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METHODOLOGICAL NOTES

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Similarity in problems related to zero-gravity hydromechanics

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Contents

Abstract. **On introduction of gravitational acceleration into the calculation of length and velocity scales, a set of dimensionless parameters appears in the mathematical model which enable the behavior of convection and heat and mass transfer under changing gravitational conditions to be predicted. By directly simulating the equations of motion and heat transfer, the effectiveness of the proposed formulation of conservation laws in treating such phenomena in liquids is demonstrated for both isothermal and non-isothermal cases and under both terrestrial and space conditions.**

1. Introduction

The mere fact that the mass force coefficient in the equations of motion decreases in the state of weightlessness does not imply the decrease of convection in fluids. Given the external micro-accelerations, Coriolis force, free fall acceleration variations and other non-gravitational sources of motion with changing direction, the motion in fluids is determined by nonlinear interaction of the above mentioned weak forces. Perhaps this is the reason why the experiments, for example, in cosmic physical metallurgy, are not repeatable [1]. Thus, accurate prediction of the behavior of liquid environments in cosmic conditions is possible only provided that the complete non-stationary three-dimensional problem is solved, and this is presently a very complex computational task with no currently known solution. In this respect, the primary goal is the determination of the fundamental mechanisms of influence of the micro-accelerations and free fall conditions on the fluid dynamics and the stability of the emerging flows. The second important task is the determination of real media and geometrical shapes to model the processes occurring in liquids in cosmic conditions on the Earth. Creation of materials with perfect predefined characteristics in space is possible only provided the above stated problems are solved.

The Oberbeck – Boussinesq system of equations is the model most commonly used for description of the motion of melts and other non-isothermal fluids. The specific character of problems under consideration is that prediction of the motion intensity inside the region, given the temperature conditions on the region's boundary, is impossible in advance. The level of convection and the structure of the flow are determined when the non-linear equations of motion and heat and mass transport are solved. (The same holds, for example, for the rate of floating of drops and bubbles and the character of the emerging flow.) This circumstance complicates the dimensional analysis. Introduction of an external value, characterizing velocity, may result in the appearance of another parameter in addition to many, already introduced in the mathematical model. The usage of the Grashof number for approximation of the predefined cosmic conditions by a model medium on the Earth is impractical because of the complex non-linear form of the Grashof number, containing physical constants of the medium, the geometrical parameter of the region and the temperature difference.

Based on the dimensional theory, the motion equations in this article are reduced to a form with a minimal set of dimensionless parameters of simple type. Then, there appears a possibility to determine the character of changes of convection and processes of heat and mass transfer, corresponding to changing gravitational conditions in a given environment, and to define the rules for setting similar environments on the Earth and in space. Direct calculations based on the equations of motion and heat and mass transport prove that the theoretical assumptions are correct.

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2. Dimensional analysis and the mathematical model

Suppose that a liquid medium has the following parameters: density ρ_0, coefficient of dynamic viscosity μ, surface tension coefficient σ_0; and let g be the gravitational acceleration. According to the dimensional theory [2], a medium with four parameters is characterized by one dimensionless combination. In this case, the following combination appears:

$$\mathrm{M} = \frac{\mu^4 g}{\rho_0 \sigma_0^3} \, .$$

The parameter $\mathrm{Fi} = \mathrm{M}^{-1}$, introduced by Kapitza [3] and later named the film number, is used for the investigation of draining films of liquids. Powers of the number Fi (for example, $\mathrm{Fi}^{1/11}$ in Ref. [14]) are used in the transition to experimentally measurable functions in specific environments. The value M [5], referred to as the Morton number [6], is used for the description of floating of bubbles in different liquids. The introduction of special coordinates, where the data for each medium is represented on a line with the slope defined by M [7], makes it possible to generalize the data for different media.

The presence of the dimensionless parameter M, containing the basic physical constants of the medium (including g), allows all the media to be ordered according to its value. For instance, some well-known liquids (see Table 1): petroleum oil, glycerin, water, mercury, melts of semiconductors, in terrestrial conditions have values of M, decreasing from 178 to the order of 10^{-15} respectively. The value $\mathrm{M} = 0$ corresponds to the model of an ideal liquid with no viscosity, or internal friction. Thus, even an elementary comparative analysis by the value of M allows one to predict the dynamics of flows in media. For instance, the melts of semiconductors are placed at the bottom of Table 1, immediately above the ideal liquid model. It is therefore natural to expect that characteristics of flows in melts are closer to those in an ideal liquid than, for example, to the characteristics of water or petroleum oil flows. (This hypothesis was verified by the generalization of the results of studies on bubbles and drops floating up in different liquids [7].) If a formal passage to the limit of ideal liquid model is taken into account, by viscosity or by the Reynolds number, this fact, in turn, indicates the increasing role of the external perturbations in melts of semiconductors, since there is a possibility that the flows in the boundary layer near a solid wall are unstable [8]. The later becomes even more important for prediction of the behavior of liquid media in cosmic conditions, modeled by a small value of g. The value of M for a specified liquid decreases with decreasing value of g and thus the hydrodynamic properties of this liquid approximate those of an ideal liquid.

Now consider the equations of motion. The Oberbeck – Boussinesq system of equations [9] proved to be efficient as applied to the description of thermo-gravitational flows [10]. If the conventional method of conversion to dimensionless variables based on region's diameter is $2L$ and velocity U is used, the equations are:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} = -\nabla\left(P + \frac{z}{\mathrm{Fr}}\right) + \left(\frac{\mathrm{Gr}}{\mathrm{Re}^2}\right)\Theta \mathbf{n}_z + \mathrm{Re}^{-1}\Delta V \, , \quad \mathrm{div}\mathbf{V} = 0 \, , \tag{1}$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{V}\nabla\Theta = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\Delta\Theta \, , \tag{2}$$

$$\frac{\partial C}{\partial t} + \mathbf{V}\nabla C = \frac{1}{\mathrm{Re}\,\mathrm{Sc}}\Delta C \, , \tag{3}$$

and the dynamic boundary conditions on the free surface are

$$P - \frac{2}{\mathrm{Re}}\mathbf{nDn} = \frac{1}{\mathrm{We}} H + P_{\mathrm{a}} \, , \tag{4}$$

$$2\boldsymbol{\tau}\,\mathbf{Dn} = \frac{\mathrm{Mn}}{\mathrm{Re}}\nabla_\Gamma \Theta \, , \tag{5}$$

where t is the time, $\mathbf{V}$ is the velocity vector, P is the pressure, Θ is the temperature, and C is the concentration;

$$\mathrm{Gr} = \frac{g\beta L^3}{\nu^2}\Delta T \, , \quad \mathrm{Re} = \frac{UL}{\nu} \, , \quad \mathrm{Fr} = \frac{U^2}{gL} \, ,$$

$$\mathrm{We} = \frac{\rho_0 U^2 L}{\sigma_0} \, , \quad \mathrm{Mn} = \frac{\sigma_0 \varkappa_\sigma L}{\rho_0 \nu^2}\Delta T$$

are the Grashof, Reynolds, Froude, Weber, and Marangoni numbers respectively; $\mathrm{Pr} = \nu/\varkappa_T$, $\mathrm{Sc} = \nu/\varkappa_C$ are the Prandtl and Schmidt numbers; ν, β, $\varkappa_T$, $\varkappa_C$, ΔT are the coefficients of kinematic viscosity, temperature expansion, thermal conductivity, diffusion and the specific temperature drop respectively; $\mathbf{n}_z$ is a unit vector, directed against gravity, the z axis being directed with gravity; $\boldsymbol{\tau}$, $\mathbf{n}$ are the tangent and normal vectors on the free surface; D is the tensor of strain; H is the surface curvature; $P_{\mathrm{a}} = \mathrm{const}$ is the pressure on the free surface; and ∇_Γ is the gradient along the free surface (it is

Table 1.

Number position	Liquid	T, °C	ρ, g cm^{-3}	ν, cm^2 s^{-1}	σ, din cm^{-1}	Pr	M
1	Glycerin	20	1.259	11.75	59.4	10^4	178
2	Petroleum oil	27.5	0.866	0.67	20.7	~ 50	1.45×10^{-2}
3	Glycerin	100	1.21	0.107	54.2	10^2	1.40×10^{-6}
4	$NaNO_3$	307	1.89	0.0146	116.6	9.24	1.90×10^{-10}
5	Glycerin	150	1.147	0.01	48.8	10.0	1.20×10^{-10}
6	Water	20	1.0	0.01	72.8	7.1	2.54×10^{-11}
7	GaSb	712	6.03	0.0038	454	0.05	4.80×10^{-13}
8	Water	80	0.97	0.0033	62.6	2.2	4.32×10^{-13}
9	GaAs	1238	5.3	0.0032	530	0.07	1.03×10^{-13}
10	Mercury	15	13.61	0.00116	487	0.028	3.86×10^{-14}
11	Germanium	937	5.51	0.00135	600	0.017	2.52×10^{-15}
12	Ideal liquid	—	—	0.0	—	—	0.0

usually assumed that $\sigma = \sigma_0\left[1 + \varkappa_\sigma(\Theta - \Theta_0)\right]$, Θ_0 is constant). It is assumed that the rest of the boundary conditions are given.

The large number of dimensionless parameters in Eqns (1)–(5), their positions the non-linear equations and the complexity of the Gr and Mn numbers do not allow verification of the predictions without solving the problem. However, if L and U assume the form

$$L = \left(\frac{\sigma_0}{g\rho_0}\right)^{1/2} = \delta_\sigma \quad \text{and} \quad U = \left(\frac{\sigma_0 g}{\rho_0}\right)^{1/4},$$
$$t = \frac{L}{U}\, t' = \left(\frac{\delta_\sigma}{g}\right)^{1/2} t', \tag{6}$$

then the dimensionless parameters, the analogues of the Reynolds, Grashof and Marangoni numbers, assume the following form:

$$\mathrm{Re} = \left(\frac{\sigma_0^3\rho_0}{g\mu^4}\right)^{1/4} = \mathrm{M}^{-1/4} = \mathrm{Re}_g,$$
$$\mathrm{Gr} = \beta\Delta T\,\mathrm{Re}_g^2, \qquad \mathrm{Mn} = \varkappa_\sigma\Delta T\,\mathrm{Re}_g^2\,. \tag{7}$$

The Prandtl and Schmidt numbers do not change, We = 1, Fr = 1. Thus, the coefficient of the mass force in Eqn (1) is essentially simplified and the form of the other coefficients allows the prediction of the properties of the solutions under changing gravitational conditions.

From the point of view of the dimensional theory [2], the introduction of g into the definition of specific size and velocity is a legitimate operation. Moreover, this particularly allows valuable qualitative assumptions to be made about the influence of different parameters with respect to changing value of g. Note that the physical patterns remain invariant. The principal advantage of using the parameters (7) is the character of the appearance of g in the dimensionless parameters and, further, in the coefficients of the motion and heat and mass transfer equations. Now g appears only in the complex with the physical constants of a medium in Re_g, and the parameter Re_g takes the position of the Reynolds number, that is, it appears in terms with the higher derivatives in Eqns (1)–(3).

With allowance made for the character of appearance of the parameter Re_g in the equations, the association between Re_g and the number M verifies the preliminary qualitative assumptions about the hydrodynamic properties of different fluids and their comparative closeness to the model of an ideal liquid. Another important implication is that now it becomes rather easy to distinguish subclasses of similar physical phenomena. The necessary and sufficient condition for similarity of two physical phenomena is the equality of the numerical values of all their dimensionless combinations [2]. Taking into account the Prandtl and Schmidt numbers and assuming that the geometry of the region is given, one may conclude from Eqns (7) that the equality of numbers $\hat{\mathrm{U}}$, $\beta\Delta T$, $\varkappa_\sigma\Delta T$, Pr, Sc is necessary and sufficient for the similarity of the two phenomena.

3. Numerical calculations

All the model calculations are made using the program package COGMA [11]. The resources of this program package are sufficient to make practically all calculations for two-dimensional stationary and non-stationary isothermal and non-isothermal flows in a given region. It also allows a number of tasks to be solved under the assumption that the free surface is flat. Thus, the resources of this package are more than sufficient for illustration of the principal conclusions of this article.

In the COGMA package the motion equations are solved relative to the vorticity and stream function. To avoid the necessity of retrieving the solutions corresponding to a given medium, the equations are solved in a dimensional form. Conversion of the flow characteristics to a dimensionless form, calculation of dimensionless parameters and comparison of the solutions for different media are made immediately after the numerical solutions are obtained. The accuracy of calculations was verified on the model tasks by calculations made on a sequence of refining meshes. Based on the experimental data, a mesh of approximately 8000 cells, evenly distributed along the axes, was chosen. This proved to be sufficient for calculation of flows with a Reynolds number greater than 1000. More than three points were in the boundary region near a solid wall.

The numerical solutions were compared by matching the flow patterns (isolines of the temperature and stream functions) and were controlled at the maximal and minimal values of the stream function.

4. Classes of invariant physical phenomena

The equality of numbers M for two similar physical phenomena means that if one considers a liquid in terrestrial conditions, and the same liquid in cosmic conditions, their behaviors are not similar. This means that a liquid medium behaves differently in free fall condition, i.e. it becomes a different medium. Strange as it seems, this fact follows from the laws of conservation (1)–(3). (If the characteristic geometry and velocity are chosen according to Eqns (6), the Froude number is identically equal to one, therefore the pressure is invariant with respect to the gravitational conditions. In other cases the equality of the Froude numbers should be required additionally, leading to the same result.) The equality of the Prandtl and Schmidt numbers is also a fundamental condition for the similarity of two media. It is further assumed that these conditions hold for all specific cases.

4.1 Similarity of the isothermal processes

For the isothermal flows in different liquids, the system (1), (2) reduces to the Navier–Stokes equations. In this case, according to Eqns (6), M is the only parameter remaining in the equations of motion. The necessary and sufficient condition for similarity of two media is the equality of the numbers M, provided there is equality of the other geometrical and kinematic parameters.

Consider a two-dimensional case and a flow in a cavity with a moving upper cover as an example. Suppose that the medium is a melt of germanium in terrestrial conditions (position 11 in Table 1). The height of the cavity is 1 cm, its width is 1.1 cm, and the velocity of the cover is 0.55 cm s^{-1}. The form of the flow in this case is shown in Figure 1. The minimal value of the stream function is -0.065 cm^2 s^{-1}; its maximal value of 0.00027 cm^2 s^{-1} is reached in the corner. It follows from Eqns (6) that $U = 18.07$ cm s^{-1}, $L = 0.333$ cm, then the dimensionless velocity of the cover is 0.03, the dimensionless value of the stream function in the vortex is -0.0108.

Consider water (position 6 in Table 1) as a second medium. Assuming $g = 0.097$ cm s^{-2}, the M numbers are equal for these two media. For this value of g, we calculate $U = 1.63$ cm s^{-1} for water from Eqns (6) and, multiplying this value by 0.03, we obtain the value 0.05 cm s^{-1} for the velocity of the cover. Considering the value of L for water (27.39 cm for the given g), the height and width of the cavity are 82 cm and 90 cm. For these parameters the flow has the same form as shown in Fig. 1. The minimal value of the stream function in vortex is -0.485 cm^2 s^{-1}, its value in the corner is 0.0019 cm^2 s^{-1}, and the dimensionless value of the stream function in the vortex coincides with the corresponding value for the melt of germanium.

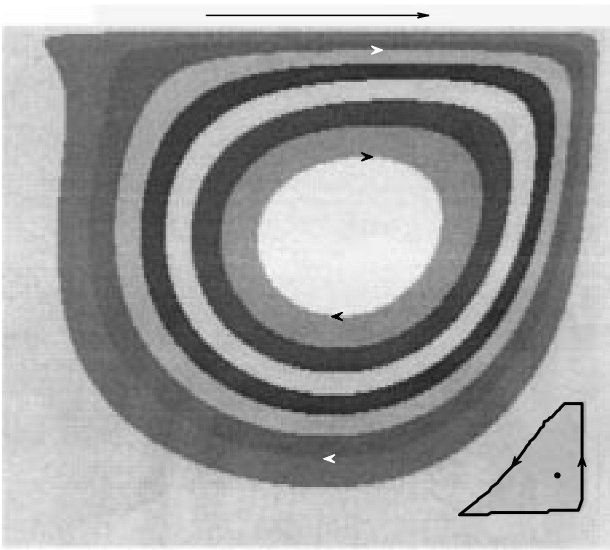

Figure 1. Structure of the flow in a cavity with a moving upper cover.

Calculation of the Reynolds and Froude numbers for these cases results in the correspondingly the same values of 448 and 0.00028 respectively (accurate to the round-off errors of the cavity width calculation for water). Therefore, the above-considered flows are similar.

At the same time, a flow in the melt of germanium in a cavity of the same size, but for $g = 0.097$ cm s^{-2}, has the same form, intensity and Reynolds number (calculated using the vorticity and stream function). However, these two flows of melts of germanium are not similar, since they have different Froude numbers.

4.2 Similarity of flows in a closed region with a heated lateral wall

Considering the non-isothermal environments, it is necessary to solve Eqns (1)–(2). Regarding Eqns (6), they contain as many as three dimensionless parameters. The definition of similar environments in this case requires the equality of the Prandtl numbers. Therefore, in order to do the calculations, we set a corresponding coefficient of thermal conductivity for one of the media. Then the necessary and sufficient condition for similarity of the two media is the equality of the numbers M and $\beta\Delta T$.

Assume that the parameters of the first medium correspond to a melt of germanium (position 11 in Table 1), and the parameters of the second medium correspond to water (position 6 in Table 1) with a modified coefficient of thermal conductivity. Consider a flow in a closed region with a rectangular section and stationary solid walls of the same size as considered in the previous section. The equality of M numbers is ensured by selection of proper values of g. Assume that the lateral walls are isothermal and the temperature of the right wall is 20 °C higher than of the left. The distribution of temperatures on the lower and upper boundaries is set linear. Given these parameters, a free convection develops in the melt and the liquid begins to rotate counter-clockwise (Fig. 2). The maximal value of the stream function in the vortex is 0.455 cm^2 s^{-1} (the minimal is -0.00027 cm^2 s^{-1} in the corners), its dimensionless value is 0.076. The velocity reaches its maximal value (1.43 cm s^{-1}) on the axis of symmetry at a distance of approximately 0.14 cm from the horizontal walls.

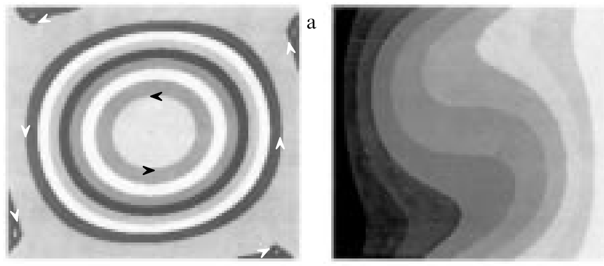

Figure 2. Structure of the flow (a) and temperature field isolines (b) in a closed region with isothermal vertical walls with different temperatures (the Prandtl number is small). The flow detaches from the walls in the corners on Fig. 2a.

Based on the similarity requirement, the temperature contrast for water should be 10 °C ($\beta = 0.0002$ for water, $\beta = 0.0001$ for germanium). Calculation of the flow and temperature fields for this temperature difference and geometry results in flow types, analogous to that shown in Figure 2, with a maximal value of the stream function of 3.38 cm^2 s^{-1} and a minimal value of -0.00189 cm^2 s^{-1} in the corners. Thus, the dimensionless value of the stream function is 0.076, i.e. the same as for the melt of germanium. The maximal value of the velocity is 0.129 cm s^{-1}.

Calculation of the Reynolds and Froude numbers using the maximal velocity and the Grashof number for the considered flows results in practically the same values of 1162, 0.0019, and 1.43×10^6 respectively. Therefore, the equality of the dimensionless parameters of the two phenomena proves them similar.

Similarity of flows can be observed not only in the case of isothermal lateral walls. If there is a linearly distributed temperature difference on the lateral walls (there is a vertical temperature difference), then the flows remain similar for the above defined geometry of the region and the temperature difference on one of the horizontal walls.

4.3 Similarity of flows in a region with a heated lower wall

Consider a rectangular region, filled with liquid. The upper and the lower walls are isothermal, the temperature of the lower wall is higher than that of the upper. The temperature distribution over the lateral walls is set linear. The liquid is in a state of unstable equilibrium, and a faintest perturbation results in the development of motion. The character and type of the flow depend on the temperature difference, geometry of the region and the medium. A one-vortex flow with stagnant zones in the corners develops in the regions with close values of width and height and a moderate temperature drop. For symmetry reasons, the vortex can rotate in any direction with the same intensity and flow structure.

Assume that the section of a region is the same as in the previous case, namely a rectangle with height of 1 cm and width of 1.1 cm, and the liquid is a melt of germanium. The temperature of the lower wall is 20 °C higher than of the upper. The structure of the steady-state flow and the temperature distribution in the region are shown in Figure 3. The liquid rotates counter-clockwise and has stagnant zones in the corners. Calculation of the Reynolds number using the maximal velocity (about 1.07 cm s^{-1} at a distance of 0.25 cm from the lower wall) and width results in a value of the order of 870. The flow stabilization time is of the order of 60 s.

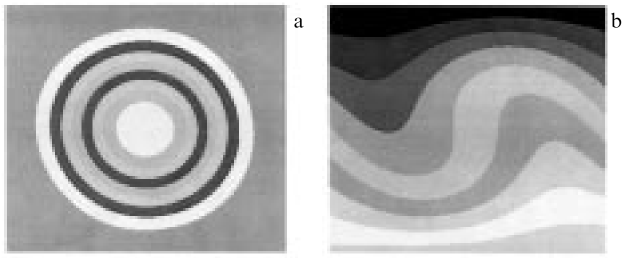

Figure 3. Structure of the flow (a) and temperature field isolines (b) in a closed region with isothermal horizontal walls with different temperatures. There are stagnant zones in the corners of Fig. 3a (not shown).

Like in the previous case, consider the water with a modified thermal conductivity coefficient as a second medium, with a region geometry of 82 by 90 cm, and gravitational conditions, ensuring the M number for water to be equal to the M number for germanium in terrestrial conditions.

The temperature difference between the lower and upper wall, necessary for similarity of these flows, is 10 °C. According to calculations, the flow structure is similar to the structure shown in Fig. 3a, with the reverse rotation direction. Moreover, the temperature field changes with the flow and is a mirror image of the field shown in Fig. 3b. The Reynolds number is equal to 870, and the maximal value of velocity is -0.097 cm s^{-1} at approximately 19.5 cm from the lower wall. The stabilization time is very large, approximately $\sim 2.1 \times 10^5$ s.

Conversion of the maximal and minimal values of the stream function to a dimensionless form according to Eqns (6) results in equal absolute values of 0.059. The ratio of the maximal value of the stream function to its minimal value is also the same for each flow and is equal to 70. Therefore, these two flows are similar.

If there is an additional horizontal temperature difference on one of the walls, then the flows remain similar, if this temperature difference satisfies the condition stated in Section 4.2. In this case the flows become similar with respect to the rotation directions too. Note the significant decrease of the stabilization time for both gravitational conditions if there is a horizontal temperature difference. For the media in terrestrial conditions the stabilization time reduces by a half for a horizontal temperature difference of 2 °C; in cosmic conditions it reduces by an order. The flow intensity increases if a horizontal temperature difference is introduced. For the cases considered the Reynolds number is equal to 950.

4.4 Similarity of flows with free and thermo-capillary convection

A free surface is another source of motion in liquids; the intensity of this motion is determined by the temperature gradient on the free surface. In this case an additional necessary condition for the similarity of two flows is the equality of the values $\varkappa_\sigma \Delta T$. However, for the temperature difference specified by $\beta \Delta T$, this equality is possible only for certain values of $\varkappa_\sigma$. Provided that the environment and the region geometry are the same as described in Section 4.2 and the free upper surface is flat, the flow structure and the temperature distribution for this case are shown in Fig. 4 ($\varkappa_\sigma = 0.000166$ for germanium; in calculations the value of $\varkappa_\sigma$ for water was assumed to be 0.000333). The flow structure is bi-vortex: the upper vortex is induced by Marangoni convection, and the lower by free convection, caused by the heated lateral walls. The motion intensity in the lower vortex ($\psi_{\max} = 0.069$) is somewhat less than in the case with no free surface and a solid upper boundary (Fig. 2). This is a result of counteraction of the thermo-capillary convection that develops on the free surface independently, inducing a counter-clockwise rotation of liquid. The vortex motion induced by Marangoni convection generates intensive motion in the surface layer with the maximal velocity value ($u = 0.354$) on the surface. This value is 4.5 times greater than the maximal velocity in the lower part of the flow. The temperature distribution in this case has a layered structure.

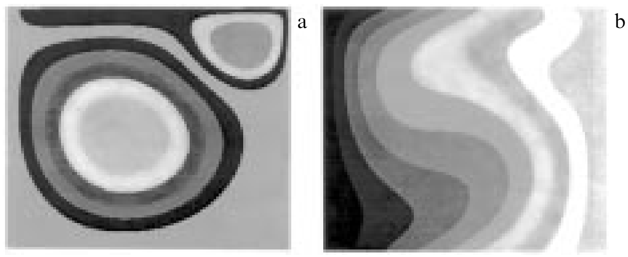

Figure 4. Structure of the flow (a) and temperature field isolines (b) in a region with an open upper boundary and isothermal vertical walls with different temperatures. The Prandtl number is small.

Calculation of the flows in a melt of germanium for the given geometry with a free upper boundary gives fundamentally different results for different values of g. The Marangoni and free convection in terrestrial conditions form a system of vortices from the free surface downward with decreasing intensity (for different melt heights). In cosmic conditions there is only a one-vortex or two-vortex flow. Identical one-vortex flows with close minimal values of the stream function can develop only in thin melts [12]. In this case the stabilization time of flows in cosmic conditions is 20 to 70 times greater than in terrestrial conditions.

4.5 Similarity by the Prandtl number

The Prandtl number notably influences both the structure and character of the flows. In all the above considered calculations the coefficient of thermal conductivity for water was changed so that the Prandtl numbers for water and for germanium were equal. With this change the flows in water were similar to the flows developing in a melt of germanium. The estimates for water with an unchanged Prandtl number and for germanium with a changed coefficient of thermal conductivity (so that its Prandtl number is equal to that for water) give practically equal temperature fields and oscillatory non-stationary behavior of the velocity field (for the case with heated lateral walls in Section 4.2) (Fig. 5). The maximal dimensionless value of the stream function is of the order of 0.0016 in both cases. The non-stationary deviations of the maximal value of the stream function reach a value of the order of 0.000015. Comparison of the maximal value of the stream function with estimates for a small Prandtl number indicates a notable decrease of motion intensity.

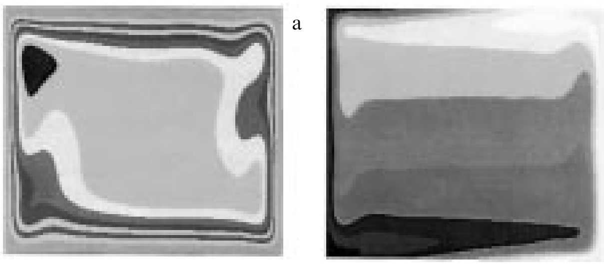

Figure 5. Structure of the flow (a) and temperature field isolines (b) in a closed region with isothermal vertical walls with different temperatures (the Prandtl number is large). There are no stagnant zones in the corners of Fig. 5a.

The flow is concentrated near the solid boundaries of the region, the motion in the center of the region is very weak and its velocity is 50 times less than the maximal velocity in the area with non-stationary fluctuations. There are no stagnant zones in the corners. The maximal value of velocity reaches 0.00373, the Reynolds number reaches a value of the order of 55. The temperature field reflects all the peculiarities of the flow and is fundamentally different from the case with a small Prandtl number. If the Prandtl number is small, the temperature field is layered and its isolines are curved along the flow direction. If the Prandtl number is large, a vertical stratification of the temperature field, relative to the gravity, forms in the extensive central region. The cold layers are placed lower than the warm layers. There are thin areas with large temperature gradients near the walls.

The structure of the flow and the temperature field for the parameters specified in Section 4.4 (Fig. 4) but with $\mathrm{Pr} = 7.1$, are shown in Fig. 6. The size of the vortex induced by Marangoni convection essentially increased. However, the maximal value of velocity on the free surface slightly decreased, $u = 0.344$. In the lower region the free convection

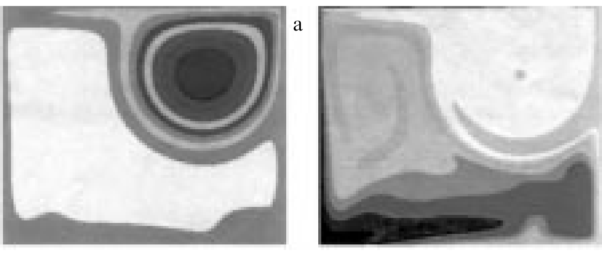

Figure 6. Structure of the flow (a) and temperature field isolines (b) in a region with open upper boundary and isothermal vertical walls with different temperatures (the Prandtl number is large).

produces essentially weak motion, as compared to the flow in Fig. 4, with a maximal velocity approximately 20 times less. The temperature fields are very different. The intensive motion in the Marangoni vortex area creates a region with an evenly distributed temperature. If the motion is weak, in the region of influence of the free convection the temperature field is stratified in its bottom part and is close to the temperature field, shown in Fig. 5. In the upper-left corner with more intensive motion there is a region with evenly distributed temperature.

5. Practical recommendations on usage of the model media

The two flows considered in Section 4.1, the melt of germanium in terrestrial conditions and water on the space station "Mir" [12], are similar, given corresponding geometrical proportions. Therefore, the flows in an opaque medium, such as a melt, can be observed, carrying out a corresponding experiment with water in cosmic conditions. A more interesting example is modeling the behavior of a melt of germanium in cosmic conditions with a medium under terrestrial conditions. Such modeling becomes possible if the number M for the medium in terrestrial conditions is 3 to 5 orders less than for germanium.

Another possibility for practical usage of similar flows is the selection of a model medium on the Earth with close numbers M (for example, positions 7 and 8 in Table 1). Then the geometry of the regions are set (in this case the regions have close dimensions). Then it is possible to study the hydrodynamic flows in the model environment 8 (water) and to make predictions of the character of flows in a melt of semiconductor 7. An analogous experiment can be carried out on a space station, thus allowing information to be obtained about the behavior of melts in cosmic conditions. Comparison of experiments carried out in terrestrial and cosmic conditions will allow the influence of cosmic conditions on the development of flows to be determined.

These recommendations also relate to the non-isothermal types of flow considered in Sections 4.2 through 4.5. In this case, however, the modeled environment should have a Prandtl number corresponding to the liquid being investigated. Further, if the region has a free surface, the values of Pr should also correspond for both environments.

6. Conclusions

The suggested form of the motion and heat and mass transport equations allows the character of convection and heat and mass transport to be predicted if the gravitational conditions change. If the coefficient of the mass force in the equations of motion decreases, the hydrodynamic properties of the liquid shift toward the properties of the ideal liquid.

A simple form of dimensionless parameters allows the classes of similar phenomena in different media to be singled out. The stabilization time of similar flows varies in different media and gravitational conditions. The stabilization time of flows is usually greater for smaller values of g.

Note that all the dimensionless parameters of similar flows coincide. This fact was verified by calculations made for the phenomena determined by the parameters M (or $\mathrm{Re_g}$), $\beta\Delta T$, $\varkappa_\sigma\Delta T$, Pr. The calculated values of the Reynolds, Grashof, Weber, and Froude numbers turned out to be equal for the liquids considered.

An attempt to determine the similarity of media by the Grashof number failed. For example, given a medium (germanium in a closed region) and assuming the Grashof numbers be equal (the temperature difference increases and the value of g decreases proportionally), we obtain the flows of the same type with the same intensity (by the maximal value of the stream function) and equal Reynolds numbers. However, these flows are not similar since the Froude numbers for them are different. Considering flows with equal Grashof numbers, but in regions with different geometry and values of g, we obtain flows of the same type with equal Reynolds and Froude numbers, but the different Weber numbers. Thus, the equality of the Grashof numbers does not imply similarity for the considered classes of flows. Since the Rayleigh number is a product of the Grashof and Prandtl numbers, the equality of the Rayleigh numbers, generally speaking, does not imply the similarity of physical phenomena. In order to make this conclusion, a number of problems were calculated and all the above-mentioned dimensionless parameters were computed after determination of the specific velocity of the flow. The approach suggested in this article allows the similarity of two environments to be determined more easily and without actually solving the equations even if the gravitational conditions change.

Inequality of the Froude numbers for stationary flows with the same structure and Reynolds numbers influences the character of the stability of the flows, relative to small perturbations, and the development of non-stationary processes in the liquid.

Comparison of a flow between heated vertical walls and a flow induced by a heated lower wall in a region with the same temperature drop on the opposite walls indicates that they differ in both the intensity of the stationary flow and the stabilization time. The flow between two heated lateral walls is better developed and has a shorter stabilization time. The presence of even a small horizontal temperature difference notably accelerates the stabilization of the flows if the lower wall is heated.

Examination of flows induced by gradients of admixture concentration in the melt and on the free surface, in the case when the gravitational conditions change, is analogous to the above-considered non-isothermal cases. The character of the flows, considering large Schmidt numbers (or diffusive Prandtl numbers), can be estimated based on the calculations made for the large Prandtl numbers.

It was assumed that the physical and thermo-physical parameters of liquid environments do not depend on the gravitational conditions. In general, it is necessary to investigate the influence of the gravitational conditions on the coefficients of momentum, heat and mass transport in liquids. The fact that the number M, characterizing liquid media, changes with changing gravitational conditions gives grounds for such investigation.

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