METHODOLOGICAL NOTES

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Superluminal waves in amplifying media

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Contents

Abstract. **In amplifying media steady-state waves can travel faster than the speed of light in vacuum without violating the principles of special relativity. The possibility of generating superluminal waves in induced elementary particle production processes is discussed.**

1. Introduction

The possibility of superluminal motion was extensively discussed in the 1960s and early 1970s. A hypothesis of the existence of superluminal particles called tachyons, was suggested [1 – 3]. In order to remain within the framework of the special relativity theory, it was necessary to assume that the tachyon rest mass is imaginary, $\mathrm{i}m_0$. However, the imaginary rest mass of tachyons did not cause anxiety because it was believed that tachyons can only exist at velocities u exceeding the speed of light in vacuum c_0. In this respect, the situation with tachyons is not exclusive because such particles as photons and neutrinos cannot exist and move at velocities less than the speed of light. For $u > c_0$, the observable tachyon mass is equal to

$$m = \frac{m_0}{(u^2/c_0^2 - 1)^{1/2}} \tag{1}$$

i.e., it is always real.

Serious complications were caused by the causality problem. Although the methods of reconciliation of the existence of tachyons with the causality principle were witty [2, 3], they have not been very successful. In addition, attempts to observe tachyons experimentally have failed so far. For this reason, the interest in the problem of tachyons, as elementary particles, gradually ceased.

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Meanwhile, in 1965 the movement of a physical object at a superluminal velocity was observed. It was found that the light pulse propagating in a laser-amplifier could have a stationary shape and a velocity exceeding the speed of light in vacuum [4 – 7].

In 1974, paper [8] was published in which the superluminal motion was related to the propagation of perturbations in unstable media. The authors of Ref. [8] considered examples of unstable media and, in particular, discussed paper [4]. The concept of superluminal motion in unstable media was further developed in paper [9]. In a recent paper [10], the possibility of tachyon-like propagation of light in an amplifying nonrelaxing medium of two-level atoms was discussed — a case that absolutely coincides with the model discussed in Ref. [8]. However, in Ref. [10] there are no references to previous experimental and theoretical papers on light amplification [4 – 7] and to paper [8], which probably were unknown to the authors [10]. Hence, one of the stimuli for writing this paper is the desire to reconstruct the historical connection of events and to discuss a number of details related to the existence of tachyon-like waves in amplifying media.

2. Superluminal radiation pulse in an amplifying medium

In the early 1960s, after the advent of lasers, the problem of generation of sufficiently powerful light pulses with durations of the order of one nanosecond (10^{-9} s) appeared. To do this, a short light pulse was generated with the help of a so-called master laser-oscillator, which should then be amplified by a laser-amplifier [3 – 6]. A schematic of the setup used for this is shown in Fig. 1. The light pulse generated by the master oscillator is split into two beams. The first, more powerful beam propagates through the amplifier. The second beam propagates in the air and serves as a reference for comparison with the amplified pulse. Both these pulses are detected by photodetectors whose output signals are fed to an oscillograph for visual observation. It was expected that the velocity of the pulse in the air would be greater than that of the pulse in the amplifier. It was assumed *a priori* that not only the intensity of the pulse propagating through the amplifier

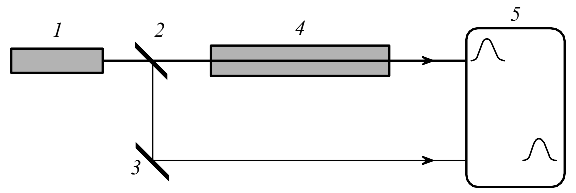

Figure 1. Schematic of the setup for amplification of short light pulses: (*1*) master laser-oscillator; (*2*) beamsplitter that splits the beam from the master oscillator into two beams; (*3*) mirror; (*4*) amplifier; (*5*) oscillograph.

should increase but its shape should also change due to nonlinear amplification. However, the actual experimental result caused astonishment and some confusion among the researchers. The shape of the pulse did not change during its propagation in the amplifier. And the main paradox was that the pulse propagated through the amplifier at a velocity greater than the speed of light in vacuum. Processing the results showed that the velocity of the light pulse in the amplifier was greater than the speed of light in vacuum by several times!

The confusion among physicists involved in these studies was brief. Nobody has the slightest doubt of the principles of the special relativity theory. That is why the correct answer was found quite rapidly: indeed, if one does not doubt the main principles of the theory of relativity, it immediately becomes clear that the amplifying medium puts the joke on the researchers.

The master laser-oscillator generates a light pulse with a leading edge that rises in the initial stage as $\exp(t/\tau)$. The characteristic time τ is determined by the parameters of the master oscillator. Figure 2 shows the time-base sweep of the light pulse generated by the laser-oscillator. Note the long initial stage of the pulse. As a rule, the duration of this stage is several tens of times longer than the duration of the pulse kernel, which is usually measured at the pulse half-maximum. This long initial stage is the leading edge of the pulse entering the laser-amplifier. In the amplifier, photons representing together the light pulse propagate at a velocity equal to the speed of light in the amplifier medium, which is often called the active medium.

In the active medium, along with amplification, processes resulting in the loss of the light pulse energy can occur. For this reason, the amplification is determined by the difference between an increase in the concentration of photons at the expense of the energy of the active medium and its decrease caused by absorption.

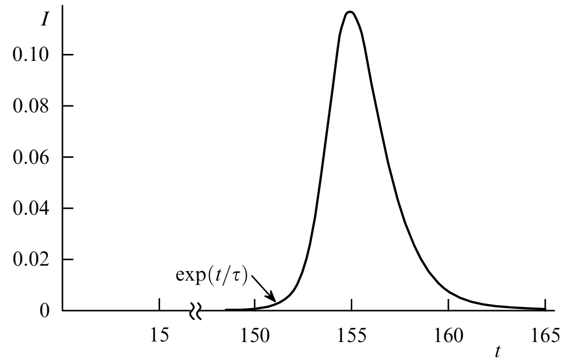

Figure 2. Shape of the pulse from a master oscillator. The concentration of photons is plotted against time in relative units.

To envision more clearly what occurs with the light pulse propagating in the amplifier, imagine that we observe the pulse with the help of a device moving at the speed of light in the amplifying medium. If the medium in which the light pulse propagates had been transparent, we would have seen a pulse invariant in time and distributed over the concentration of photons in space as a motionless picture (Fig. 3, curve *1*). However, because the medium amplifies light, the number of photons at each point varies with time. In the initial region of the pulse where the concentration of photons is not too large, the amplification is proportional to their concentration (*linear* amplification). In the region where the concentration of photons is sufficiently large, the amplification is weaker than that proportional to the concentration. Finally, in the region where the energy content in the medium is substantially depleted due to its transfer to the light pulse, the concentration of photons even decreases. In this region, the medium loses the ability to amplify light and can only absorb it. Absorption is virtually always proportional to the concentration of photons. Figure 3 (curve *2*) shows the immobile light pulse amplified in its leading edge and weakened in its trailing edge. But what has the amplifying medium done? It as if has moved the pulse forward! Since the device moves at the speed of light, while the pulse has moved forward relative to the device during the observation time, it means that the pulse propagates at a velocity exceeding the speed of light! It is this phenomenon that has been observed by experimentalists in the experiments described above. We see that there is no contradiction to the theory of relativity, because the photons themselves are only moving at the speed of light. Simply, due to amplification the concentration of photons that come out earlier proves to be higher than that of photons escaping later. In this case, the pulse envelope, or in particular its maximum, propagates at a superluminal speed rather than the photons. The pulse is as if rolling over the advanced leading exponential edge. The superluminal 'shift' also provides the stability of the propagating wave.

Once the mechanism of superluminal propagation of the light pulse has been perceived, the main problems of the experiment were solved. The matter is that the pulse

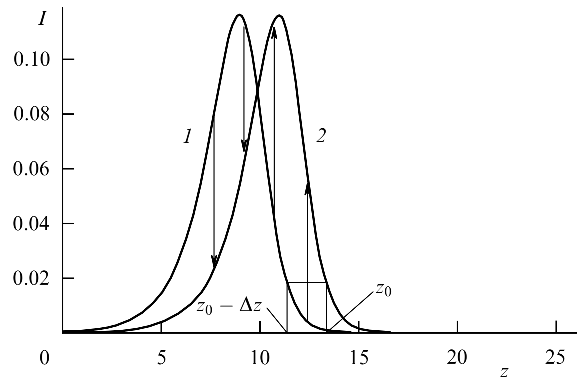

Figure 3. Light pulse in an amplifier in a coordinate system moving at the speed of light. The quantities are plotted along the axes in relative units. Curve *2* is obtained from curve *1* by increasing (due to amplification) some regions (the upward arrows) and decreasing (due to absorption) other regions (the downward arrows).

maximum, by rolling over its leading edge, should finally approach the pulse onset. From this moment on, it should be amplified and shortened. However, because of the rather long leading edge, a long amplifier length may be required for the pulse maximum to reach its starting point. The pulse can reach the end of the amplifier before approaching its starting point. In fact, this takes place in the experiments discussed at the beginning of the paper. For this reason, to solve the problem of increasing the pulse energy and shortening its duration, its long leading edge was cut off with the help of a special shutter. Figuratively speaking, the path over which the pulse could roll with superluminal velocity was cut off. The pulse with the cut-off leading edge propagates in the amplifier medium at the speed of light, is saturated by the energy (is amplified), and shortens [5, 6]. However, the generation of a powerful and short laser pulse is a special topic, which is beyond the scope of this paper.

3. Propagation velocity of the superluminal pulse

Let us calculate the propagation velocity of the superluminal wave. Experiments have shown that the superluminal wave propagates as a whole, its shape being unchanged. This means that the maximum of the pulse propagates at the same velocity as its low-intensity parts. For this reason, we will perform calculations for the low-intensity part of the leading edge of the pulse where linear amplification takes place. Let us denote the required velocity by u and consider two points separated by a small spatial interval Δz (see Fig. 3). The time in which an object propagating at the velocity u passes this interval is equal to $\Delta t = \Delta z/u$. As was already noted, the light pulse is at rest in the coordinate system under study but it is amplified due to the interaction with the active medium. In the linear region, the increase ΔI_+ in the concentration of photons at the point z_0 caused by amplification during a short time interval Δt is proportional to the length of this interval and the concentration of photons $I(z_0)$ at this point:

$$\Delta I_+ = \varkappa c I(z_0) \Delta t \,. \tag{2}$$

Here, $\varkappa$ is the linear (differential) gain and c is the speed of light. Similarly, the decrease ΔI_- in the concentration of photons caused by absorption is

$$\Delta I_- = -\alpha c I(z_0) \Delta t \,, \tag{3}$$

where α is the linear (differential) absorption coefficient. The total relative change in the concentration of photons at the point z_0 for time Δt is

$$\frac{\Delta I}{I(z_0)} = \frac{\Delta I_+ + \Delta I_-}{I(z_0)} = (\varkappa - \alpha) c \Delta z \,. \tag{4}$$

As noted above, the leading edge of the light pulse emerging from the laser increases in time as $\exp(t/\tau)$ in the region of low concentration of photons. The relative change in the concentration of photons during the short time interval $\Delta z/u$ is equal to $\Delta z/u\tau$. For this reason, the number of photons at the point z_0 at the moment $t + \Delta t$ will be equal to the number of photons at the point $z_0 - \Delta z$ at the moment t, if $\varkappa c \Delta t = \Delta z/u\tau$. The pulse appears shifted by the distance Δz for the time Δt (see Fig. 3). This means that the rate of its shift in the moving coordinate system chosen by us is $\Delta u = \Delta z/\Delta t$. The total velocity of the superluminal wave in the laboratory coordinate system is $u = c + (\varkappa - \alpha) c u \tau$, or

$$u = \frac{c}{1 - (\varkappa - \alpha) c \tau} \,. \tag{5}$$

Expression (5) shows that for the values of parameters when

$$0 < (\varkappa - \alpha) c \tau < 1 \,, \tag{6}$$

u can be noticeably higher than the speed of light. The velocity of the superluminal wave depends both on the parameters of the amplifier via its gain and the parameters of the master oscillator via its characteristic time τ. By varying τ, one can control the pulse propagation velocity without changing the parameters of the amplifying medium.

Consider a numerical example for illustration. Assume that the gain $\varkappa = 0.003$ cm^{-1}, $\alpha = 0.001$, and $\tau = 2.5 \times 10^{-8}$ s. Such values of parameters can be quite easily obtained in the experiment. The speed of light in the amplifier material is 2×10^{10} cm s^{-1} (the refractive index of the material is assumed to be 1.5). In this case, according to Eqn (4), the pulse propagation velocity exceeds the speed of light in the amplifier material by a factor of 11 and the speed of light in vacuum by a factor of 7.3. By selecting parameters, other values of the propagation velocity of the maximum of the light pulse can be also obtained.

What occurs with the light pulse if $(\varkappa - \alpha) c \tau > 1$ or $(\varkappa - \alpha) c \tau < 0$? In the first case, the amplification is so high that the maximum of the amplification wave appears at the very onset of the pulse and shifts oppositely to the direction of pulse propagation. In the second case, the above discussion has no meaning because the medium is absorbing as a whole rather than amplifying.

4. Mathematical model of a laser and superluminal stationary waves†

The quantitative features of the propagation of the superluminal wave can be described using the following quite simple model of the resonance interaction of light with matter. Assume that the interaction of the matter with the field has an electric nature. Then, the propagation of the electromagnetic field in the matter can be described by the wave equation for the electric component $\mathbf{E}(\mathbf{r}, t)$ of the electromagnetic field:

$$\frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} - c_0^2 \nabla^2 \mathbf{E}(\mathbf{r}, t) + 2\alpha c_0 \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} = -4\pi \frac{\partial^2 \mathbf{P}_{\mathrm{T}}(\mathbf{r}, t)}{\partial t^2} \,, \tag{7}$$

where $\mathbf{P}_{\mathrm{T}}(\mathbf{r}, t)$ is the polarization of the amplifier medium. The latter can be naturally divided into two parts, $\mathbf{P}_0(\mathbf{r}, t)$ and $\mathbf{P}(\mathbf{r}, t)$. The quantity $\mathbf{P}(\mathbf{r}, t)$ describes the polarization of atoms that are directly responsible for amplification in the amplifier medium (working atoms). These atoms are in the excited state and resonantly interact with the radiation field. For the description of this quantity, a dynamic model is required, which will be considered below. The quantity $\mathbf{P}_0(\mathbf{r}, t)$ describes the contribution to the polarization from the rest atoms of the medium whose concentration, as a rule, is much higher than that of the working atoms. This part of

† Here, the information reported is well known to specialists in the field of laser physics. However, the author do hope that this paper will be of interest to a wider scope of readers.

the polarization can be considered quasi-equilibrium and can be described by the relation

$$\mathbf{P}_0(\mathbf{r}, t) = \hat{\chi}_0 \mathbf{E}(\mathbf{r}, t) . \tag{8}$$

For our further purposes, the operator $\hat{\chi}_0$ can be assumed constant. The total polarization of the medium, taking into account the Lorentz correction for the local field, proves to be equal to [11]

$$\mathbf{P}_\mathrm{T}(\mathbf{r}, t) = \chi_0 \mathbf{E}(\mathbf{r}, t) + \frac{\varepsilon_0 + 2}{3}\, \mathbf{P}(\mathbf{r}, t) , \qquad \varepsilon_0 = 1 + 4\pi\chi_0 . \tag{9}$$

Finally, Eqn (6) takes the form

$$\frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} - \frac{c_0^2}{\varepsilon_0}\, \nabla^2 \mathbf{E}(\mathbf{r}, t) + 2\alpha c_0\, \frac{1}{\varepsilon_0}\, \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} = -4\pi\, \frac{\varepsilon_0 + 2}{3\varepsilon_0}\, \frac{\partial^2 \mathbf{P}(\mathbf{r}, t)}{\partial t^2} . \tag{10a}$$

The most popular model for the description of the dynamics of polarization of working atoms is the so-called two-level approximation. In an atom interacting with the field, only two energy levels with the transition between them resonant with the frequency of the acting field are considered (the working levels). It can be shown [11–13] that the polarization $\mathbf{P}(\mathbf{r}, t)$ of the atoms is described by the equations

$$\frac{\mathrm{d}^2 P}{\mathrm{d}t^2} + \frac{2}{\tau_2}\, \frac{\mathrm{d}P}{\mathrm{d}t} + \omega_0^2 P = -2\omega_0\, \frac{|\mu|^2}{\hbar}\, N E_\mathrm{L} , \tag{10b}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} + \frac{1}{\tau_1}(N - N^{(0)}) = J(t) + \frac{2}{\hbar\omega_0}\, E_\mathrm{L}\, \frac{\mathrm{d}P}{\mathrm{d}t} , \tag{10c}$$

where τ_2 is the relaxation time of polarization determining the width of the spectral line; τ_1 is the relaxation time of the level population; ω_0 is the frequency of the transition between working levels of atoms; and μ is the matrix element of the corresponding dipole moment. Equations (10b, c) contain the field acting on the particle (local field), which is related to Maxwell's field in Eqn (8) by the expression [11]

$$\mathbf{E}_\mathrm{L}(\mathbf{r}, t) = \frac{\varepsilon_0 + 2}{3} \left[\mathbf{E}(\mathbf{r}, t) + \frac{4\pi}{3}\, \mathbf{P}(\mathbf{r}, t) \right] . \tag{11}$$

One can see that Eqns (10b, c) contain, along with the polarization, the dynamic variable N, which represents the difference of populations of working levels of the active medium, which are resonant with the frequency of the electromagnetic field. $N^{(0)}$ is the population difference of working levels in the absence of pumping. In the thermodynamic equilibrium state, $N^{(0)}$ is determined by the Boltzmann distribution. As was mentioned, to obtain amplification, it is necessary (but not sufficient!) that the population of the upper working level was higher than that of the lower level, i.e., the population difference should be positive. This requires pumping of the system resulting in transitions of atoms from the lower energy level to the upper level. The pumping is described by the term $J(t)$. Equations (10) contain all the fundamental information on the laser operation and propagation of the light pulse through the amplifier. The reader can find additional information in the paper ''Laser'' and other papers on lasers in the Physical Encyclopedia [14].

The system of Eqns (10) is widely applied as a model of a laser for studying the dynamics of quantum oscillators and amplifiers [12–22]. These references are far from a complete bibliography on this topic.

Relations between the parameters entering the equations allow one to analyze them by the method of 'slow variable complex amplitudes'. In the case of the amplification wave, the field and polarization are represented as a plane wave†:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{A}(z, t) \exp(k_0 z - \omega_0 t) ,$$

$$\mathbf{P}(\mathbf{r}, t) = \frac{3}{\varepsilon_0 + 2}\, \mathbf{B}(z, t) \exp(k_0 z - \omega_0 t) , \qquad k_0 = \frac{\omega_0}{c} , \tag{12}$$

where $A(z, t)$ and $B(z, t)$ are slow functions of time and coordinate as compared to the rapidly oscillating exponential. The population difference $N(z, t)$ should also be assumed the slow function of the coordinate and time. The procedure of substitution of Eqn (11) into Eqns (10) is presented in detail, for example, in Refs [11, 14, 16, 17]. As a result, the following system of equations is obtained

$$\frac{\partial \mathbf{A}}{\partial t} + c\, \frac{\partial \mathbf{A}}{\partial z} + \tilde{\alpha} c \mathbf{A} = \mathrm{i}\beta \mathbf{B} ,$$

$$c = \frac{c_0}{\sqrt{\varepsilon_0}} , \qquad \tilde{\alpha} = \frac{\alpha}{\sqrt{\varepsilon_0}} , \qquad \beta = \frac{2\pi\omega_0}{\varepsilon_0} ; \tag{13a}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \frac{1}{\tau_2}\, \mathbf{B} = -\mathrm{i}\, \frac{\tilde{\mu}^2}{\hbar}\, N \mathbf{A} , \qquad \tilde{\mu}^2 = |\mu|^2 \left(\frac{\varepsilon + 2}{3} \right)^2 ; \tag{13b}$$

$$\frac{\partial N}{\partial t} + \frac{1}{\tau_1}\, N = \tilde{J}(t) + \frac{\mathrm{i}}{2\hbar}(AB^* - A^* B) , \qquad \tilde{J} = J + \frac{N^{(0)}}{\tau_1} . \tag{13c}$$

We neglected the term $4\pi(\varepsilon_0 + 2)\mu^2 NB/(9\hbar)$ in Eqn (13b) as compared to the term B/τ_2. In laser media commonly used in amplifiers of powerful light pulses, the ratio of these terms does not exceed 0.01.

Despite the relative simplicity of the system of Eqns (13), it can be analyzed in detail only numerically. Numerical calculations [7] show, in complete agreement with experiments, that under certain conditions a light pulse propagating in the amplifying medium acquires a *stationary* shape:

$$\mathbf{A}(z, t) = \mathbf{A}(\xi), \qquad \mathbf{B}(z, t) = \mathbf{B}(\xi),$$

$$N(z, t) = N(\xi), \qquad \xi = t - \frac{z}{u} ,$$

where u is the unknown propagation velocity of the pulse. In this case, Eqns (13) take the form

$$\left(1 - \frac{c}{u}\right) \frac{\mathrm{d}\mathbf{A}}{\mathrm{d}\xi} + \tilde{\alpha} c \mathbf{A} = \mathrm{i}\beta \mathbf{B} , \tag{14a}$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\xi} + \frac{1}{\tau_2}\, \mathbf{B} = -\mathrm{i}\, \frac{\tilde{\mu}^2}{\hbar}\, N \mathbf{A} , \tag{14b}$$

$$\frac{\mathrm{d}N}{\mathrm{d}\xi} + \frac{1}{\tau_1}\, N = \tilde{J}(t) + \frac{\mathrm{i}}{2\hbar}(\mathbf{A}\mathbf{B}^* - \mathbf{A}^* \mathbf{B}) . \tag{14c}$$

† In the case of the laser-oscillator, the active medium is placed into a cavity, and the field and polarization are usually represented by a series in eigenfunctions of the cavity.

If the pulse is stationary, then its parts where the field is strong propagate at the same velocity as the low-intensity parts of the pulse. In the low-intensity region of the pulse, Eqns (14) can be linearized. In equation (14c), the term $(\mathrm{i}/2\hbar)(\mathbf{AB}^* - \mathbf{A}^*\mathbf{B})$ can be neglected because it represents a product of two small quantities. Then, this equation can be immediately solved, and its stationary solution at constant pumping is $N = \tilde{J}\tau_1 \equiv N_0$. The substitution of this value of N into Eqn (14b) yields a system of two linear equations, because N_0 is a constant:

$$\left(1 - \frac{c}{u}\right)\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}\xi} + \tilde{\alpha}c\mathbf{A} = \mathrm{i}\beta\mathbf{B}\,,$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\xi} + \frac{1}{\tau_2}\,\mathbf{B} = -\mathrm{i}\,\frac{\tilde{\mu}^2}{\hbar}\,N_0\mathbf{A}\,. \tag{15}$$

By representing the solution of system (15) in the form

$$\mathbf{A} = \mathbf{C}_1 \exp\frac{\xi}{\tau}\,, \qquad \mathbf{B} = \mathbf{C}_2 \exp\frac{\xi}{\tau}\,, \tag{16}$$

we obtain the characteristic equation relating the characteristic rise time of the pulse leading edge with the gain, and the pulse propagation velocity:

$$\begin{vmatrix} \dfrac{1}{\tau}\left(1 - \dfrac{c}{u}\right) + \tilde{\alpha}c & -\mathrm{i}\beta \\ \mathrm{i}\,\dfrac{\tilde{\mu}^2}{\hbar}\,N_0 & \dfrac{1}{\tau} + \dfrac{1}{\tau_2} \end{vmatrix} = 0\,, \tag{17}$$

or

$$u = \frac{c}{1 - \left[\varkappa\tau/(\tau + \tau_2) - \tilde{\alpha}\right]c\tau}\,, \tag{18}$$

where $\varkappa = \beta(\tilde{\mu}^2/\hbar)\tau_2 N_0$ is the differential gain of the weak field. One can see that the more consistent theory leads to an expression for the propagation velocity of the amplification pulse coincident with Eqn (5), if $\tau_2 \ll \tau$. The factor $\tau/(\tau + \tau_2)$ can be interpreted as follows. The model of the active medium described by Eqns (10b, c) yields the Lorentzian spectral shape of the gain with the characteristic width $1/\tau_2$. This means that different spectral components of the amplified pulse have different gains. The shorter the pulse duration τ, the broader its spectrum. Because the amplification decreases with increasing detuning of the spectral component of the pulse from the resonance frequency of the amplifying medium, the average value of the gain decreases with decreasing τ, which is reflected in Eqn (17).

The shape of the propagating pulse depends on whether the pumping is continuous or the amplification is produced by a short pumping pulse. Upon continuous pumping, the amplitude of the wave increases exponentially in the linear part of the leading edge and asymptotically tends to a stationary value. In the case when $\tilde{\alpha}c/(u - c) \gg \tau_1^{-1}$, the amplitude may exhibit oscillations (Fig. 4). The time interval between the first and second peaks is determined by the pumping intensity. In systems analogous to the one studied in Ref. [3], this interval exceeds the duration of the first peak by many orders of magnitude. For this reason, the oscillatory shape of the superluminal wave was not observed in experiments. In the case when $\tilde{\alpha}c/(u - c) \ll \tau_1^{-1}$, the amplitude tends continuously to a stationary value (Fig. 5) and is described by the implicit expression

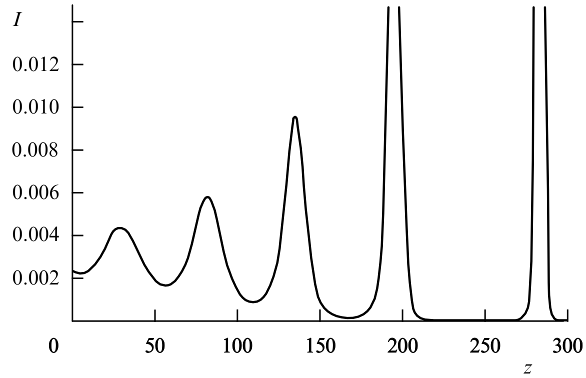

Figure 4. Oscillatory shape of the superluminal wave.

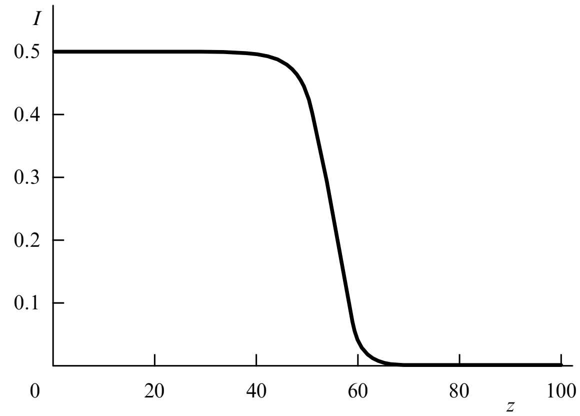

Figure 5. Smooth shape of the superluminal wave.

$$\frac{A^2}{(r - 1 - A^2/A_{\mathrm{s}}^2)^r} = A_0^2 \exp\frac{2\xi}{\tau}\,, \tag{19}$$

where A_0 is the starting value of the field, $A_{\mathrm{s}} = (\tilde{\mu}/\hbar)\sqrt{\tau_1\tau_2}$ is the so-called saturating amplitude of the field, and $r = \varkappa/\tilde{\alpha}$. The characteristic rise time of the exponential leading edge τ is not determined by parameters of the amplifier, being an additional external parameter. It is specified by the shape of the leading edge of the amplified wave, which, by entering the amplifier from outside, 'paves' a path for the superluminal wave. Therefore, the superluminal wave cannot be created in a uniformly excited amplifier due to spontaneous emission of photons. It can only appear in the case of an exponential leading edge of the pulse. If the leading edge increases more slowly than an exponential, the pulse accelerates till it reaches the stationary shape. If the shape of the leading edge is steeper than exponential (for example, a Gaussian), the pulse propagates at the speed of light, its intensity increasing and duration shortening [6].

It is obvious that to obtain amplification, the condition $r > 1$ should be satisfied. If r exceeds unity only slightly, then expression (18) allows one to write the wave shape in the explicit form

$$A^2(\xi) = A_0^2\,\frac{(r - 1)\exp(2\xi/\tau)}{1 + (A_0^2/A_{\mathrm{s}}^2)\exp(2\xi/\tau)}\,. \tag{20}$$

The study of the stability of the stationary value shows that in the region of parameters satisfying the relations

$$a > b + 1\,, \qquad r > \frac{a(a+b+3)}{a-b-1}\,, \tag{21}$$

where

$$a = \alpha c \tau_2 \frac{u}{u-c}\,, \qquad b = \frac{\tau_2}{\tau_1}\,,$$

the wave propagation becomes random [19]. This is an analog of the random lasing in a laser-oscillator. Random lasing in lasers was first predicted in papers [20, 21] and at present is studied in detail, both theoretically and experimentally [14, 21]. The first relation in inequalities (21) requires quite large absorption coefficient of photons. The laser-amplifier differs from the laser-oscillator in that the efficient absorption in the amplifier can be controlled by the propagation velocity of the superluminal wave. Figure 6 shows an example of the random mode. To emphasize the random nature of the process, Fig. 6 shows variations in the amplitude and phase.

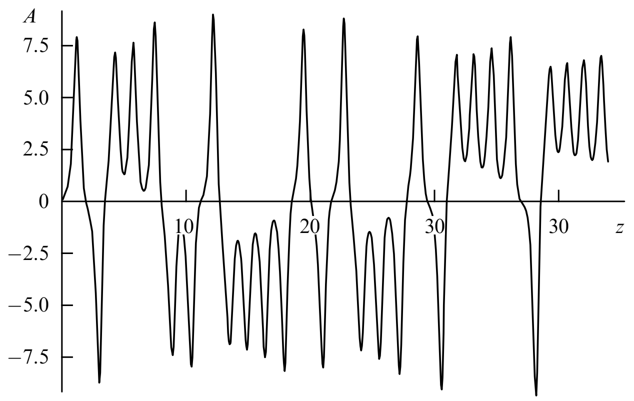

Figure 6. Random shape of the superluminal wave ($a = 3$, $b = 1$, $r = 22$).

In contrast to laser-oscillators, the random mode in laser-amplifiers has been studied inadequately due to the complicated theoretical model (equations in partial derivatives) and experimental difficulties. The most substantial experimental problem is to suppress amplified spontaneous emission (superluminescence) in an active medium with sufficiently high gain. This can be achieved by adding to the active medium the particles absorbing resonantly at the same frequency as the working atoms. The atoms added at a *low* concentration should have a *large* absorption cross section, so that they can suppress amplification of spontaneous perturbations [23, 24]. At the same time, the additional atoms will not prevent the development of perturbations with sufficiently large amplitudes, because the absorption will saturate at comparatively low radiation intensities due to the large cross section.

The model with saturable absorption contains the additional term $\alpha_1/(1+\sigma A^2)$ in Eqn (14a), so that (14a) transforms to the equation

$$\left(1-\frac{c}{u}\right)\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}\xi} + \left(\tilde{\alpha} + \frac{\alpha_1}{1+\sigma A^2}\right)c\mathbf{A} = \mathrm{i}\beta\mathbf{B}\,. \tag{14a$'$}$$

Equations (14b, c) remain unchanged. Numerical integration of the system of Eqns (14a$'$, b, c) yields the result presented in

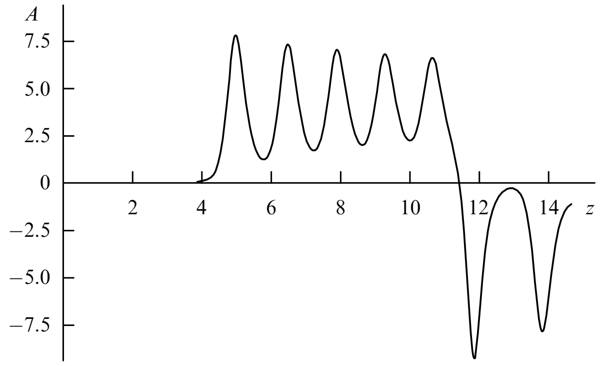

Figure 7. Random shape of the superluminal wave in a medium with saturable losses ($a = 3$, $b = 1$, $r = 22$, $a_1 = 30$, $\sigma = 1000$).

Fig. 7. One can see that after some time the wave is interrupted despite continuous pumping. Although the wave duration depends on the starting value of the amplitude, it has a random nature. This result, being unexpected at first glance, is explained as follows. One can easily see that the system (14) has a nonzero solution, if the field amplitude and polarization simultaneously vanish at some moment. In the random mode, the radiation field and polarization vanish from time to time. Nevertheless, this does not stop the process, because the polarization and the field do not vanish simultaneously†. However, in the model with saturable absorption, generation may be suppressed not only due to simultaneous vanishing of the field and polarization but also due to the simultaneous decrease in the absolute value of their amplitude below the critical value determined by the saturable absorber. Such a situation arises sooner or later, so that the development of the process ceases.

Attempts to obtain an analytic solution to Eqns (14) for pulsed pumping have failed. The numerical study of Eqns (13) shows [7] that under conditions (6) a stationary pulse is generated, as demonstrated in Fig. 8.

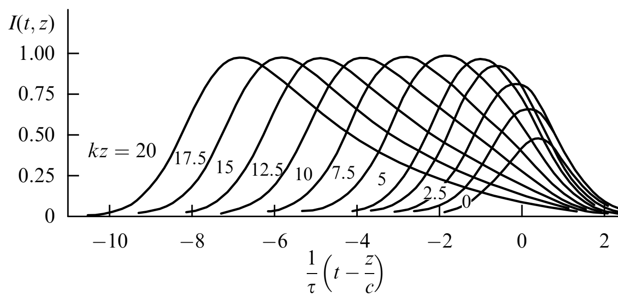

Figure 8. Passage of the pulse to the stationary propagation mode.

An analytic solution can be found, if the absorption in the active medium of the amplifier is neglected and the relaxation processes are assumed to be slow compared to the pulse duration. The pulse propagation in such a medium satisfies Eqns (13), in which $J = 0$, $\alpha = 0$ and $1/\tau_1 = 1/\tau_2 = 0$. In this case, solutions of Eqns (13a, b) can be obtained in the analytic

† If the spontaneous radiation is taken into account, the development of the process will not stop completely; however, it will be delayed in time.

form:

$$B = -\mathrm{i}N_0\tilde{\mu}\sin\psi\,, \qquad N = N_0\cos\psi\,,$$

$$\psi = \frac{\tilde{\mu}}{\hbar}\int_{-\infty}^{\xi} A(\xi')\,\mathrm{d}\xi'\,. \tag{22}$$

By using (21), Eqn (12a) can be transformed to the form

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\xi^2} = \frac{1}{\tau^2}\sin\psi\,, \qquad \frac{1}{\tau^2} = \frac{u}{u-c}\,\beta\,\frac{\tilde{\mu}^2}{\hbar}\,N_0\,. \tag{23}$$

This equation is well known in soliton theory [25]. If τ^2 is positive, the equation has a solution in the form of a solitary stationary pulse:

$$A(\xi) = \frac{2\hbar}{\tilde{\mu}\tau}\,\mathrm{sech}\,\frac{\xi}{\tau}\,. \tag{24}$$

This means that the stationary solitary pulse can propagate in the amplifying medium ($N_0 > 0$) only at a superluminal velocity ($u > c$). One can see that the pulse propagation velocity is closely related to the time τ characterizing its duration. It can be written in the explicit form:

$$u = \frac{c}{1 - \varkappa c\tau^2/\tau_2}\,. \tag{25}$$

Expression (25) can be obtained from expression (18) by neglecting absorption in the active medium and assuming that $\tau \ll \tau_2$.

The pulse described by expression (24) has an interesting property. It can be easily calculated that

$$\psi(\infty) = \frac{\tilde{\mu}}{\hbar}\int_{-\infty}^{\infty} A(\xi)\,\mathrm{d}\xi = 2\pi\,. \tag{26}$$

For this reason, the pulses (24) are called 2π-pulses. Their feature is that during propagation in the medium they do not change its state. Indeed, one can see from Eqns (22) that the difference N in populations of the energy levels first decreases with increasing ψ and becomes negative: the front part of the light pulse 'takes' all the energy stored in the active medium region of pulse propagation. Then, this energy is returned back into the active medium. Numerical studies [7, 26] have shown that single 2π-pulses are unstable in the amplifying medium. Upon entering the amplifying medium, a pulse with the area somewhat smaller than 2π transforms into two pulses: a stationary superluminal 2π-pulse and a stationary π-pulse following the first pulse. However, these details do not change the essence of the problem under study.

Equations (10a, b) represent the model of the so-called homogeneously broadened line, when the spectral parameters of the medium are only determined by the relaxation time of polarization τ_2. In practice, media with inhomogeneously broadened lines are more often encountered in which the line broadening is determined not only by relaxation but also by the scatter of the resonance frequencies of individual working atoms. In gases, the inhomogeneous broadening is determined by the dependence of the resonance frequency on the atom velocity caused by the Doppler effect. In solids, this broadening is determined by a small scatter of frequencies of working atoms caused by different shifts of different atoms relative to crystal sites. In the case of the inhomogeneously broadened spectral line, Eqns (10) are modified to the system

$$\frac{\partial\mathbf{A}}{\partial t} + c\,\frac{\partial\mathbf{A}}{\partial z} = \mathrm{i}\beta\int\mathbf{B}(\Omega,t)g(\Omega,\omega_0)\,\mathrm{d}\Omega\,, \tag{27a}$$

$$\left[\frac{\partial}{\partial t} + \mathrm{i}(\Omega-\omega_0)\right]B(\Omega,t) = -\mathrm{i}\,\frac{\tilde{\mu}^2}{\hbar}\,N(\Omega,t)A(t)\,, \tag{27b}$$

$$\frac{\partial}{\partial t}N(\Omega,t) = \tilde{J}(\Omega) + \frac{\mathrm{i}}{2\hbar}\big[\mathbf{A}(t)\mathbf{B}^*(\Omega,t) - \mathbf{A}^*(t)\mathbf{B}(\Omega,t)\big]. \tag{27c}$$

Here Ω indicates the frequency of a working atom. Analysis [26] showed that a solitary stationary wave (24) is the solution of this system of equations. However, the characteristic time τ appears in the first power in the expression for the propagation velocity of the superluminal pulse:

$$u = \frac{c}{1-\varkappa c\tau}\,. \tag{28}$$

This can be interpreted as follows. In deriving expression (28), the inhomogeneous linewidth was assumed to be appreciably greater than the spectral width of the pulse determined by the reciprocal value of τ. For this reason, the total spectrum of the pulse is amplified virtually uniformly with the gain $\varkappa$.

The energy of the electromagnetic field contained in the superluminal wave can be related to its propagation velocity. For example, the energy per unit area of the cross section of the 2π-pulse propagating in an ideal medium with the inhomogeneously broadened spectral line is

$$W = \int\frac{E^2}{8\pi}\,\mathrm{d}z = 2\hbar\omega\,\frac{N_0}{\Delta\omega}\,\frac{u^2}{u-c}\,. \tag{29}$$

Note here that the 2π-pulses, by returning the medium to the initial state, can propagate without changing their intensity even in a resonantly *absorbing* medium†. It follows from Eqns (23) that in this case their propagation velocity is less than the speed of light. For this reason, a detailed analysis of the propagation of 2π-pulses in an absorbing medium is not the subject of this paper. More detailed information on this topic can be found in Ref. [26].

In Ref. [9], the spectrum of weak excitations was studied corresponding to the linearized system of equations describing an amplifier without losses and relaxation:

$$\frac{\partial\mathbf{A}}{\partial t} + c\,\frac{\partial\mathbf{A}}{\partial z} = \mathrm{i}\beta\mathbf{B}\,, \qquad \frac{\partial\mathbf{B}}{\partial t} = -\mathrm{i}\,\frac{\tilde{\mu}^2}{\hbar}\,N_0\mathbf{A}\,. \tag{30}$$

The characteristic equation of this system is

$$(\omega-\omega_0)^2 - (k-k_0)(\omega-\omega_0) + \omega_{\mathrm{R}}^2 = 0\,,$$

$$\omega_{\mathrm{R}}^2 = \beta\,\frac{\tilde{\mu}^2}{\hbar}\,N_0\,. \tag{31}$$

The relation between the frequency ω and the wave number k determined by this equation is shown in Fig. 9. The branches for positive and negative values of $k-k_0$ correspond to 'slow' and 'fast' excitations, respectively. Their propagation rate at the merging point of the upper and lower branches is completely determined by the parameters of the system and is

† The case in point is the resonance part of the amplifier medium. As for the non-resonance absorption described by the coefficient α, it should either be absent (an ideal medium) or very weak.

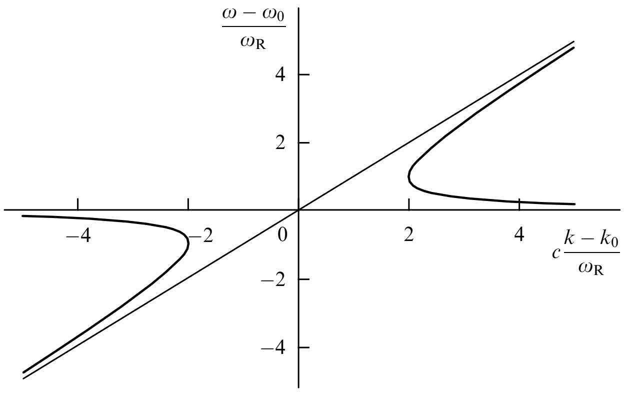

Figure 9. Dispersion curves of weak perturbations in an amplifying medium with uniform amplification of the line.

$$u = \frac{\omega}{k} \approx \frac{c}{1-\omega_R/\omega_0}\,. \tag{32}$$

This differs from the propagation rate of the solitary stationary wave. The velocity of the latter depends on the differential gain $\varkappa$ of the medium, whereas $\omega_R \propto \sqrt{\varkappa}$. The superluminal wave with small amplitude, which is determined by the branches of the left dispersion curve, has competitors. In the interval

$$-\omega_R < c(k-k_0) < \omega_R\,, \tag{33}$$

the waves of weak perturbations have complex frequencies and increase in time. The wave with $k = k_0$ has the maximum growth decrement. Its complex frequency is $\omega = \omega_0 \pm \mathrm{i}\omega_R$, so that the wave increases in time proportionally to $\exp(\omega_R t)$. In time, it will suppress superluminal excitations of small amplitudes. In contrast to the small-amplitude excitations, the above-considered solitary stationary superluminal waves are stable.

Of interest is the dispersion relation for weak perturbations in media with inhomogeneously broadened spectral lines:

$$(\omega-\omega_0)-(k-k_0) = \mathrm{i}\pi\omega_R^2 g(\omega,\omega_0)\,. \tag{34}$$

This corresponds to the linearized system of Eqns (27) [under the condition that the form factor of the inhomogeneous broadening $g(\Omega,\omega_0)$ has no poles]. As the form factor, a Gaussian

$$g(\Omega,\omega_0) = \frac{1}{\sqrt{\pi}\,\Delta\omega}\exp\left[-\frac{(\Omega-\omega_0)^2}{(\Delta\omega)^2}\right] \tag{35}$$

with an inhomogeneous linewidth $\Delta\omega$ is often used. It follows from Eqn (34) that in the case of an inhomogeneously broadened line, only weak increasing perturbations exist, whereas stationary weak perturbations are absent altogether.

5. Parametric decay of photons in a nonlinear medium and superluminal waves

It is clear that resonantly amplifying media are not unique in the field of generation of superluminal stationary waves. They can be found in almost any processes where the *induced* amplification plays the decisive role. In optics, these processes include induced Brillouin scattering, stimulated Raman scattering, Rayleigh scattering, induced temperature and enthalpy scattering, and other processes in nonlinear media [14]. In this section, we consider the process of parametric decay of photons in a nonlinear medium.

Radiation propagating in a nonlinear medium can be transformed in such a way that instead of the photon flux with frequency ω_0 two new photon fluxes with frequencies ω_1 and ω_2 appear, their sum being equal to the initial frequency: $\omega_0 = \omega_1 + \omega_2$. Such a process is called parametric decay. If $\omega_1 = \omega_2$, the process is called degenerate, and it satisfies the equations [27]

$$\frac{\partial A_1}{\partial t} + u_1\frac{\partial A_1}{\partial z} = -\gamma_1 A_0 A_1^*\,, \tag{36a}$$

$$\frac{\partial A_0}{\partial t} + u_0\frac{\partial A_0}{\partial z} = -\gamma_0 A_1^2\,. \tag{36b}$$

Here, γ_0 and γ_1 are constants determined by the nonlinear susceptibility of the medium and A_0 and A_1 are the amplitudes of the initial and transformed waves. The fields themselves have, obviously, the form $E_0 = A_0\exp\left[\mathrm{i}(k_0 z-\omega_0 t)\right]$ and $E_1 = A_1\exp\left[\mathrm{i}(k_1 z-\omega_1 t)\right]$. These equations have a solution of the form of a stationary soliton-like wave [27]:

$$A_1 = A_{1c}\,\mathrm{sech}\,\frac{\xi}{\tau}\,,\qquad A_0 = A_{0c}\tanh\frac{\xi}{\tau}\,,\qquad \xi = t-\frac{z}{u}\,. \tag{37}$$

The parameters of this wave are related by the expressions

$$\left(1-\frac{u_0}{u}\right)\left(1-\frac{u_1}{u}\right) = \gamma_0\gamma_1 A_{1c}^2\tau\,,\qquad \left(1-\frac{u_1}{u}\right) = \gamma_1 A_{0c}\tau\,. \tag{38}$$

These relations yield, in particular, expressions for the propagation velocity u and amplitude of the soliton-like wave:

$$u = \frac{u_1}{1-\gamma\tau u_1 A_{00}}\,,\qquad A_{1c}^2 = \frac{u_1-u_0+\gamma_1\tau u_1 u_0 A_{00}}{u_1\gamma_0}\,A_{00}\,. \tag{39}$$

Here, A_{00} is the amplitude of the wave with frequency ω_0 at the entrance to the nonlinear medium. One can see that the velocity of a soliton can significantly exceed not only the group velocity u_1 but also the speed of light in vacuum. If $u_0 > u_1$, for the soliton-like superluminal wave to appear, the amplitude of the initial wave should exceed the threshold value determined by the condition of positivity of the expression for A_{1c}^2. The parametric soliton has much in common with the 2π-pulse in the resonantly amplifying medium. However, a substantial difference is that there is a strict correlation between the duration and intensity of the 2π-pulse, whereas such a correlation is absent in the parametric soliton.

6. Superluminal emission of radiation

Propagation of a superluminal wave should be accompanied by radiation of additional electromagnetic waves. The mechanism of their emission in many respects resembles the mechanism of emission of Vavilov – Cherenkov waves. However, in contrast to the Vavilov – Cherenkov effect, the emission under study does not require slowing down of the phase speed of light in the medium, because the velocity of the superluminal wave can appreciably exceed the speed of light in vacuum. It is reasonable to call radiation emitted within the framework of the mechanism of superluminal waves 'superluminal radiation'.

The Hertz vector potential for superluminal radiation waves should satisfy the equation

$$\frac{\partial^2 \mathbf{\Pi}(r,\varphi,z;t)}{\partial t^2} - c^2\nabla^2\mathbf{\Pi}(r,\varphi,z;t) = \gamma \mathbf{B}_{\mathrm{T}}(r,\varphi;\xi)\exp\left[\mathrm{i}(k_0 z-\omega_0 t)\right],$$

$$\gamma = 4\pi\,\frac{\varepsilon_0+2}{3\varepsilon_0}, \tag{40}$$

where

$$\mathbf{E} = c^2\,\mathrm{grad\,div}\,\mathbf{\Pi} - \frac{\partial^2\mathbf{\Pi}}{\partial t^2}, \qquad \mathbf{H} = c^2\,\frac{\partial}{\partial t}\,\mathrm{rot}\,\mathbf{\Pi}. \tag{41}$$

In equation (40), r and φ are the transverse coordinates of the wave.

So far we have neglected transverse dependences of the field and polarization, assuming that they are described by plane waves. In the following calculations, the presence of the transverse dependence of polarization is of fundamental importance.

Assuming that the fields are cylindrically symmetric and independent of the azimuthal coordinate, we represent the Hertz vector for superluminal radiation waves in the form

$$\mathbf{\Pi}(r,z,t) = \sum_{n=1}^{\infty}\int \mathbf{\Pi}_n(p,t)J_0(q_n r)\exp(\mathrm{i}pz)\,\mathrm{d}p,$$

$$\mathbf{B}_{\mathrm{T}}(r,\varphi,\xi) = \sum_{n=1}^{\infty}\int \mathbf{B}_{\mathrm{T}}^{(n)}(\Omega)J_0(q_n r)\exp(-\mathrm{i}\Omega\xi)\,\mathrm{d}\Omega, \tag{42}$$

where $q_n = \alpha_n/a$, α_n is the nth root of the zero Bessel function, and a is an undetermined quantity having the dimensionality of length. It follows from Eqn (4) that

$$\frac{\partial^2\mathbf{\Pi}_n(p,t)}{\partial t^2} + c_0^2(p^2+q_n^2)\mathbf{\Pi}_n(p,t) = \gamma u\mathbf{B}_{\mathrm{T}}^{(n)}(\Omega)\exp\left[-\mathrm{i}(\Omega+\omega_0)t\right], \qquad p = k_0+\frac{\Omega}{u}. \tag{43}$$

Equation (43) has the obvious solution:

$$\mathbf{\Pi}_n(p,t) = \frac{\gamma u}{c^2(p^2+q_n^2)-(\Omega+\omega_0)^2} \times \mathbf{B}_{\mathrm{T}}^{(n)}(\Omega)\exp\left[-\mathrm{i}(\Omega+\omega_0)t\right]. \tag{44}$$

Thus, the wave with the wave vector k_n ($k_n^2 = p^2+q_n^2$) has the frequency $\Omega+\omega_0$. For the superluminal radiation wave to split from the main wave, by deviating from the direction of its propagation, it is necessary that

$$\frac{\Omega+\omega_0}{c} > p \equiv k_0+\frac{\Omega}{u}.$$

This immediately yields the condition $u > c$ required for the appearance of the superluminal radiation wave. It follows from Eqn (44) that radiation with $q_n \neq 0$ does not exist for the plane superluminal wave. For this reason, the appearance of the superluminal radiation wave is closely related to the finite transverse dimension of the superluminal wave. This circumstance is not a special feature of the case under study: in a transversely uniform electron beam, the Vavilov–Cherenkov radiation is also absent.

We will perform further calculations by assuming that the superluminal radiation wave is a weak perturbation of the process of propagation of the superluminal wave. Then, the polarization in Eqns (40), (42)–(44) can be considered specified. In the case of the 2π-pulse, the field of the transversely uniform wave is described by an analytic expression (24). In order to use this expression in calculations of the transversely finite wave, we will assume a Π-like dependence of the wave amplitude on the transverse coordinates†. In this case,

$$\mathbf{B}(r,\xi) = \begin{cases} \chi_0\,\dfrac{2\hbar}{\tilde{\mu}\tau}\,\mathrm{sech}\,\dfrac{\xi}{\tau}, & r \leqslant a, \\ 0, & r > a, \end{cases} \tag{45}$$

$$\mathbf{B}(r,\xi) = \begin{cases} -\mathrm{i}\tilde{\mu}N_0\sin\psi = -2\mathrm{i}\tilde{\mu}N_0\,\mathrm{sech}\,\dfrac{\xi}{\tau}\tanh\dfrac{\xi}{\tau}, & r \leqslant a, \\ 0, & r > a. \end{cases} \tag{46}$$

Now, the quantity a appearing in Eqns (42) acquires the meaning of the transverse dimension of the superluminal wave. In accordance with this‡

$$\mathbf{B}_0^{(n)}(\Omega) = 2\chi_0\,\frac{\hbar}{\tilde{\mu}}\,\frac{1}{\alpha_n J_1(\alpha_n)}\,\mathrm{sech}\,\frac{\pi\Omega\tau}{2}, \tag{47}$$

$$B^{(n)}(\Omega) = \tilde{\mu}N\,\frac{1}{\alpha_n J_1(\alpha_n)}\,F(\Omega\tau),$$

$$\mathrm{i}F(\Omega\tau) = \Gamma\left(\frac{3+\mathrm{i}\Omega\tau}{2}\right)\Gamma\left(\frac{1-\mathrm{i}\Omega\tau}{2}\right) - \Gamma\left(\frac{3-\mathrm{i}\Omega\tau}{2}\right)\Gamma\left(\frac{1+\mathrm{i}\Omega\tau}{2}\right), \tag{48}$$

where $\Gamma(z)$ is the Euler gamma function. The function $F(x)$ is shown in Fig. 10. The poles of expression (44) determine the set of emitted frequencies:

$$\Omega_n \approx \frac{c^2}{2}\,\frac{\alpha_n^2}{a^2\omega_0}\,\frac{u}{u-c}. \tag{49}$$

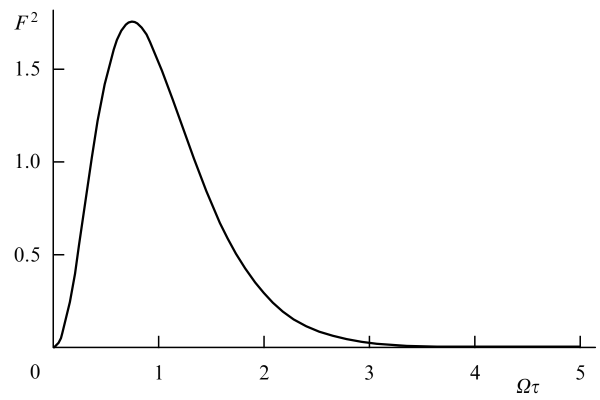

Figure 10. Plot of the function $F^2(\Omega t)$.

† A Π-like cross section solution is not a self-consistent wave, so that further calculations represent only estimates.

‡ Note the interesting fact that the Fourier transform of a hyperbolic secant is also a hyperbolic secant.

The calculation shows that the intensity of components rather rapidly decreases with increasing n. For this reason, the first component with

$$\Omega_1 \approx 2.88\,\frac{c^2}{a^2\omega_0}\,\frac{u}{u-c} \tag{50}$$

dominates in the field of the superluminal radiation wave.

One of the features of the Vavilov – Cherenkov radiation is that it is emitted within a comparatively narrow cone located at an angle to the direction of propagation of the emitting particle. For radiation with frequency ω, the angle is determined by the expression [28]

$$\cos\theta = \frac{c_0}{u}\,\frac{1}{n(\omega)}\,, \tag{51}$$

where $n(\omega)$ is the refractive index of the medium in which the emitting particles propagate. The parameters of the radiation cone are determined by the dependence of the refractive index on the frequency. In the case under study, the analogous angle is determined by the expression

$$\sin\theta_n = \frac{q_n}{k_n}\,. \tag{52}$$

For $n = 1$ and $k_0 \sim 10^5$ cm^{-1}, we obtain $\theta_1 \approx 0.14°$.

The superluminal radiation waves originating with $q \neq 0$ can be amplified at the expense of the energy accumulated in the amplifier medium if the frequency of these waves lies within the amplification band. The calculation of the amplification involves calculation of the polarization produced in the amplifying medium by the superluminal radiation wave.

To calculate the superluminal radiation wave in the case of parametric decay of photons, the polarization described by expression (45) should be used with the quantity $2\hbar/(\tilde{\mu}\tau)$ replaced by $A_{1\mathrm{c}}$.

All the calculations performed in this paper are valid only when the amplitude of the superluminal radiation wave is small compared to the amplitude of the superluminal wave. Otherwise, it is necessary to study the appearance of the superluminal wave and the superluminal radiation wave as a unified process. This rather time-consuming computational problem is beyond the scope of the present paper.

7. Superluminal waves in amplifying media and physics of elementary particles

The emission of the superluminal radiation wave shows that the superluminal wave is not simply a kinematic effect related to the movement of the pulse envelope in the amplifying medium. The superluminal wave can be manifested as a physical object and it is quite reasonable to call it the *optical tachyon*. Nevertheless, superluminal motions in amplifying (unstable) media represent a collective process, in contrast to the initial concept of a tachyon as one of the elementary particles. It would be of interest to consider the problem of superluminal motions in the physics of elementary particles from the point of view of amplifying media and the results discussed above. If some particle can decay during its propagation into particles of another type, then the beam of initial particles represents an unstable amplifying medium. Optical tachyons are a direct consequence of *induced* emission of photons. They can appear when the probability of stimulated transitions in a process exceeds that of spontaneous transitions (or is, at least, noticeable above the background). It follows from the known relation between the probabilities of stimulated and spontaneous transitions that stimulated transitions begin to dominate over spontaneous transitions if the spectral density of bosons generated in a process exceeds the spectral density of their possible states in the phase space. For this reason, optical tachyons may originate in processes involving many particles. The density of states in the phase space increases with the energy of particles. Particles with zero rest mass have the lowest density of states (Fig. 11). From this point of view, a neutrino would be an interesting object. However, a neutrino, being a fermion, cannot be directly used to stimulate the process. But perhaps a stimulated process involving two neutrinos rigidly coupled in phase is possible? This is suggested by the process of parametric decay of photons described above. Upon degenerate parametric decay, a pair of completely identical photons differing by a constant phase are born in each elementary decay event. This pair is often called a biphoton. The corresponding electromagnetic wave is in the so-called *squeezed state* [29 – 31]. Similarly to the parametric process of photon decay, a pair of strongly correlated neutrinos created in some process could be called a 'bineutrino'. Could a bineutrino be an inducing agent, and if so, under what conditions? This question requires special consideration. Similarly, an electron – positron pair forming a positronium could play the role of an inducing agent in processes of creation of pairs.

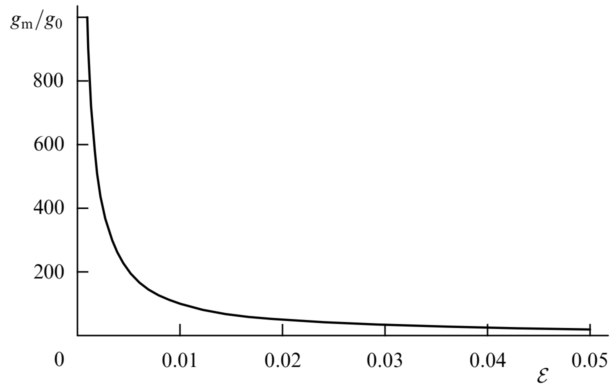

Figure 11. Ratio of spectral densities of states in the phase space for particles with nonzero (g_m) and zero (g_0) rest mass. The kinetic energy plotted on the horizontal axis is normalized to the rest energy of a particle with nonzero rest mass.

In processes of the type of parametric decay of photons, the laws of conservation of angular momentum and momentum should be satisfied [27, 32]. For this reason, the phase volume of spontaneously created photons will decrease with increasing degree of coherence of the wave of initial photons, i.e., with decreasing scatter of the initial photons over their wave vectors and frequencies. The same should be valid for creation processes of bineutrinos (and other particles), if their creation is not accompanied by the appearance of additional particles that expand the phase volume of the possible final states. By choosing the reaction and its conditions, it can probably be provided that stimulated processes play an important role in the densities of created particles achievable in nature.

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