the electronic degrees of freedom can be integrated out. What remains is an expression for the partition function in terms of an effective action for Δ, V and $\mathbf{A}$, with a saddle-point solution $|\Delta| = \Delta_0$ and $V = \mathbf{A} = 0$. We obtain

$$S_{\text{eff}} = \int \mathrm{d}^3\mathbf{r} \int_0^\beta \mathrm{d}\tau \left[\frac{|\Delta|^2}{g} + \frac{\mathbf{E}^2 + \mathbf{B}^2}{8\pi} \right] - \mathrm{Tr} \ln \widehat{G}^{-1} ,$$

$$\widehat{G}^{-1} = \left(\partial_\tau + \frac{\mathrm{i}}{2}\{\nabla, \mathbf{v}_s\} \right) \hat{1} + |\Delta| \hat{\sigma}_1 + \left(\xi(\nabla) + \frac{m\mathbf{v}_s^2}{2} - \mathrm{i}e\Phi \right) \hat{\sigma}_3 ,$$

where the superfluid velocity $\mathbf{v}_s = (1/2m)[\nabla\varphi - 2e\mathbf{A}/c]$, the chemical potential for Cooper pairs $\Phi = V - \varphi/2e$, and $\Delta = |\Delta| e^{\mathrm{i}\varphi}$ have been introduced.

3. Effective action for QPS

The effective theory is constructed by expanding up to second order around the saddle point in Φ and $\mathbf{v}_s$ to obtain the electronic polarization terms [9, 12]. A phase-slip event in imaginary time involves a suppression of the order parameter in the phase slip core (i.e. inside the space-time domain $x \leqslant x_0$, $\tau \leqslant \tau_0$), and a winding of the superconducting phase around this core. The total QPS action S_{QPS} can be presented as a sum of a core part S_{core} around the phase slip center for which the condensation energy and dissipation by normal currents are important, and a hydrodynamic part outside the core S_{out} which depends on the hydrodynamics of the electromagnetic fields and dissipation due to the presence of quasiparticles above the superconducting gap.

In what follows we will consider sufficiently thin wires with cross section $S < \lambda_L^2$, where λ_L is the London penetration length of a bulk superconductor. Due to scattering on impurities and boundary imperfections the electron mean free path l in such wires is typically much shorter than the coherence length of a clean sample $l \ll \xi_0 = v_F/2\Delta$. Here we restrict our attention to this physically important diffusive limit. Assuming that outside the QPS core the magnitude of the order parameter field is not suppressed $|\Delta| = \Delta_0$ for S_{out} we obtain [9, 12]

$$S_{\text{out}} = \int \mathrm{d}x\, \mathrm{d}\tau \left(\frac{C + C'}{2} V^2 + \frac{\widetilde{C}}{2} \Phi^2 + \frac{1}{2Lc^2} A^2 + \frac{m^2 \mathbf{v}_s^2}{2e^2 \widetilde{L}} \right) + \frac{S}{2\beta} \sum_{\omega < 1/\tau_0} \int \mathrm{d}x \frac{\sigma(\omega)}{|\omega|} |\partial_x V(\omega, x) + \frac{\mathrm{i}\omega A(\omega, x)}{c^2} , \quad (1)$$

where the integration runs over $|x| > x_0, |\tau| > \tau_0$. In general the kinetic inductance $\widetilde{L}$ and the kinetic capacitance $\widetilde{C}$ in (3) depend on the frequency ω and the wave vector k [12]. In the limit of low ω and small k we have $\widetilde{L} = 4\pi\lambda_L^2/S$ and $\widetilde{C} = Se^2 N_0 n_s/n$, where n_s and n are respectively the superconducting and the total electron density. In (3) we also introduced the capacitance $C' = Se^2 N_0 n_n/n$ which we will drop from now on in the limit $n_s \gg n_n \equiv n - n_s$ at low T.

The geometry and screening by dielectrics outside the wire are accounted for by the capacitance per length C and the inductance times length L that replace the $\mathbf{E}^2 + \mathbf{B}^2$-term. [For thin wires transverse screening is irrelevant and so we retain only one component of the vector potential]. The expressions for C and L also depend on the relevant space and time scales as well as on the wire geometry. In the ideal case of a cylindrical uniform wire for $kr_0 \ll 1$ (r_0 is the wire radius) one has $C = \varepsilon_r [2\ln(1/kr_0)]^{-1}$ and $L = 2\ln(1/kr_0)/c^2$, c is the speed of light and ε_r the dielectric constant of the substrate. In practice the details of the wire geometry can be very complicated (e.g. the cross section S is not constant along the wire, i.e. the wire is never uniform) and, on top of that, other (metallic) objects may be located in the vicinity of the wire. The above effects lead to an effective cutoff of the logarithmic dependence on k at the scale $k \sim 1/d$ with d depending on experimental details (e.g. d may be a typical scale of the wire inhomogeneity or the distance to the metallic groundplane). Here we will stick to a simplified model and assume $C = \varepsilon_r [2\ln(d/r_0)]^{-1}$ to be constant at all relevant distances. As to L, its particular form turns out to be unimportant for thin wires with $\sqrt{S} < \lambda_L$ in which case the kinetic inductance always dominates $\widetilde{L} \gg L$. In addition to the above kinetic and electromagnetic effects, expression (3) accounts for dissipative currents outside the core. The corresponding contribution is described by the last term in Eqn (3).

As to the core contribution, it consists of two terms

$$S_{\text{core}} = \frac{b}{2} N_0 \Delta_0^2 S \tau_0 x_0 + \frac{S}{\beta} \sum_{|\omega| > \tau_0^{-1}} \frac{x_0 \sigma}{|\omega|} \left| E\left(\omega, \frac{x_0}{2}\right) \right|^2 . \quad (2)$$

The first part is the condensation energy that is lost inside the core and the second part defines the energy of dissipative currents in the core during a phase slip event. Here σ is the normal state conductance of the wire: we already made use of the fact that the typical QPS frequency is sufficiently high [9] $1/\tau_0 \gtrsim \Delta_0$, therefore dissipative currents inside the core are insensitive to superconductivity. It is also important to emphasize that no gradient terms for Δ (both in space and in time) should be added to (4). Such terms can be recovered only by expanding the effective action in powers of ω and k. For fast processes (like QPS) this expansion obviously becomes incorrect and it is necessary to carry out a more careful treatment of polarization terms in the action. For the QPS event with $1/\tau_0 > \Delta_0$ this treatment yields [12] $b \sim \ln\left[1/(2\Delta_0\tau_0) + \xi^2/x_0^2\right]$, where $\xi = \sqrt{D/2\Delta_0}$ is the coherence length of a dirty superconductor, $D = v_F l/3$.

4. Variational procedure

In order to evaluate the QPS action $S_{\text{QPS}} = S_{\text{core}} + S_{\text{out}}$ we will use a variational approach which consists of several steps.

We first minimize the hydrodynamic contribution S_{out} with respect to the potentials V and A. As a result we arrive at the saddle point conditions which link the potentials to the phase variable outside the core. Making use of the fact that for thin wires one has $\widetilde{L} \gg L$, $\widetilde{C} \gg C$, we obtain in the Fourier representation

$$V_{\omega,k} = \frac{\mathrm{i}\omega\varphi_{\omega,k}/2e}{1 + \sigma(\omega,k) S k^2/\widetilde{C}|\omega|} , \quad (3)$$

$$A_{\omega,k} = -\mathrm{i}k \frac{cL\varphi_{\omega,k}}{2e\widetilde{L}} . \quad (4)$$

With the aid of (3) and (4) one can rewrite the action S_{out} in terms of only the phase variable $\varphi(\tau, x)$. Then minimizing this part of the action with respect to φ and keeping in mind the identity $\partial_x\partial_\tau\varphi - \partial_\tau\partial_x\varphi = 2\pi\delta(\tau, x)$ (which follows from

the fact that after a wind around the QPS center the phase should change by 2π) we find

$$S_{\text{out}} = \int_{|\omega|<1/\tau_0} \frac{\mathrm{d}\omega}{2\pi} \int_{|k|<1/x_0} \frac{\mathrm{d}k}{2\pi}\, \mathcal{G}(\omega, k)\,. \tag{5}$$

The general expression for the function $\mathcal{G}$ in (5) is somewhat tedious and is not presented here. In the following limits substantial simplifications can be achieved:

(1) $\omega \gg \sigma(\omega, k) S k^2/C$. The function (5) has the form

$$\mathcal{G}(\omega, k) = \frac{\pi^2/2e^2}{k^2/C + \widetilde{L}\omega^2}\,. \tag{6}$$

(2) $\sigma(\omega, k) S k^2/\widetilde{C} \ll \omega \ll \sigma(\omega, k) S k^2/C$. We find

$$\mathcal{G}(\omega, k) = \frac{\pi^2 \sigma(\omega, k) S}{2e^2|\omega|} \frac{1}{1 + \widetilde{L}\sigma(\omega, k) S|\omega|}\,. \tag{7}$$

(3) $\omega \ll \sigma(\omega, k) S k^2/\widetilde{C}$. The function $\mathcal{G}$ again acquires the form (6) but with C substituted by $\widetilde{C}$.

As a last step of our variational procedure we minimize the total QPS action $S_{\text{core}} + S_{\text{out}}$ with respect to the core parameters τ_0 and x_0. Let us first neglect dissipation by formally putting $\sigma = 0$ in (2) and (5). Then solving the equations

$$\frac{\partial S_{\text{QPS}}}{\partial \tau_0} = 0, \qquad \frac{\partial S_{\text{QPS}}}{\partial x_0} = 0$$

and treating b as a constant (i.e. neglecting its weak dependence on τ_0 and x_0) we obtain

$$x_0 = c_0 \tau_0 = \sqrt{\frac{\pi}{4e^2 \tilde{L} S b N_0 \Delta_0^2}}\,, \tag{8}$$

where $c_0 = 1/(\widetilde{L}C)^{1/2}$ is the velocity of the Mooij–Schön mode [14] which determines the space-time asymmetry of the core. The total action for a single QPS reads

$$S_{\text{QPS}}^{(0)} = \frac{\mu}{2} + \mu \ln\left(\frac{R}{2x_0} + \frac{R}{2c_0\tau_0}\right), \tag{9}$$

where $R^2 = X^2 + c_0^2\beta^2$, X is the wire length and $\mu = (\pi/4e^2)(C/\tilde{L})^{1/2}$. The first term in (9) represents the core action S_{core}, the second term defines S_{out} (for simplicity we chose the cutoff by integrating outside the ellipse $(x/x_0)^2 + (\tau/\tau_0)^2 > 1$). Substituting $\tilde{L} = 4\pi\lambda_{\text{L}}^2/S = 1/2\pi e^2 N_0 \Delta_0 D S$ into (8) at $T \ll \Delta_0$ we find $x_0 = c_0\tau_0 = \pi\xi/\sqrt{b}$, i.e. the core size x_0 is of the order of the superconducting coherence length ξ, and the QPS time $\tau_0 \sim \xi/c_0 \ll 1/\Delta_0$. This result justifies the above conjecture that the typical QPS frequency is higher than Δ_0 and demonstrates why our core action $S_{\text{core}} \propto x_0\tau_0$ is much smaller than that found within the TDGL analysis [8] which yields the QPS frequency of order Δ_0.

Let us now include dissipation. At high frequencies dissipative currents flowing both inside and outside the core are important and should be taken into account even at $T = 0$. The dissipative contribution from S_{out} is obtained from (5) and (7). After a simple integration one finds

$$S_{\text{out}}^{\text{diss}} \approx \frac{\sigma S}{e^2 x_0}\,. \tag{10}$$

This expression is nothing but the Caldeira–Leggett dissipative action of a normal conductor with cross section S and length $\sim x_0$. A similar expression defines the dissipative contribution from the core $S_{\text{core}}^{\text{diss}}$.

If σ is small one can treat the dissipative terms perturbatively. This is sufficient as long as $\sigma S \lesssim e^2\mu\xi$. It is easy to check that in the practically important Drude limit $\sigma = 2e^2 N_0 D$ the above condition would mean $\xi \gtrsim c_0/\Delta_0$. This condition is never satisfied for realistic parameters. Therefore in this limit dissipation cannot be treated perturbatively and our variational procedure should be modified. Under certain simplifying assumptions one can find

$$S_{\text{core}}^{\text{diss}} \approx \frac{\sigma S}{e^2 x_0}\left[\frac{1}{2r^6} + r^6\right] \ln\frac{c_0 x_0}{Dr}\,, \tag{11}$$

where $r = c_0\tau_0/x_0$. The strong dependence of (11) on r enforces the minimum condition $r \approx 1$, i.e. the asymmetry of the core remains approximately the same as in the underdamped limit. Under this condition the whole action $S_{\text{QPS}}^{(0)} + S_{\text{QPS}}^{\text{diss}}$ can be easily minimized with respect to x_0 and we obtain [9] $x_0 \approx c_0\tau_0 \approx \xi\sqrt{a}$ and

$$S_{\text{core}} \approx a\mu\,, \qquad a \approx \left(\frac{c_0}{\Delta_0\xi}\right)^{2/3}. \tag{12}$$

5. Metal–superconductor phase transition

The next step is to consider a gas of QPSs in a superconducting wire. We also assume that an applied current I (much smaller than the depairing current) is flowing through the wire. Substituting the saddle point solution $\varphi = \sum_i^n \tilde{\varphi}(x - x_i, \tau - \tau_i)$ into the action and keeping track of the additional term $\int \mathrm{d}\tau \int \mathrm{d}x (I/2e)\partial_x\varphi$ [11], we find

$$S_n = na\mu - \mu \sum_{i\neq j} v_i v_j \ln\left(\frac{\rho_{ij}}{x_0}\right) + \frac{\Phi_0}{c} I \sum_i v_i \tau_i\,. \tag{13}$$

The quantity $\rho_{ij} = \left[c_0^2(\tau_i - \tau_j)^2 + (x_i - x_j)^2\right]^{1/2}$ defines the distance between the i-th and j-th QPS in the (x, τ) plane, $v_i = +1$ (-1) are the QPS (anti-QPS) 'charges', and $\Phi_0 = hc/2e$ is the flux quantum. Only neutral QPS configurations with $v_{\text{tot}} = \sum_i^n v_i = 0$ (and hence n even) contribute to the partition function [9].

For $I = 0$ Eqn (13) defines the standard model of a 2D gas of logarithmically interacting charges v_i. The effective (small) fugacity y of these charges is

$$y = x_0\tau_0 B \exp(-a\mu)\,, \tag{14}$$

where B is the usual fluctuation determinant which we roughly estimate as $B \sim a\mu/x_0\tau_0$. From the Coulomb gas analogy, we conclude that a KTB phase transition [2] for QPSs occurs in a superconducting wire at $\mu = \mu^* \equiv 2 + 4\pi y \approx 2$: for $\mu < \mu^*$ the density of free QPS in the wire (and therefore its resistance) always remains finite, whereas for $\mu > \mu^*$ QPSs and anti-QPSs (AQPS) are bound in pairs and the linear resistance of a superconducting wire is strongly suppressed and T-dependent. We arrive at an *important conclusion*: at $T = 0$ a 1D superconducting wire has a vanishing linear resistance, provided the electromagnetic interaction between phase slips is sufficiently strong, i.e. $\mu > \mu^*$.

The above analysis is valid for sufficiently long wires. For typical experimental parameters, however, $X < c_0\beta$ (or even $X \ll c_0\beta$), and the finite wire size needs to be accounted for.

Here we consider the physical situation with non-vanishing (even for $\omega \gg \Delta_0$) wire conductance far from the QPS core $\sigma = \sigma_{\rm qp}$. This situation can be realized in the presence of quasiparticles above the gap due to finite temperature or non-equilibrium effects. In this case our consideration should be modified as follows.

We first apply the 2D scaling [2] $\partial_l y = (2-\mu)y$ and $\partial_l \mu = -4\pi^2\mu^2 y^2$, where μ and y depend on the scaling parameter l. Solving these equations up to $l = l_X = \ln(X/x_0)$ we obtain the renormalized fugacity $\tilde{y} = y(l_X)$.

For larger scales $l > l_X$ only the time coordinate matters. At sufficiently low frequencies the inter-QPS interaction is determined by function (7) and the problem reduces to a that of a 1D Coulomb gas with logarithmic interaction. Therefore, (for $\tilde{y} \ll 1$) further scaling is defined by [11, 13] $\partial_l \tilde{y} = (1-\gamma)\tilde{y}$ and $\partial_l \gamma = 0$, where $\gamma = \pi S \sigma_{\rm qp}/2e^2 X$ is the dimensionless 'quasiparticle' conductance of the wire. For $\gamma > 1$ the fugacity scales down to zero, which again corresponds to a superconducting phase, whereas for $\gamma < 1$ it increases indicating a resistive phase in complete analogy to a single Josephson junction with ohmic dissipation. The phase transition point again depends on S, but also on the wire length X and the value $\sigma_{\rm qp}$ (see below).

6. Wire resistance at low T

At any nonzero T the wire has a nonzero resistance $R(T,I)$ even in the 'ordered' phase $\mu > \mu^*$ (or $\gamma > 1$). In order to evaluate $R(T)$ in this phase for a long wire we proceed perturbatively and first calculate the free energy correction δF due to one bound QPS-AQPS pair. The one QPS-AQPS pair contribution δF to the free energy of the wire is

$$\delta F = \frac{Xy^2}{x_0\tau_0}\int_{\tau_0}^{\beta}\frac{{\rm d}\tau}{\tau_0}\int_{x_0}^{X}\frac{{\rm d}x}{x_0}\exp\left\{\left(\Phi_0\frac{I\tau}{c}\right) - 2\mu\ln\left[\frac{\rho(\tau,x)}{x_0}\right]\right\}, \tag{15}$$

where $\rho = (c_0^2\tau^2 + x^2)^{1/2}$. For nonzero I the expression in Eqn (15) is formally divergent for $\beta \to \infty$ and (after a proper analytic continuation) acquires an imaginary part ${\rm Im}\,\delta F$. This indicates a QPS-induced instability of the superconducting state of the wire. The corresponding decay rate $\Gamma = 2\,{\rm Im}\,\delta F$ defines the total voltage drop V across the wire (see [9] for more details). For the wire resistance $R(T,I) = V/I$ this yields $R \propto T^{2\mu-3}$ and $R \propto I^{2\mu-3}$ for $T \gg \Phi_0 I$ and $T \ll \Phi_0 I$ respectively. For thick wires with $\mu > \mu^*$, we expect a strong temperature dependence for the resistivity. For thinner wires the temperature dependence of the resistivity becomes linear at the transition to the disordered phase in which our analysis is not valid. At $T \ll \Phi_0 I/c$ we expect a strongly nonlinear $I-V$ characteristic $V \sim I^{\nu}$ in thick wires, and a universal $\nu(\mu^*) = 2$ in thin wires at the transition into the resistive state with $V \sim I$, i.e. $\nu = 1$. Note that in contrast to the KTB transition in 2D superconducting films, the jump is from $\nu = 2$ to 1, instead of $\nu = 3$ to 1.

For a short wire $X < c_0/T$ we again proceed in two steps. A 2D scaling analysis yields the 'global' fugacity $\tilde{y}$. In analogy with the resistively shunted Josephson junction [11], the voltage drop from the imaginary part of the free energy reads

$$V = \frac{2\Phi_0\tilde{y}^2}{\Gamma(2\gamma)c\tilde{\tau}_0}\sinh\left(\frac{\Phi_0 I}{2cT}\right)\left|\Gamma\left(\gamma + \frac{{\rm i}\Phi_0 I}{2\pi cT}\right)\right|^2\left(\frac{2\pi\tilde{\tau}_0}{\beta}\right)^{2\gamma-1},$$

giving $R \propto T^{2\gamma-2}$ and $R \propto I^{2\gamma-2}$ respectively at high and low T. Here $\tilde{\tau}_0$ is defined from the high frequency cutoff in (7): $\tilde{\tau}_0 \sim XC/e^2\gamma$. The above result is valid for $\gamma > 1$ and also for smaller γ at not very small T [11]. At $T \to 0$ in the metallic phase the resistance becomes [11]

$$R = \frac{S\sigma_{\rm qp}}{X}, \tag{16}$$

i.e. R is just equal to the quasipartical resistance of the wire whereas the superconducting channel is blocked due to quantum fluctuations.

7. Discussion

Let us compare our predictions with experimental results [5, 6, 10]. Taking $\sqrt{S} \sim 10$ nm and $\varepsilon_{\rm r} = 1$, for typical system parameters of $k_{\rm F}^{-1} \sim 0.2$ nm $< l \sim 1-10$ nm $\lesssim \xi \sim 10$ nm $< \xi_0 \sim \lambda_{\rm L} \sim 100$ nm we obtain the velocity $c_0/c = c_{MS}/c \approx (\sqrt{S}/10\lambda_{\rm L})$, $\mu \approx 30(\sqrt{S}/\lambda_{\rm L})$ and $a \sim 5-10$. This estimate yields the core action $S_{\rm core} \simeq a\mu \lesssim 10$ in agreement with [5].

For the quoted parameters, we predict the superconductor to metal transition at a wire thickness $\sqrt{S} \simeq \lambda_{\rm L}/15 \lesssim 10$ nm. This prediction also agrees with the results of Giordano, who finds that wires with $r_0 = \sqrt{S/\pi} \approx 8$ nm have a resistivity that saturates at a measurable level at low T, whereas the resistivity of thicker wires [5] $r_0 \gtrsim 13$ nm always decreases with T.

Another remarkable feature is that the classical-to-quantum crossover temperature T^* was found to be quite close to $T_{\rm c}$ for sufficiently thin wires [5]. Comparing our quantum action $2S_{\rm core}$ with the classical exponent [3] we immediately arrive at a simple estimate for $T^* \approx \Delta_0^{2/3}c_0^{1/3}/\xi^{1/3}$. For the above parameters it yields $T^* \sim 10\Delta(T^*)$, i.e. for thin wires one indeed expects this crossover to happen quite close to $T_{\rm c}$.

Independent measurements of $R(T)$ for superconducting wires have been carried out in Refs [6, 10], where systematic deviations from classical predictions [3] for thin wires have been also reported. Although the overall trend [6, 10] is similar to that observed in [5] the shape of some experimental curves look quite different from [5]. It was argued [8] that these quantitative differences are due to the granularity of the wires used in the experiments [5]. However, the variations of S were reported to be moderate in [5]. If so, this experimental feature can only cause a somewhat non-uniform distribution of QPSs along the wire because the QPS fugacity increases with decreasing S. With trivial modifications our theory can be applied to this situation as well. Although the agreement of our predictions with the results [5] by itself cannot rule out the 'weak link' interpretation [8], it is quite clear that the inability of the author [8] to explain the data [5] within the QPS scenario is solely due to serious drawbacks in the theory [8] and *not* due to possible experimental problems with the granularity of the wires [5].

In order to proceed with our comparison let us recall that the wires in [6, 10] were quite short $X \sim 1-2$ μm whereas the wires investigated in [5] were typically one or even two orders of magnitude longer. At not very low T one can put $\sigma_{\rm qp} \sim \sigma$ and estimate the parameter $\tilde{\tau}_0^{-1}$ to be of the order of 1 K or even bigger for the samples [6, 10]. At the same time for the samples [5] $\tilde{\tau}_0^{-1}$ is typically below 10 mK. Thus the difference between the results [5] and [6, 10] can be attributed to the different behavior of the function $\mathcal{G}$ (5) for different frequencies: the form (6) should be applied in the case [5] whereas the samples [6, 10] in the interesting temperature range should rather be described by means of (7). E.g. it

appears that the data of Fig. 1a of [10] can be (at least qualitatively) described by the dependence

$$R(T) \propto \tilde{y}^2 (T\tilde{\tau}_0)^{2\gamma-2} \tag{17}$$

both for thicker ($\gamma > 1$) and thinner ($\gamma < 1$) wires. The resistance of the latter – according to our analysis — should increase with decreasing T. This is just what has been found in [10]. The crossover from this behavior to that with decreasing $R(T)$ for thicker wires can be interpreted as an indication of the phase transition at $\gamma = 1$.

Another interesting feature to be discussed is the negative magnetoresistance of the wires observed in [10]. At first sight this feature contradicts our QPS scenario: in a (sufficiently strong) magnetic field H the gap Δ_0 is partially suppressed and the barrier for QPS should decrease. Hence, the fugacity y and the wire resistance R, in contrast to [10], should increase with H. However, if one includes dissipative effects outside the core the picture can change drastically. Indeed $\sigma_{\rm qp} = \sigma n_n/n$ strongly depends on the relation between T and $\Delta_0(T, H)$. At sufficiently low T a decrease of Δ may lead to an exponential [$\sigma_{\rm qp} \propto \exp(-\Delta_0/T)$] increase of the number of quasiparticles and, therefore, dissipation. Thus we have two effects: the field H increases the QPS fugacity y but also increases the dissipation γ which suppresses quantum fluctuations. It is quite obvious [e.g. from Eqn (17)] that the second effect may dominate in a certain parameter region and the resistance will *decrease* with increasing H. At very large H the gap Δ_0 will be suppressed and the resistance will increase again. This re-entrant behavior was observed in [10].

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