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Mesoscopic and strongly correlated electron systems "Chernogolovka 97"

Mesoscopic unification

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As recently as 20 years ago, little conceptual overlap and little or no cooperation could be found between the metal physics, superconductivity and semiconductor communities; the subject of nonlinearity seemed to be irrelevant to solid state physics; and it would take much courage to predict that specialists from such seemingly diverse fields would some day meet to discuss problems of mutual interest. But they did meet — for the conference on mesoscopic and strongly correlated electron systems held in Chernogolovka on June 16 – 23, 1997†.

Borrowed from paleontology by W van Kempen and M Azbel, the title term 'mesoscopic' refers to systems which, while containing too many particles to apply quantum mechanical equations, are still too small for statistical methods, the system's global quantities exhibiting fluctuations of the order of their expectation values.

Understandably, a conference so titled is bound to feature works on low-dimensional systems. In this area, the subjects covered are quantum dots and tunneling transport (C Marcus, T Ihn et al. [1, p. 122]), electromagnetic response (H Bouchiat et al.), optics (A Forchel, V D Kulakovskiĭ et al. [1, p. 115]), laser applications (D Bimberg), and also transport and luminescence in 2D electron gas systems (I Bar-Joseph et al. [1, p. 112], Yu Dubrovskiĭ et al.), including coupled quantum wells (V B Timofeev et al. [1, p. 109], L V Butov [1, p. 118], thin wires (A Finkelstein [1, p. 171], M Devoret, D Esteve, W Hansen ([1, p. 175) and quantum rings (Yu Galperin et al. [1, p. 178]. Added to this list should be the study by H S J van der Zant et al. [1, p. 167] of charge density waves in thin films. The studies cited have all evolved from metal as well as semiconductor physics, and indeed the very term 'semiconductor' now sounds somewhat out-of-date and household for those involved. Many would simply shrug the off distinction by saying that conducting substances are divided into metals and insulators, of which both conduct at finite temperatures and only differ in how.

A very important and practically interesting spin-off of mesoscopics is 'single-electronics', a group of phenomena that involve the Coulomb blockade of conductivity. If the size d of metal (semiconductor) particles is small, so is their capacitance $C \sim d$, and transferring an electron from one particle to another produces an energy change of order $E_C = e^2/2C$. At sufficiently high temperatures $T \leqslant E_C$, electron transfer may become suppressed, and it is precisely this situation which has been termed a 'Coulomb blockade'. The magnitude of charge energy may be controlled by the voltage at an additional electrode, and such tunable Coulomb blockade devices have come to be known as one-electron transistors. This subject was covered in a theoretical talk by J König et al. [1, p. 159] and in two experimental talks by V A Krupenin et al. [1, p. 204] and E S Soldatov [1, p. 202] (in the latter, a Coulomb blockade at room temperature was reported).

One further new field of purely mesoscopic research, systems of a large number of mesoscopic elements arranged in regular or random arrays (M Pascaud and G Montambaux [1, p. 182], L Molenkamp and A Chaplik), also holds promise for numerous applications. The most ambitious of possible applications is in the development of quantum computation systems. The theory of quantum computation and the practical requirements it imposes on such mesoscopic systems is discussed by A Kitaev, who also advances a totally novel idea of using systems with anyons (see below in connection with the Quantum Hall Effect) in developing quantum computers.

The dividing line between mesoscopic and macroscopic systems is the characteristic length L_ϕ over which the wave function of a diffusing particle conserves its phase. For processes on a scale $L < L_\phi$, it is necessary to consider the interference of electron waves coming by various possible paths, and it is in this way that quantum optics came into solid state physics. Today, this is a rapidly developing field, whose impact on solid state physics will certainly grow in the very near future. The advent of interference in low-temperature electron physics was fore-run by weak localization [2] and low-temperature quantum corrections due to the electron – electron interactions in a dirty metal [3] (which are much more important than in a 'pure' Fermi liquid, which has no impurity scattering).

† In the conference proceedings that follow this introductory paper, a list of the session talks precedes the corresponding section and, wherever possible, references to journal papers or Internet (condmat@xxx.lanl.gov) e-prints are given throughout for studies not submitted for the present special *Physics – Uspekhi* issue, to be referred to as [1].

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Weak localization effects have in fact bridged the gap between mesoscopics and the physics of macrosystems, the dephasing length L_ϕ identifying the 'mesoscopic regions' with the property that interference within them controls the behavior of the macrosystem as a whole. On the other hand, the gradual reduction in the size of experimental mesoscopic systems has required the 'fine structure' of the spectra (due to the finite volume V of the system) to be incorporated into the theory; the energy scale being the one-electron level separation $\delta = 1/\nu(0)V$ (where $\nu(0)$ is the average density of states at the Fermi level). If the essential energy interval ΔE does not exceed δ too much, the description of the spectrum in terms of the average density of states alone becomes too crude implying that level position correlations must be included. Problems of this kind were first encountered in theoretical physics more than half a century ago in connection with complex nucleus spectra and were approached phenomenologically [4] by treating the nucleus Hamiltonian as a certain high-rank random matrix and then investigating its eigenvalue and eigenfunction statistics (Wigner – Dyson matrix ensembles). Wigner – Dyson (WD) theory seems first to be applied to disordered metals [5] in analyzing microwave field absorption by an ensemble of small metallic particles whose energy spectra were taken to be random in the sense of Wigner and Dyson. The advent of mesoscopic physics spurred the development of the theory of matrix ensembles. In particular, a powerful analytical technique known as the 'supersymmetric matrix σ model' was developed [6] which led to a rigorous result that for a dirty metal still far away from the localization threshold, the spectrum and wave function statistics are indeed described quite accurately in terms of WD ensembles. The fact that, in contrast to the atomic nucleus, mesoscopic Hamiltonian parameters — say, the magnetic field — can be varied experimentally has raised the problem of how this affects the spectra of random matrices (the so-called 'parametric level statistics'). Surprisingly, the problem of the parametric statistics of WD ensembles is equivalent [7] to that of calculating dynamic correlation functions in a strongly interacting one-dimensional Fermi gas (the Calogero – Sutherland model [8]). Thus, strong correlations not only appear in mesoscopic physics in connection with the real electron – electron interaction (whose effect is enhanced by disorder [3]) but also come from entirely different quarters, namely as a spin-off of the mathematics of random matrices. Similar to the situation with the study of phase transitions in the mid-70s, the interplay of the theory of condensed matter and quantum field theory is again becoming a factor of life. Secondly, but no less important, this 'grand unification' was brought to life by advances in the theory of the Fractional Quantum Hall Effect (see below) with its field-theoretic concept of the 'topological' Chern – Simons interaction (see Ref. [11]) that destroys time-reversal invariance.

An experimental study of level position correlations in a GaAs quantum dot was conducted by L Viña et al. [1, p. 153] by measuring magneto-exciton spectra. An obvious alternative, to measure fluctuations in the tunneling differential conductivity dI/dV against the voltage V and magnetic field B, was analyzed theoretically by V I Fal'ko [1, p. 156].

The inclusion of weak-localization-type interference corrections leads to a deviation from pure Wigner – Dyson spectral statistics and produces correlations between the energy-eigenvalue and wave-function fluctuations. This line of research was presented by A Mirlin and V Kravtsov. Ya Fedorov et al. and K. Efetov report on the fairly recent results that some physical problems of interest can be formulated as an extension of the theory of WD ensembles to non-hermitian random matrices.

Owing to their ability to model the chaotic spectra of finite systems, WD ensembles provide a good description of small pieces of a dirty metal. Suppose, however, that the disorder is not too large (if, say, the mean free path is large compared to the size of the system and predominant scattering is by boundaries — as is the case with very-pure-GaAs quantum dots). The questions which then arise are, to what extent and on what energy (E) scale can the spectra be considered chaotic, or, in other words, how does quantum chaos develop and what is its relation to classical chaos? This is discussed by A Larkin, who demonstrates for the first time that weak-localization corrections to spectral statistics are linked with Lyapunov's exponent which describes the chaotization of the classical motion of the same system. An entirely different approach to the development of quantum chaos was taken by Yu Gefen et al. who analyzes how an ordinary Fermi-liquid spectrum of collective excitations (quasiparticles) develops in a quantum dot (i.e., in a system with a discrete one-particle spectrum) due to the interparticle interaction, as the excitation energy $E \geqslant \delta$ increases. It turns out that purely formally this transition is similar to the problem of the Cayley tree localization threshold. This work (already published, see Ref. [9]) opens an intriguing new field in the theory of interacting mesoscopic systems. U Sivan et al. treat strongly interacting two-dimensional electrons in a somewhat more conventional manner in their experimental and theoretical study of how the exchange and correlation components of the ground-state energy behave in the very strong Coulomb repulsion regime.

There is in principle one further reason why weak localization may bring mesoscopics and macrophysics together. In pure metals, the mean free path for impurity scattering may be as large as 0.1 cm, and the inelastic phonon scattering time at $T \approx 1$ K is about 10^{-7} s. For the Fermi velocity of order 10^8 cm/s this means that the dephasing length is $L_\phi \approx 1$ cm and that macroscopic single crystals of super-pure metals (a typical experimental object of the 60s) are mesoscopic samples. Not that this circumstance has thus far been employed in any essential way, though.

Another important area of mesoscopic research are current and voltage fluctuations, which may not be small compared to the expectation values due to the relatively small number of conductance channels. A theory of shot noise in multi-contact metallic systems was presented by Ya M Blanter et al. [1, p. 149]. Further, at very low temperatures it is possible to measure noise at frequencies $\omega \gg k_B T/\hbar$, i. e., quantum noise ('zero oscillations'). A theoretical analysis of quantum noise detection in various experimental conditions is presented by G Lesovik [1, p. 145].

An important point about weak localization theory is that localization has in fact no physical reason always to be small so may well cross over to strong localisation. M Gershenzon [1, p. 186] achieved such a crossover by varying the temperature of one and the same sample. Since a satisfactory theory of the weak to strong localization crossover is not yet available, semiquantitative scaling-type analyses are often employed. The concept of scaling was widely used in solid state physics in developing the fluctuation theory of phase transitions back in the 70s (when both the basic features and very existence of scaling were proved rigorously), but it is often a good starting point for major developments in many other fields as well. For example, it is due to scaling that metal-insulator transitions began to be treated as a variety of second-order phase transitions [10]. Here, however, a transition is caused by varying the amount of disorder ($T = 0$), electron density, or the magnitude of the magnetic field, and the fluctuations depend both on time and space coordinates, thus increasing the effective dimensionality of the system. Near the phase transition, the fluctuation correlation length R_c increases, and within regions with $L < R_c$ the situation is controlled by fluctuations, both dynamic and 'frozen-in' (the latter term referring to impurity distribution). So here again we face the problem of a large number of interacting mesoscopic regions.

Although the peak of metal–insulator transition studies was in the 80s, since when many works on this problem have become classic, the field is still far from realizing its full potential. Some of the results presented at the conference were totally unexpected from the point of view of scaling theory as given in Ref. [10]. A good case in point is the metal-insulator transition in the Si-MOS structure, i.e., in a two-dimensional no-magnetic-field electron system (V Pudalov [1, p. 211]). This is a further reminder that a complete theory of metal-insulator transitions is still a long way off. In a number of talks, an analogy between metal-insulator transitions and some related phenomena was established. One such phenomenon, D Shahar showed, is the transition from the Quantum Hall state to an insulator in the presence of a magnetic field.

Needless to say, the Quantum Hall Effect, a spectacular new state of 'electron matter' in two dimensions, was also well within the scope of the conference. The Integer Quantum Hall Effect is due to the localized wave functions of non-interacting electrons in a disordered 2D system (R Ashoori), Quantum Hall conductance arising from the only states that remain delocalized, namely those at the centres of the Landau levels. The Fractional Quantum Hall Effect is accounted for by strong Coulomb electron-electron correlations in the lowest Landau level; the Coulomb interaction in this case is far more effective than in an ordinary Fermi liquid because the kinetic energy of the electrons is 'frozen' by the magnetic field.

Perhaps the most unusual strong interaction aspect of the Fractional Quantum Hall Effect is the nature of the quasiparticle excitations above the QHE ground state. While undeniably made up of electrons, they have a fractal charge and, in terms of statistics, are intermediate between Bose and Einstein. Theoretically, this striking effect has a rather long history (and is best described in field theory terms by invoking the Chern–Simons interaction [11, 12] mentioned above), but it is only recently that its existence has been confirmed experimentally. An example is an experiment by C Glattli et al., who measured shot noise to determine the excitation charge.

A surprising recent theoretical prediction is the collective nature of the lowest spin excitation in QHE: it turns out, specifically, that an energetically favorable spin 'texture' is one with a full spin much in excess of unity. This object, known as 'the skyrmion,' is discussed by many workers, both theoretically (S Iordanskiĭ [1, p. 131], W Apel, Yu A Bychkov [1, p. 134], Yu Nazarov) and from the experimental detection viewpoint (I Kukushkin et al., B Goldberg, D K Maude et al.). In the talks by S Dorozhkin et al. [1, p. 127] and V Dolgopolov [1, p. 138], capacitance spectroscopy measurements of the excitation spectra above the FQHE state were reported.

FQHE states with filling factors $\nu = 1/3, 2/3, 2/5$ etc. are incompressible and differ profoundly from the $\nu = 1/2$ and similar compressible Hall states. This is another remarkable example of particle-quasiparticle transformation: when in a strong magnetic field, strongly interacting electrons behave as 'nearly' weakly interacting ones in the absence of the field, that is, the interaction and magnetic field compensate each other in some bizarre way. The resulting object acts as an elementary excitation and has been termed the 'composite fermion.' Composite fermions may also be useful in describing other Hall states, but it is for $\nu = 1/2$ that they come closest to the way usual Fermi particles behave in the absence of a magnetic field. The theory of composite fermions was presented by L S Levitov et al. [1, p. 141] and E Rashba, and experiments on compressible FQHE states, by J Eisenstein and Z D Kvon [1, p. 164]. The latter work shows, incidentally, that mesoscopic fluctuations in conductivity as a function of the magnetic field (which are well known in non-interacting mesoscopics theory) have a similarity to the fluctuations as function of the gate potential, which have always been viewed as indicating the presence of strong Coulomb correlations. This observation is another indication of the dominant role strong interactions play in Hall systems.

We turn our attention next to superconductivity. It has been obvious since the early Ginzburg–Landau times that one of the key quantities in the theory of superconductivity is the phase φ of the condensate wave function $\Psi = |\Psi|\, e^{i\varphi}$. BCS theory showed that superconductivity is due to the Cooper interaction of electrons, which is strong when the total electron momentum is small. The discovery of the Josephson effect gave rise to superconductivity studies on single contacts, bottlenecks, thin wires, very thin films, and similar low-dimensional objects which later found a wide range of mesoscopic applications. Furthermore, the phase coherence effects discussed above have much in common (at least mathematically) with the Cooper instability phenomenon, which is the cause of superconductivity. It is for these reasons that the whole of the conference material, except perhaps for QHE, repeats itself in the 'plus Cooper interaction' version.

In discussing superconducting contacts, the phenomenon of Coulomb blockade quite naturally deserves mention. The theory of weak Coulomb blockade in the S–S–S 'transistor' with one-channel quantum contacts was presented by D Ivanov and M Feĭgel'man [1, p. 197], the opposite extreme of strong Coulomb blockade in the tunneling Josephson junction was analyzed by L Kuz'min, and the practical realization of 'Cooper–Coulomb' effects on atom-sized objects was the subject of D Müller's talk.

Systems of a large number of mesoscopic superconducting elements, — such as regular, regular-fractal, and disordered networks of weak Josephson bonds — are currently enjoying no less popularity than are networks of normal elements (P Martinoli, A Ustinov et al.).

Very general symmetry problems related to the Bose condensation and Cooper pairing phenomena in mesoscopic systems were analyzed in the talk by A Andreev.

As is the case for 'normal' dirty metals, dirty mesoscopic superconductors also exhibit large fluctuations in thermodynamic and kinetic quantities; the relevant talks are those by A Frydman [1, p. 220] and A Geim (experiment) and by B Spivak [1, p. 195] (theory). A question of fundamental interest here is, how far should an island of superconducting metal be reduced in size to prevent superconducting pairing in it? A relevant theory was presented by K Matveev. A study of high-dissipation Josephson junctions should also be noted (M Palaanen).

The experimental manifestation of superconductor-insulator transitions was discussed by V Gantmakher [1, p. 214] and A Lavrov [1, p. 223]. Theoretically, these experiments may be related to the crossover from BCS pairing to Bose – Einstein condensation in highly disordered low-electron-density systems. No reliable theory of such systems is yet available.

Although the problem of treating localization, Coulomb interaction, and superconducting pairing concurrently in a consistent way is far from being solved, some progress has been made in a number of interesting special cases. One example is transport in S – N and S – N – S structures, where N designates a metal which, although dirty, is still far from the localization threshold. The dominant role is played here by the events in which an electron becomes a hole after undergoing Andreev reflection from the superconductor surface. [13]. Electron interference in a normal region near the superconductor makes the conductivity phase sensitive and, at low temperatures, substantially non-local (the familiar series circuit resistance addition law does not hold here!). Talks on this topic included those by V Petrashov (experiment) and D Averin and A Volkov [1, p. 191] (theory). A Shitov et al. [1, p. 207] presented an example of how the Andreev reflection may lead to 2D localization in a thin normal-metal film deposited on a superconductor. A similar system was discussed by I Imry, who showed that the orbital motion of an electron near the NS boundary may cause a paramagnetic response.

Standing somewhat apart is the topic of transport in an S – N – S contact with a small number of transverse conducting channels (i.e., when the transverse contact dimension is comparable to the electron wavelength). Such a situation arises very naturally in two-dimensional low-density electron gas problems (in quantum dots, for example). For superconducting systems this condition, although much more problematic, still proves achievable (M Sanquer). The theory of transport in atom-sized quantum S – N – S contacts was discussed by A Matrin-Rodero. The superconductor – insulator transition was treated by A Gold [1, p. 217] in terms of self-consistent-field theory and by A Zaikin [1, p. 226] for the case of very thin wires.

To conclude, such were the basic topics of the June 1997 Chernogolovka conference whose proceedings are presented below and which, for all its compactness, covers virtually all the active areas of the modern physics of electrons in solids. The conference material in fact presents the state-of-the-art of the low temperature physics of systems with electron – electron interactions and is therefore totally appropriate for publication in a fundamentally review-oriented journal like ours.

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