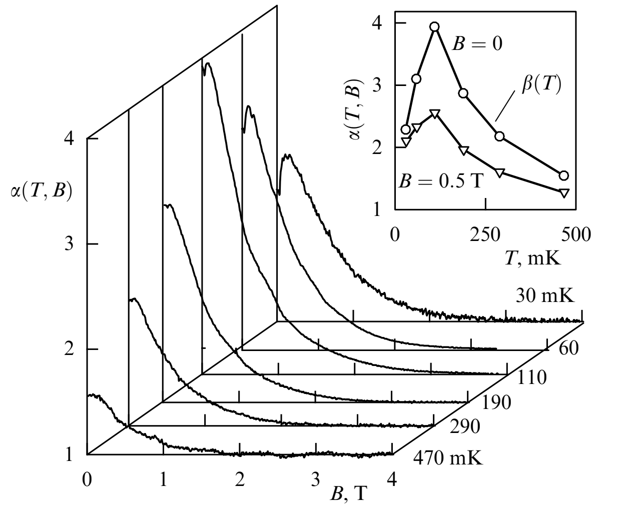

Figure 5. Normalized magnetoresistance $\alpha(T, B)$ of the Cd – Sb sample as a function of B at different temperatures. Inset: α as a function of T at two values of B; $\alpha(T = 9) \equiv \beta(T)$.

bringing the value of α to unity. So, the resistance values at a field of 4 T, used for normalizing, are taken from the field-independent region. Note that $\beta(T) = \alpha(T, 0)$. In the temperature range between 490 and 190 mK the value of $\beta(T)$ increases with decreasing temperature, in qualitative agreement with Eqn (6) (inset to Fig. 5). This corresponds to what has been observed previously at higher temperatures [12]. However, in the low-temperature limit the behavior of $\beta(T)$ changes drastically. It is maximum at a temperature of about 100 mK and then decreases with further lowering of the temperature. In weak magnetic fields an initial increase appears on the dependence. However, at any fixed magnetic field $B < 2$ T the value of α behaves similarly (see inset to Fig. 5). This contrasts sharply with the temperature dependence of the normal-state resistance which is monotonous in the range of temperatures used (see Fig. 4).

The observed decrease of the ratio β with decreasing temperature unambiguously indicates that at low temperatures the conductivity σ_1, which originates from single-particle tunneling, is shunted by the conductivity σ_2 of another kind:

$$\sigma = \sigma_1(T) + \sigma_2 \,. \tag{7}$$

We believe that σ_2 is due to incoherent pair tunneling (coherent, i.e. Josephson, pair tunneling is supposed to be absent in this insulating state; probably the maximum of $\alpha(B)$ at $B \approx 0.1$ T at the lowest temperatures designates the destruction of the remnants of the coherent scattering by the magnetic fields). The single-particle tunneling current i_1 is described in the first-order approximation by the barrier transparency t: $i_1 \propto t \exp(-\Delta/T)$. It is proportional to the product of two small factors, one of which is temperature dependent. Since the Cooper pairs are at the Fermi level, the two electrons forming a pair do not need to be excited above the gap for simultaneous tunneling. Hence $i_2 \propto t^2$, without the exponential temperature-dependent factor. When the temperature is sufficiently low, so that

$$t > \exp\left(-\frac{\Delta}{T}\right), \quad \text{i.e.} \quad T < \frac{\Delta}{|\ln t|}\,, \tag{8}$$

the single-particle tunneling is frozen out, and the two-particle tunneling current comes into play.

The two particles bound into a pair in the initial state may come to be unbound in the final state. Such a process of pair tunneling looks similar to the two-particle contribution to the tunnel current through a superconductor — the normal-metal junction (SIN junction) [14]. The latter may prove to be very important in high-resistance granular superconductors.

4. Conclusion

Both experiments described above can be interpreted as confirmation of the existence of localized pairs. But they do not give any information about how this localization is realized. The one-particle localization radius ξ may turn out to be either larger, or smaller than the coherence length ξ_{sc}. The case $\xi \gg \xi_{sc}$ is an extreme limit of granular superconductors, with only one pair in a grain. The opposite case $\xi \ll \xi_{sc}$ is assumed, for instance, in the model of localized bipolarons [15]. To distinguish these two possibilities, other experiments are required.

This work was supported by Grants RFFI 96-02-17497 and INTAS-RFBR 95-302 and by the "Statistical Physics" program of the Russian Ministry of Sciences.

References

1. Fisher M P A *Phys. Rev. Lett.* **65** 923 (1990)
2. Hebard A F, Paalanen M A *Phys. Rev. Lett.* **65** 927 (1990)
3. Yazdani A, Kapitulnik A *Phys. Rev. Lett.* **74** 3037 (1995)
4. Paalanen M A, Hebard A F, Ruel R R *Phys. Rev. Lett.* **69** 1604 (1992)
5. Shahar D, Ovadyahu Z *Phys. Rev. B* **46** 10917 (1992)
6. Gantmakher V F et al. *Zh. Eksp. Teor. Fiz.* **109** 1765 (1996) [*JETP* **82** 951 (1996)]
7. Gantmakher V F et al. *Pis'ma Zh. Eksp. Teor. Fiz.* **64** 713 (1996) [*JETP Lett.* **64** 767 (1996)]
8. Liu Y et al. *Phys. Rev. B* **47** 5931 (1993)
9. Dynes R C, Garno J P, Rowell J M *Phys. Rev. Lett.* **40** 479 (1978)
10. Adkins C J, Thomas J M D, Young M W *J. Phys. C* **13** 3427 (1980)
11. Gantmakher V F et al. *Zh. Eksp. Teor. Fiz.* **104** 3217 (1993) [*JETP* **77** 513 (1993)]
12. Gantmakher V F et al. *Zh. Eksp. Teor. Fiz.* **105** 423 (1994) [*JETP* **78** 226 (1994)]
13. Gantmakher V F, Esipov S E, Teplinskiĭ V M *Zh. Eksp. Teor. Fiz.* **97** 373 (1990) [*Sov. Phys. JETP* **70** 211 (1990)]
14. Hekking F W J, Nazarov Yu V *Phys. Rev. B* **49** 6847 (1994)
15. Alexandrov A S, Mott N F *Rep. Prog. Phys.* **57** 1197 (1994)

Superconductor – insulator transition in the disordered Bose condensate: a discussion of the mode-coupling approach

A Gold

1. Introduction

In the last 20 years the Anderson transition [1] in disordered Fermi systems has been studied extensively. The Anderson transition is a disorder induced metal – insulator transition in a non-interacting electron gas at temperature zero. It is widely believed that a disordered non-interacting Fermi gas can be described by the scaling theory [2]: according to this theory one expects that in two dimensions and at temperature zero a metallic phase does not exist, due to weak-localization corrections, and for vanishing temperature the static conductivity should scale to zero. For the disordered interacting

Fermi gas in two dimensions it was recently shown that a metal-insulator transition exists and that the scaling theory does not apply [3]. Apparently, the metallic phase is stabilized by interaction effects. This point of view has already been used a long time ago in the mode-coupling approach for an interacting disordered electron gas (disordered Jellium model), where a metallic phase in two dimensions was found for weak disorder and an insulating phase for strong disorder [4]: screening effects reduce the effects of disorder. Analytical results for the static conductivity of a disordered two-dimensional electron gas with a long-range random potential have been given in Ref. [5]. However, we note that the mode-coupling approach was criticised because it cannot describe weak-localization effects.

An interesting question in the context of disordered quantum liquids is: what happens for a disordered Bose condensate? The following two scenarios are at least possible: (i) a transition from a superfluid phase to a metal or (ii) a transition from a superfluid phase to a insulator.

2. Results of the theory

I predicted a disorder induced superfluid – insulator transition for an interacting Bose condensate in three dimensions [6]. I found that at a critical amount of disorder a transition from a superconductor to an insulator takes place. At the transition point I found a phase with a finite static conductivity. In the insulating phase the Bose condensate is localized: in this phase a condensate density still exists but the macroscopic wave function is localized (pinned) by the disorder. I treated the interaction with the random-phase approximation (RPA) and the disorder within a mode-coupling approximation. Multiple-scattering effects are treated within this approximation in an approximate scheme. I have also studied the dielectric properties of the disordered Bose condensate [7].

Recently, I calculated the transport properties of a disordered Bose condensate in a superlattice [8] and in a two-dimensional system [9] with a long-range interaction potential (Coulomb potential) [10] and in three dimensions [11, 12] with a short-range interaction potential (Bogolyubov model). As in three dimensions I found a disorder induced superconductor – insulator transition, which depends on the condensate density.

We note that Ramakrishnan [13] suggested that a two-dimensional disordered Bose condensate might be a realistic model for superconducting ultra-thin films. Experiments on homogeneous ultra-thin films showed some evidence for a superconductor – insulator transition [14]: the onset of superconductivity in thin films of Bi or Pb takes place when the normal state sheet resistance was below a certain value (near 6.45 kΩ). For a recent review on the superconductor – insulator transition, see Ref. [15].

In the mode-coupling approach [16] the current-relaxation kernel, which corresponds to a frequency dependent relaxation rate (scattering time), determines the dynamical conductivity. The current-relaxation kernel is calculated within the mode-coupling approach, which is, essentially, Fermi's golden rule expression: the matrix element (disorder) times the density of final states (for density modes). The density of final states is given by the density-density relaxation function. In the density – density relaxation function disorder effects enter via the current-relaxation kernel (using a conserving theory), which give a closed equation in order to calculate the current-relaxation kernel in a self-consistent way [16].

In the case of a disordered Bose condensate the density of final states is described by the collective modes of the system [6]. The absence of particle – hole excitations in a Bose condensate is the origin of the superfluid transport properties of the disordered Bose condensate if the amount of disorder is small.

Different kinds of impurities (disorder) and different kinds of interaction effects (long-range or short-range interaction) and different dimensions d can be studied within the mode-coupling approach. The mode-coupling theory gives results for weak disorder, in agreement with perturbation theory, but also for intermediate and strong disorder. Only for weak disorder is the condensate a superfluid. For large disorder an insulating phase is found. The transition point shows a finite static conductivity. The superconductor – insulator transition occurs if the disorder is equal to a critical amount of disorder (for fixed condensate density) or if the condensate density N is lower than a critical density N_c (for fixed disorder). The frequency-dependent conductivity for a disordered Bose condensate has been calculated for three dimensions [7] and two dimensions [10].

The instability of the Bose condensate in the presence of disorder can be regarded as a disorder-induced softening of the collective modes (plasmons). Note, in the superfluid in the long wave-length limit one finds that the collective modes are well defined, however, the energy is reduced due to disorder. The transition point, characterized by a metallic conductivity, can be interpreted in terms of the Ioffe – Regel criterion. For the metal – insulator transition in electron systems the Ioffe – Regel criterion is well established. The localization criterion for Fermi systems and for Bose condensates are very similar and this indicates that in both systems the same kind of physics is working: disorder versus screening. At low carrier density (or for a large amount of disorder) disorder wins over screening (the condensate is localized), while at high carrier density (or for a small amount of disorder) screening wins over disorder (the condensate is non-localized).

In the insulating phase the macroscopic wave function of the Bose condensate is pinned by the potential fluctuations of the random potential. This phase corresponds to a new state of matter and one might call it Bose glass.

For a long-range random potential [10] the predictive power of our theory has been demonstrated by presenting analytical and numerical results. For weak disorder the frequency-dependent conductivity can be derived in an analytical form. A transcendental equation for the current-relaxation kernel has been given for strong disorder [10]. The transcendental equation has been solved analytically and numerically: the frequency-dependent conductivity for low frequencies and the stiffness in the insulating phase have been calculated.

The similarity [17] of the calculated dynamical conductivity of a disordered two-dimensional Bose condensate and recent experimental results found for the measured dynamical conductivity in high-temperature superconductors indicate that a disordered Bose condensate might be a relevant 'effective' model in high-T_c materials.

3. Disorder: mode-coupling approximation

One of the problems concerning the mode-coupling approximation was the fact that within this approach [16] the weak-localization corrections cannot be described: the current-relaxation kernel was proportional to the density – density relaxation function and the squared gradient of the random potential (gradient coupling). Weak localization corrections occur due to a certain symmetry of the density – density correlation function [18], and within this symmetry the current-relaxation kernel is proportional to the squared random potential and the density-density relaxation function (potential coupling). For non-interacting electrons it was shown that the potential coupling, responsible for the weak-localization corrections, can be obtained within the mode-coupling approximation [19].

It should be remembered that the mode-coupling theory [16] was formulated for non-interacting electrons, where the density – density relaxation function is determined by electron – hole excitations: the only decay channel for the current is the decay into particle-hole excitations.

In the case of a Bose condensate such one-particle excitations do not exist and the current only decays into collective modes. This means, on the one hand, that the modes (particle – hole excitations) responsible for the weak-localization corrections are not present in the case of a Bose condensate. On the other hand, it is not known whether the above mentioned symmetry of the density – density relaxation function is also present in case of an interacting system. A disordered interacting quantum liquid can be a disordered interacting electron gas (decay of current into particle – hole excitations and decay into collective excitations) or a disordered interacting Bose condensate (decay of current into collective excitations).

4. Interaction: random-phase approximation

The instability point (transition point) of the superconductor in the mode-coupling theory is described by the parameter A and A is proportional to the squared random potential screened by the Bose condensate. The instability occurs for $A_{3D} = 1$ and the superfluid phase is characterized by $A < 1$. In the insulator phase one finds $A > 1$. We have calculated the dependence of A on the strength of interaction effects and we found that A increases with decreasing interaction strength. This dependence has been shown for a long-range interaction potential [9] and for a short-range interaction potential (Bogolyubov model) [12]. We conclude that the superfluid phase of the Bose condensate without interaction effects is not stable in the presence of weak disorder: interaction effects (screening effects) are needed to stabilize the superfluid phase. The detailed behavior of screening effects have been studied [12] for a disordered Bogolyubov model. It was shown that for dimension $d < 4$ interaction effects are crucial for the existence of a superfluid phase even for weak disorder. However, interaction effects are irrelevant for the existence of a superfluid phase for $d > 4$ and weak disorder. Of course, by including many-body effects via the local-field correction into the screening function one can improve the random-phase approximation. In general, one can say that many-body effects beyond the mean-field approximation decrease the screening properties and, therefore, increase the effects of disorder [9].

5. Comparison with other theories

In the theoretical work [20] a superfluid-insulator transition with a finite conductivity at the transition point was proposed for the charged Bose condensate with uncharged impurities. Scaling assumptions were used to argue that in two-dimensional systems the conductivity at the transition point is finite and universal (independent of all microscopic details). We also found [9] that the conductivity at the transition point is finite (as already found long ago for three dimensions [6]), but the conductivity at the transition point depends on the form of the random potential and is not universal.

In the scaling approach [20] it was assumed that a superconducting correlation length ξ diverges at the transition point as $\xi \sim \Delta^{-\nu}$ with $\Delta = (N - N_c)/N_c$. A dynamic critical exponent z for a characteristic frequency $\Omega_c \sim \xi^{-z} \sim \Delta^{z\nu}$ was introduced [20]. It was argued that for a two-dimensional charged Bose condensate $z = 1$ and $\nu = 1$.

The effective superfluid density N_s in our theory, which appears in the $\Delta(\omega)$-peak of the static conductivity, behaves as $N_s \sim (N - N_c)$ and the characteristic frequency is given by $\Omega_c \sim (N - N_c)$. The critical exponents and the dynamic instability of the superfluid – insulator transition were originally derived in Ref. [6]. Contrary to claims that the importance of critical dynamics was first observed in Ref. [20] we claim that this observation was first made in Ref. [6]. We also would like to mention that a diagrammatic approach for the three-dimensional disordered Bogolyubov model [21] has confirmed our predictions concerning a finite condensate density at the instability point and the existence of a superfluid-insulator transition. We note, however, that in the mode-coupling approach the condensate density is an input parameter. We assumed that the condensate density N is finite and independent of disorder (a disorder induced depletion of the condensate density was neglected).

6. Conclusion

The importance of interaction effects for experiments concerning the metal – insulator transition in a two-dimensional electron gas was recently pointed out [3]: the metallic phase is stabilized by interaction effects. The importance of interaction effects (screening effects) for a disordered Bose condensate and a disordered electron gas [4] was emphasized a long time ago. I would like to emphasize that the mode-coupling approach gives real predictions for the static and dynamic conductivity and predictions for the instability point in terms of parameters which characterize the random potential and the condensate density.

The mode-coupling approach is not an exact theory. However, I believe that one part of the essential physics is included in this approach. The important issues in this approach are, on the one hand, disorder effects, described by Fermi's golden rule, and, on the other hand, screening effects, described by the mean-field type random-phase approximation. The superconductor – insulator transition in a disordered Bose condensate was predicted in 1983 [6] and only much later were conductivity measurements [14] interpreted as a superconductor-insulator transition with an insulating phase in which the condensate is pinned.

References

1. Anderson P W *Phys. Rev.* **109** 1492 (1958)
2. Lee P A, Ramakrishnan T V *Rev. Mod. Phys.* **57** 287 (1985)
3. Kravchenko S V et al. *Phys. Rev. B* **51** 7038 (1995)
4. Gold A, Götze W *Phys. Rev. B* **33** 2495 (1986)
5. Gold A *Phys. Rev. B* **44** 8818 (1991)
6. Gold A *Z. Phys. B* **52** 1 (1983)
7. Gold A *Phys. Rev. A* **33** 652 (1986)
8. Gold A *Z. Phys. B* **81** 155 (1990)
9. Gold A *Z. Phys. B* **83** 429 (1991)
10. Gold A *Z. Phys. B* **87** 169 (1992)
11. Gold A *Phys. Rev. Lett.* **70** 1563 (1993)
12. Gold A *Z. Phys. B* **91** 501 (1993)
13. Ramakrishnan T V *Phys. Scr.* **T 27** 24 (1989)
14. Haviland D B, Liu Y, Goldman A M *Phys. Rev. Lett.* **62** 2180 (1989)

15. Wallin M et al. *Phys. Rev. B* **49** 12115 (1994)
16. Götze W *J. Phys. C* **12** 1279 (1979)
17. Gold A *Physica C* **190** 483 (1992)
18. Vollhardt D, Wölfle P *Phys. Rev. B* **22** 4666 (1980)
19. Belitz D, Gold A, Götze W *Z. Phys. B* **44** 273 (1981)
20. Fisher M P A *Phys. Rev. Lett.* **65** 923 (1990)
21. Huang K, Meng H-F *Phys. Rev. Lett.* **69** 644 (1992)

Mesoscopic phenomena in disordered superconductors

A Frydman, E P Price, R C Dynes

1. Introduction

The superconductor – insulator (SI) transition in 2D ultrathin films has been thoroughly investigated during recent years using two different morphologies; a *uniform* film in which the morphology is homogeneous down to atomic scale [1] and a *granular* film in which the morphology consists of grains of about one hundred angstrom in diameter [2]. In both cases it is found that as the normal-state sheet resistance, R_{N}, is increased, the superconductivity is weakened and eventually the sample behaves as an insulator. However, the nature of SI transition is very different in the two morphologies. In the uniform case the amplitude of the superconducting gap, Δ, and the critical temperature, T_{c}, decrease simultaneously as R_{N} increases (so that $\Delta/T_{\mathrm{c}} = \mathrm{const}$), implying that the magnitude of the superconducting order parameter decays with increasing resistance [1]. In granular films, on the other hand, T_{c} and Δ remain at bulk values throughout the entire SI transition. In this case the superconducting transition as a function of temperature becomes broader as R_{N} increases until the sample becomes insulating (see Fig. 2). This behavior implies that in granular films, the individual grains are large enough to support a bulk superconductor order parameter. However, as R_{N} increases, phase fluctuations appear between the grains and long range phase coherence is destroyed; thus the sample no longer exhibits global superconducting properties [2]. Even though the general mechanism for the SI transition in granular superconductors is understood, the nature of the resistive ‘tail’ on the superconducting side is still a topic of interest as is the detailed role that spatial phase fluctuations play in the destruction of superconductivity. For this reason we have studied samples with sub-micron dimensions that have a relatively small number of grains along the length of the sample. We investigated samples with lengths ranging from 0.1 – 2 μm and, by applying *in situ* quench-condensation techniques we were able to go through the SI transition using a single sample with only a minor change in its morphology. We present data from samples having different lengths and at different stages of the transition. In all cases, samples which show signs of superconducting behavior exhibit a series of discontinuous voltage jumps in the $I - V$ curves, which we interpret as sequential destructions of dc supercurrents in individual weak links between grains. In addition, these samples are characterized by a rich profile of conductance fluctuations as a function of bias voltage or magnetic field. The amplitude of these fluctuations scales with the sample conductance and may reach values much larger than the normal-metal universal conductance fluctuation value of e^2/h. The latter features are ascribed to interference effects of the superconducting wavefunction within quantum-coherent regions. This interference is modulated by a magnetic flux penetrating the sample due to an external magnetic field or to self induced flux from the current flowing through the granular system.

2. Experimental

We prepared the samples by thermally evaporating a strip of Pb, connected to four leads, on a Si substrate. Next, we cut a slit in the Pb strip, 0.1 – 2 μm wide, using e-beam lithography and dry plasma etching. In order to prevent oxidation of the Pb we coated the sample *in situ* with a 20 A layer of Ag. The sample was then immersed in a cryogenic evaporator which was equipped with a Sn source. We evaporated sequential ultrathin layers of Sn onto the slit in the Pb while the substrate was held at $T = 10$ K, and measured the transport properties after each evaporation stage. This set-up allowed us to change the resistance *in situ* and go through the entire insulator-superconductor transition using a single sample. Figure 1 illustrates the sample geometry and shows an STM picture of a similar quench-condensed film. It is seen that the grain sizes are typically 100 A in diameter. An estimate based on transport data yields a similar value [3]. We expect, therefore, that our samples consist of about 10 – 150 grains along the current direction. Standard dc and lock-in based ac methods were used to measure resistance versus temperature ($R - T$), current-voltage ($I - V$), dynamic resistance-voltage ($\mathrm{d}V/\mathrm{d}I - V$) and magnetoresistance ($R - H$) curves for different R_{N}. All measurements were performed in an rf shielded room.

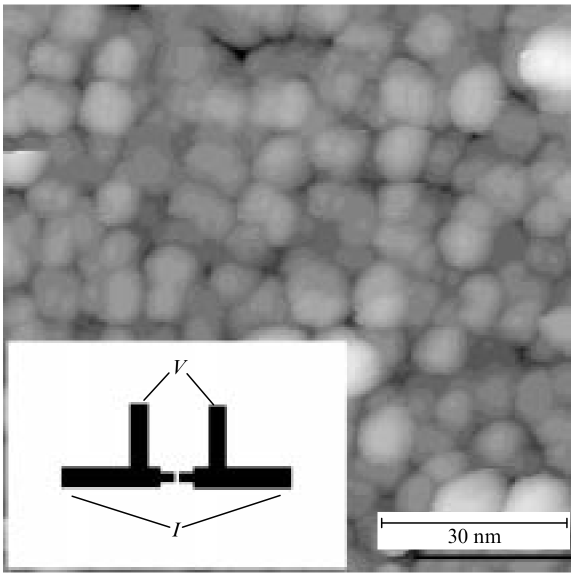

Figure 1. STM scan of a 100 A film of quench condensed Pb taken at $T = 8$ K. Inset: A schematic representation of the sample.

3. Results

In Figure 2 we show $R - T$ curves for a 1.5 μm long sample. Decreasing R_{N} corresponds to increasing the film thickness. These curves are similar to those seen in large granular 2D samples [2]. Note that ‘T_{c}’ barely changes throughout the SI transition, while the ‘tails’ of the $R - T$ curves become broader as the normal state resistance is increased. This implies that the destruction of superconductivity is caused by phase fluctuations between the grains rather than by suppression