long range correlations $\langle \delta I_{ij}(\mathbf{R}_1, \mathbf{R}_2)\delta I_{ij}(\mathbf{R}_3, \mathbf{R}_4)\rangle \sim R^{-4(d-1)}$ when $|\mathbf{R}_1 - \mathbf{R}_3| \sim |\mathbf{R}_2 - \mathbf{R}_4| = R \gg l$. As a result they give the main contribution to the correlation function of the interlayer exchange energy $\langle \bar{I}_{ij}\bar{I}_{kl}\rangle$ at $L \gg l$. As a result, we have:

$$\langle \delta\bar{I}_{ij}\delta\bar{I}_{kl}\rangle = \frac{2}{\pi} I_0^2 E_{\mathrm{F}}^2 T \sum_m \omega \int \mathrm{d}\mathbf{R}_1\,\mathrm{d}\mathbf{R}_2\,\mathrm{d}\mathbf{R}_3\,\mathrm{d}\mathbf{R}_4$$
$$\times \left[\hat{\sigma}_i \hat{P}^c_\omega(\mathbf{R}_1, \mathbf{R}_2)\hat{\sigma}_k \hat{P}^c_\omega(\mathbf{R}_2, \mathbf{R}_3)\hat{\sigma}_j \hat{P}^c_\omega(\mathbf{R}_3, \mathbf{R}_4)\hat{\sigma}_l \hat{P}^c_\omega(\mathbf{R}_4, \mathbf{R}_1)\right.$$
$$\left. + \hat{\sigma}_i \hat{P}^d_\omega(\mathbf{R}_1, \mathbf{R}_2)\hat{\sigma}_j \hat{P}^d_\omega(\mathbf{R}_2, \mathbf{R}_3)\hat{\sigma}_k \hat{P}^d_\omega(\mathbf{R}_3, \mathbf{R}_4)\hat{\sigma}_l \hat{P}^d_\omega(\mathbf{R}_4, \mathbf{R}_1)\right]. \quad (8)$$

Here $\omega = \pi(2m+1)T$ is the Matsubara frequency, m is an integer, T is the temperature and $\hat{\sigma}_i$ are spin operators. Integration over $\mathbf{R}_1$, $\mathbf{R}_3$ and $\mathbf{R}_2$, $\mathbf{R}_4$ in Eqn (8) is performed over volumes of the first and the second ferromagnetic layers respectively. The results of calculation of Eqn (8) depend on the ratio between the lengths L, L_2, $L_T = \sqrt{D/T}$, $L_{\mathrm{so}} = \sqrt{D\tau_{\mathrm{so}}}$ and on the boundary conditions for Cooperons and Diffusons, which are shown in Fig. 2c. Here L_{so}, τ_{so} are the spin-orbit relaxation length and time, respectively, and D is the electron diffusion coefficient in the N layer. In the case of the 'open' geometry of the N layer shown in Fig. 1a and $L_T, L_{\mathrm{so}} \gg L > l$; $L, L_2 \gg L_1, L_3$, we have

$$\langle \delta\bar{I}_{ij}\delta\bar{I}_{kl}\rangle = \frac{5 \times 2^{7/2}\zeta(5/2)}{3^2\pi^{9/2}} X \frac{I_0^2}{(p_{\mathrm{F}}l)^2}(p_{\mathrm{F}}L_1)^4 \delta_{ij}\delta_{kl}\,. \quad (9)$$

Here X is a factor, which is of order unity when $L \sim L_2 \leqslant L_T$ and $\zeta(x)$ is the zeta-function. In different limiting cases we have:

$$X = \begin{cases} \left(\dfrac{L_2}{L}\right)^4, & L_T > L_2 > L\,, \\ \dfrac{L_2 L_T^3}{L^4}, & L_2 > L_T > L\,. \end{cases} \quad (10)$$

It is interesting that in the case $L \sim L_2 < L_T$, Eqs (9), (10) turn out to be independent of L. In the case $L > L_T$ the expression for X acquires an additional exponentially small factor $\exp(-L/L_T)$. In the case $L_{\mathrm{so}} > L$ the minimum of the exchange energy corresponds to a parallel or antiparallel orientation of the layer's magnetizations (θ equals zero or π). In the opposite limit $L_{\mathrm{so}} \ll L$ we get the same formula as Eqn (9) but without the factor $\delta_{ij}\delta_{kl}$. This means that the exchange interaction between the F layers is of the Dzialoshinski-Moria type and a minimum of the exchange energy corresponds to a sample specific angle $\theta(\widehat{\mathbf{n}_1, \mathbf{n}_2})$ distributed randomly over the interval $(0, \pi)$. While deriving the results presented above we neglected the sensitivity of the boundary conditions for Cooperons and Diffusons shown in Fig. 2c to the change of magnetization directions in F-layers. In the case of the open sample geometry Fig.1a this is correct, provided $Ap_{\mathrm{F}}^3 L_1/v_{\mathrm{F}} \ll 1$. To get an estimate for $\langle \delta\bar{I}_{ij}\delta\bar{I}_{kl}\rangle$ in the opposite limit one has to substitute E_F for the factor $A(L_1 p_{\mathrm{F}})$ in Eqn (5). For example, in the case $L_T > L \sim L_2 > L_{\mathrm{so}}$ we have

$$\langle \delta\bar{I}_{ij}\delta\bar{I}_{kl}\rangle \sim E_{\mathrm{F}}^2 (p_{\mathrm{F}}l)^{-2} \sim \frac{\hbar}{\tau}\,. \quad (11)$$

Here τ is the elastic mean free path in the metal. We would like to stress again that the origin of Eqs (9)–(11) is the long range correlation of the signs of $I_{ij}(\mathbf{R}_1, \mathbf{R}_2)$ and $I_{kl}(\mathbf{R}_3, \mathbf{R}_4)$ which survive over distances much larger than l.

As is usual in the physics of mesoscopic metals [15, 16], the external magnetic field changes the electron interference pattern and thereby $\delta\bar{I}_{ij}$, and $\theta(\widehat{\mathbf{n}_1, \mathbf{n}_2})$ turns out to be a random sample-specific oscillating function of the magnetic field H. Another way to change the relative orientations of the F-layers is demonstrated in Fig. 1b. Namely, $\theta(\widehat{\mathbf{n}_1, \mathbf{n}_2})$ is a random sample specific function of the order parameter phase difference $(\chi_1 - \chi_2)$ in superconductors S_1 and S_2 shown in Fig. 1b. The reason for this is that some diffusive paths connecting points *1* and *2* in Fig. 1b can visit superconductors (line '*b*' in Fig. 1b) and the corresponding amplitude of the probability of traveling along these paths acquires the additional phase $(\chi_1 - \chi_2)$ [17]. Another consequence of the phase dependence of the exchange energy is that the critical Josephson current of the device shown in Fig. 1b depends on the angle θ between the magnetizations of the F layers.

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Coulomb effects in a ballistic one-channel S-S-S device

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1. Introduction

Coulomb effects in several different types of three-terminal devices consisting of an island connected to external leads by two weak-link contacts, and capacitatively coupled to an additional gate potential, have been extensively studied during recent years [1–3]. In the present paper we develop a theory for a system consisting of two almost ballistic one-channel QPC's connecting a small SC island with two SC leads. The fabrication of such a system might become possible, due to recent technological progress [4–6]. We derive the dependences of the average Josephson current across the system, and its fluctuations (noise power) as functions of the SC phase difference between the leads α, and of the electric gate potential V_g. We show that such a system realizes a tunable quantum two-level system (pseudo-spin 1/2) which may be useful for the realization of quantum computers (cf. e.g. [7, 8]).

2. Model of a one-channel nearly ballistic S-S-S junction

Consider a small superconducting island connected to two external superconducting leads by one-channel nearly ballistic quantum point contacts [9, 10] (see Fig. 1). Following [9] we assume that each contact is much wider than the Fermi wavelength so that the transport through the constriction may be treated adiabatically, but much smaller than the coherence length $\xi_0 \equiv \hbar v_F/\pi\Delta$ (where v_F is the Fermi velocity, Δ is the superconducting gap).

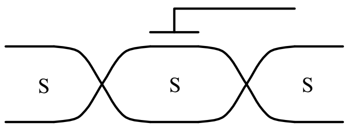

Figure 1. Double-contact S-S-S system.

Our assumption of low temperature is that the average number of one-electron excitations on the island is much less than one. Then they cannot contribute to the total charge of the grain and we may restrict our Coulomb blockade problem to the evolution of the superconducting phase only. The condition of low temperature is then $T < \Delta/\log[V\nu(0)\Delta]$, where V is the volume of the grain, and $\nu(0)$ is the density of electron states at the Fermi level.

We neglect phase fluctuations in the bulk of the island and describe the whole island by a single superconducting phase χ. At a fixed value of the phase on the island, the spectrum of each of the two junctions consists of the two Andreev states localized on the junction and the continuum spectrum above the gap Δ [10]. The energies of the Andreev states lie below the gap:

$$E_{\pm}(\delta\phi) = \pm\Delta\sqrt{1 - t\sin^2\left(\frac{\delta\phi}{2}\right)}\,, \tag{1}$$

where $\delta\phi$ is the phase difference at the contact, and t is the transmission coefficient. We set the superconducting phase on one of the leads to be zero; the phase on the other lead α is assumed to be fixed externally. Then the total Josephson energy of the two contacts is (Fig. 2):

$$U(\chi) = U_1(\chi) + U_2(\alpha - \chi)\,, \tag{2}$$

where $U_i(\delta\phi) = E_-(\delta\phi)$.

The potential of the grain V_g determines the balance between electrostatic energies $E(Q) = (Q - V_g C)^2/2C$ for different charges Q on the grain. C is the capacitance of the grain. We shall further assume that the capacitance C is not very small, so the charging energy $E_C \equiv (2e)^2/C \ll \Delta$.

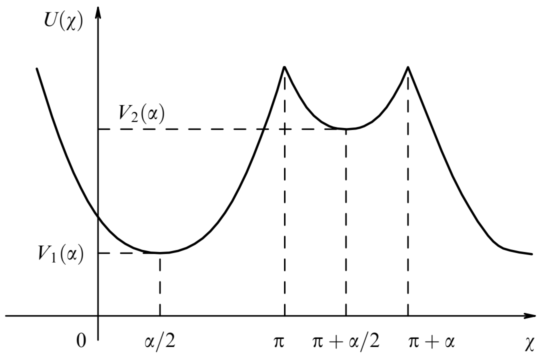

Figure 2. Potential $U(\chi)$.

The adiabatic Hamiltonian for the double junction looks like this:

$$H(\alpha, N) = U(\chi) + U(\alpha - \chi) + E_C\frac{(\pi_\chi - N)^2}{2}\,. \tag{3}$$

Here $N = V_g C/2e$ is the rescaled dimensionless gate voltage, and α is the superconducting phase difference at the external leads. The number of Cooper pairs at the grain π_χ is the momentum conjugate to χ, $[\chi, \pi_\chi] = \mathrm{i}$.

We must comment on the applicability of the adiabatic approximation (3), which implies that the contacts follow their ground states and that transitions between Andreev levels (and to the continuous spectrum) are negligible. In the small backscattering limit $r \equiv 1 - t \ll 1$ the contacts spend most of the time far from the resonance points $\delta\phi = \pm\pi$ (the probability of finding the phase differences at the contacts close to the resonant value is exponentially small at T/Δ, $E_C/\Delta \ll 1$), and therefore the gap in the excitation spectrum is always of order Δ, except in processes of phase tunneling. Thus, at the bottom of the potential well of (2) we may neglect non-adiabatic transitions because $|\dot\chi| = E_C|\pi_\chi - N| \ll \Delta$. However, during phase tunneling processes, the phase χ must cross a point where the gap in the single-contact excitations (1) collapses. We shall see that Coulomb effects are described precisely as phase tunneling processes. Therefore we need a more careful treatment of phase tunneling, which we develop below.

3. Breakdown of adiabatic behavior in a single superconductive quantum point contact

In a ballistic quantum point contact at $t = 1$ the spectrum of Andreev states (1) has a level crossing point at $\delta\phi = \pi$. At this point, the left and right Andreev states have equal energies, but in the absence of backscattering ($t = 1$) the transitions between them are impossible. Therefore, we expect that the ideal ballistic contact cannot adiabatically follow the ground state as the phase $\delta\phi$ changes, but remains on the same left or right Andreev state as it passes the level-crossing point $\delta\phi = \pi$. Now we study the crossover from this limit to the opposite adiabatic limit and find the crossover scale for the reflection coefficient r.

For this purpose we consider a simplified system: a small superconducting grain connected to only one superconducting lead by a nearly ballistic single-channel Josephson junction. As before we assume that the reflection probability $r \ll 1$ (almost unity transmission), that the charging energy $E_C \ll \Delta$ and that the temperature is sufficiently low to prohibit single-electron excitations on the grain.

To probe the degree of adiabaticity, we study the periodic dependence of the ground state energy E_0 on the gate voltage. Because of the weakness of charging effects, this dependence will be sinusoidal:

$$E_0(N) = \varepsilon \cos(2\pi N)\,, \tag{4}$$

and we are interested in the amplitude ε of these oscillations, which provides a good measure of adiabaticity in the phase dynamics. Since we are restricting our attention to low lying excitations, it is only necessary to include the two Andreev levels on the junction. As in the discussion in the previous section, the dynamic variable is the phase on the grain, the potential term is the Josephson energy of the Andreev levels, and the kinetic term is the charging energy. The final form of the Hamiltonian including *both* Andreev levels is:

$$H(\chi) = \Delta \begin{pmatrix} -\cos\frac{\chi}{2} & r^{1/2}\sin\frac{\chi}{2} \\ r^{1/2}\sin\frac{\chi}{2} & \cos\frac{\chi}{2} \end{pmatrix} + \frac{1}{2}E_C\left(\mathrm{i}\frac{\partial}{\partial\chi} - N\right)^2. \tag{5}$$

Here χ is the phase difference across the contact, and r is the reflection coefficient. Obviously, the eigenvalues of $H(\chi)$ at $E_C = 0$ reproduce result (1).

This Hamiltonian loses its validity at the top of the upper band at $\chi = 2\pi n$, where the upper Andreev state mixes with the continuous spectrum. However, the probability of the phase χ reaching the top of the upper band of $H(\chi)$ is exponentially small at $E_C \ll \Delta$ (smaller than the tunneling probability). The adiabatic-diabatic crossover is determined by the properties of the system near the minimal-gap point $\chi = \pi$. Therefore, we may neglect the transitions to the continuous spectrum at $\chi = 2\pi n$. There are two opposite limits of the problem: small and 'large' reflection. At zero reflection, the Hamiltonian splits into lower and upper components. Within each component the potential is periodic with period 4π. As explained above, we must neglect the next-nearest-neighbor tunneling via the top of the bands. Therefore, the potential minima of $H(\chi)$ are disconnected and cannot tunnel to each other, $\varepsilon = 0$. In the opposite limit of 'large' reflection the gap opens in the spectrum of Andreev states, and the system adiabatically follows the lower state. We can replace the two-level Hamiltonian $H(\chi)$ by its lowest eigenvalue and arrive at the quantum-mechanical problem of a particle in a periodic potential. The quasiclassical limit of this problem is solved in textbook [16]. In our notation the answer reads as follows:

$$\varepsilon_{\mathrm{ad}} = \mathrm{const}\sqrt{E_C\Delta}\exp(-S_{\mathrm{cl}}), \qquad \text{where}$$
$$S_{\mathrm{cl}} = B_1\sqrt{\frac{\Delta}{E_C}} - \frac{1}{4}\log\frac{\Delta}{E_C} + O(1) \tag{6}$$

is the classical action connecting two adjacent minima (or more precisely the two return points). The numerical constant B_1 is of order one (at $r \to 0$, $B_1 = 4.69 + 1.41 r\log r + \ldots$).

To study how the adiabaticity is destroyed it is useful to introduce the dimensionless 'coherence factor' $f(r)$ defined by $\varepsilon = f(r)\varepsilon_{\mathrm{ad}}$, where $\varepsilon_{\mathrm{ad}}$ is the amplitude of oscillations of the ground-state energy derived in the adiabatic approximation. The perturbation theory with respect to virtual non-adiabatic transitions shows [15] that the first correction to the coherence factor looks like $1 - f \sim (1/r)\sqrt{E_C/\Delta}$, which allows an estimate of the crossover scale $r_{\mathrm{ad}} \sim \sqrt{E_C/\Delta}$. Consider now the limit of weak backscattering ($r \ll r_{\mathrm{ad}}$), take the unperturbed wavefunction to be the ground state of the Hamiltonian with zero r (at a given wavevector N), and then compute the first-order correction in $r^{1/2}$ to the energy: the wavefunction is of 'tight-binding' type and is generated by the 'ground-state' wavefunctions Ψ_i localized in the potential minima (diabatic terms):

$$\varepsilon = 2\langle\Psi_i|H_{12}(\chi)|\Psi_{i+1}\rangle = 2r^{1/2}\Delta\int \mathrm{d}\chi\,\Psi_i^*(\chi)\Psi_{i+1}(\chi)\sin\frac{\chi}{2}\,. \tag{7}$$

Here the (normalized) wavefunctions Ψ_i and Ψ_{i+1} are the ground-state solutions for different potentials ($-\Delta_0\cos(\chi/2)$ and $\Delta_0\cos(\chi/2)$) and the overlap integral (7) has a saddle point at the minimal-gap point $\chi = \pi$, which reduces the effective region of integration to $|\chi - \pi| \leqslant (E_C/\Delta)^{1/4}$. The normalization of the quasiclassical tail of the wavefunctions $\Psi_i(\chi)$ yields $\Psi(\chi = \pi) = \exp(-S_{\mathrm{cl}}(\chi = \pi))$ (up to a numerical factor independent of E_C/Δ). Thus we obtain

$$\varepsilon \sim r^{1/2}\Delta\left(\frac{E_C}{\Delta}\right)^{1/4}\exp(-S_{\mathrm{cl}})\,, \tag{8}$$

i.e. $f(r) \sim (r^2\Delta/E_C)^{1/4}$, which confirms that the crossover scale for reflection probability is given by $r_{\mathrm{ad}} \sim \sqrt{E_C/\Delta}$.

4. Josephson current

Suppose that the backscattering in each of the contacts is larger than the adiabatic crossover scale r_{ad} found in the previous section. Then we may use the adiabatic model (3) to compute the low-energy spectrum of the double junction and the Josephson current as a function of the phase difference across the junction α and of the gate voltage N. Even if the reflection in the contacts is smaller than the adiabatic crossover scale, we may account for non-adiabatic effects by using the 'coherence factors' discussed in the previous section.

Accepting the simplifying condition $E_C \ll \Delta$ allows us to treat the Coulomb term in the Hamiltonian perturbatively. First, neglecting the Coulomb term, we obtain a classical system on the circle in the potential (2) with two minima. Weak backscattering in the contacts only smoothes out the 'summits' of the potential, but leaves the bottom of the potential practically unchanged. The energies of the two minima are $V_1(\alpha) = -2\Delta|\cos(\alpha/4)|$ and $V_2(\alpha) = -2\Delta|\sin(\alpha/4)|$ (see Fig. 2). At zero temperature, our classical system prefers the lowest of them. Thus the energy of the S-S-S system in the absence of the Coulomb term is given by $E(\alpha) = -2\Delta\cos(\alpha/4)$ for $-\pi < \alpha < \pi$. Differentiat-

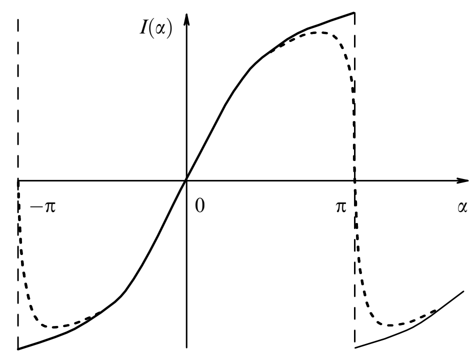

Figure 3. Josephson current. The dotted line shows smearing of the singularity due to the Coulomb term.

ing this energy with respect to the phase α gives the Josephson current $I(\alpha) = 2e[\partial E(\alpha)/\partial\alpha] = \Delta \sin\alpha/4$ for $-\pi < \alpha < \pi$. Notice that the current has large jumps at the points of level crossing $\alpha = \pi + 2\pi n$. Qualitatively this picture is very similar to the case of a single S-S ballistic junction [6, 9, 10], but the shape of the current-phase dependence $I(\alpha)$ is different.

The singularities at $\alpha = \pi + 2\pi n$ are smeared out by the level mixing due to quantum tunneling between two potential minima. Due to the shift in the 'angular momentum' by N, the wave functions in the two potential wells acquire an additional factor $\exp(iN\chi)$. This results in a relative phase of the two tunneling amplitudes of $2\pi N$. The net tunneling amplitude (defining the level splitting) may be written as

$$H_{12}(N) \equiv \Delta\gamma(N) = \Delta\left[\gamma_1 \exp(i\pi N) + \gamma_2 \exp(-i\pi N)\right], \quad (9)$$

where γ_1 and γ_2 are the two amplitudes of phase tunneling in the two different directions. Below we assume that these amplitudes are computed at the level-crossing point $\alpha = \pi$, where they are responsible for level splitting.

The amplitudes γ_1 and γ_2 obey the asymptotes derived in the previous section (except for numerical factors):

$$\gamma_{1,2} \approx f(r)\left(\frac{E_C}{\Delta}\right)^{1/4} \exp\left(-B_2\sqrt{\frac{\Delta}{E_C}}\right) \ll 1,$$
$$\exp\left(-B_2\sqrt{\frac{\Delta}{E_C}}\right) \ll 1, \quad (10)$$

where $B_2 \sim 1$ is determined by the classical action connecting the two potential minima (at $r \ll 1$, $B_2 \cong 1.45 + 2.20 r \log r + \ldots$). For the best observation of Coulomb oscillations, γ_1 and γ_2 must be of the same order, but not very small. In the ideal case $\gamma_1 = \gamma_2 = \gamma$ the total amplitude $\gamma(N) = 2\gamma\cos(\pi N)$. The characteristic scale for the r-dependence of B_2 is $\delta r \sim \sqrt{E_C/\Delta}$, therefore for γ_1 and γ_2 to be of the same order, the transparencies of the two contacts must differ by no more than $\sqrt{E_C/\Delta}$.

Here we should comment on the difference of our result (9), (10) from the normal two-channel system discussed in [2]. In the normal system the two tunneling amplitudes multiply, and the net ground-state energy oscillations are proportional to $r\ln r$ at small r. In the superconducting system, the external leads have different superconducting phases, and the tunneling in the two contacts occurs at different values of the phase on the grain. Therefore, the tunneling amplitudes add with some phase factors and give the asymptote of $\sqrt{r}$ at $r \to 0$. In fact, the oscillations in the superconducting system will be proportional to r (similarly to the normal system [2]) in a different limit — at the phase difference $\alpha = 0$, when the potential $U(\chi)$ has a single minimum and a single barrier.

The hybridized energy levels in the vicinity of $\alpha = \pi$ are given by the eigenvalues of the 2×2 Hamiltonian

$$H(\alpha, N) = \begin{pmatrix} V_1(\alpha) & H_{12}(N) \\ H_{12}(N) & V_2(\alpha) \end{pmatrix}. \quad (11)$$

Diagonalization (and expanding near $\alpha = \pi$) gives the two energy levels:

$$E_{1,2}(\alpha, N) = -\Delta\left[\sqrt{2} \pm \sqrt{\frac{(\alpha-\pi)^2}{8} + \gamma^2(N)}\right]. \quad (12)$$

Differentiating the energy of the lowest level E_1 over α, we find the Josephson current (cf. Fig. 3):

$$I(\alpha) = \frac{e\Delta}{\sqrt{2}} \frac{\pi - \alpha}{\sqrt{(\alpha-\pi)^2 + 8\gamma^2(N)}}. \quad (13)$$

The width of the crossover at $\alpha = \pi$ depends periodically on V_g: $|\alpha - \pi| \sim |\gamma(N)|$. At a nonzero temperature these Coulomb effects will compete with the smearing by temperature so that the width of the singularity at $\alpha = \pi$ is given at nonzero temperature $T \ll \Delta$ by $|\alpha - \pi| \sim \max[\gamma(N), T/\Delta]$. Therefore, in order for Coulomb effects to dominate the thermal fluctuations, we must have $T \leqslant \gamma\Delta$.

5. Supercurrent noise

Now we calculate the low-frequency spectrum of the fluctuations of the Josephson current in our model. We shall be interested in frequencies much lower than the oscillator energy scale $\sqrt{\Delta E_C}$, thus we consider only transitions between the eigenstates of the reduced ground-state Hamiltonian (11). We also assume that the temperature is lower than $\sqrt{\Delta E_C}$, then we may disregard the excited oscillator states and the internal noise in the contacts (discussed in [11–13, 17]). Obviously, under these assumptions we can observe current fluctuations only in the close vicinity of the resonance point $\alpha = \pm\pi$, where the energies (12) of the two low-lying states are close to each other. We expect to observe two peaks in the noise spectrum — one at zero frequency (due to the thermal excitations above the ground state), and the other at the transition frequency $|E_1 - E_2|$ (from the off-diagonal matrix elements of the current operator). In this section we compute the integral weights of these peaks and postpone the discussion of their width (determined by dissipative processes) until elsewhere.

Firstly, we shall discuss the zero-frequency peak. In our approximation it is just the thermal noise of a two-level system with Hamiltonian (11). The spectral weight is then given by a simple formula:

$$S_0(\alpha, N, T) \equiv \langle I^2\rangle - \langle I\rangle^2 = \frac{I^2(\alpha, N)}{\cosh^2(E_1 - E_2)/2T}.$$

Substituting $I(\alpha, N)$ and $E_{1,2}(\alpha, N)$ from the previous section, we obtain the noise intensity near the resonance:

$$S_0(\alpha, N, T) = \frac{\Delta^2}{2}\frac{\beta^2}{\beta^2 + \gamma^2(N)} \cosh^{-2}\left[\frac{\Delta}{T}\sqrt{\beta^2 + \gamma^2(N)}\right], \quad (14)$$

where $\beta = (\alpha - \pi)/2\sqrt{2}$. For the effect of the Coulomb interaction to be observable, the temperature must be smaller than the Coulomb gap: $T \leqslant \gamma\Delta$. At constant T and N, the noise decreases exponentially as α goes away from its critical value $\alpha = \pi$, and at $\alpha = \pi$ the noise is suppressed in the interval $|\alpha - \pi| < \gamma(N)$ (Fig. 4).

The interplay between these two factors results in a strong dependence of the peak value of the noise on the potential of the grain. The peak value of the noise $\max_\alpha S(\alpha, N, T)$ is plotted against N in Fig. 5. Most favorable is the case of identical contacts, when $\gamma_1 = \gamma_2 = \gamma$ and, therefore, $\gamma(N) = 2\gamma\cos(\pi N)$. In this case, when $\cos(\pi N) \ll T/\gamma\Delta$ (small gap limit) the noise takes its maximal value $S \approx \Delta^2/2$. In the opposite limit of a large gap ($\cos(\pi N) \gg T/\gamma\Delta$) the noise decreases exponentially:

$$S \approx \Delta^2\left[\frac{T}{\Delta\gamma|\cos\pi N|}\exp\left(-4\frac{\Delta\gamma|\cos\pi N|}{T}\right)\right].$$

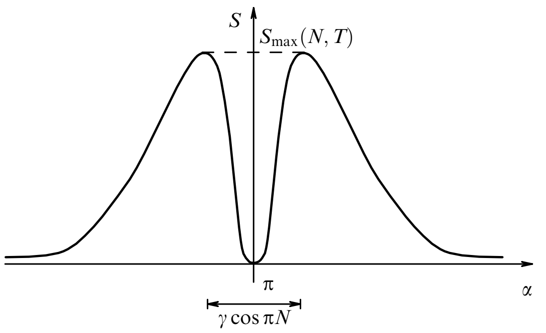

Figure 4. Zero frequency noise as a function of α.

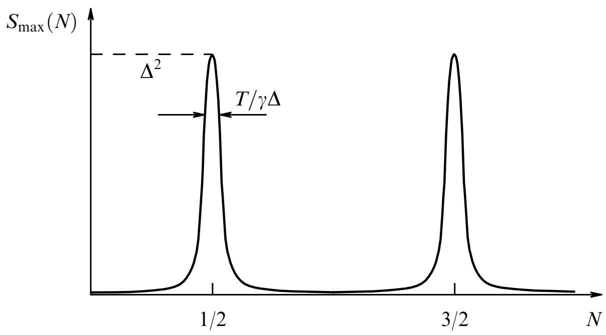

Figure 5. Maximal value of the noise $S_{\max}$ versus the gate potential V_g.

The noise has a sharp peak at the resonance point $\cos \pi N = 0$, where two levels on the grain with different electron numbers have equal energies. Now we turn to the noise peak at the interlevel frequency $\omega_{12} = |E_1 - E_2| = 2\Delta[\beta^2 + \gamma^2(N)]^{1/2}$. In our approximation of a two-level system such a noise is temperature independent, and its weight is determined purely by the off-diagonal matrix element: $S_{\omega_{12}} = 1/2\left|\langle 1|I|2\rangle\right|^2$. A straightforward computation for the Hamiltonian (11) gives (for $\alpha \approx \pi$):

$$S_{\omega_{12}}(\alpha, N) = \frac{\Delta^2\gamma^2(N)}{4[\beta^2 + \gamma^2(N)]} \,. \tag{15}$$

This result contrasts the corresponding noise intensity in the single quantum point contact (found in [11–13, 17]), where the corresponding noise intensity S_ω is temperature-dependent, because that system has four possible states (or, alternatively, two fermion levels).

6. Conclusions

We have developed a theory of Coulomb oscillations for the Josephson current and its noise power via the S-S-S system with nearly ballistic quantum point contacts. The period of Coulomb oscillations as function of the gate potential is $V_g^0 = 2e/C$. These oscillations arise from the quasiclassical tunneling of the superconducting phase on the grain and are, therefore, exponentially small in $\sqrt{E_C/\Delta}$. In addition, we predict a crossover from adiabatic to diabatic tunneling at the backscattering probability $r_{\text{ad}} \sim \sqrt{E_C/\Delta}$. For backscattering below r_{ad}, the amplitude ε of the Coulomb oscillations is proportional to the square root of the smallest (of the two contacts) reflection probability $\sqrt{r_{\min}}$. This contrasts the case of a normal double-contact system [18] where ε is proportional to the product $\sqrt{r_1 r_2}$.

The average Josephson current-phase relation $I(\alpha)$ is shown to be strongly non-sinusoidal and roughly similar to that known for a single nearly ballistic QPC, in the sense that it contains a sharp 'switching' between positive and negative values of the current as the phase varies via $\alpha = \pi$. A new feature of our system is that it is possible to vary the width of the switching region $\delta\alpha$ by the electric gate potential V_g; in the case of equal reflection probabilities $r_1 = r_2$ this electric modulation is especially pronounced, $\delta\alpha \propto |\cos(\pi C V_g/2e)|$. The noise spectrum of the supercurrent is found to consist mainly of two peaks: the 'zero-frequency' peak due to rare thermal excitations of the upper level of the system, and another one centered around the energy difference ω_{12} between the two levels. The widths of these peaks are determined by the inverse life-time τ of the two states of our TLS, which is due to electron-phonon and electromagnetic couplings. Both these sources of level decay are expected to be very weak in the system considered, but the corresponding quantitative analysis is postponed for future studies, so we presented here the results for *frequency-integrated* (over those narrow intervals $\sim 1/\tau$) noise power.

The S-S-S device with almost ballistic contacts represents a new type of a system which may be used as a realization of an artificial 'spin 1/2' — an elementary unit for quantum computations. In comparison with normal Josephson systems with tunnel junctions which were proposed for use in adiabatic quantum computations [8], the advantage of our system is that it may operate at considerably higher values of the Josephson critical currents; moreover, the current-phase characteristics of such a system are almost universal in the sense that they are determined mainly by the microscopic parameters of the SC materials and only weakly depend on the specifics of the contact fabrication.

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