

The anisotropy of properties of the Earth's inner core

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Abstract. The anisotropy of properties of the Earth's inner core can be seen from the fact that the time a seismic wave takes to traverse the core along the rotation axis is 1 % less than the time taken in the equator plane. Another piece of evidence comes from the splitting of the Earth's eigen oscillation spectra. In addition to the velocity difference between traversing the core along and normal to the rotation axis, a lateral core anisotropy has been discovered both from function splitting and travel time data. This phenomenon is discussed.

1. Introduction

Sixty years ago in 1936 Inge Lehman discovered that the Earth has an inner core. In her paper on the subject [1] she wrote that new properties of matter should be taken into account to describe the very depths of the Earth. How has the view of the nature of the inner core changed over the sixty years, since it was considered to be made of iron even before Lehman? In the last ten years an anisotropy of the inner core has been revealed but it is explained by the crystalline properties of iron, i.e., practically in the same way as was done when Lehman made her discovery.

However some seismologists remain unsure whether the core is solid since the analysis of wave propagation in the core does not provide decisive proof. These seismologists are ready to believe that the inner core (G-core in the Bullen model) is solid but they want the final decisive proof. A direct proof would be the detection of a shift wave propagating through

the G-core (the so-called PKJKP wave) (Fig. 1). The only publication on the detection of the wave [2] has not been accepted by seismologists. Jeroen Tromp (with reference to P M Shearer) called the problem of the discovery of PKJKP waves and the direct proof of the solid state of the inner core the Holy Grail of seismology [3]. (Here the Holy Grail is used rather as a symbol of the unsuccessful quest of many generations of medieval knight-errants than the vessel in which one of the followers of Jesus Christ gathered His blood.)

Most probably the inner core is solid but there is still no unquestionable answer. In recent years analysis of a huge amount of seismic traces through the core (310 000 traces according to Ref. [4]) revealed surprising features. The point is that the wave properties of the inner core are anisotropic, i.e., the wave travels faster through the Earth when it propagates along the rotational axis of Earth than when it propagates in the equatorial plane. The remarkable fact is that the results on the PKIKP wave velocities are in good accord with the results of the analysis of natural oscillation spectra.

As in any elastic body, natural oscillations can be induced in the Earth. Examples of natural oscillations are a bell, the string of a violin or an air column in the tube of an organ, etc. In Earth free oscillations are excited as a result of a strong earthquake. These oscillations can last for many hours or even for many days. For example, the seismograph on the Isabella earthquake-detection station in California recorded oscillations for 16 000 min after the earthquake in Chile in 1960 [5]. The periods of free oscillations are very different. The slowest oscillations penetrate all the thickness of the Earth and carry information about the composition and properties of the crust and mantle as well as of the outer and inner cores. These oscillations are recorded by longwave acceleration detectors, gravimeters and clinometers. Complex mathematical methods are used to calculate the natural oscillation spectra.

In 1954 Benoff discovered free oscillations of the Earth by analysis of the Kamchatka earthquake seismograms of 1952. He identified the fundamental spheroidal Earth

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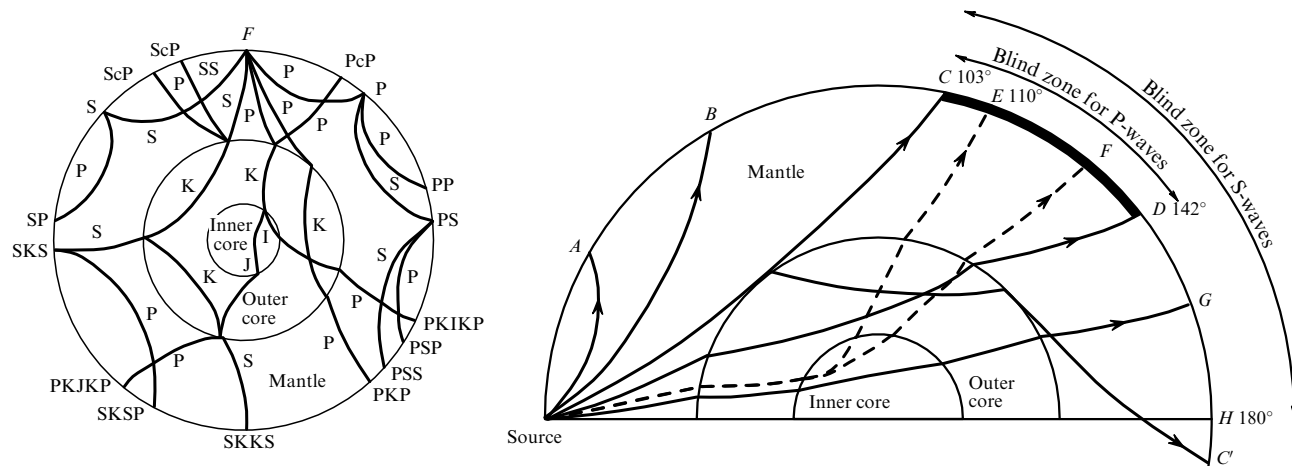


Figure 1. Propagation of seismic waves (and their notation) inside the Earth from the source F (a). Selected beams of P-waves [6] (b).

oscillation of period 57 min from analysis of the seismo-grams.

2. Free oscillations of the Earth

Free oscillations of the Earth can be described by a function with the associated natural frequency ${}_n\omega_l^m$. The number of overtones n (principal quantum number by analogy with quantum mechanics), angular (orbital) number l , and azimuthal number m are integer and are used to identify a specific mode of oscillation. For any l there are $2l + 1$ associated values of m : $m = -l, \dots, m = 0, \dots, m = l$. A multiplet ${}_nS_l$ (spheroidal mode) or ${}_nT_l$ (toroidal mode) represents all the $2l + 1$ natural oscillations (singlets) with the same quantum numbers n and l . In a spherically symmetric model of Earth all singlets for a given multiplet have the same free frequency ${}_n\omega_l$. Singlets $2l + 1$ are degenerate. Any deviation from a spherical shape cancels the degeneration and the singlets are split so that each has its own specific eigenfrequency ${}_n\omega_l^m$.

Figures 2 and 3, and Tables 1 – 3 present the structure and periods of some types of natural oscillations for major tones and overtones.

Oscillations of the ${}_0S_1$ -type are not excited. Otherwise the Earth would move back and forth but this seems impossible

Table 1. Periods of free oscillation of the Earth [6].

Spheroidal types of oscillations		Toroidal types of oscillations	
Type	Period, min	Type	Period, min
${}_0S_0$	20.46	${}_0T_2$	43.94
${}_0S_2$	53.83	${}_0T_3$	28.37
${}_0S_3$	35.56	${}_0T_4$	21.72
${}_0S_4$	25.76	${}_0T_{10}$	10.31
${}_0S_{10}$	9.67	${}_0T_{20}$	5.993
${}_0S_{20}$	5.792	${}_0T_{40}$	3.333
${}_0S_{40}$	3.538		

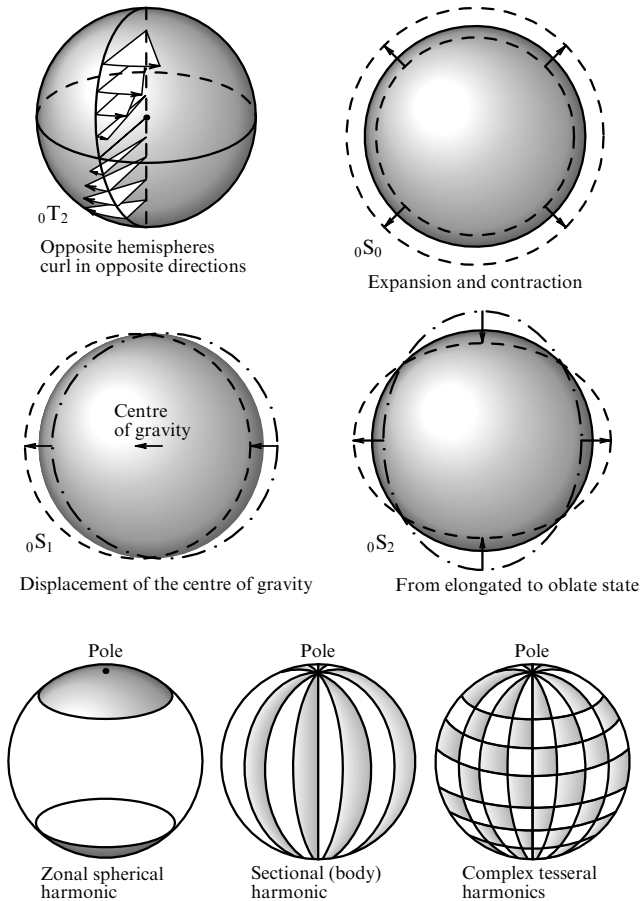


Figure 2. Motion of different areas of Earth's surface as a result of some free oscillations of the Earth [6].

Table 2. Observed periods of major tones of free oscillations of the Earth and their depths of penetration [7].

Type	${}_0S_0$	${}_0S_2$	${}_0S_3$	${}_0S_4$	${}_0S_5$	${}_0S_6$	${}_0S_{10}$	${}_0S_{20}$	${}_0S_{40}$
Period, s	1227.7	3233.1	2139.2	1546.0	1188.4	962.3	579.3	347.3	212.2
Depth, km	6370	5850	5500	5270	5090	4630	3940	2600	1400

Table 3. Observed periods of some overtones of spheroidal free oscillations of the Earth [7].

Period, s	2477.9	1470.8	904.2	804.2	724.9	613.6	398.5	305.8	243.6
Type	${}_1S_1$	${}_1S_2$	${}_2S_2$	${}_2S_3$	${}_2S_4$	${}_1S_0$	${}_2S_0$	${}_3S_0$	${}_4S_0$

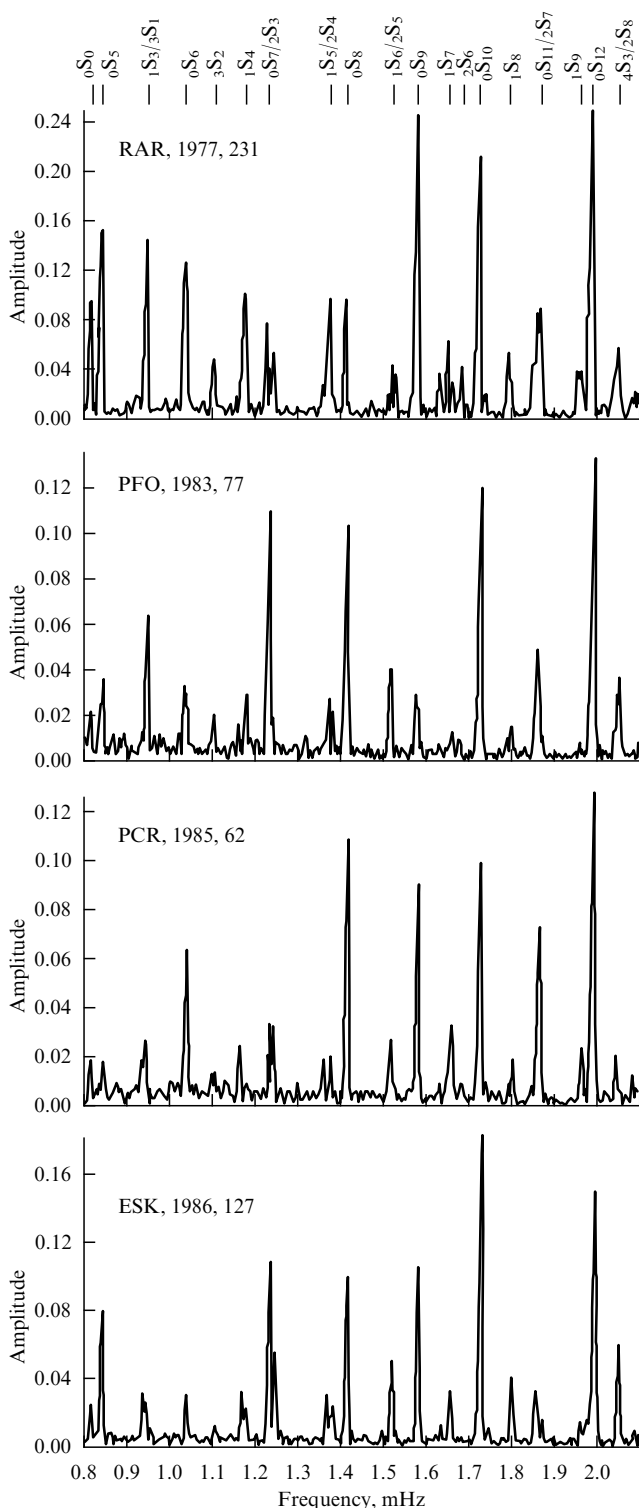


Figure 3. Modes of free oscillations of the Earth for four earthquakes [24]. Oscillations were recorded for 75 h.

as it is impossible to lift yourself by pulling your shoe-strings [6]. According to the opinion of other authors these oscillations have not been detected because surface displacements

on the earth are almost horizontal and relatively small. Theoretically the period of ${}_0S_1$ is about 42 min [7].

Table 2 represents the depths to which some oscillations of major tones penetrate.

Alterman, Yarosh, and Pekeris [8] described a class of spheroidal oscillations they called core oscillations. Theoretically this type of oscillation is especially sensitive to any changes in structure of the central core. The amplitudes of the oscillations are significant in the core and negligible in the mantle. Note that the core oscillation is present only in Bullen's 'B' model and absent in all other models. I want to remind the reader that in the 'B' model the density of the inner core is about one and a half of the conventional value. This model was not accepted but the idea of core oscillation was, though in a changed form. As will be shown later, among the large set of modes of free oscillations of the Earth some are more sensitive to the properties of the core (including the inner core) than to the properties of the mantle.

3. Splitting of oscillation modes

The first splitting (doubling) of frequencies of natural oscillations of the Earth was discovered by analysis of seismograms of the disastrous Chilean earthquake of May 22, 1960. Simultaneously the proposition was made that the splitting effect is caused by the rotation of Earth. Using the classical results a hypothesis was set forth that in a rotating circular basin waves have longer periods if they travel in the direction of rotation. The groundwork of the theory of this phenomenon, as applied to splitting of modes of free oscillations of the Earth, were laid by Pekerson, Alterman, and Yarosh [9]. Later it was shown that the rotation of Earth is not the only reason for splitting. In particular, the elliptical shape of the Earth and core should be taken into account.

Earth is not a sphere primarily because of its rotation and elliptical shape. The associated phenomenon of splitting can be described by the equation

$$\omega^m = \omega_l(1 + bm + cm^2),$$

where the indices n and l are dropped for the sake of simplicity. The ω_l parameter is the central frequency of a multiplet. The b parameter represents the first order effects associated with the rotation of Earth whereas the function m represents linear splitting. The c parameter and quadratic splitting m^2 represent the ellipticity of Earth and second order effects.

Masters and Gilbert observed [10] that the actual splitting exceeds the theoretical estimate which is derived considering the rotation and ellipticity of the Earth. In recent years about 20 similar observations have been made [11]. At first it was supposed that the reason for the discrepancy between the actual splitting and theoretical estimate is the peculiarities of the structure of the core-mantle interface (the interface was supposed to be symmetric about the axis of rotation). Three mechanisms were proposed to explain this abnormal splitting. The first mechanism is based the lateral inhomogeneity of the core proportional to the spherical harmonic Y_2^0 . This model reproduces the observed splitting for the most part but its

author thinks that the required inhomogeneity does not exist in reality. The second mechanism considers the lateral inhomogeneity on the surface of the inner core and on the core–mantle interface [11]. The third mechanism accounts for the anomalies in the time of propagation of PKIKP waves and assumes that the related anisotropy of the inner core corresponds to cylindrical symmetry about the rotational axis of the Earth. The author [11] believes that lateral inhomogeneity cannot explain the observed splitting effects.

We shall consider the cylindrical anisotropy of the inner core as a possible cause of the mode splitting of free oscillations of the Earth. The simplest type of anisotropy to possess a cylindrical symmetry about the rotational axis of Earth is the lateral isotropy with five elastic parameters A , C , F , L , and N [12]. These parameters are related to the field tensor with Cartesian components. The third axis is the rotational axis of the Earth.

The parameters C and A are related to the velocities ($v_P^2\rho$) of P-waves which propagate, respectively, in parallel and transversely to the rotational axis of the Earth. In turn, the parameters L and N are related to the velocities ($v_S^2\rho$) of S-waves which again propagate in parallel and transversely to the rotational axis. The parameter F is associated with the velocities of waves propagating at other angles to the rotational axis. As rotation and ellipticity, this type of anisotropy brings about the splitting of modes of oscillations. Allowing for anisotropy changes somewhat the form of ω^m :

$$\omega^m = \omega_l'(1 + bm + c^2m + c'm^2 + dm^4). \quad (1)$$

The coefficients c' and d describe effects related to the anisotropy of the inner core. They depend on the three parameters:

$$\alpha = \frac{C - A}{A_0}, \quad \beta = \frac{L - N}{A_0}, \quad \gamma = \frac{A - 2N - F}{A_0},$$

where $A_0 = k + (4/3)\mu$ is a function of the volume modulus k and the shear modulus μ ; A_0 has the meaning of the square of the P-waves velocity. For $\alpha > 0$ in the inner core P-waves propagate more slowly in the equatorial plane than along the rotational axis of the Earth. Similarly, for $\beta > 0$ in the inner core S-waves propagate more slowly along the equator than along the rotational axis. The third parameter γ affects P-waves and S-waves when they propagate in other directions.

Using data on seven multiplets Woodhouse et al. [13] obtained values of the coefficients: $\alpha = 6.7\%$, $\beta = 0.7\%$, $\gamma = -2.7\%$. The adopted model predicted a difference of about 8 s in travel times between the equatorial and polar paths. The real value is about 2.2 s. This gap can be reduced by introducing the dependence of the anisotropy on the radius of the inner core into the model. Using the dependence on r^2 the authors obtained $\alpha = 10.4\%$, $\beta = 1.9\%$, $\gamma = -3.3\%$ on the surface of the inner core and, as a consequence, 4.1 s instead of 8 s.

Figure 4a shows the radial distribution of the three modal parameters α , β , and γ in the inner core [11]. Tromp justified the behaviour of these parameters in the depth of the G-core by new data on travel times for P-waves, especially, at angles 130 – 136° . Note that in his next paper Tromp presents a different distribution of practically the same parameters (Fig. 4b). In fact the most interesting parameters $\alpha = (C - A)/A_0$ and $\varepsilon = (C - A)/2A_0$ differ by two times

and their radial distributions inside the inner core are somewhat different but in both cases they exhibit peculiar behaviours near the boundary ($R = 1200$ km). It seems that, on one side, the anisotropy of wave properties of the inner core in fact has a singularity near the boundary and, on the other hand, the knowledge of the wave velocity distribution in the inner core plays a decisive role in understanding the phenomenon of anisotropy.

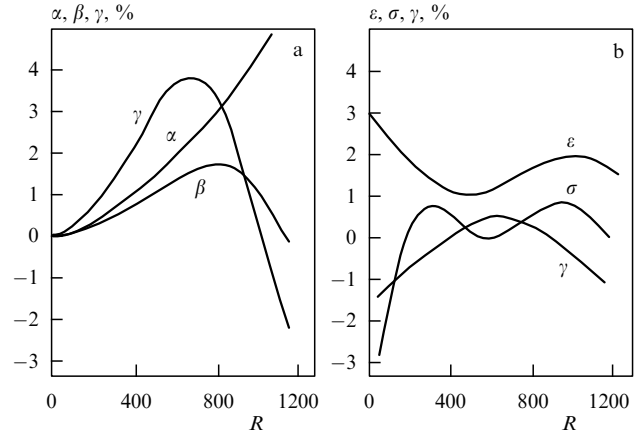


Figure 4. Radial distribution of modal parameters in the inner core: (a) $\alpha = (C - A)/A_0$; $\beta = (L - N)/A_0$; $\gamma = (A - 2N - F)/A_0$ [11], (b) $\varepsilon = (C - A)/2A_0$; $\sigma = (L - N)/2A_0$; $\gamma = (1/2A + 1/2C - 2L - F)/4A_0$ [3].

4. Anisotropy of velocities

The velocity of a P-wave can be obtained from the expression:

$$v_P^2\rho = A \sin^4 \xi + C \cos^4 \xi + 2(2L + F) \sin^2 \xi \cos^2 \xi, \quad (2)$$

where ξ is the angle between the direction of the P-wave and the N–S axis. In the case of the isotropy $A = C = 2L + F$. Morelli et al. [14] considered the anisotropy of velocities of seismic waves as a perturbation of the isotropic field. This approach is grounded on the fact that the perturbation does not exceed several percent. Using the notation ε and σ for the perturbation we have

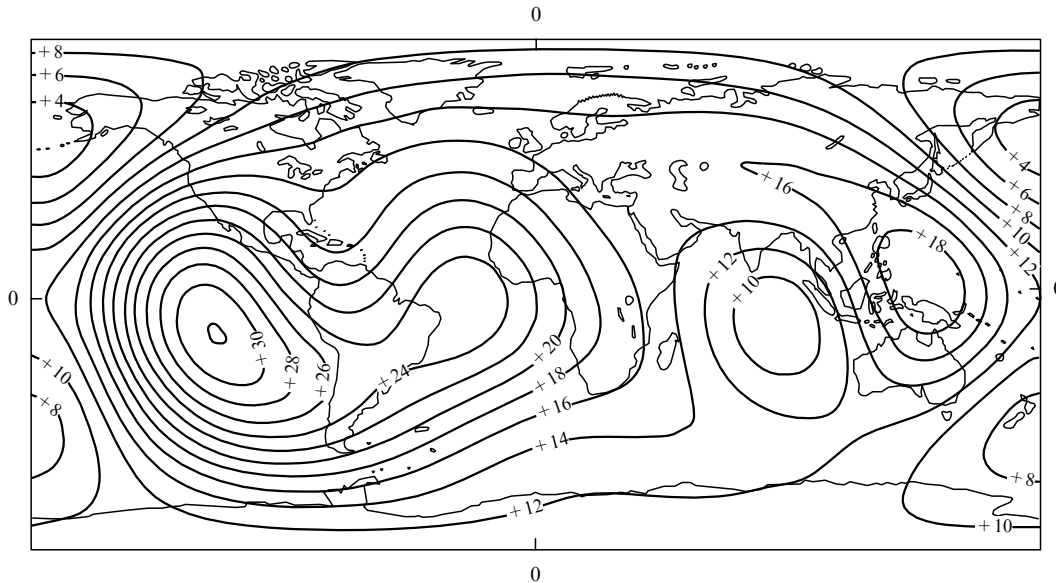
$$C = (1 + 2\varepsilon)A, \quad 2L + F = (1 + \sigma)(AC)^{1/2}.$$

As a consequence the expression for the velocity of a P-wave takes the form

$$v_P = v_{eq}(1 + \varepsilon \cos^2 \xi + \sigma \cos^2 \xi \sin^2 \xi), \quad (3)$$

where $v_{eq} = (A/\rho)^{1/2}$ is the equatorial velocity. In this model the authors assume that the perturbation increases proportionally to r^2 . Thus, it peaks on the boundary of the inner core. The peak values of the parameters of anisotropy are $\varepsilon = 0.032 \pm 0.005$ and $\sigma = -0.064 \pm 0.015$ for angles $\xi = 170$ – 180° .

In studying the anisotropy of velocities of P-waves in the inner core the authors of one of the first papers from this cycle [20] calculated the difference in travel time for PKIKP-waves and P-waves (see Fig. 1). Analysis of 400 records of 143° -traces of PKIKP-waves has shown that the difference in travel



conclusion that the anisotropy is uniformly distributed on a radius in the inner core, but as is shown by other authors this is not quite true.

5. Splitting function

In a number of papers [3, 13, 15] and, especially, in Ref. [16], which we will primarily follow, Tromp, Woodhouse, and Giardini developed a technique for synthesis of oscillation spectra and evaluation of the coefficients c_{st} [these coefficients fully describe the splitting of a given multiplet ($s = 0, 2, 4, \dots, 2l$; $-s \leq t \leq s$)]. These coefficients are presented by a special splitting function $\eta(\theta, \varphi)$. Below we shall make clear the physical meaning of this function and the associated coefficients c_{st} .

The contribution of a separate multiplet to the observed seismogram can be written as a function of time t :

$$u(t) = \text{Re}[\exp(i\omega t)\mathbf{r}\exp(i\mathbf{H}t)\mathbf{s}], \quad (4)$$

where \mathbf{r} is a receiver function (vector), \mathbf{s} is a source function (vector), and ω is the multiplet frequency. The vectors \mathbf{r} and \mathbf{s} are expressed by means of the formulae

$$\mathbf{r}_m = R_k^m(\theta_r, \varphi_r), \quad \mathbf{s}_m = S_k^m(\theta_s, \varphi_s),$$

where k is the multiplicity index (combination of l , m , and n); θ_r , φ_r ; θ_s , φ_s are the co-latitude and longitude of the receiver and source. The vector \mathbf{s} depends on the moment tensor of the source (of the earthquake) while \mathbf{r} depends on the seismograph orientation.

The splitting matrix \mathbf{H} can be written as:

$$\mathbf{H}_{mm'} = \Omega\beta_m\delta_{mm'} + \omega_0 \sum_{s=0} \sum_{t=-s} \gamma_{es} c_{st}. \quad (5)$$

Here the first term in the right-hand side of the equation represents the contribution of the Coriolis force to the splitting process: Ω is the angular velocity of rotation of the Earth and β_m is the Coriolis multiplet splitting parameter. The coefficient c_{st} depends linearly on the inner inhomogeneity of the Earth; γ_{es} is fully specified by the spherical harmonic $Y_s^t(\theta, \varphi)$ of degree s and of order t . According to Ref. [16] the coefficient c_{st} can be expressed in the form:

$$c_{st} = \delta_{s2}\delta_{t0}c^{\text{ell}} + \int \delta m_{st}(r)M_s(r)dr + \sum \delta h_s H_s, \quad (6)$$

where $M_s(r)$, H_s are known distribution functions for the multiplet intensity of free oscillations of the Earth on the Earth's radius (kernels), and $\delta m_{st}(r)$ are harmonic coefficients of the Earth inhomogeneity. In papers on the anisotropy of the core, the Earth's inhomogeneity is understood to be relative perturbations of P-wave speed, S-wave speed, and density ρ :

$$\delta m_{st}(r) = \left(\frac{\delta v_{Pst}}{v_{Po}}, \frac{\delta v_{Sst}}{v_{So}}, \frac{\delta \rho_{st}}{\rho_o} \right), \quad (7)$$

where the subscript 'o' refers to the standard model of the Earth. The first term in the expression for c_{st} represents the contribution of the Earth's hydrostatic ellipticity in the splitting. The last term represents the contribution of the discontinuity of parameters in the standard model of Earth to the splitting. Note that both terms are axially symmetric and they cannot be the cause of a lateral anisotropy in splitting.

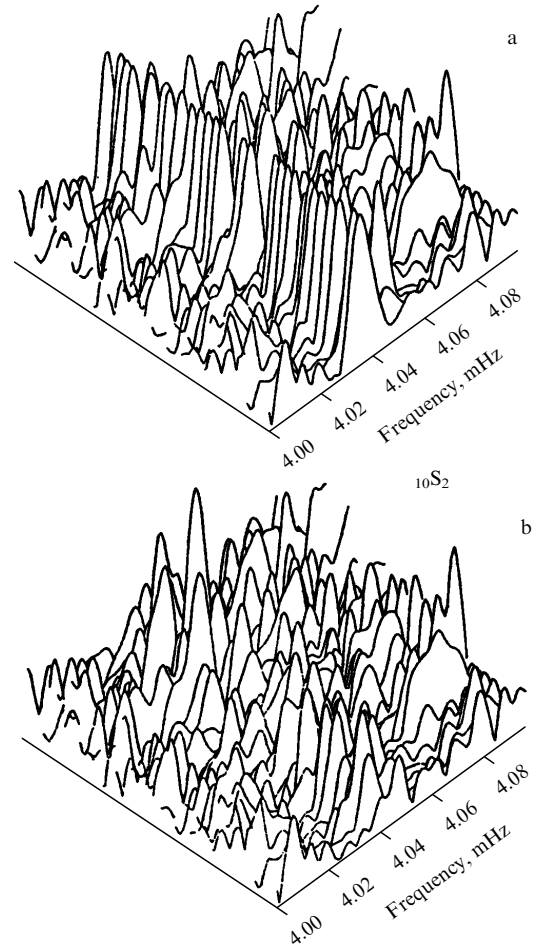


Figure 7. Amplitudes of initial spectrum of the oscillations of mode $10S_2$ (a); the same amplitude after subtraction of the synthesised spectrum of oscillations of this mode (b) [24].

The study of the anisotropy of multiplets splitting for free oscillations of the Earth is finally reduced to a determination of $\delta m_{st}(r)$ through calculation of spectra using seismograms. Given $\delta m_{st}(r)$ and the source and receiver parameters the coefficients c_{st} can be determined. For example, let us assume that the inhomogeneity gives birth to a maximal number of multiplets $s_{\text{max}} \leq 2l$. Then the number of coefficients is $s_{\text{max}} = (1/2)(s_{\text{max}} + 1)(s_{\text{max}} + 2)$. For example, for the mode of degree 2, for which $s_{\text{max}} = 4$, there are only 15 coefficients c_{st} and they represent all the spectra of this mode (Fig. 7).

The representation of the splitting coefficients c_{st} is given by the splitting function:

$$\eta(\theta, \varphi) = \sum_{s=0} \sum_{t=-s} c_{st} Y_s^t(\theta, \varphi). \quad (8)$$

6. Data selection

For the purpose of analysis, the modes of splitting of natural oscillations of the Earth were selected so that their frequencies did not overlap with the frequencies of other modes. In addition, in the cited papers the authors strove to select modes so that the peak intensity distribution of modes on the Earth's radius (kernels) covered all the depths as much as possible, including the mantle, and the outer and inner cores.

The periods of modes varied from 200 to 2000 s. Table 4 shows data on six spheroidal modes (there are data on 27 modes in Ref. [16]), including the frequency f and mode quality Q as well as the splitting coefficients which are associated with the ellipticity (A) and rotation (B) of the Earth which are calculated from the standard model of Earth (PREM). The coefficients A and B are determined as follows:

$$A = \frac{\alpha \varepsilon_k + \alpha' \Omega^2 / \omega^2}{2\pi}, \quad B = \frac{\beta \Omega}{2\pi},$$

where α, β are the splitting parameters. The modes cover the interval 0.5 to 5 mHz and they show the splitting dominance at the expense of rotation (B is higher than A) at low frequencies and at the expense of ellipticity ($A > B$) at high frequencies.

Table 4. Parameters of some multiplets (see Ref. [16]).

Mode	f , μHz	Quality	A , μHz	B , μHz
${}_0S_7$	1231.8	342	0.585	0.227
${}_1S_8$	1799.3	379	0.960	0.771
${}_2S_4$	1370.2	380	0.873	0.392
${}_5S_3$	2703.3	502	1.734	0.654
${}_{10}S_2$	4032.3	192	1.807	0.788
${}_{11}S_4$	4766.8	702	2.787	0.055

Analysis of longwave free oscillations of the Earth is possible only when there are long-time records of events and they are recorded by high-output geophones. A very sparse network of digital seismic stations started to record seismic data in 1976. High-intensity seismic events, appropriate for the analysis of mode splitting, as a rule occur not more than once a year.

The accelerometers of the International Deployment of Accelerometers network were mainly used to record natural oscillations of the Earth. They can detect natural oscillations of the Earth at frequencies lower than 1 mHz for catastrophic earthquakes. In addition the data of the Global Digital Seismograph Network and the Geoscope network were used. However, many authors prefer to use the data of the IDA network. As a rule analysis is applied to earthquakes without repetitive frequent powerful pushes since in this case the pattern of natural oscillations of Earth becomes very complicated. Finally twenty pairs ('source-destination' traces) [16] were selected for analysis among all the bulk of data which sufficed for analysis of the splitting functions of all the modes of interest. Each seismic event is tracked for six days before and for six days after (to clear out foreshocks and aftershocks). Each trace is presented as a train of oscillations 192 h of full duration and is then subjected to Fourier analysis. To eliminate any possible distortion synthesised seismograms and partial derivatives in time are constructed for each mode. The synthesised functions are subjected to the same filtration and Fourier analysis as the observed functions. In Ref. [16] much consideration is given to the nature of noise (both natural, and computational).

7. Results

The value of a splitting function at the geographic point with coordinates (θ, φ) can be written as follows:

$$\eta(\theta, \varphi) = \mathbf{m} \mathbf{a}(\theta, \varphi), \quad (9)$$

where \mathbf{m} is the vector of the c_{st} coefficients and the vector $\mathbf{a}(\theta, \varphi)$ includes the spherical harmonics for the point (θ, φ) . As is done to prepare the map of the anisotropy of velocities (see Figs 5 and 6) the splitting function demonstrates the inhomogeneity of structure of the Earth upon summation over depth and multiplication by suitable kernel coefficients. The authors of works on the anisotropy of splitting of modes of free oscillations of the Earth use a relative (normalised) scale in which the maximal perturbation is $\pm 0.2\%$ for any mode. An estimate for the maximal error shows that it does not exceed 40% of the peak value of a splitting function. The value of any error is only a fraction of a mHz (the reader may compare it with the values of A and B in Table 4). Note that an error of $0.1 \mu\text{Hz}$ can lead to a phase disbalance of 10° after accumulation of the synthesised signal for 10 hours. Consideration for errors makes it possible to measure the central frequency and to determine the splitting function more precisely.

The distribution of the splitting function in depth (kernels) and over the Earth's surface reflects the three-dimensional structure of the Earth very closely. Today distribution patterns are built for many sets of multiplets, each of which has its own intensity distribution in depth (its

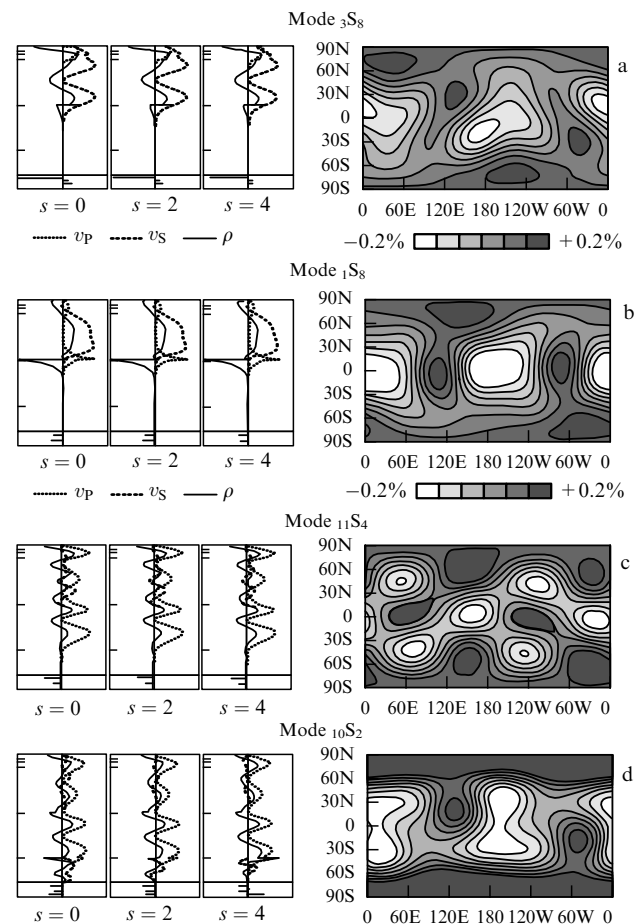


Figure 8. Splitting functions for different modes of free oscillations of the Earth are shown on the right. Intensity changes from -0.2% (white) to $+0.2\%$ (black). Distribution of intensity of free oscillations in depth (in Earth radii) for three different S (0, 2, 4) (kernels) are shown on the left. The kernel function is presented as changes in velocities of P-waves and S-waves and in density ρ with depth. Splitting and kernel functions of the mantle (a) and (b), splitting and kernel functions of outer core (c), and splitting and kernel functions of inner core (d).

own kernel). Figure 8 shows several patterns, each of which presents variations of the splitting function at depths of the mantle, inner and outer cores. The function itself is normalised by the 1% perturbations of P-wave speed, S-wave speed, and density (ρ). To the left of the patterns there are distributions of the perturbations $d\rho/dr$, dv_P/dr , and dv_S/dr in depth (kernels). The following lines are marked: the free (day) boundary, the discontinuity in the mantle at a depth of 670 km, the core–mantle interface and the boundary of the inner core.

Studies have shown that the modes ${}_4S_3$, ${}_5S_4$, ${}_5S_5$, ${}_5S_6$ with periods in the range 488 to 332 s are highly sensitive to the inhomogeneity of P-wave speed. The modes ${}_1S_5$, ${}_1S_6$, ${}_2S_4$, ${}_3S_8$ behave in the same manner though their kernels extend to a larger depth in mantle. The authors conclude that the mantle plays the decisive role in the splitting of modes, rather than any of its boundaries. This conclusion is supported by an analysis of the multiplets ${}_0S_6$, ${}_0S_7$, ${}_1S_7$, ${}_1S_8$, the periods of free oscillations of which range from 963 to 555 s. Most of their kernels are highly sensitive to an inhomogeneity of S-wave speed across the whole depth of the mantle and to inhomogeneities of the distributions of v_P and ρ on the boundary of the outer core. A comparison of the patterns of spatial distributions of splitting functions which are constructed based on the splitting data for these multiplets shows clearly that they are alike (Fig. 8a and 8b). This visual resemblance can be verified by the correlation coefficients for the modes ${}_0S_6$, ${}_1S_7$, ${}_1S_8$ which are higher than 0.9. This fact supports the conclusion that the whole mantle is ‘at work’.

The pattern is quite different for multiplets whose kernels fall within the outer core (Fig. 8c). Analysis of the spectra of modes ${}_6S_3$, ${}_{11}S_4$, ${}_{13}S_2$, ${}_{13}S_3$ of the PKIKP type, which have a weak sensitivity in the mantle and extend into the outer core, shows that the distribution of the splitting functions of these multiplets differ essentially from similar patterns typical for the mantle. Their periods range from 354 to 192 s. Figure 8c shows that there are six ‘light’ and ‘dark’ symmetric spots in the outer core. These light and dark spots represent the density distribution in the outer core and their strict order can bear witness to the twelve-cell convective structure of the tesseral harmonic type T_4^3 .

The splitting functions and kernel modes of ${}_{10}S_2$ are shown in Fig. 8d. This mode takes a special place in the history of research of the structure of the inner core [10]. The value of the splitting and the distribution pattern for mode intensity in depth depend to a greater extent for this mode than for other multiplets on which model of the Earth is used. (We have mentioned earlier this feature of so called core oscillation.) In fact the observed central frequency and quality of mode are much greater than theoretical values, for example, the observed quantity is $Q = 800$ while the calculated value is $Q = 192$ (see Table 4). There are at least two different explanations of this mismatch. We have touched one of the explanations before — a good agreement between experiment and theory is observed when the density of the inner core is taken to be about 1.5 times greater than the conventional value (as follows from the relevant B-model of the Earth by Bullen), or, as is assumed in Ref. [16], in the model of Earth the P-wave speed should be somewhat greater in the inner core. It turns out that the 2% increase in the P-wave speed causes the theory to be consistent with the spectral observations. It is not yet quite clear which solution is adequate to the real situation. There is only one sure fact that the splittings for the P-wave velocity and density of the

inner core are different from the structure of the outer core and they are much more like the structure of the mantle (Fig. 8b, d).

One of the ways to check how well the method of construction of the splitting function represents the spatial distribution of inhomogeneity in Earth’s structure is to compare its results with the pattern obtained by analysis of the topography of an inversion of the data on P-wave travel time. Since the splitting properties of natural modes depend on inhomogeneities of ρ , v_P , and v_S combined, if we want to construct a splitting function, then a law of proportionality should be proposed between their aspheric perturbations

$$\frac{\delta v_S}{v_{S0}} = \alpha_{SP} \frac{\delta v_P}{v_{P0}}, \quad \frac{\delta \rho}{\rho_0} = \alpha_{\rho P} \frac{\delta v_P}{v_{P0}}. \quad (10)$$

According to laboratory experiments the values of the factors of proportionality are $\alpha_{SP} = 1.25$, $\alpha_{\rho P} = 0.5$ [16]. Numerical modelling of splitting for several modes has shown that the model and experimental results agree better when these factors are about two times greater. Note that the study of the influence of aspheric perturbations in density and in the speeds of seismic waves has shown that in general a change in the speed of an S-wave has an advantage over perturbations in ρ and v_P . However, this is not true for some specific modes. Some modes of free oscillations are very sensitive to perturbations in density, other modes are highly sensitive to perturbations in P-waves, etc. Parameter matching makes it possible to achieve a good similarity between the measured and synthesised splitting functions for some modes. Correlation coefficients approach 0.9.

8. Discussion of the problem

J Tromp, the author of several papers on splitting [3, 11] attaches much importance to the data obtained after two very strong earthquakes in 1994 in Bolivia on June 9 and in the Kuril islands on October 4. High-quality data on the speeds of PKIKP-waves and on splitting functions, obtained after these earthquakes, made it possible to refine and check all the previous results. Abnormal splitting of the modes of natural oscillations of the Earth, which are sensitive to the structure of the inner core, was supported by data on the speed of propagation of PKIKP-waves and explained in the framework of the so called cylindrical anisotropy of the inner core. The explanation of the phenomenon observed is based on the concept of the chemical and mineralogical composition of inner core. According to the conventional view the core consists of hexagonal close-packed iron. In the pT -conditions of the inner core this iron can supposedly manifest properties of cylindrical anisotropy similar to those that seismic observations reveal. It is absolutely clear that iron can exhibit these properties if it is in a single-crystal or similar state. Others think that there is convection in the inner core which is responsible for the anisotropic properties of the substance of the core. In this case the substance of the core should move in the centre along the rotational axis so that it ‘flows out’ of one pole and then ‘flows in’ at the other pole. These authors believe that the matter (i.e. single-crystal iron) is in a partially molten state in the core. Another author supposes [18] that iron can acquire properties of cylindrical anisotropy if it experiences the action of the Earth’s magnetic field. Clearly, Karato assumed that the iron in the inner core had a magnetic (paramagnetic) susceptibility for this action to

be possible. Finally, a third explanation was proposed. It is that iron minerals have a preferred orientation because of rotation and self-gravitation in the process of crystallisation and growth of the inner core. J Tromp thinks that 'it does not seem impossible that the inner core is a single huge crystal'.

Note that most attempts to explain the observed phenomenon focus upon the cylindrical anisotropy. It seems that almost all authors do not notice the lateral anisotropy and, moreover, its similarity in the inner core and mantle. This proposition does not quite refer to paper [19] in which the authors discuss the problem of the absence of a lateral inhomogeneity in outer core, naturally, with respect to the observed aspheric structure of Earth in the whole range of depth from the centre to the surface. Here the special role of the core–mantle and inner core — outer core interfaces (in Ref. [19] the authors call it dramatic) is emphasised as peculiar discontinuities in anisotropy. However, the authors give most attention to the peculiarities of wave propagation on the first boundary. In their opinion a large-scale lateral inhomogeneity is observed in the core–mantle interface which is absent in the outer core. This result supports earlier conjectures that large-scale inhomogeneities of density are absent in the outer core. Creager [25] emphasises that the anisotropic properties of the inner core manifest themselves mainly near its boundary and, moreover, except for the cylindrical symmetry, the anisotropy has a definite longitudinal, i.e., lateral inhomogeneity. Creager bases his conclusion on so called hand-picked data since they give better results than machine-picked data. His results were verified in Refs [26–28]. In later papers [29, 30] the authors changed their views somewhat on the degree of anisotropy of the inner core. In Ref. [30] W J Su and A M Dziewonski used the data on the travel time for 313 422 seismic traces of PKIKP-waves recorded by 2 335 seismographs after 26 377 earthquakes. Here a three-dimensional image of the anisotropy of the inner core is presented for the first time. The image shows that the anisotropy is only about several percent and that it is centred in a 200–300 km layer on the core boundary†. The lateral distribution of anisotropy, obtained in Ref. [30], agrees more or less with the data in Ref [25] (Fig. 9b). In either case the anisotropy is observed in opposite latitudes with respect to the location of the Pacific ocean (in Fig. 8d this can be observed in the distribution of the splitting function).

In recent years the trend seems to be to pin down some previously revealed features of the anisotropy of the inner core in the field of research of the splitting of free oscillations in the core. For example, R Widmer et al. [31] came to the conclusion that the observed splitting could be localised in the outer core (rather than in the inner core according to the conventional view). Later these doubts were rejected and in Ref. [32] F Gilbert shows that the hydrodynamic flows in the outer core cannot be the cause of the observed splitting.

In a number of papers [33, 34 and others] a very interesting problem of the value of the quality of oscillations for P-waves and S-waves was discussed. In Ref. [33] the value of Q for P-waves was estimated to be $Q = 360$ in the upper 320 km layer of the inner core. In this case the quality is about 50 for S-waves. R. Widmer et al. [34] evaluate this quantity to be

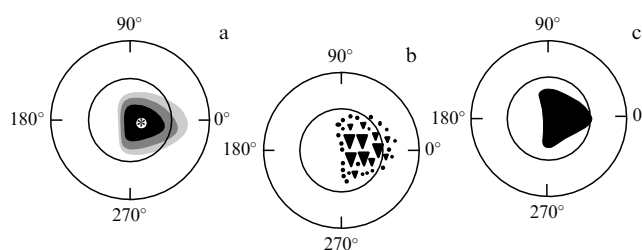


Figure 9. Lateral anisotropy of velocities of PKIKP-waves in the inner core (the view from the northern hemisphere). The darker is the colour, the higher is the wave velocities: the black colour matches to the 70% difference from the cylindrical anisotropy, while the white colour matches to the 40% difference; each interim colour is incremented by 10% [25]. The point (*) is the 'centre of gravity' of anisotropy (a). The cylindrical anisotropy of PKIKP-waves (this view is the same as in case of a). Differences in travel-times: the larger is triangle, the higher is the velocity (the difference is -3 s for larger triangles, -2 s for smaller triangles, and 0 s for points) [30] (b). Location of continents on Earth's surface (this view is the same as in case of a). The areas of continents (in percents) are laid off on the angle (with the 15° step in longitude) (c).

110 ± 25 . Other authors evaluate the value of quality of the inner core to be about 940 for P-waves while the quality for the S-waves is always lower. Different explanations of such discrepancies are proposed but there is no common opinion.

In Ref. [35] S Kaneshima et al. emphasise the lateral inhomogeneity of the inner core near its boundary in the 300 km range as well as the high homogeneity of the outer core. The comparison was based on the study of differences in travel time for traces BC and DF ($BC = 148^\circ$, $DF = 152^\circ$, see Fig. 1), and also on the ratio of the amplitudes A_{BC}/A_{DF} . Most of the authors of papers on the anisotropy of the inner core point out that this property is peculiar to a relatively thin layer near the boundary of the core. We recall that the results of studies of spatial distribution of splitting functions show that the major role belongs to the whole bulk of the mantle and core rather than to their boundaries. This question has not been studied yet and, hence, there is no clear answer.

The most important issues of the density of the inner core and of the existence of PKJKP-waves have not received an adequate consideration in the papers we reviewed.

9. Conclusions

According to the opinion of many authors who have studied the anisotropic properties of the inner core the major result of their works is the discovery of discrepancies in the velocities with which seismic waves propagate along the rotational axis and in the equatorial plane. This difference is not large (about 1%), however, it is cross-checked by many statistics on travel times for seismic traces and by analysis of the splitting of modes of free oscillations. We call the reader's attention to the fact that in the first case the researcher deals with oscillations in Hertz range while in the analysis of free oscillations of Earth the frequencies are lower by about two to three orders of magnitude. However, the spatial structure of the anisotropy of the inner core obtained by synthesis of the splitting function and by pattern plot of travel times for PKIKP-waves bear a very close resemblance. Examination of the results of these studies shows that the authors of all these papers without exception notice only the fact that the inner core exhibits anisotropic properties along the rotational axis and in the equatorial plane. Other, no less striking results are not discussed at all. First of all, except for the anisotropy above,

† According to recent estimates of Russian geophysicists [Adushkin V V et al. *Dokl. Akad. Nauk* **354** 382 (1997)] the thickness of the layer is significantly less. They used data recorded for small epicentral distances on PKIKP waves, which originated from nuclear explosions.

there is a clear peculiarity in the spatial structure of the inner core: both the splitting function and the travel time in the inner core are somewhat different for continents and oceans (this is especially true for the Pacific ocean, see Fig. 9). Secondly, a similar situation is observed in the mantle. Thus, the Earth's geography is as if imprinted in the inner core as well as in the mantle (!). In either case this fact refers to the boundaries rather than to the bulk of the earth shells. The situation becomes even more intriguing if thirdly we consider the splitting functions of the outer core: they present no 'earth geography' but exhibit a structure which can be treated as convective cells (Fig. 8c). A less sensational, but important fourth result is that in the inner core the best match between the theoretical and the observed splitting function is achieved when the density is about 1.5 times higher than the conventional value. And, finally, the theory agrees with experiment with respect to splitting of the modes of 'core' oscillations (in the inner core) when the model accounts for S-waves (i.e., for PKJKP-waves, the search of which is compared to the quest for the Holy Grail). It is quite possible that these waves propagate in the inner core but have not been detected.

Now we shall turn our attention to attempts to find a plausible explanation for the anisotropy of the inner core. The majority of authors [3, 10, 11] and others believe that this phenomenon is related to a peculiar structure of the inner core, in which a single crystal of iron is oriented along the rotational axis of Earth. A Morelli et al. think that this anisotropy 'is not physically impossible' though it is unclear how it can arise. Other authors try to explain the existence of such an anisotropic single crystal made, as is believed, of iron in a hexagonal close-packed phase by convection in the inner core (convection in a single crystal?) [21] or by the influence of the magnetic field on its growth [18]. The so called true polar wander is invoked quite incorrectly for the purpose of explanation [21] because it supposedly affects the flow of iron along the axis, etc. These attempts seem quite unconvincing so there is no sense in dwelling on them any more. The more so as not one of these five results is discussed specifically in any other work (except, possibly, [19, 25]). Note that our review covers more than 25 works on the anisotropy of the inner core (apparently, the majority) performed in the last 15 years [2, 3, 10, 11, 13–35] and some others. All, without exception, show that the acoustic properties of the boundary layer of the inner core differ from the similar properties of deeper layers of the inner core and from the properties of the outer core. A similar situation exists on the core–mantle interface. In spite of a large number of works and the unambiguity of results almost none of the authors discuss the lateral anisotropy, restricting themselves to the cylindrical anisotropy. However there is a way by which the cylindrical anisotropy can be linked to a lateral anisotropy.

As is known, continents occupy about 30% of the overall area of Earth while oceans cover about 70%. If we 'cut out' a cylinder, parallel to the rotational axis of Earth, with the base encircling the latitude 75° , from the Earth sphere, then about 50% of the area of its ends (with regard for Antarctica and Arctic) will be occupied by continents. If we 'cut out' a disk near the equator so that its side surface is approximately equal to the overall area of the bases of the cylinder then the continents will occupy no more than 20% of area (oceans will cover the remaining 80%). Thus, the relationship between the areas (continents–oceans) will be twice as high in the first case (in the cylinder along the rotation axis) than in

the second (in the equatorial region). This picture becomes even more convincing if we construct the longitudinal dependence of the continental areas (Fig. 9c). Figure 9c shows the ratio of the continental area to the overall area of Earth for each 15° in longitude. Clearly, this ratio is at a minimum in the Pacific ocean and at a maximum in Africa. The obvious resemblance between Fig. 9a and 9c (as well as Fig. 8b and 8d) emphasises one more our conclusions: that the 'Earth geography' (though not entirely clearly) shows up both in the mantle and in the inner core.

As noted above and follows from Fig. 8 the maximum difference of the splitting functions in laterally is 0.4%. With the above estimate we can conclude that the anisotropy of the inner core along the rotational axis and across it should be about 1% ($0.4\% \times 2.5$). Naturally this fact restates the problem rather than explains the effect we discuss in this review. However it seems that in this new statement the problem can be solved, at least, qualitatively.

Let us suppose that single-type processes, for example phase transitions of the first kind, occur in a region of the mantle near its interface with the outer core and also on the boundary of the outer core. It can also be supposed that in the region where the phase transitions occur the aspheric disturbance ratio (α_{sp}) could be greater (or smaller) than that in the nearby region. In this case the differences in the value of α_{sp} could be inherent to the mantle substance (on the interface with the core) and to the inner core (on the interface with the outer core) and should not be present in the substance of the convective outer core. The existence of exothermic phase transitions could probably lead to the observed cylindrical and lateral anisotropies. The acoustic properties of both phase transitions could be identical and in principle this could be the reason for the similarity of patterns in Fig. 8b and 8d and the reason for their 'geographical' features embodied in Fig. 9a and 9c.

At present the only correct model is considered to be the so called 'cool' model of the Earth, the inner core of which is a ball made of crystalline iron with a small admixture of other elements. The outer core is also made of iron, but of molten iron. In such a severe phantasy-prohibiting framework it seems impossible to find a plausible explanation to the facts we set forth in this paper.

We shall try to approach this problem in another way and imagine that the Earth, as well as other planets, was formed along with the Sun in the same process and by the same scenario via self-gravitation and rapid contraction of matter. This, seemingly, the most obvious explanation of the formation and evolution of the solar system, was declared to be false after a series of works by O Yu Schmidt. For the last fifty years it has been neglected though in earlier times many great scientists such as Descartes, Kant, Laplace and others made their contributions to the 'hot' model of Earth. Not ranking myself with these scientists I have tried to develop the 'hot' model of Earth for the last fifteen years [36].

The essence of the 'hot' model of Earth is that if its gravitational energy ($E \sim GM/R$) was 'used' to heat up the matter of the Earth at the moment of its formation, then its temperature $T \sim E/c_p$ would be of the order of 3 eV (G is the gravitational constant, M is the mass, R is the radius of Earth, and c_p is specific heat). The matter of the Earth at this temperature would have to be in a gaseous state (a weakly ionised plasma). The density of plasma compressed by a megabar pressure at the centre of Earth could reach ten grams per cubic centimeter and more, matches quite well the

density of matter in the inner core. The matter in the state of an 'astrophysical plasma' [37] when the de Broglie wavelength becomes comparable with the distance between atoms may possess quantum properties. Here a two-dimensional Wigner electron crystal [37, 38] can appear in the form of a 2D layer with anomalous properties on the compressed plasma boundary (i.e. on the boundary of the inner core of the Earth) provided that $\Gamma = E_e/E_k > 178$ ($E_e = Ze^2/r$ is the Coulomb interaction energy $E_k = kT$). It is known that although the Wigner crystal is in fact a plasma it possesses crystalline properties typical for solids and, in particular, P-waves and S-waves can propagate through the crystal. Possibly, this state of the matter of the inner core could help to explain its anisotropy†. Possibly, a two-dimensional Wigner crystallisation on the boundary of the inner core would give a new principal possibility to solve the No.1 problem in the physics of the Earth — the generation of its magnetic field, for example, with the use of the Hall quantum effect (as is known this effect appears when there is a Wigner crystal in the structure [39]). The creation of a new principal model of the generation of the magnetic field of the Earth and other planets is especially important because the idea of a magnetic dynamo, though around for 50 years, cannot explain many features of the magnetic field.

In support of our opinion that not all is quite right with dynamo, we want to draw the reader's attention to the results on the structure of the inner core shown in Fig. 8c. If interpreted as a tesseral harmonic type convection, then this is not the kind of convection the magnetic dynamo model postulates.

In conclusion of the discussion on the problem of anisotropy of the inner core we could not help but note the high level of accuracy of computations in the reviewed papers. Note also that the effect revealed is about or less than one percent. The authors discuss different properties of the Earth in the inner core with accuracies of a fraction of percent! They revealed that the differences responsible for the anisotropic properties of the inner core are confined in its boundary. The thickness of this layer is evaluated. Similarly it is shown that the anisotropic properties of the mantle also manifest themselves on the core-mantle interface. Extensive work is done on the synthesis of seismograms; their spectral analysis; correction of velocity and density models of the Earth; the correlation of synthesised signals with those the observations present; the selection and filtration of results of these observations, etc. By and large this series of works tries to make an exact science out of the descriptive-contemplative science of the Earth, in which state it still abides.

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† I Lehman perhaps referred to the quantum properties of the matter of the inner core when she wrote about the use of new properties of matter for an explanation of the very deep parts of our planet.

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