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Self-action of a heat-releasing admixture in a liquid medium

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Contents

Abstract. A heat-releasing admixture in a liquid or gaseous medium gives rise to and is transferred by convective flows and gravitational density flows, making it possible to speak of its self-induced transfer. In contrast to classical convection problems, however, the heat releasing field is not specified in this case and is solved self-consistently as also are the temperature, velocity, and admixture concentration fields. Owing to its strata stabilization and turbulence suppression effects, the admixture may be 'self-closed' in a turbulent flow. Exact solutions of relevant nonlinear problems are presented and analyzed.

1. Introduction

In classical convection problems heat is generally brought into a fluid (gaseous) medium through the boundaries of the region under investigation. Meanwhile it may be well to consider qualitatively different problems, where the source of heat is an admixture transferred by the motions in the medium, including the motions (convection) induced by the admixture itself. An example of such an admixture are aerosols in the atmosphere, adsorbing short-range solar radiation. The aerosol can be considered as a bulk source of heat in the air, which induces convection and, hence, the transfer of the aerosol itself. It has been suggested that an admixture of this type (coal dust, soot) could be used to affect some atmospheric processes [1 – 4]. Full-scale experiments [2] provided support for this idea. Some other possible applications refer to the hydrodynamic effects of natural and anthropogenic aerosols of varied origin in the atmosphere [4]. It is well-known, for example, that poorly transparent air over large cities and industrial regions can give rise to considerable thermal effects. It would be interesting to study the influence of heat sources (sinks) of this type on convection and relevant atmospheric circulations. Other examples are dust storms in the atmosphere, the conceivable dynamic effects of volcanic aerosols, and atmospheric influences of forest fires. In meteorological literature the formation of tropical cyclones over the Atlantic was hypothesized to relate to huge dust clouds over the ocean brought from the Sahara Desert. A very important, though specific admixture is water vapor in the atmosphere. Below we show that nontrivial and diverse effects arise even in the simplest case when heat-release is proportional to the admixture concentration.

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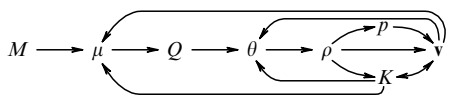
2. The main mechanisms of self-action of a heat-releasing admixture

The occurrence of a heat-releasing admixture in a medium makes the convection problem much more complicated than in the classical case, since the heat-release field is not known in advance and should be found from the solution to the problem together with the fields of temperature, density, pressure, velocity and concentration of the admixture. Below we show schematically the main interrelations in such a system:



Here M is the field of the source of the heat-releasing admixture, μ is the fractional density of the admixture (in kg m^{-3} units), Q is the heat-release intensity, θ, ρ, p are perturbations of temperature, density and pressure, respectively (in the physics of atmosphere 'potential temperature' is conveniently used instead of temperature [5]); $\mathbf{v}$ is the perturbation of the average (non-turbulent) velocity; and K is the effective coefficient of turbulent exchange.

As is seen from the scheme, the self-influence of the admixture can, in general, develop of three mechanisms. The first deals with self-lifting [2] of the admixture, i.e. floating of the admixture medium due to the buoyant force produced by the heat-releasing admixture. The second mechanism relates to the appearance of horizontal density and pressure gradients and, hence, gravitational density flows in the

medium which transfer, in addition to other substances, the admixture inducing them. It should be emphasized that in this case convective instability does not necessarily arise and the density stratification may be quite stable. The third mechanism is concerned with the influence of the admixture on the turbulent exchange. To be more precise, there are two mechanisms of such an influence, i.e. the generation of shear flows during heat-release and the resultant changes in the density stratification†.

The above scheme is rather complicated for every mechanism, and it may appear that the relevant self-consistent hydrodynamical problems could be studied only numerically. Nevertheless, we present below some analytical solutions, which enable us to reveal some significant regularities in nonlinear phenomena of this type. Similarly to classical convection problems, the mathematical problems under consideration involve the Navier–Stokes equations, and the equations of continuity and heat transfer (in many cases one can restrict oneself to the Boussinesq approximation [6]). The heat transfer equation includes, in addition, a bulk heat source associated with the admixture, which in simple cases is proportional to the concentration (fractional density) of the admixture. Besides, the set of equations contains the equation for the admixture transfer. When changes in the turbulence are considered (the third mechanism of self–action), the problem should be closed with respect to turbulent flow.

3. Example of the nonlinear dynamics of 'self-lifting' of the admixture

As an example of the first mechanism let us consider the dynamics of a stable axially symmetric free-rising convective jet induced by a point source of a heat-releasing admixture in a medium exhibiting stable stratification. In the approximation of the boundary layer extended along the jet axis [7–9] the set of equations can be presented as

$$u\,\frac{\partial w}{\partial r}+w\,\frac{\partial w}{\partial z}=\frac{1}{r}\,\frac{\partial}{\partial r}\left(Kr\,\frac{\partial w}{\partial r}\right)+\alpha g\theta-g\,\frac{\mu}{\rho}\,, \tag{1}$$

$$\frac{\partial(ru)}{\partial r}+\frac{\partial(rw)}{\partial z}=0\,, \tag{2}$$

$$u\,\frac{\partial\theta}{\partial r}+w\,\frac{\partial\theta}{\partial z}=-\gamma w+\frac{1}{r}\,\frac{\partial}{\partial r}\left(Kr\,\frac{\partial\theta}{\partial r}\right)+\varkappa\mu\,, \tag{3}$$

$$u\,\frac{\partial\mu}{\partial r}+w\,\frac{\partial\mu}{\partial z}=\frac{1}{r}\,\frac{\partial}{\partial r}\left(Kr\,\frac{\partial\mu}{\partial r}\right). \tag{4}$$

Here r and z are the radial and the vertical coordinates (the source of the admixture occurs at the point $r=0$, $z=0$); u and w are the corresponding velocity components; θ is the deviation of the temperature from the background; μ is the fractional density of the admixture; α is the thermal expansion coefficient; ρ is its average density; g is the free fall acceleration; γ is the vertical background gradient of the potential temperature (we consider the case of stable stratification, i.e. $\gamma>0$); $\varkappa$ is a dimensional coefficient (K s^{-1} m^3 kg^{-1}) characterizing the 'heating capacity' of the admixture. The exchange coefficient K is believed to be the same for all substances, as is usual, for example, in the description of turbulent exchange in the atmosphere.

It is assumed that away from the jet axis all the perturbations are damped:

$$u=w=\theta=\mu\xrightarrow[r\to\infty]{}0\,. \tag{5}$$

In view of symmetry, the following conditions should be met on the jet axis:

$$u=\frac{\partial w}{\partial r}=\frac{\partial\theta}{\partial r}=\frac{\partial\mu}{\partial r}=0\quad\text{at}\quad r=0\,. \tag{6}$$

Plus the integral condition of conservation of the admixture flux along the jet,

$$2\pi\int_0^\infty \mu w r\,\mathrm{d}r=M\,, \tag{7}$$

where M is the intensity of the heat-releasing admixture source.

The last term in Eqn (1) takes account of the dependence of the buoyancy on the admixture weight. In many cases it is irrelevant. On the whole, Eqns (1)–(7) naturally generalize the available statements of the problems on free-rising convective jets [7–9].

As is easy to check, the problem has a self-similar solution [9]

$$w=\frac{24Kz}{\pi R^2}\left(1+\frac{3r^2}{\pi R^2}\right)^{-2},\qquad \mu=\frac{3M}{8\pi Kz}\left(1+\frac{3r^2}{\pi R^2}\right)^{-2},$$

$$u=\frac{12Kr}{\pi R^2}\,\frac{1-3r^2/(\pi R^2)}{\left[1+3r^2/(\pi R^2)\right]^2}\,,\qquad \theta=\alpha^{-1}\,\frac{\mu}{\rho}\,,$$

where the quantity $R=8Kz\left[\gamma/(\varkappa M)\right]^{1/2}$ means the radius of the rising convective current.

As distinct from many problems on convective jets [7–9], in this solution the heat source is not concentrated at the point $r=0$, $z=0$, but distributed throughout the jet and is not known *a priori*. The medium volume containing the heat-releasing admixture floats up. Interestingly, in this solution a decrease in the buoyancy of each element of the flow as a result of heat release is exactly canceled by an equal decrease in its buoyancy due to rising in less dense layers of the medium. Therefore the buoyancy is zero everywhere. Such behavior with neutral buoyancy has appeared to be rather typical for convection induced by bulk heat-release in a medium exhibiting stable stratification [3, 4, 9]. With regard to real boundary conditions the solutions can depart slightly from this behavior, but they tend to approach it.

4. Gravitational density flows induced by the admixture

By way of example let us consider a situation arising upon the interaction of a floating cloud of a heat-releasing admixture with a horizontal jump in the medium density. The latter can serve as a model of an inhibiting layer in the atmosphere. Such layers, where the air density more or less sharply decreases with height, are widespread and have a profound effect on the vertical transfer in the atmosphere.

† As is the convention in atmospheric physics and geophysical hydrodynamics, by density stratification we mean the distribution of density with height. If the density of the medium decreases with height rather fast, an element of the medium displaced upwards turns out to be heavier than the surroundings and is therefore acted upon by a returning force. Such a stratification is called stable. Evidently, vertically heterogeneous heating of the medium affects the stratification which, in turn, influences the turbulence [5].

Figure 1 shows schematically the geometry of the problem. The jump in density (potential temperature) takes place at the level $z = 0$. Over and above the jump the medium is assumed to be neutrally stratified. Since the cloud floats in the lower medium (in the region $z < 0$), its density ρ_3 is less than the density ρ_2 of the lower medium. The cloud continues to release heat and its density further decreases but it is not expected to have time to fall to values less than the density of the upper medium ρ_1. Thus, the admixture cloud occurs at the interphase between two media and has an intermediate density: $\rho_1 \leqslant \rho_3 \leqslant \rho_2$. Similar problems are encountered, for example, in oceanology [10]. A simple consideration shows that in such cases horizontal density gradients originate in the liquid with intermediate density giving rise to horizontal divergence, i.e. spreading of the liquid over the interphase ('intrusion' of the liquid with intermediate density over the interphase). To describe the nonlinear dynamics of the intrusion, we will use the Barenblatt model [10]. However, our situation is more complex than in oceanology, since the cloud releases heat, and, hence, its buoyancy is not constant, but increases with time. Therefore model [10] should be generalized.

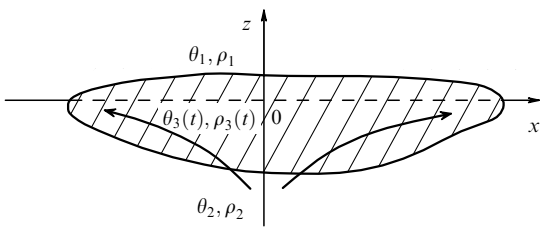


Figure 1. Scheme of the effect of a heat-releasing admixture cloud with a horizontal density jump. The latter takes place at $z = 0$. The cloud region is dashed; the arrows denote current lines.

As in Ref. [10] we consider an integral model dealing with values averaged in the horizontal direction. We neglect mixing of the cloud with the surroundings, but take account of friction, i.e. vertical exchange of momentum. We deal with the 'viscous' stage of the cloud evolution, when its horizontal dimension greatly exceeds the vertical one and the horizontal pressure gradient is, for the most part, balanced by friction.

The excess pressure arising in the liquid with intermediate density, as is easy to see [10], is expressed through the thickness $h(x, y, t)$ of the liquid layer and the media densities as

$$p = \frac{gh(\rho_2 - \rho_3)(\rho_3 - \rho_1)}{2(\rho_2 - \rho_1)} .$$

The proportion of the liquid with intermediate density, which has penetrated into the upper layer is

$$\sigma = \frac{h_1}{h} = \frac{\rho_2 - \rho_3}{\rho_2 - \rho_1} ; \qquad 0 \leqslant \sigma \leqslant 1 \qquad \text{for} \qquad \rho_1 \leqslant \rho_3 \leqslant \rho_2 .$$

We assume that the admixture is uniformly distributed throughout the cloud. Temperature and density perturbations upon heat release linearly increase in modulus with time,

$$\Delta\theta_3 = \varkappa\mu t , \qquad \rho_3 = \rho_2 - \bar{\rho}\alpha\varkappa\mu t ,$$

where ρ is the average density.

Model [10] considers two equations for two unknown functions of the horizontal coordinates and time, i.e. the thickness of the cloud $h(x, y, t)$ and its horizontal velocity of spreading $\mathbf{v}(x, y, t)$. The former equation is the balance between the vertical pressure gradient forces $\mathbf{F}_p \sim -\mathrm{grad}(ph)$ and the friction force $\mathbf{F}_r \sim \mathbf{v}/h$. The latter is the equation for the conservation of the cloud mass:

$$\frac{\partial h}{\partial t} + \mathrm{div}(h\mathbf{v}) = 0 .$$

Eliminating one unknown, we can easily reduce the set to a single equation of the type of the nonlinear heat capacity equation [10, 11]:

$$\frac{\partial h}{\partial t} = k \cdot \Delta(h^4) . \tag{8}$$

Here Δ is the horizontal Laplace operator,

$$k = \frac{g(\rho_2 - \rho_3)(\rho_3 - \rho_1)}{8c\nu(\rho_2 - \rho_1)} ,$$

ν is the dynamic viscosity of the medium, and c is a dimensionless empirical constant.

In our case all the above equalities are also valid, but the coefficient k depends on time, since the cloud density ρ_3 depends on time. This makes the problem not self-similar since the exponential asymptotes are absent. But the introduction of a new time-dependent variable $\tau = \int_0^t k(t')\,\mathrm{d}t'$ formally reduces Eqn (8) to the form obtained in Ref. [10]. Solutions generalizing self-similar solutions of Ref. [10] were found in Ref. [12]. They describe gravitational spreading of the admixture cloud over the interphase. But, distinctly from Ref. [10], the time dependence of the spreading is more complex than the exponential spreading, and the spreading time is finite. At a certain moment the cloud density, due to heat-release, becomes less than ρ_1, the cloud breaks through the inhibiting layer and the entire admixture penetrates into the region $z > 0$.

Note that full-scale experiments [2] are very similar to the situation considered. In the experiments the clouds of coil soot did indeed break the inhibiting layers in the atmosphere, thus demonstrating the conceptual possibility of active influences on some atmospheric processes. However the measurements performed in Ref. [2] are insufficient for a quantitative comparison of the theory with experiment.

5. Example of self-action of an admixture related to its influence on the turbulence of the medium

Suppose a turbulent flow (for example, a horizontally homogeneous atmospheric layer near the ground) contains a weightless heat-releasing admixture. The heat release increases the medium's temperature. If the temperature at the low boundary remains unchanged (the temperature of soil or water surface is more conservative than that of the air) the medium becomes increasingly warmer than the low boundary $z = 0$. This means the appearance of a stable density stratification in the medium and the suppression of turbulence, which , in turn, influences the transfer and distribution of the admixture.

In the simplest case of horizontally homogeneous plane-parallel flow, the stationary set of hydrodynamics equations

and heat and admixture transfer equations have the form

$$\frac{\mathrm{d}}{\mathrm{d}z}\,K\,\frac{\mathrm{d}u}{\mathrm{d}z}=0\,, \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\alpha_T K\,\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)+\varkappa\mu=0\,, \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\,\alpha_\mu K\,\frac{\mathrm{d}\mu}{\mathrm{d}z}=0\,. \tag{11}$$

Here u is the horizontal velocity, K is the turbulent viscosity coefficient, and α_T, α_μ are dimensionless quantities describing the difference between the coefficients of turbulent heat and admixture transfer and the coefficient of viscosity. To self-close a turbulent current, we will use a stationary equation for the balance of the turbulent energy in the Kolmogorov–Monin form [13, 11, 14]:

$$K\left(\frac{\mathrm{d}u}{\mathrm{d}z}\right)^2-\alpha g\alpha_T K\,\frac{\mathrm{d}\theta}{\mathrm{d}z}-\frac{K^3}{c_1^4 l^4}=0\,, \tag{12}$$

$$K=l\sqrt{b}\,. \tag{13}$$

Here l is the turbulence scale, b is the specific kinetic energy of turbulent pulsations, and c_1 is a dimensionless empirical constant. Recall that the first term in Eqn (12) describes the generation of turbulence, the second presents its damping (at $\mathrm{d}\theta/\mathrm{d}z>0$) due to the buoyant force, and the third term stands for the turbulence dissipation. The diffusion of turbulent energy is not taken into account in Eqn (12). This simplification is justified in situations with fairly stable stratification. In our case its validity can be checked *a posteriori*.

As to the turbulence scale, in the case considered of a fairly stable stratification, the vertical scale of turbulent pulsations is relatively small and limited mainly by the buoyant force. If the characteristic velocity of the turbulent motion is of the order of $b^{1/2}$, it is easy to assess that a particle of the medium, moving in the vertical direction at this velocity, will travel a distance of the order of $\left[b/(\alpha g\,\mathrm{d}\theta/\mathrm{d}z)\right]^{1/2}$ before it is brought to stop by the buoyant force. We will take this value as the turbulence scale

$$l=s\left[\frac{b}{\alpha g(\mathrm{d}\theta/\mathrm{d}z)}\right]^{1/2}, \tag{14}$$

where s is a dimensionless constant. This hypothetical 'self-closure' is in line with the dimensions and similarity considerations [4, 14]. Now the set (9)–(14) is closed. We will dwell upon the boundary conditions.

Equations (9) and (11) correspond to constant vertical diffusive flows through the medium layer under consideration (the atmospheric layer near the ground is commonly referred to as the 'layer of constant flows' [13, 11]). We are dealing with stable stratification when the current of the heat-releasing admixture moves from the top downwards:

$$\alpha_\mu K\,\frac{\mathrm{d}\mu}{\mathrm{d}z}=M_1=\mathrm{const}>0\,. \tag{15}$$

At the lower boundary $z=0$ we can take, for example, the admixture to be of absorbence $\mu=0$ and adherence $u=0$. The momentum flux through the layer under study can be expressed, as is customary in the theory of the layer near the ground, via the 'friction velocity' u_* as

$$K\,\frac{\mathrm{d}u}{\mathrm{d}z}=u_*^2\,. \tag{16}$$

The temperature at the lower boundary can be taken to be constant, $\theta=\theta_0$. The boundary condition at the upper layer is not so obvious. Since in this case a general solution to the nonlinear set (9)–(14) can be found, we need not preset all the boundary conditions. After the general solution has been found, we can consider various boundary conditions.

It follows from Eqns (13) and (14) that

$$b=\frac{1}{s}\left(\alpha g\,\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^{1/2}K\,,\qquad l=(sK)^{1/2}\left(\alpha g\,\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^{-1/4}.$$

In view of these relations, the third term in Eqn (12) appears to be proportional to the second and Eqn (12) takes the form

$$K\left(\frac{\mathrm{d}u}{\mathrm{d}z}\right)^2-\lambda\alpha_T\alpha g K\,\frac{\mathrm{d}\theta}{\mathrm{d}z}=0\,,$$

where $\lambda=1+(s^2c^4\alpha_T)^{-1}$. At $K\neq 0$ it results in the unambiguous relation between vertical temperature and velocity gradients,

$$\frac{\mathrm{d}u}{\mathrm{d}z}=\left(\lambda\alpha_T\alpha g\,\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^{1/2}=(\lambda\alpha_T)^{1/2}N\,, \tag{17}$$

where $N=(\alpha g\,\mathrm{d}\theta/\mathrm{d}z)^{1/2}$ is the 'buoyant frequency' (Brent–Vyasyalay frequency).

In the absence of a heat-releasing admixture the set of equations has a solution with linear profiles of velocity and temperature:

$$\frac{\mathrm{d}u}{\mathrm{d}z}=\frac{\lambda\alpha g P}{u_*^2}\,,\qquad \frac{\mathrm{d}\theta}{\mathrm{d}z}=\frac{\lambda\alpha g P^2}{\alpha_T u_*^4}\,,\qquad K=\frac{u_*^4}{\lambda\alpha g P}\,,$$

$$b=\frac{u_*^2}{s(\alpha_T\lambda)^{1/2}}\,,\qquad l=\left(\frac{\alpha_T s^2}{\lambda^3}\right)^{1/4}\frac{u_*^3}{\alpha g P}\,.$$

Here the normalised heat flow $P=\alpha_T K\,\mathrm{d}\theta/\mathrm{d}z$ is constant (in the absence of bulk heat-release). This solution is in agreement with traditional views of the structure of a turbulent layer of constant flows of a stable stratification [14].

In the presence of an admixture, it follows from Eqns (10), (16) and (17) that

$$\mu=-\frac{u_*^4}{\alpha g\varkappa}\,\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\lambda K}\right). \tag{18}$$

Equations (17) and (18) can be reduced to a single equation linear with respect to the function K^{-1},

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\left(\frac{1}{K}\right)+\frac{\lambda\alpha g\varkappa M_1}{\alpha_\mu u_*^4}\,\frac{1}{K}=0 \tag{19}$$

(α_T, and hence λ are assumed to be constant). If α_μ is also taken to be constant, Eqn (19) is readily integrated and all the other unknowns are easily found. For the above-specified boundary conditions the solution is written as

$$\mu=\frac{P_0}{\varkappa H}\sin\frac{z}{H}\,;\qquad u=\frac{\lambda\alpha g H P_0}{u_*^2}\sin\frac{z}{H}\,;$$

$$\theta = \theta_0 + \frac{\lambda\alpha g H P_0^2}{2\alpha_T u_*^4}\left[\frac{z}{H} + \frac{1}{2}\sin\left(2\,\frac{z}{H}\right)\right]; \tag{20}$$

$$K = \frac{u_*^4}{\lambda\alpha g P_0 \cos(z/H)}, \qquad l = \left(\frac{\alpha_T s^2}{\lambda^3}\right)^{1/4} \frac{u_*^3}{\alpha g P_0 \cos(z/H)};$$

$$b = \frac{u_*^2}{s\sqrt{\lambda\alpha_T}}.$$

Here P_0 is the constant of integration, which has the meaning of the normalised heat flow $\alpha_T K \mathrm{d}\theta/\mathrm{d}z$ at $z = 0$; the vertical scale

$$H = u_*^2\left(\frac{\alpha_\mu}{\lambda\alpha g \varkappa M_1}\right)^{1/2}. \tag{21}$$

The quantities μ and K cannot be negative in conception. Meanwhile, in the solution found they are, in general, sign-variable. This means that a physically meaningful stationary solution is not possible for any parameters of the problem. In particular, it can be determined only in the layer, whose thickness does not exceed a quarter-period of the sinusoid $\sin(z/H)$, i.e. in the region

$$0 \leqslant z \leqslant \frac{\pi}{2}\left(\frac{\alpha_\mu}{\lambda\alpha g M_1}\right)^{1/2} u_*^2.$$

Note, that a similar restriction occurs for any reasonable choice of boundary conditions. This can be interpreted as follows. In the medium layer under study heat is released and diffuses downwards to the surface $z = 0$. As the heat release in the medium intensifies (i.e. the flux of the heat-releasing admixture M_1 enhances), a stable vertical temperature gradient related to the overheating of the medium with respect to the lower boundary $z = 0$ increases. This nonlinear model describes suppression of the turbulent exchange upon enhanced stable stratification. As M_1 grows, the amount of admixture and heat to be withdrawn from the medium at the level $z = 0$ increases, however the possibilities for withdrawal, on the contrary, decrease as a result of reduced turbulent exchange. Therefore in a turbulent medium an admixture can self-close. Stationary solutions are possible only in the case of not too large values of the admixture flux and (or) not very thick atmospheric layers.

6. Conclusions

Below we have given some examples illustrating various mechanisms of self-action of a heat-releasing admixture in a liquid medium. In the literature some other possible situations are considered. For example, if the admixture is introduced into the medium not from above (as in Section 5) but from below, the heat-release can enhance turbulence [15]. In this case a self-induced transfer takes place instead of 'self-closing' of the admixture, as discussed in Section 5. There is no question that various manifestations of the effects concerned are not restricted to problems of atmospheric physics, but are of more general interest.

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