

METHODOLOGICAL NOTES

PACS numbers: **44.25.+f**, 47.20.Bp

'Anticonvection'

L Kh Ingel'

Contents

1. Introduction 741
2. Convective instability can also arise upon stable stratification. Welander's anticonvection 741
3. Efficiency of simple physical reasoning in describing anticonvection 742
4. Anticonvection related to phase transitions at the interphase 744
5. Air over salt water – another mechanism of anticonvection 745
6. Thermocapillar anticonvection 745
7. Conclusions 745
References 745

Abstract. **A convective instability in a system of two immiscible fluids under gravity may arise not only from heating from below but also as a result of a horizontally uniform heating from above, i.e., with either fluid exhibiting a stable stratification. Some recently discovered mechanisms of such instability are discussed.**

1. Introduction

It is well known that in a fluid (gas) under gravity a convective instability (Rayleigh – Taylor instability) can arise upon homogeneous heating from below or cooling from above. The physical origin of this phenomenon is quite clear: the colder, denser fluid sinks while the light, heated one rises due to the buoyancy force.

In this paper we deal with the opposite situations (heating from above, cooling from below) when there is seemingly no reason to expect that an instability should arise.

2. Convective instability can also arise upon stable stratification. Welander's anticonvection

Welander [1] pioneered a theoretical result, which might at first sight appear paradoxical. The state of mechanical equilibrium in a horizontally homogeneous system consisting of two immiscible fluids heated from above (exhibiting stable stratification) can be unstable, giving rise to convection in both the fluids near the interphase. Later on a similar result was independently obtained in Refs [2, 3]. Welander called this phenomenon 'anticonvection'. The physical mechanism of anticonvection significantly differs from that of Rayleigh instability and can be outlined as follows:

Suppose, for example, that in a certain region of an upper (lighter) fluid near the interphase an arbitrarily weak temperature perturbation arises and this region turns out to be warmer and less dense than the neighbouring fluid layers. The weight of the bulk column there will be less than that of the surroundings. Therefore in the upper fluid near the interphase a horizontal pressure gradient will originate with the horizontal component of the pressure force directed from the periphery to the centre of the perturbed (heated) region.

Hence, in the considered region of the upper fluid convergent flows (and in view of continuity, upward vertical flows) will arise (Fig. 1). Due to viscosity the convergent horizontal motion will involve, to a certain extent, the upper layer of the lower fluid. On account of continuity these convergent horizontal flows in the lower fluid give rise to vertical downward motions. Note that, according to estimates, the deformation of the interphase can be neglected in many important cases. Therefore the particles of the convergent horizontal flow in the upper fluid can move, in the vertical sense, only upwards, while the corresponding particles in the lower fluid — only downwards (see Fig. 1). Downward flows of the lower medium induce a positive temperature perturbation below the interphase (since the

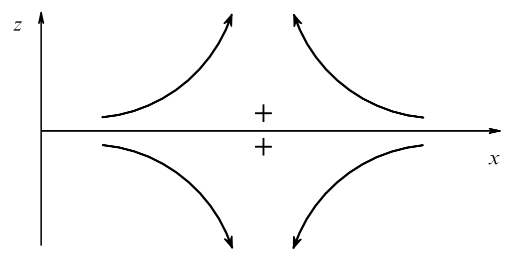

Figure 1. Schematic representation of the flow lines in the upper and lower fluids when convergent anticonvection develops. The interphase coincides with the x-axis. The signs of the arising temperature perturbations are indicated. In the case of a divergent anticonvection the flow lines are reversed, the signs of the temperature perturbations remaining unchanged.

L Kh Ingel Institute of Experimental Meteorology, SPA 'Typhoon', prosp. Lenina 82, Obninsk, Kaluga Region 249020, Russia
Tel. (7-08439) 7-13 21
Fax (7-08439) 4-09 10
E-mail: typhoon@storm.iasnet.com

Received 3 December 1996
Uspekhi Fizicheskikh Nauk **167** (7) 779–784 (1997)
Translated by G N Chuev; edited by S D Danilov

system is heated from above, downward flows bring some heat). Due to heat exchange through the interphase this temperature perturbation enhances the initial temperature perturbation in the upper medium. For certain relations between the medium parameters this positive feedback can dominate over the negative feedbacks of the system [1, 2], making the mechanical equilibrium of the system unstable.

In the case under discussion the lower fluid plays the role of a source of temperature perturbations inducing flows in the upper fluid, which, in turn, amplify temperature perturbations in the lower fluid, etc.

Depending on the relation between the medium parameters this instability can develop in another scenario.

Suppose, for example, that as a result of an chance perturbation an element of the upper fluid moves down towards the interphase. In the new position this element appears to be warmer than the surroundings, since the system is homogeneously heated from above. If the thermal conductivity and the thermal expansion of the upper fluid is rather small, the element will cool quite slowly and have a rather low buoyancy. However, due to heat exchange through the interphase an element warmer than the surroundings will also arise in the lower, heavier fluid near the interphase. If the thermal expansion of the lower fluid is rather high, the heated element there will be substantially lighter than the surroundings. As a result, it will move upwards and spread over the interphase. Due to viscosity the horizontal motion will, to a certain extent, involve the lower layer of the upper fluid. (In this case the direction of the flow lines in Fig. 1 should be reversed, though we stress that the sign of the temperature perturbations remains unchanged). In view of continuity, the divergent horizontal flows appearing in the upper fluid must give rise to downward flows in it, thereby enhancing the initial perturbation.

The linear analysis of hydrodynamic stability [1, 2] indicates that for a certain relation between the medium parameters the positive feedback can be rather intense and can lead to the development of instability.

In the latter case the upper fluid is a source of temperature perturbations inducing flows in the lower medium. The flows, in turn, enhance the temperature perturbations, etc. We call this anticonvection 'divergent' (positive deviations of the temperature correspond to a positive horizontal divergence of the flows arising near the interphase). In the former case above the anticonvection in this sense can be called 'convergent'.

According to the linear analysis of stability performed in Refs [1, 2] anticonvection certainly does not arise in every bilayered system. For example, it can develop in a water-mercury system but cannot occur in a water – air system, which is more interesting for geophysical applications. But despite some attempts, anticonvection has not yet been identified experimentally under favourable conditions. This failure is probably associated with technical problems (see Ref. [4] and references therein).

It was inferred [1] that the theoretical restriction on the appearance of anticonvection in a water – air system is not necessarily extended to turbulent natural media such as the ocean and atmosphere. The point is that if we use effective turbulent exchange coefficients (instead of molecular ones) the conditions for anticonvection development change significantly. Besides, in the case of such media, the system involves one more degree of freedom — a substantial varying parameter, i.e. a horizontally homogeneous source (sink) of heat near the interphase associated with radiation effects and phase transitions on the water surface.

Models [1, 2] assumed the vertical background heat flow to be the same in the upper and lower media and not to change at the interphase. This means that the vertical background temperature gradients in both the media are unambiguously related to each other. But if we suppose the existence of a background heat source (sink) of intensity Q [W m^{-2}] at the interphase associated with radiation effects or phase transitions, then the vertical background heat flow should experience a temperature jump equal to $|Q|$ in passing through the interphase. Since in nature the value of Q may vary over a wide range (for example, with the time of day), the vertical background temperature gradients in both the media can also change significantly.

Some problems concerned with the above were considered in papers by Perestenko and the author (see, for example, Refs [4, 5] and the references therein). It was shown that the probability for anticonvection arising in a bilayered system depends on the Q value and could considerably increase. In particular, convection of both types is conceptually possible in water – air system.

This conclusion becomes clear from the following simple reasoning. Varying the heat source (sink) intensity Q at the interphase we can always make the temperature (density) stratification in one of the media be nearly neutral. Such a medium is easily set into motion by temperature inhomogeneities arising near the interphase in the other medium. Being rather intense, the motion in the former medium straightforwardly generates and enhances thermal inhomogeneities in the latter, etc. In other words, anticonvection particularly readily arises in a system, where one medium is stratified weakly while the other one exhibits stable stratification. If the upper medium is weakly stratified, then, it is easy to see that convergent anticonvection develops. The opposite conditions are favourable for the appearance of divergent anticonvection.

3. Efficiency of simple physical reasoning in describing anticonvection

In Refs [1, 2, 5] linear problems of the stability of mechanical equilibrium between two fluids homogeneously heated from above were studied in a rigorous formulation. In Refs [1, 5] these fluids occupied the upper and the lower half-spaces, Ref. [2] dealt with horizontal fluid layers of finite depth. Linearized sets of the Navier – Stokes equations, the equations of continuity and heat transfer in the Boussinesq approximation were considered for each medium. In the simplest two-dimensional case the studied set of equations has the form:

$$\partial_t u_i = -\frac{\partial_x p_i}{\bar{\rho}_i} + \nu_i \Delta u_i \,, \tag{1}$$

$$\partial_t w_i = -\frac{\partial_z p_i}{\bar{\rho}_i} + \nu_i \Delta w_i + \alpha_i g \Theta_i \,, \tag{2}$$

$$\partial_x u_i + \partial_z w_i = 0 \,, \tag{3}$$

$$\partial_t \Theta_i + \gamma_i w_i = \varkappa_i \Delta \Theta_i \,. \tag{4}$$

Here subscript $i = 1, 2$ means the quantities relating to the upper and lower media, respectively; u and w are the components of velocity along the horizontal x-axis and the vertical z-axis, respectively; Θ is the temperature perturbation (more precisely, the potential temperature, i.e. the quantity used in geophysics with regard to medium compressibility); p is the pressure perturbation, $\bar{\rho}$ is the average density of the medium; $\gamma \geqslant 0$ is the vertical background gradient of the potential temperature; α is the thermal expansion coefficient; g is the free fall acceleration; ν and $\varkappa$ are the coefficients of viscosity and thermal conductivity, respectively.

It is assumed that away from the interphase (at $|z| \to \infty$) all the perturbations are damped. Deformations of the interphase are neglected (this can be easily justified for a water – air system and for many other cases). Tangential stresses, temperatures and perturbations of the heat flow at the interphase are supposed to be continuous. The vertical background heat flow, determined by the temperature gradients γ_i, may break at the interphase (in other words, a horizontally homogeneous heat source (sink) is assumed at the interphase).

Broadly speaking, the stability problem under discussion is rather complicated and, in general, requires a numerical analysis (though some asymptotics can be studied analytically). Noteworthy is the fact that despite the complexity of the system, many of the results can be understood and reproduced immediately from simple physical reasoning [4, 6] without solving differential equations. By way of illustration let us dwell upon the estimate of increments of growing perturbations in a water – air system [6].

Let us take the situation most favourable for the development of a convergent anticonvection, when the lower fluid in the initial state exhibits stable stratification while the upper one has a neutral ($\gamma_1 = 0$, $\gamma_2 > 0$) stratification. Suppose that at the interphase $z = 0$ a temperature perturbation is specified in the form:

$$\Theta|_{z=0} = \Theta_0 \cos(kx) \exp(\omega t) , \tag{5}$$

where $\Theta_0 > 0, \omega, k = 2\pi/L, L$ are constants whose meaning is evident. Horizontally inhomogeneous heating must give rise to pressure perturbations and flows near the interphase. Let us estimate the amplitudes of the pressure perturbations, assuming them to depend similarly on x and t and omitting hereafter, for brevity, the multipliers of the type $\cos(kx)\exp(\omega t)$. Let us denote the height to which the temperature perturbation penetrates into the upper fluid by H_1. We suppose that H_1 is much less than the wavelength L, i.e. perturbations are 'pressed' to the interphase (this assumption, as well as many others can be checked *a posteriori*). In this case we can, first, neglect the horizontal exchange [in Eqns (1), (2), (4), $\partial^2/\partial x^2 \ll \partial^2/\partial z^2$], and second, use the hydrostatics approximation, or consider only the first and the last terms from the right-hand side of Eqn (2). Whence we can write:

$$|p_1| \sim g\bar{\rho}_1 \alpha_1 H_1 \Theta_0 , \quad \left| \frac{\partial_x p_1}{\bar{\rho}_1} \right| \sim g\alpha_1 H_1 \Theta_0 k . \tag{6}$$

Suppose that in Eqn (1) both the first terms are essential (this means that they must be of the same order). Therefore, in view of relations (6) the typical horizontal velocities of the flows arising in the upper fluid are:

$$|u_1| \sim \frac{kp_1}{\bar{\rho}_1 \omega} \sim \frac{g\alpha_1 H_1 \Theta_0 k}{\omega} , \tag{7}$$

and the momentum flux from the upper fluid to the lower one is:

$$\bar{\rho}_1 \nu_1 \left| \frac{\partial u_1}{\partial z} \right| \sim \bar{\rho}_1 \nu_1 \frac{|u_1| - \left| u|_{z=0} \right|}{H_1} . \tag{8}$$

Assume that motions arising due to viscosity are much slower in the lower fluid than in the upper one, so that $\left| u|_{z=0} \right| \ll |u_1|$. Then:

$$\bar{\rho}_1 \nu_1 \left| \frac{\partial u_1}{\partial z} \right| \sim \frac{\bar{\rho}_1 \nu_1 |u_1|}{H_1} \sim \frac{\bar{\rho}_1 \nu_1 g \alpha_1 \Theta_0 k}{\omega} . \tag{9}$$

It is notable that the last quantity is independent of H_1 and should be equal to the absolute value of the momentum flux from the interphase deep into the lower fluid:

$$\bar{\rho}_2 \nu_2 \left| \frac{\partial u_2}{\partial z} \right| \sim \frac{\bar{\rho}_2 \nu_2 \left| u|_{z=0} \right|}{H_2} , \tag{10}$$

where H_2 is the characteristic depth of penetration of perturbations into the lower fluid. Whence:

$$\left| u|_{z=0} \right| \sim |u_1| \frac{\bar{\rho}_1 \nu_1 H_2}{\bar{\rho}_2 \nu_2 H_1} . \tag{11}$$

In view of continuity

$$|w_2| \sim kH_2 \left| u|_{z=0} \right| . \tag{12}$$

Suppose that the first two terms of Eqn (4) in the lower fluid are of the same order. Then the temperature perturbation arising in the lower fluid is:

$$|\Theta_2| \sim \frac{\gamma_2 |w_2|}{\omega} . \tag{13}$$

It is this temperature perturbation that must maintain the flows in the upper fluid. It follows from the continuity of the perturbed heat flow at the interphase that:

$$\bar{\rho}_2 C_2 \varkappa_2 |\partial_z \Theta_2| = \bar{\rho}_1 C_1 \varkappa_1 |\partial_z \Theta_1| ,$$

where C is the thermal conductivity at constant pressure. Replacing the derivatives by the relations of finite differences we have:

$$\frac{\bar{\rho}_2 C_2 \varkappa_2 (\Theta_2 - \Theta_0)}{H_2} \sim \frac{\bar{\rho}_1 C_1 \varkappa_1 \Theta_0}{H_1} .$$

It follows that

$$\Theta_2 - \Theta_0 \sim \Theta_0 \frac{\bar{\rho}_1 C_1 \varkappa_1 H_2}{\bar{\rho}_2 C_2 \varkappa_2 H_1} . \tag{14}$$

If a water – air system is considered, the right-hand side of (14) contains a small multiplier and we may suggest that the dimensionless coefficient at Θ_0 is small. It follows that:

$$\Theta_2 - \Theta_0 \ll \Theta_0 , \qquad \Theta_2 \sim \Theta_0 . \tag{15}$$

Substituting the latter relation into (13), in view of (12), (11), and (7) we have:

$$\frac{\bar{\rho}_1 \nu_1 \alpha_1}{\bar{\rho}_2 \nu_2 \alpha_2} \frac{N_2^2 k^2 H_2^2}{\omega^2} \sim 1 , \tag{16}$$

where $N_2^2 = \alpha_2 g \gamma_2$ is the squared 'buoyancy frequency' (Brunt – Väisälä frequency) in the lower fluid. Suppose that the depth of penetration of the perturbation into the lower fluid is determined by the diffusion law:

$$H_2 \propto \sqrt{\frac{\nu_2}{\omega}}\,. \tag{17}$$

Substituting (17) into (16) we get the dispersion relation:

$$\omega \sim \left(\frac{\bar{\rho}_1 \alpha_1}{\bar{\rho}_2 \alpha_2} \nu_1 N_2^2 k^2\right)^{1/3}. \tag{18}$$

The increment of the exponential growth of perturbations depends mainly on the wavelength $L = 2\pi k^{-1}$; it decreases as $L^{-2/3}$. As might be expected, it grows as the stratification increases in the lower fluid. Note that it is independent of $\varkappa_1$, $\varkappa_2$, and ν_2.

The above speculations involve many assumptions, which might appear rather arbitrary at first sight. But detailed tests show that they have quite a wide field of application. Numerical solutions of the stability problem [4, 6] indicate that the two thirds law is typical for long-wave asymptotics of dispersion curves.

It is notable that similar simple non-rigorous physical reasoning has enabled us to obtain correct results in treating more complicated mechanisms of instability associated with surface evaporation, and some other phenomena [4, 7, 8] (see the following sections).

4. Anticonvection related to phase transitions at the interphase

In the foregoing discussion we did not take account of evaporation effects at the interphase. Meanwhile these effects can result in one more mechanism of positive feedback [4, 7, 9], which will be considered in examining the water – air system.

If surface evaporation is allowed for, the air is stratified with respect to the specific humidity q. In the absence of perturbations the initial humidity q decreases with height. Since wet air is lighter than dry air, this humidity stratification destabilises the density stratification of the air. Therefore the density stratification remains stable (in the limiting case — neutral) in both media. Suppose that a thermal perturbation of arbitrarily small amplitude arises at the interphase, making the local water surface area slightly colder than the surroundings. Due to thermal exchange the air layer over this area will also be colder than the surroundings. Cooled denser air will spread over the surface and settle down, carrying drier air from above to the surface. Drying of the air near the water surface can enhance evaporation. This leads to further cooling of the surface area under consideration. Thus, there is a feedback which can intensify the initial temperature perturbation which, in turn, enhances the downward airflow, etc.

This mechanism of positive feedback, as presented above, includes a point which might appear paradoxical. On cooling an area on the water surface, evaporation does not decrease, as one might expect, but increases. This calls for explanation. As the water surface cools, the saturation vapor content immediately above this surface decreases (in the analysis of stability [4, 9] the specific air humidity is assumed to be equal to its saturated value at $z = 0$). All other factors being equal, the decreased humidity near the surface leads to a decrease in the diffusive moisture flux coming up from the surface, which means a reduction of evaporation. But the point is that, in the presence of a humidity background stratification by ('humidity non-equilibrium' of the system), there is one more factor responsible for changes in air humidity at $z > 0$. This is the vertical flows induced by horizontal thermal inhomogeneities. These flows are directed downwards over the cooled area of the water surface and bring drier air from above to the region at $z > 0$. Thus, as an area on the water surface cools, the air moisture at different heights decreases under the action of two different mechanisms. Firstly, due to a decrease in the saturation at $z = 0$, which is of most concern near the surface, and secondly, owing to the occurrence of downward flows in the air, which is more intense at great heights. Depending on the relation between the amplitudes of these two components of the humidity perturbation (which is determined, in particular, by the background stratification with respect to humidity and the horizontal scale of the perturbation under consideration) the vertical humidity gradient at $z = 0$, i.e. evaporation from the water surface, may not only increase but also decrease. The 'evaporative' instability mechanism described above occurs only in the latter case, i.e. for sufficient 'humidity non-equilibrium' of the initial state of the system.

The linear analysis of stability has shown that this mechanism of positive feedback can be quite efficient. As distinct from the mechanism of Welander's anticonvection, the evaporative mechanism can lead not only to monotonic but also to oscillating modes of instability development. It is readily seen that this mechanism also takes place when the air is limited from below not by a water surface but by a wet solid surface [7].

The described mechanism is analyzed in considerable detail in Refs [4, 7]. In this case some substantial qualitative and sometimes quantitative results can be found directly from simple physical reasoning. We have revealed, in particular, that the oscillating mode of instability is associated with a specific interaction between the 'evaporative' mechanism and Welander's divergent type anticonvection (see Section 2). To see this let us recall the mechanism of amplification by evaporation of a 'cold spot' on the water surface. It should be commented that downward flows, in the vertical sense, and convergent flows, in the horizontal sense, occur naturally in water colder and denser than the surroundings (Fig. 2). It is seen that near the interphase the horizontal components of the flows can be directed along the air flows. Estimates and exact calculations show that sometimes the air flows can be reversed due to viscosity. It is in these cases that the oscillating

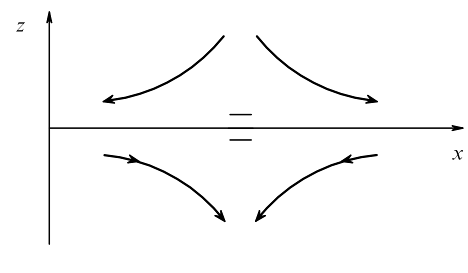

Figure 2. Schematic representation of possible flows near a cooled area on the water surface. Downward motions in both the media upon their cooling give rise to convergent motions near the interphase.This can lead to oscillatory mode of the instability development.

mode of instability development can take place. The reversed direction of the air flows gives rise to upward motions in the air. As a result, the air humidity near the surface increases, the evaporation reduces and, as a consequence, the cold surface is heated. The heating reverses the direction of the water flows, which, in turn, can make the horizontal flows in the air 'move backwards', etc. Obviously, in the air over a solid wet surface this mechanism of oscillations could not occur. Indeed, the rigorous analysis of stability reveals in this case only monotonic growth of perturbations [7, 9].

5. Air over salt water – another mechanism of anticonvection

In geophysical applications salinity effects in water can be of considerable importance.

Let us turn back to the 'evaporative' mechanism of instability described in the previous section. As was mentioned, this mechanism can occur only for rather strong 'humidity non-equilibrium' of the system, i.e. sufficient initial vertical moisture flux, or strong initial stratification with respect to humidity. If this is not the case, on cooling of an area on the water surface, evaporation from the area decreases rather than increases and the positive feedback described does not take place.

But if the water is salty, the decreased evaporation gives rise to a negative perturbation of salinity below the 'hot spot' on the surface. Less salty water is less dense. True, in the region of the 'cold spot' there is a factor having the opposite effect: the liquid density increases as it cools down. In the case under study the cooling of the water and its 'desalination' change the water density in opposite directions. According to analysis of Ref. [8], at real values of the sea water salinity, in a certain range of horizontal perturbations the desalination of water affects its density more significantly. The water density in the region of the 'cold spot' may be less than initially, giving rise to upward motions which bring colder water from below to the surface. This leads to further cooling of the 'cold spot'. As a result, the evaporation decreases, the salinity further reduces, and upward motions below the 'cold spot' enhance, etc. The analysis of hydrodynamic stability [8] shows that this mechanism of positive feedback can be quite efficient.

6. Thermocapillar anticonvection

In conclusion let us dwell upon the mechanism which can provide a considerable thermocapillar effect at the interphase. Suppose, there is again a 'cold spot' at the interphase. In view of the thermocapillar effect, convergent horizontal flows will arise around the spot both in the lower medium (due to viscosity) and in the upper one. Because of the continuity, upward flows will appear in the upper medium over the 'cold spot'. Upward motions in the system cooled from below decrease the temperature. This, in turn, means intensification of the 'cold spot', amplification of the convergent horizontal flows, etc. This anticonvection mechanism and its interaction with Welander's mechanism are studied in Ref. [10].

7. Conclusions

Thus, there are a number of non-trivial mechanisms inducing convective instability in bilayered systems upon heating from above, i.e. upon stable stratification of both media.

Note the fundamental difference from Rayleigh's classical convection at which the conditions for instability development only qualitatively depend on the perturbation wavelength. In a bilayered system heated from above, various physical mechanisms of instability become of importance for waves of various length. For some modes the 'thermal' non-equilibrium of the system is more significant, for others — the 'humidity' non-equilibrium or salinity effects. Therefore, the dynamics of various unstable modes can differ qualitatively. For example, the instability may develop either monotonically or by oscillations.

In summary it may be said that the study of anticonvection so far has been purely theoretical, involving the linear analysis of hydrodynamic stability.

The author is very grateful to O V Perestenko, whose collaboration enabled most of the problems considered to have been clarified.

References

1. Welander P *Tellus* **16** 349 (1964)
2. Gershuni G Z, Zhukhovitsky E M *Izv. Akad. Nauk SSSR. Mekhanika Zhidkosti i Gaza* (6) 28 (1980)
3. Ingel' L Kh, Stogova I V *Izv. Akad. Nauk SSSR. Fizika Atmosfery i Okeana* **23** 1120 (1987)
4. Perestenko O V, Ingel' L Kh *J. Fluid Mech.* **287** 1 (1995)
5. Perestenko O V, Ingel' L Kh *Izv. Akad. Nauk SSSR. Fizika Atmosfery i Okeana* **27** 408 (1991)
6. Ingel' L Kh, Perestenko O V *Izv. Akad. Nauk SSSR. Fizika Atmosfery i Okeana* **28** 45 (1992)
7. Perestenko O V, Ingel' L Kh *Bound.-Layer Meteorol.* **78** 383 (1996)
8. Ingel' L Kh, Perestenko O V *Dokl. Akad. Nauk* (1997) (in press)
9. Perestenko O V, Ingel' L Kh *Dokl. Akad. Nauk* **333** 92 (1993)
10. Perestenko O V *Pis'ma Zh. Tekhn. Fiz.* **18** 5 (1992)