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Thermonuclear detonation

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Contents

Abstract. The characteristics of, and energy transfer mechanisms involved in, thermonuclear detonation are discussed. What makes the fundamental difference between thermonuclear and chemical detonation is that the former has a high specific energy release and can therefore be employed for preliminarily compressing the thermonuclear mixture ahead of the burning wave. Consequently, with moderate (megajoule) initiation energies, a steady-state detonation laboratory experiment with unlimited energy multiplication becomes a possibility.

Dedicated to the memory of my first teachers
Yu B Khariton, Ya B Zel'dovich,
D A Frank-Kamenetskiĭ

1. Introduction

In the early 1950s, the author was dispatched to the Arzamas-16 nuclear center, which is presently known around the world, and assigned to the theoretical group directed by Ya B Zel'dovich and D A Frank-Kamenetskiĭ. This was the time when the first stage of the development of nuclear weapons had been successfully completed, and rumours were circulating about a more powerful weapon — an H-bomb. Under the thick shroud of secrecy, various tracks to this goal were tested, including Sakharov's well-known 'layer cake', which was described in detail in *Priroda* (*Nature*) [1]. Zel'dovich's group investigated a 'refined', as one could say, all-hydrogen version of the device that would not contain heavy fissionable elements, in particular, a propagating combustion in deuterium, a heavier hydrogen isotope. It should be mentioned that this scheme would not have ultimately translated into a real device, nor would it have found a military application. Nonetheless, the range of problems closely related to this model stimulated intense physical research.

It is important, both with a view to establishing priority in this field of research and irrespective of these considerations, to declassify and publish the works by Ya B Zel'dovich and other researchers of that period. Now, before the time is ripe for this undertaking, let us briefly review the difficulties we ran into. They turned out quite serious and almost insurmountable, as a result, the project was ultimately abandoned.

There is a widely known theorem formulated by Yu B Khariton [2] which asserts that any exothermal material is capable of detonating if its characteristic dimension is larger than a certain critical value. The rate of thermonuclear reaction strongly depends on temperature. In the process of energy release, some energy is lost as heat transmitted through the outer surface, so the effective source of energy can be described by the equation

$$q = q_+(T) - q_-(T)\,.$$

By equating the positive and negative sources, one can calculate the characteristic temperature $T_{\max}$ and subsequently derive the combustion time. On the other hand, the plasma lifetime determined by its hydrodynamic expansion is approximately expressed by the formula

$$t_{\rm p} = \frac{r}{c_{\rm sound}} \cong \frac{r}{\sqrt{T_{\max}}}\,.$$

By comparing these two times, one can find the critical dimension. There is an important note *a propos*. Since the reaction rate is proportional to the material density ρ, the similarity among different systems is based on the parameter ρr. This means that the dimensionless parameters, such as burn-out, are functions of ρr. In particular, the critical dimension $r \sim 1/\rho$ determines the mass capable of detonating: $M \sim 1/\rho^2$ for a sphere and $M \sim 1/\rho$ (g cm^{-1}) for a cylinder. The situation is very similar to the case of neutron-

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triggered fission chain reactions, since the basic physical processes in systems of both types are controlled by the same parameters: the generated energy is proportional to the mass of materials, and the lost energy is proportional to the area of the outer surface.

At the same time, in studying thermonuclear detonation in deuterium, the researchers encountered another difficulty, namely, an upper limit on the tube radius, which had not arisen in the detonation of chemical explosives. Physically, this limit was determined by the following process. It is well known that a photon scattered by an immobile electron loses a fraction of its energy, whereas its energy can increase if the electron has a high energy and the photon is, on the contrary, soft. These two processes, termed the direct Compton effect and inverse comptonization†, lead, in the long run, to thermodynamic equilibrium between the plasma composed of particles and the electromagnetic radiation (planckization of the radiation).

Since the main source of photons in a hydrogen medium is a very soft bremsstrahlung, there is an undesired energy drain from particles to radiation. In order to limit this loss, the designers have to reduce the tube diameter so that the photons escape from it before they receive a considerable amount of energy from electrons. This means that the tube radius cannot be much larger than the Compton free path of the photons ($l = 5/\rho$, $\rho = 0.14$ g cm^{-3} for liquid deuterium). All calculations indicated that the lower bound of the radius was very close to the upper limit, and the entire system was highly susceptible to parameters of secondary importance. For example, the thickness and material of the wall surrounding the liquid hydrogen had a considerable effect on the processes inside. (An important fact that should be taken into account to avoid confusion is that the existence of the upper limit on the tube radius does not contradict Khariton's fundamental theorem, since there is such a radius for deuterium at which thermonuclear detonation will also be sustainable under conditions of full thermodynamic equilibrium. The radius, however, is so large that this configuration is of little practical significance.)

Before ending this historical review, an effect should be mentioned which Zel'dovich told me about in the early 1950s. It was known by that time that the rate of the reaction between deuterium and tritium nuclei is a factor of 100 higher than between deuterium nuclei. The combination of two factors, namely, the minimal electric charge of hydrogen ($Z = 1$) and the high rate of the D + T reaction, leads to a situation when the reaction time is too short to establish a Planck distribution in the radiation subsystem. As a result, the radiation effective temperature remains relatively low, and a larger fraction of the released energy is translated into the energy of particles.

After many years, this effect was detected in experiments. By measuring the Doppler spread of 14-MeV neutrons generated in the DT mixture during an explosion, a record temperature of above one billion degrees was registered.

Chemical and thermonuclear detonations have much in common. As long as their similarity holds, many calculation techniques are directly translatable from one field of research into the other. On the other hand, there are considerable differences, in addition to the conspicuous difference between their caloric values. In what follows, we will largely discuss the DT reaction, which proceeds at the highest rate. It is the deuterium-tritium mixture that allows one to minimize the diameter of the detonation pinch. Only along this line of development does a transition from military to peaceful utilization of thermonuclear detonation become plausible, when the energy needed to trigger the reaction is somewhere near the level acceptable for a 'laboratory' experiment.

It has turned out that a self-sustaining reaction occurs at $\rho r \geqslant 0.35$ g cm^{-2}. Hot plasma with a temperature of the order of 10 keV is controlled by a variety of energy transfer mechanisms, including hydrodynamic detonation, electronic thermal conductivity, energy transfer by charged α-particles and neutrons, and radiation effects. In different conditions different processes dominate; in particular, in an infinite medium, most of the released energy can be transported by neutrons, which take up 80% of the energy generated in the reaction $\mathrm{D} + \mathrm{T} \to \alpha + \mathrm{n}$. In pinches of the smallest radius, on the contrary, the contribution of neutrons to the 'forward' transfer of energy is insignificant because the pinch for them is transparent (the neutron free path $l_{\mathrm{n}} = 5/\rho$ is a factor of several tens longer than the pinch radius). As was mentioned above, in pure hydrogen the radiation transfer is also insignificant. The energy transfer by α-particles is also inefficient owing to their small free path, as compared with that involved in the electronic thermal conductivity. In essence, the competition between two process is important: the Jouguet – Zel'dovich hydrodynamic detonation and high-speed (thermoconducting) combustion.

The energy loss to radiation emitted through the side surface is usually insignificant in chemical detonation, whereas in thermonuclear detonation one cannot neglect this loss of energy in the most interesting cases. For example, under certain conditions the energy lost to α-particles, rather than the plasma expansion, determines the critical radius. Some problems of thermonuclear combustion waves were discussed in Refs [4, 5].

2. Jouguet – Zel'dovich's hydrodynamic detonation

Conceptually, all problems of detonation, including energy losses, were comprehensively described in the monograph [6]. Below is an alternative derivation of the formula for the detonation velocity, which is fairly simple and easily understandable. It is based on the equations of a planar, steadily propagating detonation in the dimensionless form in the reference frame moving with the detonation front:

$$\rho u = 1\,, \qquad \rho = [\rho_0]\,,$$
$$p + \rho u^2 = 1\,, \qquad u = [D]\,,$$
$$\frac{\gamma}{\gamma - 1}\,\frac{p}{\rho} + \frac{u^2}{2} = \frac{1}{2} + E\,, \qquad p = [\rho_0 D^2]\,,\ E = [D^2]\,.$$

The function $E = \int q\,\mathrm{d}t = \int_0^z [q_+ - q_-] D^{-1}\,\mathrm{d}z$ is the energy of the effective source. Its approximate shape is shown in Fig. 1 (the origin $z = 0$ is in the detonation front plane).

The characteristic feature of function $E(z)$ is its peak, whose existence is attributed to the fact that the energy released by the positive source decays as a result of burn-out. All the released energy is transferred, in the long run, from the reaction zone through the side walls, thus we have the limiting formula $E_{z\to\infty} = 0$. Therefore, the correct boundary condition for all the hydrodynamic parameters, namely, the pressure, temperature, and longitudinal velocity

† This effect has been studied by many researchers, including Ya B Zel'dovich, A S Kompaneets, and others. An updated analysis of the problem can be found in the monograph by V L Ginzburg [3].

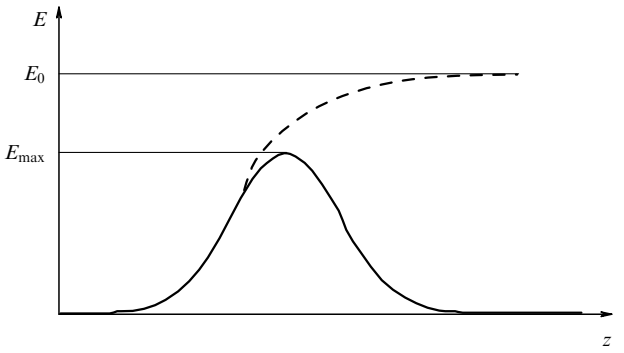


Figure 1. Effective energy yield as a function of coordinate (the dashed line corresponds to zero energy losses, $q_- = 0$).

$v = 1 - u$, is that they should all become zero at long distances from the front.

By resolving the equation system, we obtain

$$u = \frac{1}{\gamma + 1}\left[\gamma \pm \sqrt{1 - 2(\gamma^2 - 1)E}\right].$$

On the shock front, where $z = 0$, $E = 0$, $u_f = (\gamma - 1)/(\gamma + 1)$, the lower sign should be taken. In the limit $z \to \infty$, E again becomes zero and $u = 1$. This is possible only on the branch with the upper sign. Consequently, a way should be found for a transition from one branch to another. One can easily find out that the only possibility is to set the expression under the radical sign to zero at the peak, where $E = E_{max}$. Hence, we obtain $2(\gamma^2 - 1)E_{max} = 1$, or in the dimensional form

$$D_J^2 = 2(\gamma^2 - 1)E_{max}.$$

The resulting formula is in accord with Jouguet's well-known expression for $E_{max} = E_0$, which is equal to the fuel caloric value, and is similar to corresponding formulae in the monograph [6].

Note that the equality $E = E_{max}$ holds at the point where $q_+(\rho, T) = q_-(\rho, T)$ and, at the same time, at the Jouguet point, where the density is known $[\rho_J = (\gamma + 1)/\gamma]$. Thus, the equation under consideration can be used in determining the temperature at the Jouguet point. Given the temperature, one can easily calculate the speed of sound and the detonation velocity:

$$D = \frac{\gamma + 1}{\gamma} c(T_{max}).$$

The numerical value of E_{max} can be employed in calculating the effective width of the reaction zone. One should keep in mind that, although the reaction is not terminated at the Jouguet point, it does not change our calculations of D because this zone of energy release is outside the region where perturbations have a notable effect on the shock front.

3. Supersonic combustion

In chemical reactions, energy is transferred from atom to atom via collisions. Since the particle velocity in the process of diffusion cannot be higher than the atomic thermal velocity, which is, in turn, of the order of the sound velocity, the thermoconducting combustion in a chemically reacting medium is always slow (subsonic). The velocity of an agent transmitting heat in a high-temperature ionized plasma (electrons, photons, etc.) can be much higher than the thermal ion velocity, and the detonation front velocity can be higher than the speed of sound: $D > c \cong D_J$. It is clear on the intuitive level that the role of gas-dynamic processes in this case is less important. It directly follows from Euler's equation that

$$\frac{du}{dt} = D\frac{du}{dx} = \frac{1}{\rho}\frac{dp}{dx},$$

$$Du \cong \frac{p}{\rho} = c^2,$$

$$\frac{u^2}{c^2} = \frac{c^2}{D^2} \cong \frac{D_J^2}{D^2}.$$

It is clear that the ratio between the kinetic and thermal energy reduces with increasing D. Thereby we obtain a unified model of the subsonic and supersonic combustion based on the heat (transfer) equation. In this section, we will generalize the widely known Zel'dovich – Frank-Kamenetskiĭ (ZFK) formula [7] for the combustion rate with a view to taking account of thermal losses.

The steady-state thermal conductivity equation

$$\frac{\partial\varepsilon}{\partial t} = D\frac{d\varepsilon}{dx} = \frac{d}{dx}\left(\varkappa\frac{dT}{dx}\right) + q(T)$$

(where ε is the internal energy and $\varkappa$ is the thermal conductivity, which is a function of temperature) reduces to a first-order equation if the coefficients $\varkappa$ and q are not explicit functions of the coordinate:

$$Q = Q\frac{dQ}{d\varepsilon} + q_0(\varepsilon).$$

The quantity $Q = D^{-1}\varkappa\, dT/dx$ is proportional to the thermal flux, $q_0 = \varkappa q/(D^2\, d\varepsilon/dT)$ is a generalized source, and T and ε are related to one another via the specific heat. On the leading edge $\varepsilon \to 0$, $q \to 0$. Suppose that $q_0 \approx \varepsilon^n$. It is clear that at $n < 1$ a local self-ignition takes place, i.e., $(D \to \infty)$ rises from zero to a finite value in a finite time interval. For $n > 1$ and $\varepsilon \to 0$ the source intensity is low, and the solution in this limit is $Q|_{\varepsilon\to 0} = \varepsilon$.

Suppose that q_0 is a fast increasing function of ε. Then most of the energy is released in a narrow region of the maximal value $\varepsilon = \varepsilon_{max}$. But at the maximal temperature the flux $Q = 0$. After rejecting the term with the specific heat in the equation, we obtain the solution

$$Q^2 = 2\int_{\varepsilon}^{\varepsilon_{max}} q_0\, d\varepsilon.$$

The integral is little affected by a change in the lower integration limit, and we can set $\varepsilon \to 0$. On the other hand, the solution should be joined to another solution in the region $q_0 = 0$, $Q = \varepsilon \approx \varepsilon_{max}$. With all this taken into account, we have a solution in the dimensional form

$$D^2 = \frac{2}{\varepsilon_{max}^2}\int_0^{T_{max}} \varkappa q\, dT,$$

where ε_{max} is assumed to be equal to the caloric value.

The reasoning and result are in full agreement with the logic of Zel'dovich and Frank-Kamenetskiĭ, who suggested this formula for determination of the combustion front velocity in chemical reagents.

As concerns thermonuclear combustion, some modifications in the equations are necessary. In the DT reaction, the derivative of the reaction rate with respect to temperature drops abruptly beyond a temperature of 10 to 15 keV, whereas the temperature deriving from the caloric value is much higher. With due account of the burn-out, the reaction rate can even drop with the temperature. If the source intensity is not a monotonic function of temperature, and this may be the case if thermal losses are taken into account, the ZFK formula applied directly can lead to a paradoxical result, namely, the detonation velocity may drop with increasing caloric value. In order to get out of this impasse, let us analyze the shapes of integral curves of the original equation at different values of D. At large D the sought-for temperature tends to infinity, at small D it has a peak and turns to zero at a finite distance x. Obviously, the physically justified and limiting value of the velocity is that separating these two solutions. In other words, we need a solution such that at $\varepsilon = \varepsilon_{\max}$ both Q and $Q\,\mathrm{d}Q/\mathrm{d}\varepsilon$, and consequently $q(\varepsilon_{\max})$, simultaneously become zero. Then the equality $q_+ = q_-$ determines the maximal temperature. It is clear that this condition is identical to the rule that applies to the gas-dynamic detonation. (Note also that the limiting solution is nothing but a 'solitary wave', which has come into fashion recently.)

Let us turn again to the original equation and transform it to the integral form, which is convenient for iteration as $\varepsilon \to 0$:

$$Q = \varepsilon - \int_0^{\varepsilon} \frac{q_0}{Q}\,\mathrm{d}\varepsilon\,,$$

$$Q_0 = \varepsilon\,, \qquad Q_1 = \varepsilon - \int_0^{\varepsilon} \frac{q_0}{Q_0}\,\mathrm{d}\varepsilon \quad \text{and so on}\,.$$

At the same time, at $\varepsilon = \varepsilon_{\max}$ we have $Q \cong q_0$, and this allows us to use the simplest interpolation:

$$Q = \begin{cases} Q_{\varepsilon\to 0}\,, & \varepsilon < \varepsilon_0\,, \\ q_0\,, & \varepsilon > \varepsilon_0\,, \end{cases}$$

where $0 < \varepsilon_0 < \varepsilon_{\max}$ is a certain intermediate number. By letting $\varepsilon \to \varepsilon_{\max}$, we obtain

$$1 = \frac{1}{\varepsilon_0}\int_0^{\varepsilon_0} \frac{q_0}{Q|_{\varepsilon\to 0}}\,\mathrm{d}\varepsilon \cong \frac{2}{\varepsilon_0}\int_0^{\varepsilon_0} \frac{q_0}{\varepsilon}\,\mathrm{d}\varepsilon\,.$$

In the latter equation, the simple expression $Q = \varepsilon$, which is the upper estimate, has been used. In order to offset, in a sense, this high estimate and obtain a correct asymptote for a δ-source (the ZFK solution), a numerical factor of 2 has been introduced. Since the solution sought among all physically sensible solutions is that with the highest velocity, we obtain a procedure for the determination of ε_0. The ultimate formula in the dimensional form is

$$D^2 = \max_{(T_0)}\left[\frac{2}{\varepsilon(T_0)}\int_0^{T_0} \frac{q\varkappa}{\varepsilon(T)}\,\mathrm{d}T\right].$$

This formula is approximate, and its adequacy should be checked by analyzing specific examples. For a step-function source ($\varepsilon = T$, $\varkappa = 1$)

$$q(T) = \begin{cases} 0\,, & T < T_0\,, \\ 1\,, & T_0 < T < 1\,, \\ 0\,, & T > 1 \end{cases}$$

the equation solution has the form

$$T = Q + q_0 \ln\frac{q_0 - Q}{q_0 - T_0}\,, \quad q_0 = \frac{q}{D^2}\,.$$

At $T = 1$, $Q = 0$, and this yields the transcendental relationship $q_0 \ln[q_0/(q_0 - T_0)] = 1$, which is equivalent to $T_0 = [1 - \exp(-D^2)]/D^2$.

In Table 1 the exact solution (D^2), the solution by the ZFK formula [$\mathcal{D}^2 = 2(1 - T_0)$], and the solution obtained using the suggested procedure:

$$\mathcal{D}^2 = \begin{cases} \dfrac{2}{T_0 \mathrm{e}}\,, & T_0 < \dfrac{1}{\mathrm{e}}\,, \\ 2\ln\dfrac{1}{T_0}\,, & T_0 > \dfrac{1}{\mathrm{e}}\,. \end{cases}$$

are compared.

There is another example. One may prescribe a solution of the original equation and derive an expression for the source. In particular, if

Table 1. Combustion rates calculated by different methods: D^2 is the exact solution, $\mathcal{D}^2$ is the ZFK solution, and $\mathcal{D}^2$ is the solution obtained by the suggested procedure.

T_0	D^2	$\mathcal{D}^2$	$\mathcal{D}^2$
1	0	0	0
0.91	0.2	0.18	0.19
0.79	0.5	0.42	0.47
0.63	1	0.78	0.92
0.43	2	1.14	1.7
0.2	5	1.6	3.7
0.1	10	1.8	7.4

$$Q = T(1 - T^k)^m\,,$$

then

$$q_0 = T(1 - T^k)^m\Big\{1 + (1 - T^k)^{m-1}\big[(mk + 1)T^k - 1\big]\Big\}\,.$$

Calculations of D^2 by approximate formulas should be compared with unity:

$$m = 1\,, \quad \mathcal{D}^2 = \frac{k}{k+2}\,, \quad \mathcal{D}^2 = \frac{2k+1}{2k+2}\,;$$

$$m = 2\,, \quad \mathcal{D}^2 = \frac{k^2}{k+2}\,,$$

$$\mathcal{D}^2 = \max\left[2\left(2x - \frac{6k+5}{2k+1}x^2 + \frac{6k+4}{3k+1}x^3 - \frac{2k+1}{4k+1}x^4\right)\right]_{x=T^k}.$$

For $k \to \infty$, all the formulas yield correct values. At small k the discrepancy is considerable:

$$k = 1\,, \quad m = 1\,, \quad \mathcal{D}^2 = \frac{1}{3}\,, \quad \mathcal{D}^2 = 0.75\,;$$

$$k = 1\,, \quad m = 2\,, \quad \mathcal{D}^2 = \frac{1}{6}\,, \quad \mathcal{D}^2(x \cong 0{,}5) = 0.73\,;$$

$$k = 2\,, \quad m = 2\,, \quad \mathcal{D}^2 = \frac{1}{3}\,, \quad \mathcal{D}^2 = 0.81\,.$$

In all the tested examples, the suggested method for calculating the front velocity proved more accurate over a wider range of parameters.

4. Heat release and compression

As was noted above, the similarity in thermonuclear systems is controlled by the parameter ρr, but not the material density and characteristic dimension taken separately. An important property of the thermonuclear detonation, as compared with the chemical one, is the enormous compression ratio of the matter, which is caused by the much higher energy release in the nuclear reaction than in chemical reactions. In normal detonation, the material is compressed on the shock front by a factor of $(\gamma+1)/(\gamma-1)$. Nuclear detonation leads to compression ratios of several hundreds.

The material is prepared for triggering the reaction not only by the transmitted heat, but also by the strong compression, which makes the medium reactive. In order to visualize the process using a specific example, imagine a volume divided into two sections. The first section contains a ball-shaped thermonuclear target. Driven by an external source of energy E_s (laser), the target is compressed and explodes, and the energy E released in the process propagates through both sections. The target is so small that is does not confine neutrons, therefore, about 80% of the energy is lost completely. Suppose that half of the remaining 20% is the inefficient kinetic energy of the target fragments blown apart. The remaining 10% of the energy is distributed between the two sections, but, since the first section is 'hotter', assume that this energy is distributed in the proportion 2:1. If there is a similar target in the second section, it takes up a fraction of about 1% of the total energy released by the first target, proportionally to its surface. Although this estimate is far from accurate, it is clear that there is a minimal energy gain $K = E/E_s$ in the first target, which is of the order of 100 so that, if $K \geqslant K_{\min}$, the ignition conditions in the second target should be no less favorable than in the first. Thus, there is a possibility of doubling the released energy by building a more complex structure (namely, adding the second section with a target). But, once this configuration has proved to be efficient, it can be extended by adding a third, fourth, etc. ball and using the energy of the previous explosion for triggering the next. After exploding a sufficiently large number of balls after the initial triggering process, one obtains a specific steady-state detonation, which can propagate not only in the longitudinal, but also in the transverse direction. In order to distinguish this type of detonation, let us label it by TD, which means transverse detonation.

Below estimates for such a configuration are given. A tube contains balls of radius r with equal separations between them, and the separation between neighboring ball centers $h = 4r$. If the time of the process is controlled largely by the ball compression time, the detonation front velocity is, obviously, $D = 4v$, where v is the compression velocity. Suppose that the ball structure is the simplest. A thin shell from a heavy material with mass M contains a hydrogen mixture (DT) with mass m. Initially the shell is imparted a kinetic energy $E_0 = Mv^2/2$, which is, in turn, $1/K$ of the energy released by one target (K equals, as was mentioned above, approximately 100). If the shell remains thin throughout the compression process, its entire kinetic energy is translated into the energy of the gas inside the shell:

$$AmT_f = E_0 ,$$

where $A \cong 10^{15}$ erg g^{-1} keV^{-1} is the specific heat, and T_f is the final gas temperature at the maximal compression.

The compression ratio δ_f of the lighter material is derived from the assumption that entropy is brought by a shock wave with the particle velocity V equal to the initial shell velocity. For a one-atom gas, $(\gamma = 5/3)$ $\delta_f = 4(M/m)^{3/2}$. If the ratio M/m is too high, one should take into account the energy of shell compression, since it is no longer thin. Then

$$AmT_f = SE_0 ,$$
$$\delta_f = 4\left(\frac{SM}{m}\right)^{3/2} ,$$

where S is the fraction of energy transmitted to the gas.

In solving the equation of state for a 'cold' shell, $p = B\delta_h^\gamma$ (where $\delta_h = \rho_h/\rho_{0h}$ is the compression ratio), we directly derive, from the energy conservation and equality between the pressures in the hot gaseous core and shell, the relationships

$$1 = S + vS^{5(\gamma-1)/2\gamma} ,$$
$$v = \frac{1}{\gamma-1}\left(\frac{4B}{A\rho_{0h}T_{\max}\delta_{\max}}\right)^{1/\gamma}\left(\frac{\delta_{\max}}{4}\right)^{5/3}\left(\frac{8}{3}\frac{\rho_{of}}{\rho_{oh}}\right)^{(\gamma-1)/\gamma} ,$$

where $T_{\max} = E_0/Am$ and $\delta_{\max} = 4(M/m)^{3/2}$ are the temperature and compression at $S = 1$, i.e., without taking account of losses in the shell.

For typical values of the constants ($\gamma = 2.5$, $B = 10^{12}$) we have

$$v = 0.5\left(\frac{\delta_{\max}}{1000}\right)^{5/3}\left(\frac{T_{\max}}{10}\frac{\delta_{\max}}{1000}\right)^{-2/5} .$$

The fraction of losses is expressed fairly accurately by the formula

$$S = \frac{1}{(1+v)^{2/3}} .$$

If liquid hydrogen is contained in a shell from a heavy material (such as gold), $S = 0.5$ at $M/m = 80$ and $\delta_f = 1000$. In addition to the inefficient energy consumption on the shell compression, the efficiency is also limited by breaking of the spherical symmetry of compression.

Typical values of the parameters are

$$m = 10^{-3}\ \text{g},\quad \delta_f = 800 ,\quad T_f = 6\,\text{keV},\quad K = 70 ,$$
$$M = 5\times10^{-2}\ \text{g},\quad r_0 = 1\,\text{mm},\quad v = 2\times10^{7}\,\text{cm s}^{-1},$$
$$E_0 = 1\ \text{MJ} ,\quad D \cong 10^3\ \text{km s}^{-1}.$$

The problem is closed and the dimensions are selected using the condition $K \cong 100$, which is necessary for self-sustaining propagation of combustion from one ball to another. The energy gain K, in turn, depends on the ball mass m (or dimension r_0), compression velocity, and whether the triggering condition is satisfied, i.e., a certain temperature $T \sim 6$ keV is achieved and $\rho r \cong 1$ g cm^{-2}, which is sufficient for the required burn-out level ($\eta \cong \rho r/6$).

The reasoning presented above is not fully self-consistent, namely, the separation between ball centers has not been specified. I hope that the reader clearly understands that some details are omitted for simplicity. For example, the neutron heating caused by previous ball explosions can play a significant role. Its intensity is higher, the smaller the separation between ball centers. When the balls are located

closer to one another, it is more difficult to organize a configuration that would ensure spherically symmetrical compression of the balls. On the other hand, if the separation between balls is too large, there is too much 'idle' space, which leads to additional energy losses. The full optimization of the design requires fairly accurate and cumbersome calculations [8]. At the same time, it seems interesting to consider another limiting case obtained through a transition to a continuous configuration, namely, the 'cylinder-within-a-cylinder' scheme.

Thus, suppose that a cylinder of radius $r_{\rm out}$ and with an infinitely long side wall from a heavy material contains another cylinder of radius r_0, which is composed of a thin heavy shell of mass M (g cm^{-1}) per unit length and a DT gas mixture of mass m per unit length inside the shell. The space between the two cylinders is filled with a light material, such as Be, which is transparent for radiation.

A fraction of the released energy propagates in the form of radiation through the light material in the gap between the cylinders and forms a jet downstream of the reaction zone. This energy, traveling ahead of the front, preliminarily compresses the thermonuclear fuel. In this case, two regimes can be realized. The first is conceptually similar to that of isolated balls, when the high temperature needed to ignite the mixture is generated by compression, i.e., the front velocity is controlled by the transverse compression (TD regime).

The process, however, can develop in a different manner, when the energy propagating along the gap compresses but does not ignite the mixture. Since the cylinder has a simply connected configuration, the heat (high temperature) needed to trigger the reaction is transmitted along the inside cylinder by a hydrodynamic or heat-conducting mechanism. This detonation is almost identical to the natural one, the only, albeit very important, difference being that the material density is much higher than that of common solids. Since the latter regime combines both the longitudinal and transverse motion, let us dub it longitudinal-transverse detonation (LTD).

Which of the two regimes is realized depends on the specific configuration (the ratio between $r_{\rm out}$ and r_0, the wall thickness of the inside cylinder, the density and nature of the material in the gap between the cylinders). In our opinion, the LTD regime is the most efficient, but, at the same time, its implementation requires the finest tuning of the parameters. Undoubtedly, there is a wide range of parameters where both these regimes are possible. In reality, the regime with the higher velocity is set up.

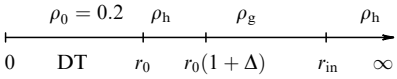
4.1 Closure of the solution in time

By taking a specific example, let us try to understand characteristic features of cylindrical detonation and outline the reasoning leading to determination of the propagation velocity. Note that the front propagation is possible only at $\rho r > 0.35$ g cm^{-2} [8].

All numerical estimates will refer to $\rho r = 0.4$ g cm^{-2}, i.e., we will investigate a regime with a certain margin of thermodynamic parameters. The combustion efficiency is set to $\eta \cong 0.2$, and the detonation velocity (without matching) to $D = 2 \times 10^8 = 20$ (the time unit is 10^{-7} s, the energy unit 10^{14} erg). Let us assume, by the symmetry condition, that $M/m = 10$, then the limiting compression ratio $\delta_{\max} \cong 4(M/m)^{3/2} = 125$ (or approximately 11 in terms of radius).

Finally, from the condition $(\rho r)_{\max} = (\rho r)_0 \delta_{\max}^{1/2} = 0.4$, we derive the inside cylinder radius $r_0 = 0.18$ cm.

4.2 Cross section of the double cylinder



$\rho_{\rm h}$ is the density of the heavy shell (M) and the outside shell ($\rho_{\rm h} \cong 20$ g cm^{-2});
$\rho_{\rm g}$ is the density in the gap (it is determined by the condition that evaporated matter of the walls do not close the gap);
Δ is the relative shell thickness, at $M/m = 10$ $\Delta = 0.05$ (5%);
$r_{\rm in}$ is the inside radius of the outer cylinder;
$r_{\rm in}/r_0$ is an optimization parameter.

4.3 Temperature inside the gap

The total caloric value of the DT fuel is $q_0 = 3.4 \times 10^{18}$ erg g$^{-1} = 3.4 \times 10^4$, in terms of the energy carried by α-particles ($q_\alpha = 6.8 \times 10^3$). With due account of the reaction efficiency $\eta = 0.2$, the released energy is $\varepsilon = 1.4 \times 10^3 m$ ($m \cong 2 \times 10^{-2}$ g cm^{-1}).

In what follows, we assume that the energy is, in the long run, distributed over the cross section, and the established temperature is determined by the specific heat. Then

$$A_{\rm DT} m T + A_{\max} M T + A_{\rm g} M_{\rm g} T + S\sigma T^4 = \frac{\varepsilon}{2}$$

where $S = \pi r_{\rm in}^2$ is the cross section area. The thermal capacities are denoted by A, and the last term on the left is the energy per unit length lost to radiation. On the right-hand side of the thermal balance equation, we have arbitrarily introduced the factor 1/2 to take account of non-uniformities in the temperature distribution and losses to the kinetic energy of matter. The calculations are listed in Table 2.

Table 2. Temperature $T_{\rm g}$ (keV) at the point of energy release as a function of density $\rho_{\rm g}$ in the gap and the ratio between the radii of the outer ($r_{\rm in}$) and inner (r_0) cylinders.

$\rho_{\rm g}$	$r_{\rm in}/r_0$		
	1.5	2	3
0.5	2.4	2.0	1.55
1	2.25	1.9	1.3
2	2.15	1.6	0.95

In the left-hand upper corner of Table 2 the radiation energy dominates, and in the right-hand lower corner the material specific heat is most important.

4.4 Evaporation of walls

If the material density in the gap is sufficiently high, the wall motion in the process of evaporation is impeded. In this approximation, the thermal conductivity equation

$$(A\rho)_{\rm h} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\frac{lc}{3}\frac{\partial \sigma T^4}{\partial x}\right)$$

can be solved as described below. A 'cushion' of evaporated matter whose pressure equals that of its environment (the layers are 'sakharized') is formed between the gap material and the heavy wall: $(A\rho)_{\rm h} = (A\rho)_{\rm g}$. The 'cushion' thickness increases with time, as if a thermal wave travels through a preprocessed heavy matter with a 'sakharized' density.

Automodeling considerations apply to the free path expressed by a power function $l = aT^n/\rho^m$, and an exact solution can be found. Let us resort to simpler dimensionality considerations, which are sufficient for estimates.

The flux is given by

$$q = -\frac{lc}{3}\frac{\partial \sigma T^4}{\partial x} = \left(\frac{ac}{3t}\right)^{1/2}(\sigma T^4)^{1/2}(A\rho T)_{\mathrm{h}}^{1/2}\sqrt{\frac{4}{4+n}}\,\frac{T^{n/2}}{\rho_{\mathrm{h}}^{m/2}}\,,$$

and the evaporation length

$$x = \frac{qt}{(A\rho T)_{\mathrm{h}}}\,.$$

By substituting numerical values (which have to be refined, as a matter of fact), we obtain the following estimates:

$$a = 10^{-3}\,,\ t = 10^{-8}\ \mathrm{s} = 0.1\ (\text{in units of } 10^{-7}),\ n = 3\,,\quad m = 1\,,$$
$$(A\rho)_{\mathrm{h}} = 10\rho_{\mathrm{g}},\quad q = 6.5T^4\ (\text{in units of } 10^{21}),\quad \frac{x}{t} = 0.65\frac{T^3}{\rho_{\mathrm{g}}}\,.$$

The latter parameter is the growth rate of the evaporated layer. From the viewpoint of gap filling, it should be compared with the total velocity of the matter driven by shock:

$$v_{\mathrm{s.\,w.}} = \sqrt{\frac{3}{4}\frac{2}{3}\frac{10\rho_{\mathrm{g}}T}{\rho_{0\mathrm{h}}}} = \frac{(\rho_{\mathrm{g}}T)^{1/2}}{2}\,.$$

The gap does not change with time under the condition $v_{\mathrm{s.\,w.}} > x/t$ or $T^{2.5}/\rho_{\mathrm{g}}^{3/2} < 0.8$.

In the estimates to follow, we assume

$$T_{\mathrm{g}} = 1.4\,,\quad \rho_{\mathrm{g}} = 2\,,\quad \frac{r_{\mathrm{in}}}{r_0} = 2.2\,.$$

The flux of radiation emitted by a black body is

$$q_{\mathrm{b.b.}} = 10^{24}T^4 = 10^3T^4\ (\text{in units of } 10^{21})\,,$$

and the ratio $\alpha = q/q_{\mathrm{b.b.}} = 6.5\times 10^{-3} \approx 10^{-2}$ is the wall reflectivity, which is a helpful auxiliary parameter to be used in what follows.

4.5 Compression time

We have determined all the characteristic values of parameters. In particular, the pressure in the gap compressing the inside cylinder $p_{\mathrm{in}} = 5\rho_{\mathrm{g}}T_{\mathrm{g}} = 14$, the average velocity of the shell M motion is $\bar{v} = (p_{\mathrm{in}}m/\rho_0 M)^{1/2} = 2.6$, and the compression time (in terms of the inside cylinder radius) is determined by the formula $t_{\mathrm{c}}/r_0 \cong 0.4$. Finally, we can derive the maximal compression temperature from energy considerations:

$$T_{\max} = \frac{p_{\mathrm{in}}}{A_{\mathrm{DT}}\rho_0} \cong 7\ \mathrm{keV}\,.$$

The resulting temperature is sufficiently high to ignite the DT mixture (even though a fraction of the energy is lost to the shell compression). In reality, the reaction ignition time is derived directly from the solution of the equation (see also Ref. [9])

$$A_{\mathrm{DT}}\frac{\mathrm{d}T}{\mathrm{d}t} = q_\alpha\frac{1}{\tau_{\mathrm{DT}}}\,,$$

where q_α is the caloric value in terms of α-particles, $1/\tau_{\mathrm{DT}}$ is the thermonuclear reaction rate, which can be expressed over a temperature range of 1 – 5 keV by a power function:

$$\frac{1}{\tau_{\mathrm{DT}}} = 2\theta^4\rho_{\mathrm{DT}}\,,\quad \theta = \frac{T}{10}\,.$$

Hence follows the flash duration required to increase the temperature to infinity, when the flash is triggered by the initial temperature θ_0:

$$t_{\mathrm{fl}} = \frac{5}{3}\frac{A_{\mathrm{DT}}}{q_\alpha\rho_{\mathrm{DT}}\theta_0^3} = \frac{1}{400}\frac{1}{\rho_{\mathrm{DT}}\theta_0^3}\,.$$

This time should be compared to the hydrodynamic time during which the DT mixture compression is near its maximum:

$$t_{\mathrm{hydro}} = 0.5\,\frac{r_{\min}}{v}\,.$$

The profile factor of 0.5 is derived from the exact solution and corresponds to a reaction rate equal to half the maximal value.

Let us assume that a flash has been triggered if $t_{\mathrm{fl}} < t_{\mathrm{hydro}}$, i.e.,

$$\frac{200(\rho r)_0\theta^3}{\bar{v}} > 1\,,\quad \text{or}\quad \theta^3 > \frac{1}{30}\quad (\theta > 0.3\,,\quad T > 3\ \mathrm{keV})\,.$$

Thus, we have come to the conclusion that, at certain parameters of materials, a TD regime triggered by compression can be realized. The 'natural' detonation velocity is unessential if it is lower than that of induced detonation.

4.6 Problem of outside thermal conductance or propagation of heat in the gap

The heat equation in the gap is

$$A_{\mathrm{g}}\rho_{\mathrm{g}}\frac{\partial T}{\partial t} = -A_{\mathrm{g}}\rho_{\mathrm{g}}D\frac{\mathrm{d}T}{\mathrm{d}x}$$
$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(\varkappa\frac{\mathrm{d}\sigma T^4}{\mathrm{d}x}\right) - \alpha\frac{c}{4}\sigma T^4\frac{2\pi(r_{\mathrm{in}} + r_{\mathrm{out}})}{\pi(r_{\mathrm{in}}^2 - r_{\mathrm{out}}^2)}\,.$$

The thermal conductivity in the gap

$$\varkappa = \frac{lc}{3} = \frac{2(r_{\mathrm{in}} - r_{\mathrm{out}})c}{3}$$

is virtually independent of the material filling the space between the cylinders and is fully determined by the geometrical factor (the geometrical photon free path is ten times as small as the particle free path).

The heat equation does not contain an insignificant term due to the radiation heat capacity, and this allows us to transform the equation to the fully dimensionless form:

$$-\frac{\mathrm{d}\theta}{\mathrm{d}y} + \theta^4 = \frac{\mathrm{d}^2\theta^4}{\mathrm{d}y^2}\,,$$
$$y = \frac{x}{x_0}\,,$$
$$x_0 = \sqrt{\frac{4}{3\alpha}}(r_{\mathrm{in}} - r_{\mathrm{out}})\,,$$
$$T_0^3 = \frac{A_{\mathrm{g}}\rho_{\mathrm{g}}D/c}{\sigma\alpha^{1/2}}\,,$$
$$\theta = \frac{T}{T_0}\,.$$

On the leading edge $\theta = -\mathrm{d}\theta^4/\mathrm{d}y = 0$. The numerical calculation is terminated at $\theta = \theta_{\max}$ (see Section 4.3, where

the temperature in the gap is determined). By substituting $q = -\,\mathrm{d}\theta^4/\,\mathrm{d}y$, we obtain the first-order equation

$$q + 4\theta^7 = q\frac{\mathrm{d}q}{\mathrm{d}\theta}\,.$$

At $\theta \to 0$, $q = \theta$. Without introducing a large error, we can set $q = \theta$ on the left of the equation:

$$q\frac{\mathrm{d}q}{\mathrm{d}\theta} = \theta + 4\theta^7\,, \quad q^2 = \theta^2 + \theta^8\,,$$

$$y = \int\frac{\mathrm{d}\theta^4}{q} = \frac{4}{3}\int_0^\theta \frac{\mathrm{d}\theta^3}{\sqrt{1+\theta^6}} = \frac{4}{3}\ln\left(\sqrt{1+\theta^6} + \theta^3\right).$$

For the parameters selected previously, $T_\mathrm{g} = 1.4$, $\rho_\mathrm{g} = 2$, and $r_\mathrm{in}/r_0 = 2.2$, we determine at various D the length x/r_0 of the radiation zone along the gap (ahead of the combustion front) and its duration $t/r_0 = x/(Dr_0)$ (see Table 3).

Table 3. Detonation velocity as a function of various parameters

D	T_g^3/T_0^3	x/r_0	t/r_0
20	2.28	28.9	1.44
30	1.52	22.3	0.74
40	1.14	18.0	0.45
50	0.91	15.1	0.3

By comparing the last column in Table 3 with the compression time t_c/r_0, we find that the propagation velocity $D \approx 40$ is really much higher than the combustion velocity: at $(\rho r)_\mathrm{c} = 0.4\,D = 20$. The aim of our approximate analysis was to outline the procedure of the closure of our problem in time and to estimate numerical values that could be used in the first iteration of a detailed numerical calculation.

It is clear that the problem could be closed at a slightly lower energy release (somewhere in the range $0.35 < \rho r < 0.4$) and $D \cong 30$, i.e., we have a margin with respect to the moment of maximal compression.

An important point, however, is that, owing to multiple constraints, the problem of the 'cylinder-within-the-cylinder' configuration can be solved only numerically on modern high-performance computers, and this solution has not been completed as yet. On the contrary, the configuration with multiple balls has been fully analyzed theoretically, moreover, it has been tested in an experiment, although its scale was far from that affordable in laboratory conditions.

5. Stability and possible applications

Another advantage of detonation under conditions of compression is its stability (in chemical detonation, the stability is ensured by the full burn-out of fuel), which is achieved if a simple condition is satisfied. Any compressing system passes through three stages: implosion, termination of motion at the moment of maximal compression, and expansion, i.e., its motion directed outwards. It can be stated that a propagation process is stable if it takes place at a stage immediately before the maximal compression. In other words, since the optimal combustion conditions are realized at the maximal compression, this regime cannot be fully implemented in real conditions, and one should scale down the reaction parameters, i.e., a certain power margin is needed.

Suppose that a deviation has occurred at some place, and there the energy release is lower than on average. This means that the radiation energy transmitted to the next section is lower than expected. Hence, a flash is triggered there with a delay and at a higher compression, since the ignition is largely controlled by the temperature. The LTD regime is analysed in similar terms, because the temperature drops when the released energy decreases, alongside the flame propagation velocity along the inside tube. The stabilizing factor is that the released energy increases with the density, therefore, if the energy release is lower at some point, it increases in the downstream region, and the instability is averaged out.

In a tube containing a lot of balls with a sufficiently large separation between them, the compression of a ball is largely controlled by the previous one, but if they are set closer to one another, the effect of other balls becomes notable. The stability of this configuration is higher since a deviation of one ball is averaged out by others. For this reason, it seems that a continuous tube should have the highest stability.

A stable detonating system has a positive energy balance, and it can be used for efficient energy generation. Once a reaction is initiated, the generated energy can be arbitrarily high. According to our estimates, the required triggering energy is several megajoules. This conclusion indicates that inertial systems have considerable advantages over magnetic, stationary devices. The most feasible ignition sources are, undoubtedly, lasers since their technology is most advanced. We reiterate, however, that a power margin is necessary. At the same time, a margin over the threshold energy yield, which is needed to offset various deviations (tolerances in the system design), can be used in designing really exotic configurations.

The point is that this margin would allow one to exploit not only cylindrical structures, but also tapered configurations, whose flare angle is proportional to the power margin. But, irrespective of the margin value, the released energy should double sooner or later, and it will be possible to conduct energy through two channels. Thus, a network of energy generating tubes with a two- or three-dimensional configuration can be constructed.

Now the closing remarks. We would like to end this paper in the same key as we began it. The released energy per unit length in a tapered system increases exponentially with the distance. At some point the energy is so high that it can trigger not only the DT reaction, which requires the relatively rare tritium, but also other reactions, first the DD or D^3He reaction, then more exotic ones. Note that the issue of neutron-free reactions has been mooted in literature many times [10]. The point is that the reactions which are easier to realize are not quite 'clean'. They generate intense neutron flows, and 14-MeV neutrons from the DT reaction can interact with almost all materials via the (n, 2n), (n, γ), (n, p) and other reactions. So, one cannot completely get rid of induced radioactivity, but can only limit it by selecting construction materials. This factor is the more significant as the neutron yield in reactions with hydrogen isotopes is a factor of five higher than in nuclear fission (in this context, it is appropriate to recall the neutron bomb). So, let us discuss neutron-free reactions. One widely known example is

$$^{11}\mathrm{B} + \mathrm{p} \to 3\alpha\,.$$

A remarkable feature of this reaction is the absence of radioactive species among its initial and final components. It

is inconceivable how short-range α-particles interacting with walls could generate radioactive materials with a notable yield. One can even say that this reaction, whose product is helium, a noble gas, is more environmentally friendly than chemical reactions. In this context, one could possibly treat such an explosion as an improved version of a chemical reaction, rather than a nuclear one, since it does not produce radioactive species.

In addition to this reaction, there are more interesting examples:

$$^6\mathrm{Li} + \mathrm{p} \rightarrow {}^3\mathrm{He} + {}^4\mathrm{He}\,,$$

$$^9\mathrm{Be} + \mathrm{p} \rightarrow \begin{cases} 2\,{}^4\mathrm{He} + \mathrm{D}\,, \\ \alpha + {}^6\mathrm{Li}\,. \end{cases}$$

These reactions are even faster than that involving boron, nonetheless, their rates are three to four orders of magnitude (depending on the temperature) lower than that of the DT reaction. Since the plasma lifetime should be proportional to the reaction time, one needs a larger dimension of the reaction zone or, to be more accurate, the optical thickness

$$\rho r \sim \frac{1}{(1/\tau)_{\mathrm{r}}}\,.$$

Calculations indicate that, with the exception of the DT reaction, nuclear fusion cannot be performed in laboratory conditions. Moreover, the rate of exotic reactions more strongly depends on the temperature. Therefore, a more rational approach to their triggering is in adding small quantities of deuterium and tritium:

$$\mathrm{Li}_6\mathrm{H}_{1-x}(\mathrm{DT})_{1/2x}\,, \qquad x \leqslant 0.1\,.$$

If such a mixture is compressed to a density of only 100 g cm^{-3}, the reaction is impractical (the consumption of tritium is hundreds of kilograms and the output energy is equivalent to tens of megatons of TNT). But if the density can be scaled up to 1000 g cm^{-3}, the required amount of tritium can be reduced to 100 g and the output scales down to tens of kilotons.

It is probable that such explosive devices, whose underlying idea is credited to Yu B Khariton, Ya B Zel'dovich, and D A Frank-Kamenetskiĭ, might be useful in limiting the technological impact on the environment.

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References

1. Romanov Yu A *Priroda* (8) 20 (1990)
2. Zel'dovich Ya B *Izbr. Trudy. Khimicheskaya Fizika i Gidrodinamika* (Zel'dovich Ya B Selections, Vol. 1. Chemical Physics and Hydrodynamics) (Moscow: Nauka, 1984) p. 23 [Translated into English (Princeton, N.J.: Princeton Univ. Press, 1992)]
3. Ginzburg V L *Teoreticheskaya Fizika i Astrofizika* (Theoretical Physics and Astrophysics) (Moscow: Nauka, 1987) [Translated into English (Oxford: Pergamon Press, 1979)]
4. Chu M S *Phys. Fluids* **15** 413 (1972)
5. Nastoyashchiĭ A F, Shevchenko L P *Atomnaya Energiya* **32** 451 (1972)
6. Zel'dovich Ya B, Kompaneets A S *Teoriya Detonatsii* (Theory of Detonation) (Moscow: Gostekhizdat, 1955) [Translated into English (New York: Academic Press, 1960)]
7. Zel'dovich Ya B, Frank-Kamenetskiĭ D A *Zh. Fiz. Khim.* **12** 100–105 (1938)
8. Avrorin E N et al. *Fiz. Plazmy* **10** 514 (1984) [*Sov. J. Plasma Phys.* **10** 298 (1984)]
9. Avrorin E N, Feoktistov L P, Shibarshov L I *Fiz. Plazmy* **6** 965 (1980) [*Sov. J. Plasma Phys.* **6** 527 (1980)]
10. Belokon' V A, Il'inskiĭ Yu A, Khokhlov R V *Pis'ma Zh. Eksp. Teor. Fiz.* **24** 569 (1976) [*JETP Lett.* **24** 525 (1976)]