

LETTERS TO THE EDITORS PACS number: 41.60.Bq

On the stimulated Cherenkov effect

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Abstract. The authors of Ref. [1] drew incorrect conclusions when extending the results of Refs [2 – 14] on the stimulated one-photon Cherenkov effect and linear electromagnetic wave amplification, both for an infinite medium and for surface Cherenkov radiation in dielectric guides.

In *Uspekhi Fizicheskikh Nauk* [*Physics-Uspekhi* **37** (10) 1005 (1994)] V M Arutyunyan and S G Oganesyan published the paper "The stimulated Cherenkov effect" under the heading "Reviews of Topical Problems" [1]. The paper includes serious mathematical and physical errors. As a consequence, some results on the stimulated Cherenkov effect (SCE) are interpreted inadequately and, moreover, they contradict many physical notions and principles. The authors would have avoided many errors if they had discussed at least the familiar results on the subject, along with the presentation of their own results.

In this note we do not intend to discuss the issues considered in Ref. [1] in much detail. We will present only the principal errors because of which the authors arrived at contradictory conclusions in Ref. [1] and their other papers [2 – 14].

1. In the review "The stimulated Cherenkov effect" all the results are obtained from perturbation theory in the first-order approximation with respect to the field of an outer wave, though it is known [15] that the SCE has a specific nonlinearity in the presence of an arbitrary small wave near the Cherenkov cone. There is a critical value of the wave intensity and it depends on the Cherenkov resonance width. If the wave intensity exceeds the critical value, then the wave becomes a potential barrier, or potential well, for the electron and in this case the linear theory cannot be applied to describe the interaction. Hence, perturbation theory is applicable to the SCE problem only if the wave intensity is much less than the critical value and the electron moves in phase with the wave.

Ignoring this fundamental feature of the SCE the authors [1] calculate the classical change in the electron energy $\Delta\mathcal{E}$ from the perturbation theory in the presence of a plane monochromatic wave. In Section 2.1 (Ref. [1]) they analyse the well-known result for $\Delta\mathcal{E}$ using the linear theory [see Eqn (7); the motion of an electron in phase with the decelerated wave is equivalent to the motion in a constant electric field]. Then they specially discuss the case of the exact Cherenkov resonance $\omega - \mathbf{k}\mathbf{v}_0 = 0$ [see Eqn (10)]. However, in this case perturbation theory is not applicable even in principle. The exact solution of this problem [15] shows that the Cherenkov resonance has a minimal width $(\omega - \mathbf{k}\mathbf{v}_0)_{\min} = \Delta\xi$, which depends on the field strength ($\xi = eA/mc^2$, A is the amplitude of the wave potential vector), and that the electron gets into an essentially nonlinear mode of interaction ('reflection' or electron capture by wave) when the width is less than the critical value. However, even if the electron is not in this mode, i.e., if $\omega - \mathbf{k}\mathbf{v}_0 > \Delta\xi$, the SCE shows a nonlinear behavior and the results of Section 2.1 are invalid. These results are applicable only if $\omega - \mathbf{k}\mathbf{v}_0 \gg \Delta\xi$ or if the wave field is [16]:

$$\xi \ll \frac{1}{2}\,\frac{c}{v_0}\,\frac{\left[1 - n(v_0/c)\cos\theta\right]^2}{(n^2-1)\sin\theta}\,\frac{\mathcal{E}_0}{mc^2}\,. \qquad (*)$$

Here the quantity in the right-hand side of Eqn (*) is the aforementioned critical value of the field (v_0 is the initial velocity of the electron, $\mathcal{E}_0$ is the electron energy, n is the refractive index, θ is the Cherenkov angle).

In Section 2.2 (Ref. [1]) the authors calculate the change in the electron energy $\Delta\mathcal{E}$ [Eqns (18), (19)] in an artificial field of finite diameter [see Eqns (11), (12)]. As the comparison of Eqns (7), (10), (19) shows, the width of the Cherenkov resonance $\omega - \mathbf{k}\mathbf{v}_0$ is replaced by v_x/d in the case of the beam of a finite diameter $2d$ along the x axis. Hence in Eqn (*) in this case $1 - n(v_0/c)\cos\theta$ must be replaced by $v_x/\omega d$ and Eqns (14), (19) can be applied in the field

$$\xi_x \ll \frac{1}{8\pi^2}\,\frac{v_0}{c}\,\frac{\mathcal{E}_0}{mc^2}\,\frac{\sin\theta}{n^2-1}\,\frac{\lambda^2}{d^2}$$

(λ is the wave length of laser radiation, $\lambda/d \ll 1$). In Ref. [1] the condition, from which Eqns (14) and (19) are derived, is $|\Delta\mathcal{E}| \ll \mathcal{E}_0$ and it yields:

$$\xi_x \ll \frac{1}{2\pi\sqrt{\pi}}\,\frac{\mathcal{E}_0}{mc^2}\,\frac{\lambda}{d}\,.$$

The comparison of these conditions shows that in Ref. [1] the probable fields exceed the legal values by a factor of d/λ ($d/\lambda \gg 1$) (the typical Cherenkov parameters are $n - 1 \sim 5 \times 10^{-4}$ for CO_2, $\mathcal{E}_0 \sim 100$ MeV, $\theta \sim 6 \times 10^{-3}$ rad, see Ref. [17]). However, the authors claim: "The quantity $\Delta\mathcal{E}_0$ is very important in the theory of interaction between free electrons and laser radiation. As will be shown later, the characteristics of all processes in which the electron is involved depend on $\Delta\mathcal{E}$" (p. 1008).

2. We will now to make some notes about the field (11), (12), in which different aspects of the SCE are considered in Sections 2.2 – 2.9 (Ref. [1]).

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(a) This field is attributed to "an electromagnetic wave propagating along the z axis and which has a finite width in the x direction only". However, such an artificially constructed field presents a set of waves propagating basically in opposite directions along the z axis, i.e., it presents a standing wave $A_x \sim f(x,z)\cos\omega t$.

(b) The authors claim that "the Fourier transform of the vector potential is chosen so that, in the plane $z = 0$, the amplitude of the field is attenuated on an increase in $|x|$ as the Gaussian distribution of width $2d$". However, in the plane $z = 0$ the field constructed is identically zero! Moreover, how can the field be given at one intermediate point in the interaction region when z varies from $-\infty$ to $+\infty$ and when there are no boundary conditions (the interaction proceeds in an unbounded medium, the wave is turned on at $t = -\infty$ and it is turned off at $t = +\infty$)?

(c) The diffraction divergence of the beam is assumed to be small: $\lambda/d \ll 1$. As a result the z projection of the field $A_z = (-q_x/q_z)A_x \approx 0$ is neglected. However, a δ-function is artificially introduced into the Fourier component of the field to allow for the exact law of dispersion $q_x^2 + q_z^2 = n^2\omega^2/c^2$. In other words the small quantity of the first order q_x/q_z (or λ/d) is neglected while the small quantity of the second order q_x^2/q_z^2 (or λ^2/d^2) is retained in the expression for the field.

Thus, if the beam divergence is considered to be small because of its finite width in the transverse direction so that the wave is assumed to propagate along the z axis and that the z projection of the field is neglected, then in this approximation q_z has a fixed value $q_z = n\omega/c$ and the field of the wave should be described by the vector potential

$$A_x = A_{0x}\exp\left(-\frac{x^2}{d^2}\right)\sin\left(n\frac{\omega}{c}z - \omega t\right),$$

$$A_y = A_{0y}\exp\left(-\frac{x^2}{d^2}\right)\cos\left(n\frac{\omega}{c}z - \omega t\right) \qquad (**)$$

rather than by Eqns (11) and (12) (see Ref. [1]). Clearly that in this case the dispersion equation can hold only approximately to within a small quantity $q_x^2/(n^2\omega^2/c^2) \sim \lambda^2/d^2 \ll 1$ $[(q_x)_{\max} \sim 1/d]$. Then instead of Eqns (14)–(17) (see Ref. [1]) we have

$$\Delta\mathcal{E} = 2\pi\sqrt{\pi}mc^2\frac{d}{\lambda}\xi_x\exp\left[-\frac{\omega^2 d^2}{4v_{0x}^2}\left(1 - n\frac{v_{0z}}{c}\right)^2\right]$$
$$\times\cos\left[n\frac{\omega}{c}z_0 + \frac{\omega}{v_{0x}}\left(1 - n\frac{v_{0z}}{c}\right)x_0\right].$$

However, it should be stressed that the field $(**)$ bounded in the x axis and infinite in the y axis does not describe a real laser field because it has at least to have a Gaussian profile in the beam cross-section.

3. Starting in 1975 the SCE was systematically and comprehensively studied in laser radiation fields with a real transverse profile by numerical integration of the exact equations of motions using the Monte Carlo algorithm in Stanford University. In particular, results on the modulation and rearrangement of an electron beam show that the depth of rearrangement at the second or third harmonic of laser radiation is of the same order as that at the first harmonic [18]. Consequently, the classical perturbation theory used in Ref. [1] to describe the beam rearrangement at the first harmonic presents the rearrangement process for a klystron type beam inadequately because small changes in the electron velocity in the interaction region entail large changes in density and in current after the interaction in the drift region.

In regard to the quantum modulation of the electron probability density the perturbation theory is applicable if $\Delta\mathcal{E} \ll \hbar\omega$. This condition is opposite to that, at which the classical consideration is valid ($\Delta\mathcal{E} \gg \hbar\omega$). The problem of the quantum modulation of an electron beam for the SCE in the presence of a plane wave was solved using the perturbation theory in Ref. [16], where the author shows that the modulation depth at the Nth laser radiation frequency depends precisely on the aforementioned critical field strength $\Gamma_N \sim (\xi/\xi_{\text{cr}})^N$. Mentioning neither Ref. [16] nor the work [18] on the classical modulation and rearrangement of the electron field Arutyunyan and Oganesyan in Sections 2.3–2.4 set forth "classical and quantum theory of electron beam modulation" at the first harmonic.

On this basis it seems that there is a little sense to discuss the results of Sections 2.2–2.9 for the field (11), (12) but, independently of the field, the authors [1] make several mistakes in their calculations, in definitions of classical and quantum notions, in passage from the classical formulae to the quantum formulae, and in their consideration of spin interactions. As a result they give an incorrect quantitative description of the classical rearrangement (Section 2.3), of quantum modulation (Sections 2.4 and 2.5), of electron beam magnetization (Section 2.6), of classical and 'quantum' theory of the Cherenkov klystron (Sections 2.8, 2.9) as well as misinterpret the physical nature of these phenomena. Therefore, we shall dwell on Sections 2.2–2.9 in somewhat more detail and show the principle mistakes especially as they recur in the subsequent discussion of the monochromatic wave (Sections 2.11, 2.13).

(a) The density of an electron beam, i.e., the electron probability density in quantum theory is defined as $\rho = \mathrm{i}\hbar\psi^*\partial\psi/\partial t +$ c.c.!! [see Eqn (44)].

As a result of this non-physical definition of the probability density of particles an extra term $\Delta\mathcal{E}/\mathcal{E}$ appears in the expression for ρ [Eqn (44)] which leads to contradictory results from the physical standpoint. This term is considered to be a classical rearrangement "remaining in the drift region $x > d$" (p. 1011). And this is a formula for quantum modulation!

It should be emphasized that the violation of the definition of such a fundamental notion as the probability density is related to the inappropriate use of the Klein–Gordon equation for electrons (see Ref. [14]) by the authors of the paper in question. It is known that the squarable Dirac equation in an outer electromagnetic field is reduced to the Klein–Gordon equation with additional terms associated with the spin interaction. By neglecting these terms each component of the bispinor wave function of an electron ψ_D satisfies the Klein–Gordon equation and $\rho = \psi_D^+\psi_D \to |\psi_k|^2$, where ψ_k is a solution to the Klein–Gordon equation. In other words, in the Klein–Gordon equation the spin interaction is neglected as a small correction rather than the electron spin as the authors assume. Consequently, $\rho = |\psi_k|^2$.

(b) Arutyunyan and Oganesyan deduce the formulae, supposedly, in the classical limit from the quantum formulae for one-photon interaction (46), (49) and identify the former with the formulae of the classical theory (29), (30)! They ought at least to consider the condition $\Delta\mathcal{E} \gg \hbar\omega$, under which the classical consideration is valid. This condition is exactly opposite to the one $\Delta\mathcal{E} \ll \hbar\omega$, under which the one-photon description is true. How can any formulae be the same, even if they look alike, when the quantity $\Delta\mathcal{E}/\mathcal{E}$ is of a different order of magnitude in the classical and quantum approximations? In the last case it is a very small: $\Delta\mathcal{E}/\mathcal{E} = (\Delta\mathcal{E}/\hbar\omega)(\hbar\omega/\mathcal{E})$ (here $\Delta\mathcal{E}/\hbar\omega \ll 1$ and $\hbar\omega/\mathcal{E} \ll 1$). If the accuracy of classical perturbation theory is the same as that of the quantum theory, then in the quantum case $\Delta\mathcal{E}/\mathcal{E}$ is at least a factor of 10^7 less than in the classical case ($\mathcal{E}/\hbar\omega > 10^7$ for the SCE). As a result the essence of the klystron type classical modulation is distorted as well as that of the quantum modulation of the electron probability density. As the authors conclude: "In the region $\Delta q_x x \ll 1$ the modulation is classical in nature and the expression for the density of electrons [Eqn. (46)] coincides with Eqn (29). Since the second term is proportional to x in this limit, the associated modulation can be called klystron modulation. In the region $\Delta q_x x \sim 1$ the difference between the amplitudes of emission and absorption reaches a maximum, and the classical modulation becomes quantum at the depth $2\Delta\mathcal{E}/\hbar\omega$." It remains only to note that the quantum modulation is particularly the wave property of a particle. Therefore, it is true for an electron (superposition of partial waves of an electron with different energies and momenta leads to oscillation in the electron probability density). The one-particle consideration of the quantum modulation problem is possible especially because of this property. In contrast, classical modulation and rearrangement have a meaning for a particle beam only.

(c) If $\Delta\mathcal{E}/\hbar\omega \ll 1$ is a parameter in the perturbation theory, then the terms $\Delta\mathcal{E}/\mathcal{E} = (\Delta\mathcal{E}/\hbar\omega)(\hbar\omega/\mathcal{E})$, where $\hbar\omega/\mathcal{E}$ is much less than unity as well as than the perturbation parameter, are present in the final formulae (46), (49) together with the terms of the order of $\Delta\mathcal{E}/\hbar\omega$. And this term is interpreted as in the quotation in item **3**(a). At the same time, upon derivation of the formula [Eqn (46) on p. 1010] the authors explain that "the terms of order of $\hbar\omega/\mathcal{E}$ are omitted". Hence, the terms of order of $\hbar\omega/\mathcal{E}$ are retained while the terms of order of $(\Delta\mathcal{E}/\hbar\omega)(\hbar\omega/\mathcal{E})$ are omitted [see Eqn (44)]? The reader should be reminded that the omitted terms $(\Delta\mathcal{E}/\hbar\omega)^2$ in the expansion of the perturbation theory (see Ref. [16]) in the one-photon approximation are much greater than the retained ones. For example, for the modulation depth expected (about 10%, see Section 2.16 in Ref. [1]) the terms of order of 10^{-2} are omitted while the terms of order of 10^{-8} are retained.

Thus, there are no terms $\Delta\mathcal{E}/\mathcal{E}$ in the expression for the electron probability density when the spin interaction is neglected even if we assume that the definition of ρ the authors give is true. Terms of this type ($\propto 1/\mathcal{E}$) can appear because of the spin interaction but it is incorrect to consider them along with small terms of the first order in the perturbation theory with respect to the parameter $\Delta\mathcal{E}/\hbar\omega \ll 1$ due to the aforementioned reasons since the spin interaction is proportional to the same parameter $\hbar\omega/\mathcal{E} \ll 1$. But even if its consideration is correct, the associated radiation of the magnetic moment of an electron is so small that it has hardly any influence on the Cherenkov radiation, or on the beam modulation, etc. So, there is no sense in drilling down into any analysis of the results of the spin interaction in the linear theory, i.e., in the first order of the perturbation theory. However, the spin effects in the stimulated Cherenkov effect are what is new in the review discussed. Moreover, the authors claim that they can be observed experimentally (see Section 2.16)! To this end we think it is worthwhile to analyse the quantitative description of the spin interaction in the review [1] in which the authors use the results of Refs [2–4, 7].

Restricting themselves to the linear approximation with respect to the field, the authors [1] do not consider when the approximation is applicable, neither in the classical theory, nor in the quantum theory. They just omitted the terms $\propto A^2$ from the start. Therefore, besides the aforementioned conditions there is another one: $\sin\theta \gg eA/\mathcal{E}$. It limits the Cherenkov angle from below in the classical case. It is also true in the quantum case if the spin interaction is neglected and the Klein–Gordon equation is considered. If the spin interaction is considered to be described by small terms of the first order, then yet another condition has to be satisfied: $eA \ll \hbar\omega\sqrt{n^2-1}$. To this end the Dirac equation has to be solved in a form, in which the bispinor component and the spin interaction are separated from each other. In this way, provided that the parameter of the perturbation theory is selected correctly and all the small terms of the same order are accounted for, the results are obtained, according to which the amplitude of the probability of Cherenkov absorption–radiation decreases by a factor of $(\hbar\omega/\mathcal{E})\sqrt{n^2-1}$ and, thus, it damps the effects considered. In this case the condition $\sin\theta \sim (\hbar\omega/\mathcal{E})\sqrt{n^2-1}$ is automatically imposed on the value of the Cherenkov angle. Thus, there is no sense to consider all the subsequent problems (wave amplification, beam modulation, etc.)

However, once the Klein–Gordon equation is solved, the authors solve the Dirac equation and then the Pauli equation "to extract the spin contribution", and they make the opposite conclusion that the terms $\Delta\mathcal{E}/\mathcal{E}$ are associated with "the orbital motion" (p. 1012) rather than to the spin interaction. The authors have to solve the Schrödinger equation to check that there are no such terms in the expression for the electron probability density $|\psi|^2$. This conclusion is a direct consequence of the errors the authors make when they extract the spin interaction and pass from the Dirac and Pauli equation to the limiting case of 'the absence' of a spin interaction. In defining the initial state of an electron in the polarization matrix the authors assume that the spin interaction vanishes for $a^\mu = 0$ (a^μ is the four-dimensional electron polarization vector) and the expressions for the probability density and beam current (62), (63) coincide with the similar expressions (46), (49) where the Klein–Gordon wave function is used. Note that $a^\mu = 0$ means the absence of beam polarization rather than the absence of a spin interaction.

As for the conclusion on the contribution of the magnetic moment of the electron into the beam modulation, based on the fact that the Planck constant is eliminated from the spin part of the interaction ($\Delta\mathcal{E}/\hbar\omega \sim \mu H/\hbar\omega$; $\mu = e\hbar/2mc$), we will only cite one paragraph from Ref. [1], in which the ideology the authors develop in [2–4, 19–21] and summarize in [1] is expressed most clearly. This is the classical interpretation of the quantum formulae when $\hbar$ is eliminated (though the contribution of the quantum loss is retained); or the passage to the classical limit as $\hbar \to 0$ in the expressions which are obtained under the condition $\Delta\mathcal{E}/\hbar\omega \ll 1$; or the derivation of the classical rearrangement from the formulae for one-photon interaction. Thus, we cite: "Since the quantity $\Delta\mathcal{E} \sim \mu H$ in the case of a spin interaction (here $\mu = e\hbar/2mc$ is the magnetic moment of the electron and H is the magnetic field strength), the amplitudes of the terms responsible for the klystron modulation are classical in nature. Interestingly enough, the Planck constant $\hbar$ enters only into the asymmetric part of the loss and has no effect over distances $x \sim x_1$'' (p. 1011). It remains only to note that by the authors' definition x_1 is just the distance (purely quantum),

$$x_1 = \frac{1}{\Delta q_x} = \frac{\lambda}{\pi} \frac{\mathcal{E}}{\hbar\omega} \frac{v_x/c}{n^2 - 1} ,$$

over which ''the classical modulation goes into a quantum one with depth $2\Delta\mathcal{E}/\hbar\omega$'' (p. 1010).

4. In Sections 2.11 and 2.12 the authors [1] discuss the possibility of a spin laser in the direction in which electrons travel. By formally solving the equation for the self-consistent field the authors obtained a non-zero coefficient for the linear Cherenkov gain at the angle $\theta = 0$ due to the electron spin: ''...in the spin laser, electrons and the amplified wave travel in the same direction with almost the same speed. Therefore they interact over the prolonged period of time so that the effect can be observed'' (p. 1022).

This result contradicts the Vavilov – Cherenkov effect since the probability of stimulated emission in the linear approximation is proportional to the intensity of spontaneous emission whereas the intensity of the Cherenkov emission is zero at the angle $\theta = 0$. Therefore, this issue is reduced to the spontaneous emission of the magnetic moment (or spin) of an electron in the forward direction during the Cherenkov process. However, it is well known that the Cherenkov emission of a magnetic dipole has no quantum features (see, for example, Ref. [23], pp. 145, 146 accounting for the spin of an electron in Cherenkov emission of a magnetic dipole). It is also known that emission is accompanied by a spin flip in the forward direction ($\theta = 0$). As a result the matrix element of transition is non-zero [22] but the emission intensity at the process threshold ($\theta = 0$), as expected, is zero and then builds up gradually (the non-zero matrix element is insufficient for a real emission, so a finite phase volume has to exist) [23].

The authors base their consideration on the fact that the law of conservation for Cherenkov emission admits the frequency $\omega \neq 0$ at the angle $\theta = 0$ (in the absence of dispersion) when the quantum loss is accounted for. However, as follows from the Cherenkov condition with regard for the quantum loss and, in the general case, from the dispersion of the medium $n = n(\omega)$, the frequencies associated with the angle $\theta = 0$ are boundary frequencies (the first new frequency appearing at the spectrum boundary with the smooth build-up of the electron velocity corresponds to the angle $\theta = 0$), at which no emission is possible (the strict inequality in the limit of integration over ω in the formula for the Cherenkov emission intensity matches this conclusion: see, for example, Ref. [24]). Consequently, the gains (169) and (175) calculated in Ref. [1] (see also Ref. [7]) must be zero independently of the electron beam polarization.

On the other hand the condition $\sin\theta \gg eA/\mathcal{E}$ from item **3**(c), under which the linear theory is applicable, is violated for $\theta = 0$ ($\mathbf{pA} = 0$). In this case, as the exact solution of the Dirac equation for an electron in the presence of a circularly polarized wave in a medium shows [32] (the case from Ref. [1]) the state of the electron is the superposition of four waves with different energy-momenta (four solutions). In calculations of the gain of a 'spin laser' the authors used an incomplete wave function of the electron in the field: in this case two solutions (formula (173) from Ref. [1]) that follow from the Dirac equation according to perturbation theory do not describe a true state of the electron since it is in fact a superposition of four partial waves [32]. All this applies equally to ''spin and polarization effects in the Cherenkov laser'' (Sections 2.5, 2.6, 2.11, 2.13) since they are obtained using the same wave function (the occurrence of these effects would mean, as for the 'spin laser', that the Cherenkov emission intensity is non-zero at the process threshold: $\theta = 0$). So the results on quantum modulation (Section 2.5) and on magnetization of electron beam (Section 2.6) are obtained using the wave function (56) in a field of the form (11) and (53) while the results on the quantum theory of the Cherenkov laser (Section 2.11) and on a rotation of the polarization plain (Section 2.13) are obtained using the wave function (156) [cf. Eqn (173)]. So the results on the spin interaction in Ref. [1] are erroneous.

We shall return to the discussion of the results on the SCE in a constant magnetic field (Section 2.14) when we discuss the role of the strong longitudinal magnetic field in the stimulated Cherenkov process because it is the same for ordinary and surface Cherenkov effects (Section 3).

5. In Section 3 the stimulated surface Cherenkov effect (SSCE) is considered. The new concept of the SSCE underlying Refs [9 – 14] is as follows: there are no stimulated effects when an electron beam moves in vacuum over a dielectric medium. ''The depth of the klystron modulation, the overpopulation of the electron beam, and the gains for the Cherenkov laser and Cherenkov klystron are zero'' (p. 1029). Laws of conservation are the physical rationale for this concept but these laws are derived using an erroneous wave function — it depends exponentially on the x coordinate and the factor of this coordinate enters into the argument of the δ-function, i.e., into the law of conservation, as the component of the photon momentum perpendicular to the waveguide surface ($\hbar k_x$). And it is an imaginary quantity $i\hbar q_x$! As a result, in the laws of conservation [see Eqns (236), (237), (244)] the x component of the electron momentum becomes imaginary. The application of a law of conservation for this component of momentum in Refs [9 – 14] leads to zero quantum loss in emission and absorption of a photon during the SSCE [see Eqn (235)]. The authors interpret this result as a ''peculiar generation that can be removed in three ways: (1) by placing the waveguide in a gaseous atmosphere; (2) by applying a constant magnetic field along the waveguide; and (3) by considering amplification by particles whose velocities lie outside the Cherenkov cone'' (p. 1029). In other words the SSCE can proceed if there is an additional fourth body to provide for 'the asymmetry' in laws of conservation of absorption and emission of a photon by an electron.

However, it is well known [25, 26] that the major difference between the problem with a boundary between media and the problem with an infinite medium is the violation of the law of conservation of the x component of the electron momentum. Moreover, it follows from the laws of conservation of energy and the z component of momentum that by absorbing (emitting) a photon the electron acquires an the x component of the momentum p_x:

$$p_x^2 = \pm 2 \frac{\mathcal{E}_0}{c^2} \hbar(\omega - k_z v_{z0}) - \hbar^2 q_x^2 , \quad q_x^2 = k_z^2 - \frac{\omega^2}{c^2} > 0 .$$

Hence photon absorption in the SSCE is possible when

$$\omega - k_z v_{z0} \geqslant \frac{\hbar c^2 q_x^2}{2\mathcal{E}_0} ,$$

while emission is possible when

$$\omega - k_z v_{z0} \leqslant -\frac{\hbar c^2 q_x^2}{2\mathcal{E}_0} ,$$

i.e., the conditions for absorption and emission in the SSCE differ by the quantum loss and there is no 'peculiar degeneration'. From this standpoint the SSCE does not differ from the SCE.

6. If the classical problems for amplification by the SSCE are solved correctly, then non-zero gains would be obtained without an additional fourth body and it would be a signal that the concept set forth and the quantum results are erroneous. The reasons that the classical gains vanish in the works [1, 9 – 14] are as follows:

(a) In consideration of the SSCE problem, in which the electron beam moves over a waveguide, the authors select a spatially homogeneous electron distribution function as the initial one. As a consequence, they do not take into account the change in the beam density (ρ_1), for which the spatial inhomogeneity of the initial density [$\rho_0(x)$] in the perpendicular direction to the waveguide surface is responsible:

$$\rho_1 \left[(\omega - k_z v_{z0}) \frac{\partial}{\partial x} \left(\rho_0(x)\, v_{1x} \right) \right]^{-1} .$$

This inhomogeneity introduces a term, proportional to the overpopulation $\partial f_0 / \partial \mathbf{p}$, into the expression for the gain and it is a major factor in the amplification process of a weak wave in the SSCE.

(b) The poles in the expression for the gain are calculated mistakenly. For example, the gain for a plane waveguide in the absence of a gaseous medium ($\varepsilon_1 = 1$) does not vanish even if the terms from item **6**(a) are not taken into account provided that the contribution of the Cherenkov poles into expression (287) in Ref. [1] is calculated correctly. This can be seen even from formula (295) for the gain over a finite part of the waveguide L. As $L \to \infty$ the factor

$$L\alpha \frac{\mathrm{d}}{\mathrm{d}\alpha} \frac{\sin^2 \alpha}{\alpha^2}$$

in this formula becomes $\omega' \,\mathrm{d}\delta(\omega')/\mathrm{d}\omega'$ (to within a factor $\omega' = \omega - k_z v_{z0}$) and, on averaging over the initial electron distribution function, the gain is proportional to $f_0(\mathbf{p})$, i.e., it is non-zero.

Thus, the errors made are systematized and summarized over all the class of problems on the SSCE to fix that an additional fourth body is required for the dynamic effects (modulation, etc.) as well as for the amplification processes of emission to proceed.

7. The Cherenkov laser problem was comprehensively studied both theoretically and experimentally in the famous works by Walsh, the results of which were set forth in many reviews in the late 70s and in the early 80s (see, for example, Ref. [27]). Walsh had started his research from the stimulated Cherenkov radiation in an unbounded gaseous medium and calculated the linear gains for various modes (see, for example, Ref. [28]). In this case two negative factors affect the amplification process simultaneously: the influence of the angular spread of the electron beam and the side effects of multiple scattering as well as those of ionization losses of particles in the medium. To eliminate the first factor Walsh applied a strong magnetic field and calculated the gains of the Cherenkov laser by a one-dimensional overmagnetized electron beam in the hydrodynamic instability mode (cool beam) as well as in the kinetic instability mode (hot beam). To eliminate the negative influence of the medium a scheme of the stimulated surface Cherenkov radiation (SSCR) was proposed and the gains of the Cherenkov laser in the total inner reflection mode for waveguides of different geometries were calculated(the finite length of a waveguide was taken into account [29]). Then papers [8 – 13], the results of which are presented in the review [1], solve the same problems in linear mode in the same formulation. At best they obtain the same results as those of Walsh (there are many inconsistencies, incorrect or unnecessary propositions on the way to the final formulae). However, the authors [1] then dare to conclude that "the principle distinction between Walsh's work and our work is that he does not consider the angular spread of the particle beam. To justify his model, Walsh supposes that an infinitely large magnetic field is applied along the electron beam. Clearly, the magnetic field does not eliminate the angular spread of an electron beam" (p. 1025). Meanwhile Eqns (205) and (309) are the same as those of Walsh and they do not depend on the angular spread because this dependence is eliminated by this strong magnetic field. The required strength of magnetic field is specified by the angular spread of the beam: $\Omega \gg k_\perp v_\perp$ ($\Omega = ecH_0/\mathcal{E}$ is the cyclotron frequency). Under this condition the magnetic field freezes the transverse degrees of freedom, i.e., the translational motion of electrons in the transverse direction, and, as a consequence, this component of the velocity falls out the Cherenkov resonance condition ($\omega - \mathbf{kv} \to \omega - k_z v_z$). This condition has a simple physical meaning: the Larmor radius of rotation of an electron $R = v_\perp / \Omega$ has to be much less than the characteristic change of the wave field in the perpendicular direction to the magnetic field $1/k_\perp$ for the phase synchronization of the Cherenkov process not to be violated (the negative effect of the angular spread of the beam is in violation of the phase synchronization and the strong magnetic field maintains the synchronization by freezing the perpendicular motion of electrons). Note that for the sake of mathematical rigour the condition $\Omega \gg \max\{k_\perp v_\perp, k_z \Delta v_z\}$ (here $k_z \Delta v_z$ is the Cherenkov resonance width) is required to eliminate the cyclotron resonance totally (in real situations the condition is, in fact, the same). Thus, the problem becomes one-dimensional and there is no need to include the ('infinitely strong') magnetic field nor the transverse component of the electric field of a weak wave (linear theory) into the equation, and this is what Walsh has done.

Let us show how the Cherenkov amplification and Cherenkov surface amplification problems in the presence of a magnetic field are solved in the review [1]. On the grounds of Refs [8 – 13] the authors assume that the magnetic field has an arbitrary strength (and then they consider the Cherenkov amplification!). As is shown in Refs. [30, 31] the perturbed distribution function f_1 [see also Eqn (194)] is a series, the Nth term of which represents the cyclotron resonance at the Nth harmonic ($N = \pm 1, \pm 2 \ldots$) while the zero harmonic represents the Cherenkov resonance. The authors left only zero harmonic in this series without any explanation. The aforementioned condition is required to omit the other terms of the series. As a direct consequence the dependence on the magnetic field strength would fall out the expression for the perturbed distribution function of electrons and, hence for the gain. Instead the authors [1] obtain other erroneous conditions to limit the range of electron momenta, for which the gains by the overmagnetized beam both for the SCE and for the SSCE, i.e., the results of Walsh are valid. In fact there is no other condition except the one we cited, in full accord with the results of Walsh.

As for the amplification problem in a tubular waveguide considered in Section 3.6, it is quite clear that the final result (320), (327) is erroneous (there is even no need to compare it with the Walsh formula) since these expressions are not proportional to the spontaneous Cherenkov radiation intensity (to the term $\sin^2\theta = 1 - c^2/n^2 v_0^2$) and expression (327) is completely incorrect. By the way, once the plain waveguide has been considered there is no real need to solve the problem for a tubular waveguide since under the condition (326) a tubular waveguide is equivalent to a plane one with double thickness [the value $a = (d - b)/2$ should be substituted into expression (309) for the plane waveguide, in which case the authors [1] would not arrive at the incorrect formula (327)]. In Refs [11, 12] formula (327) is a final result. In the review discussion the authors present, after formula (327), yet another formula (329) but here the gain is proportional to the spontaneous Cherenkov radiation (however it is two times less than for a plane waveguide?). But why? Unexpectedly, there is a need for a final formula in a particular approximation: "We also cite the expression for G in the case where the second terms are retained" (p. 1036).

Further, as is noted in item **6**(a) of these notes the initial electron distribution function is spatially inhomogeneous for the SSCE and, as a consequence, the electron density and electron current experiences additional changes in the amplification process but this fact is not taken into account in Refs [1, 9 – 13]. These changes are negligible only in the strong magnetic field limit because the transverse motion of the electron is frozen. Therefore, all the results on the SSCE in Section 3 are erroneous, except formula (309) for a plane waveguide, which is the same as that of Walsh (up to the factor $c^2/n^2 v_0^2$ in the second term in braces). The conclusion the authors come to is very interesting: "The feasibility of amplification of electromagnetic radiation in plane and tubular waveguides was also considered by Walsh et al... The role of the magnetic field is also different. Walsh et al. consider it as the leading field, whereas we introduce it to create asymmetry in emission and absorption of photons by electrons, i.e. to create the mechanism of amplification... Note that, strictly speaking, the magnetic field strength does not enter the original equations given by Walsh et al. as a parameter: they simply postulate that the one-dimensionality of an electron beam is equivalent to a very strong magnetic field. Therefore, the results obtained on the basis of this model can be considered as qualitative" (pp. 1036 – 1037).

We have dwelled here only on the principal errors made in Ref. [1], not to leave the reader in delusion. It is impossible to discuss the issues set forth in Ref. [1] in more detail within the present note. The reader can understand the true state of affairs if he or she read Ref. [1] along with the papers cited.

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