Localization of quasiparticles in an NS structure

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We consider excitations in a thin normal metal film on the surface of a superconductor. We assume that electrons with energies well below the superconducting gap Δ are reflected from the NS interface by the Andreev mechanism, which is the case if the NS interface is clean and smooth (see Fig. 1). If Andreev reflection occurs *exactly* backwards [1], the electron orbit will be closed. Such an electron will bounce between the metal–vacuum and metal–superconductor interfaces, after each two reflections returning exactly to its original position. In such a model, all trajectories appear to be localized. This simple fact can be interpreted as the integrability of classical Andreev billiards [2].

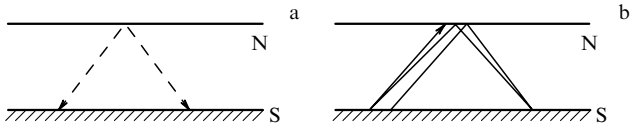

Figure 1. (a) Idealized picture of localization. The quasiparticle bounces between interfaces without spreading over the whole system. (b) At finite quasiparticle energy there is no perfect self-retracing in Andreev scattering. This results in a slow drift along the interface.

Below we discuss quantum localization in this system. We add details to the oversimplified picture discussed above and correct it. It turns out that in classical Andreev dynamics, localization (and integrability) is not a universal behavior. We discuss complications arising in the quantum problem, such as the role of smooth disorder versus short–range disorder. We compare the situation with that in a perfectly uniform metal film, where localization is absent. However, in the presence of smooth disorder with a large correlation length localization can arise. We derive the criterion for localization of this type and discuss the robustness of this phenomenon.

Recently, the effect of Andreev reflection was discussed in the context of the problem of quantum chaos in NS cavities [3, 4]. It was found that in the presence of Andreev scattering, chaotic dynamics give rise to an energy gap centered at the Fermi level. This gap can serve as a benchmark of chaotic dynamics. In contrast with the cavity problem, we study an infinite system. In this case, a natural equivalent of ergodic (chaotic) dynamics in cavities is the delocalization of states over the entire system. Accordingly, in the infinite system, an equivalent of regular dynamics in cavities is the localization of states.

We begin the discussion by reviewing the properties of the states in a uniform film of constant thickness. In this system electronic states are plane waves and thus are not localized. To see that, one can solve Bogolyubov–de Gennes equations [5] for this problem and derive the spectrum of electrons [1]:

$$\varepsilon_n(p_x) = \frac{\pi(n+1/2)v_F}{L}\left(1-\frac{p_x^2}{p_F^2}\right)^{1/2}. \qquad (1)$$

Here n is the quantum number ($n = 0, 1, 2, \ldots$), L is the thickness of the film, p_x is the momentum of the particle parallel to the NS interface, p_F is the Fermi momentum, and v_F is the Fermi velocity (here we set $\hbar = 1$). The spectrum (1) has a dispersion $[v(\varepsilon) = \partial\varepsilon/\partial p_x \neq 0]$, and therefore the states are not localized. However, the dispersion is much weaker than for free electrons and thus the Andreev states are much closer to becoming localized.

One can qualitatively understand this dispersion as follows. When Andreev reflection occurs, an electron is converted into a hole, and its energy (measured from ε_F) changes sign. Therefore, the momenta of electron and hole, p_e and p_h, are related by

$$\frac{p_e^2}{2m} = \varepsilon_F + \varepsilon\,; \qquad \frac{p_h^2}{2m} = \varepsilon_F - \varepsilon\,. \qquad (2)$$

Here ε_F is the Fermi energy, and m is the effective mass. From Eqn (2) it is clear that for an energy not exactly at the Fermi level, i.e. for $\varepsilon \neq 0$, the momenta p_e and p_h are different. On the other hand, the component of momentum parallel to the interface is conserved for Andreev reflection. Consequently, the reflection angle must change. For $\varepsilon \ll \varepsilon_F$ the change in the angle is small, and the Andreev reflection law reads:

$$\frac{\sin\theta_i}{\sin\theta_r} = -\left(1 \pm \frac{\varepsilon}{\varepsilon_F}\right), \qquad (3)$$

(θ_i and θ_r are the angles of incidence and reflection measured from the normal to the interface). Thus, after two subsequent reflections the particle will not return to the starting point. Instead, it will be displaced along the interface (see Fig. 1a). The drift velocity derived from this argument is the same as that obtained above from dispersion in (1). Note that the same effect is responsible for the suppression of spectral flow in the superconducting vortex core [6].

Evidently, even though there is no localization in a uniform system, by making the interface rough one can reach the situation when classical Andreev trajectories will become localized, with no drift along the interface. Thus our next step is to introduce disorder into this model.

We consider here a model of disorder in which the thickness of the normal film is slowly varying. More precisely, we assume that the normal metal–vacuum interface has some roughness, whereas the NS interface is flat. The main effect of interface curvature is that it acts like a focusing mirror, counteracting the dispersion. To overcome the dispersion, the curvature must exceed a certain threshold which will be estimated below.

Note that the localization effect we are considering is based on the semiclassical picture of a particle (nearly) retracing its trajectory after being Andreev–reflected. Therefore, the scattering by surface roughness must also be semiclassical in order to preserve the trajectory. The point is that quantum effects in scattering, i.e. diffraction, can destroy the localization. Indeed, due to diffraction, quantum scattering is stochastic and thus it violates the reversibility of individual trajectories. (Because two scattering events on the same disorder configuration may not lead to identical results.) Thus, we consider only sufficiently smooth surface fluctuations and formulate below a quantitative condition on the degree of smoothness.

Suppose that the mean thickness of the normal metal film is L, the variation of the thickness is Δ, and the spatial scale on which the thickness varies is r_c (see Fig. 2). The criterion for localization can be expressed in terms of these parameters.

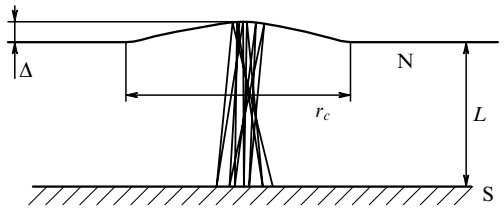

Figure 2. Localization of the quasiparticle trajectory by surface fluctuation. The curvature of the surface eliminates the average drift.

To begin with, let us ignore diffraction, and consider a purely classical motion. Without any loss of generality, we can limit the discussion of the problem to a two-dimensional space. We will use a coordinate system in which the NS interface is the line $y = 0$, and the metal – vacuum interface is given by $y = L(x)$. It is very instructive to consider a metal–vacuum surface of constant curvature, i.e. of spherical shape.

First, suppose that the NS interface is in the equator plane. Consider a trajectory which hits the NS interface exactly at the center of the sphere (see Fig. 3). Note that any such trajectory retraces itself even if Andreev scattering does not occur exactly backwards. Thus, for this trajectory the drift is indeed eliminated by the curvature. One can also show geometrically that if the center of hemisphere is within the normal region, i.e., $L > R$, there will be no net drift (see below). Thus, the situation when the center of curvature lies exactly on the NS interface is critical.

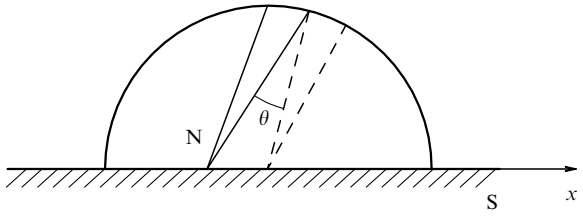

Figure 3. Particle bouncing inside the hemisphere. When the center of curvature lies on the NS interface, any trajectory which passes through the center of curvature will be self-retracing.

The conventional approach to dynamical problems of this kind involves mapping the Poincaré section [7] in the phase space. For the $D = 2$ problem the phase space is four-dimensional, but energy conservation reduces the dimensions to three. Hence, the Poincaré section is two-dimensional. To construct it, we consider the points on the metal – vacuum interface hit by an electron (and disregard the points due to holes). These points are characterized by their x-coordinates x_i. Also, we characterize the momentum of an electron by the angle θ_i between the momentum direction and the normal to the surface at the collision point. Thus, each collision is give a pair (x_i, θ_i), and the trajectory of the particle is represented as a sequence of points in the (x, θ) plane. The Poincaré section for a particular metal – vacuum surface shape is shown in Fig. 4. This section exhibits typical Kolmogorov – Arnold – Mozer features: stable periodic islands representing finite motion, i.e., non-escaping trajectories, and regions around the islands representing escaping

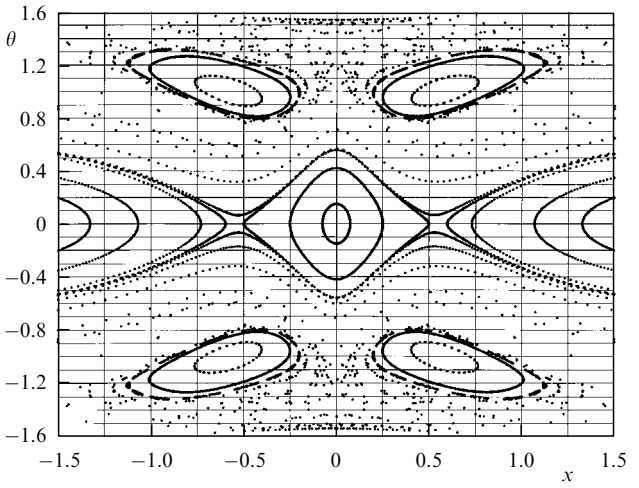

Figure 4. Poincare section for $y(x) = 10 + 1/(1 + x^2)$, $\varepsilon = 10^{-3}\varepsilon_F$. The central island is formed by almost vertical trajectories while the others represent orbits hitting the metal – vacuum interface four times per period. If one denotes each collision on the NS interface by S, and on the metal – vacuum interface by V, then the central island corresponds to a VSVSVS... sequence of collisions, while the lateral ones correspond to a VVSVVS... sequence.

trajectories. The islands correspond to localized states, whereas the outer regions correspond to delocalized states.

To find the region of stability of localized orbits, one should consider the stability of a self-retracing trajectory, like the one in Fig. 3. Suppose that the particle starts from the metal-vacuum interface at a point x and its momentum direction is characterized by the angle θ defined above. Suppose also that $\theta \ll 1$. Using the reflection law (3) and simple geometrical considerations, one can write down the linearized equations for the phase space coordinates x' and θ', describing the state of the quasiparticle after reflection from the NS interface and returning to the metal – vacuum interface:

$$\begin{pmatrix} x' \\ \theta' \end{pmatrix} = \hat{M}(\alpha) \begin{pmatrix} x \\ \theta \end{pmatrix}, \tag{4}$$

where $\alpha = \varepsilon/\varepsilon_F \ll 1$, and

$$\hat{M}(\alpha) = \begin{pmatrix} 1 - \alpha\dfrac{L}{R} & \alpha L \\ -\dfrac{\alpha}{R}\left(1 - \dfrac{L}{R}\right) & -1 + \alpha\left(1 - \dfrac{L}{R}\right) \end{pmatrix}. \tag{5}$$

After one Andreev reflection the sign of ε changes, because the electron turns into a hole. Since in the construction of the Poincaré section we are only interested in electrons and not holes, the matrix describing the motion in the (x, θ) plane is $\hat{M}(\alpha)\hat{M}(-\alpha)$. In the first order in α its eigenvalues are

$$\lambda_{1,2} = 1 \pm 2\mathrm{i}\alpha\sqrt{1 - \frac{L}{R}}. \tag{6}$$

To the same accuracy in α, it is more natural to write

$$\lambda_{1,2} = \exp\left(\pm 2\mathrm{i}\alpha\sqrt{1 - \frac{L}{R}}\right), \tag{7}$$

to ensure phase volume conservation. The eigenvalues λ_1 and λ_2 are complex for $L < R$ and real otherwise. This means that the self-retracing trajectory is stable for $L > R$.

To derive this relation in a more intuitive way, note that for each orbit the maximal value of x corresponds to $\theta = 0$, i.e., to normal reflection. The attentive reader will notice that here we have the same situation as in the example above with the hemisphere. When x is maximal, which corresponds to the trajectory turning point, the trajectory is almost self-retracing, like that in Fig. 3. The center of curvature at the point x lies on the NS interface. We conclude that localized states exist if the center of curvature is above the NS interface.

The previous discussion shows that the localization criterion is $L > R$, where R is the curvature radius. By estimating $R \sim r_c^2/\Delta$, one arrives at

$$r_c < r_c^* \sim \sqrt{L\Delta}\,. \tag{8}$$

We call this condition the *classical criterion* of localization.

Criterion (8) is not the only constraint on r_c. In this system, localized and delocalized classical trajectories can coexist at the same energy. (Indeed, for any value of ε there are trajectories going along straight lines parallel to the NS interface.) Hence, any perturbation which mixes these two types of states can destroy localization. In particular, due to the finite size of surface fluctuations which focus electrons, *quantum diffraction* takes place. The diffraction changes the orientation of the kinetic momentum in a random fashion, which leads to a spreading of the states over the whole system. We shall discuss this effect below.

For smooth disorder, one can derive the criterion of delocalization via diffraction using (1) and employing an adiabatic approximation. For that, we make L in (1) position–dependent and interpret the energy (1) taken at $p_x = 0$ as a spatially dependent potential energy. Also, we expand the square root in (1) and replace p_x by $-\mathrm{i}\partial/\partial_x$. This gives the effective kinetic energy. Thus, one gets the Hamiltonian:

$$\hat{\mathcal{H}}_{\text{effective}} = \frac{\pi(n+1/2)v_F}{L}\left\{-\frac{1}{2p_F^2}\frac{\partial^2}{\partial_x^2} - \frac{\delta L(x)}{L}\right\}. \tag{9}$$

Here $\delta L(x) = [L(x) - L]$ is the deviation of the film thickness from its mean value. Since the particle is localized near the thickest place, where $L(x)$ is maximal, one can write: $\delta L(x) \approx \Delta(1 - x^2/r_c^2)$. Then one can estimate the width d of the ground state wave function by comparing (9) to the harmonic oscillator problem:

$$d \sim \left(\frac{\lambda_F^2 r_c^2 L}{\Delta}\right)^{1/4}. \tag{10}$$

Quantum effects do not destroy a localized state if its smearing given by (10) is much less than the potential well width, i.e., if $d \ll r_c$. Thus, one arrives at another condition:

$$r_c > \sqrt{\frac{L\lambda_F^2}{\Delta}} \sim \frac{\lambda_F}{\Delta}\, r_c^*\,, \tag{11}$$

where r_c^* is defined in (8). This is the *quantum criterion* of localization.

The classical and quantum criteria (8) and (11) determine when localization can take place. Note that (8) and (11) are compatible only when $\Delta > \lambda_F$. This is expected, because if the thickness fluctuations are less than λ_F, the disorder cannot separate a group of states out of the continuum and localize these states. (In the case $\Delta < \lambda_F$, there is no room for an extra wavelength between the NS and metal – vacuum interfaces).

Can one satisfy (8) and (11) in a real system? The film thickness L must be larger or of the order of the superconducting coherence length ξ_0. Otherwise, due to the proximity gap induced in the normal layer there will be no excitations with the energies of interest. Besides, the NS interface must have a width $\geqslant \xi_0$ in order that the reflection is fully Andreev. Thus, for a superconductor with $T_c \sim 10$ K one gets $L \sim 1000$ A. Since $\lambda_F < \Delta$ is required, let us take $\Delta \simeq 10$ A. Thus, the criteria (8) and (11) give $10\,\text{A} < r_c < 100\,\text{A}$. This means that the surface must have a certain degree of smoothness. At present, it is difficult to say how realistic this condition is. One can imagine a situation where all abrupt jumps of the surface are screened by conducting electrons so that the resulting potential is sufficiently smooth.

Note that localization of the type described above is quite different from the usual localization. First, it occurs only if the scattering is classical, in contrast with the usual Anderson localization which is due to the quantum nature of scattering. A manifestation of this is the *suppression* of localization in the system by short range disorder. Note, however, that for a sufficiently high impurity concentration the electrons are again localized, now by the Anderson mechanism. Thus, localization is reentrant with respect to disorder strength. We think that at high impurity concentration Andreev reflection should enhance the Anderson localization effect. However, this question certainly needs more attention.

Secondly, in this system there is no mobility edge: the energies of the localized and delocalized states are not separated. This apparently contradicts the standard ergodicity argument by Mott [8] about the absence of the coexistence of localized and delocalized states with the same energy†. The reason that there is no ergodicity in our problem is that the disorder is smooth. Due to this smoothness, adiabatic barriers appear dividing the phase space into domains with very different dynamical characteristics (see Fig. 4).

Finally, due to the presence of the superconductor, the localization in the NS structure is less sensitive to interaction effects. In fact, in this problem we are dealing with charged quasiparticles in a highly conducting and thus well – screening medium. Usually, the metal – insulator transition in a disordered system is controlled by the effects of the Coulomb interaction. The reason for the importance of the Coulomb interaction is that due to poor conductivity near the transition the screening of the interaction is very slow. In contrast, in the NS structure the interaction is screened by the superconductor. Therefore, the screening is always fast, no matter how slow the electrons are.

Finally, we discuss how localization of this unusual type could be observed. Perhaps, it cannot reveal itself through the conductivity, because the superconductor will always shunt the electrical conductivity of the normal film. Instead, one can measure the local tunneling density of states, which could be probed, for example, by an STM.

† In our problem the matrix elements between states localized at different maxima of $L(x)$ are exponentially small due to the adiabaticity of the motion. Since they are non-zero, we are not dealing with true localization here, and in a strict sense the Mott argument still holds. Thus, the states we discuss are only nearly localized with an exponentially large decay time.

If the normal film is flat, the spectrum of electrons is described by (1). The average density of states corresponding to (1) has the well-known sawtooth structure. This structure should be independent of the position of the STM tip.

If localized states are present, they will add spatially dependent features to the tunneling density of states. Each localized state will give rise to a peak in the density of states if the STM tip is close to the place where the state is localized. Thus, one can study the spatial correlation of peaks (and other features) in the local tunneling density of states. If the peaks are really due to localized states, they should be spatially uncorrelated. This measurement scheme is insensitive to the supercurrent.

To summarize, we have studied the localization of quasiparticles in a normal metal film bordered by an Andreev mirror. This system exhibits a new type of localization caused by self-retracing due to Andreev reflection. We derived the criteria for this localization, and discussed its manifestation in the tunneling density of states.

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