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The Ranque effect

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Contents

Abstract. **The existing theories of the Ranque effect are reviewed and their inherent inconsistencies in the interpretation of some experimental data discussed. A new approach to the vortex effect is formulated, which provides a unified explanation of all the experimental data available.**

1. Introduction

In the gas dynamics of vortex flows, there is a non-trivial phenomenon, the Ranque effect (the Ranque-Hilsch effect or the vortex effect), which is remarkable. In vortex tubes of a relatively simple geometry (Fig. 1), the inlet gas flow divides into two outlet flows, one of which, the *peripheral* flow, has a higher temperature than the initial gas while the other, the *central* flow, has a lower temperature. The effect is all the more strange given that, as in the case of vortex stabilization of gaseous discharges [1], the buoyancy forces should cause a hotter gas to 'float up' at the vortex center.

Ranque discovered the effect of temperature separation of gases in 1931 when he was studying processes in a dust separated cyclone [2]. An intensive experimental and theoretical study of this effect began after World War II and continues today. The technical simplicity of the effect stimulated the activity of inventors. Using doubtful theories or trial-and-error methods they found a multitude of ways in which the first vortex tubes could be improved, and also a host of new applications. The range of designed and working devices, in which the vortex effect is used, is extremely wide

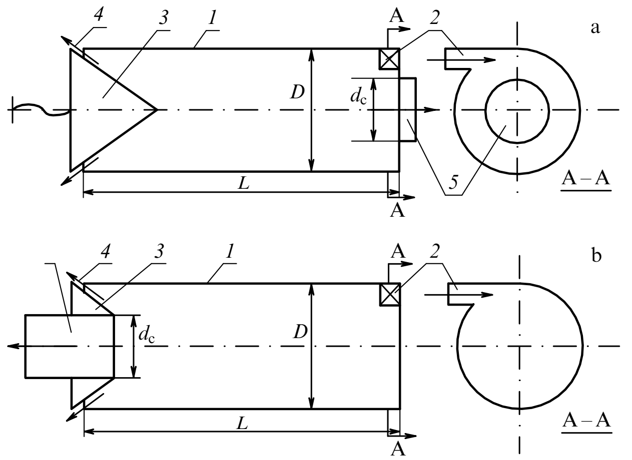

Figure 1. The schematic design of a vortex tube: (a) of the counter flow type; (b) of the uni-flow type. *1* — smooth cylindrical tube, *2* — tangential or helical swirler to feed compressed gas, *3* — throttle (valve), *4* — ring slot for hot gas output, *5* — orifice for cold gas output.

[3 – 5], and their abilities are very impressive. For example, 'in the best designs for refrigeration the temperature at the axis reaches about −200 °C from room temperature' [6]. Because of the simplicity of the devices, the activity of inventors in the field has died out for the most part, though patents pertaining to the Ranque effect are still applied for from time to time [7]. As for attempts to find an indisputable scientific explanation of the effect, papers on the subject continue to be published in series, as a rule, to support a new explanation. For example, in the last fifteen years in Russia, a doctoral dissertation was defended [8] and three monographs were published [4, 9, 10]. The Ranque effect has also been discussed in books on vortex motion problems [11 – 14] and in dissertations and papers published in Russia [15 – 21] and in other countries [22 – 29].

Thus, on one side there is an unceasing interest in the subject from a number of researches, engineers, and inventors. On the other side, most physicists have not even heard of this bright phenomenon, yet there is no doubt that it should

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be studied in a general physics course (on the face of it, a vortex tube could be the Maxwell demon, you know!). All in all, these facts indicate that there is no indisputable explanation of the Ranque effect. The introductory article of the editor-in-chief of collection [5], in which echoes of scientific battles can be heard, presents a bright illustration of the current situation. The absence of a clear theory leads some scientists into the temptation of creating perpetual motion of the second kind and refuting the second principle of thermodynamics using a vortex tube [30–32]. The Ranque effect is 'a surprising phenomenon' the nature of which 'seems mysterious even now', in the opinion of prominent specialists in the aerodynamics of vortex flows [6].

The new approach to the Ranque effect, recently set forth in Refs [33, 34], on one side seems so simple that even an amateur in the field can understand it (the idea, in fact, can only appear in the head of an amateur, who has not previously taken in a traditional explanation [33]). On the other side this approach seems very efficient since it gives a tool for drawing qualitative conclusions as well as making quantitative estimates about the processes in vortex tubes. It therefore seems worthwhile to familiarize the physics society both with the previous approaches, part of which have acquired the status of theory, and with the new explanation of the Ranque effect. However, in the second section we shall first set forth the fundamental notions of the hydrodynamics of vortex flows, because they are used more or less in all the theories. In the same section we shall introduce the major experimental results which were reproduced by different researchers. In the third section we shall show the inherent contradictions in the most advanced modern theories of the Ranque effect and the discordance between their conclusions and experimental data. Then in the fourth section we shall qualitatively consider the mechanism proposed, which explains the energy separation of gases in vortex tubes. In the fifth section we shall show what conclusions and quantitative estimates can be derived from this model of the energy separation. In the final section we shall indicate some topical problems that this approach presents.

2. Fundamental notions of the hydrodynamics of vortex flow and the characteristics of vortex tubes

The simplest example of a vortex motion of the gas or fluid is the rotation of its bulk as a solid body around an axis with a constant velocity ω. In this case the velocity v of the circular motion of a fluid element is related to the distance r from the axis of rotation by means of the formula

$$\omega = \frac{v}{r} = \text{const}\,. \tag{1}$$

This motion is usually called quasi-solid rotation or a forced vortex. The most illustrative and adequate example of such a motion is probably the rotation of a fluid together with its container when all the transient processes have died down. As for the widely used notion of 'vorticity' or 'velocity circulation' Γ,

$$\Gamma = \oint \mathbf{v} \cdot \mathrm{d}\mathbf{l}\,, \tag{2}$$

[35] we should note that it is hardly justified to use it to describe quasi-solid motion because in this case the value of Γ depends on the selection of the contour of integration. Note also that in a forced vortex the fluid is separated radially in kinetic energy. If an intensive forced gaseous vortex could be created in some way, then the gas would be separated radially in temperature because of the adiabatic compression of the outer layers and the expansion of the inner layers. Together with the radial distribution of the kinetic energy it would lead to an energy separation like that in the Ranque effect. However, this simple explanation, implicitly present in Ref. [36], cannot be considered plausible for vortex tubes, where the process, because of its continuity, clearly has a different nature.

Another extreme case of rotation of a fluid or gas is 'free' or 'potential' vortex described by the formula

$$vr = \Gamma = \text{const}\,. \tag{3}$$

In this case the value of the circulation Γ [see formula (2)] is the same for any closed contour, which encloses the axis of rotation, and is zero for any other contour [35]. Free vortices appear as the laws of conservation of angular momentum and mechanical energy dictate. Therefore, the typical velocity distribution for free vortices appears when the fluid (or gas) elements change their radii of rotation rapidly enough. Since the velocity of rotation of the fluid should increase infinitely as $r \to 0$, this motion is observed in its pure form only in the cases when there is no fluid at the vortex center, for example, in the hydraulic centrifugal atomizer or in containers of fluid with a discharge channel at the bottom (baths).

To describe plane transient flows between free and forced vortices, the so called Rankine vortex is used (a compound free-forced vortex), in which the velocity distribution has the form [14]:

$$v = \frac{C}{r}\left[1 - \exp\left(-\frac{r^2}{r_0^2}\right)\right]. \tag{4}$$

Here the constants C and r_0 characterize the vortex intensity and radial coordinate, by which the free vortex (2) is conventionally divided from the forced vortex (3). A flow close to Rankine flow can be observed when water is discharged from a bath up until a central gaseous funnel forms.

Occasionally a simpler formula is used to describe a real system:

$$vr^n = \text{const}\,, \tag{5a}$$

$$n = -1 \quad (0 < r < r^*)\,, \tag{5b}$$

$$n \leqslant 1 \quad (r^* \leqslant r). \tag{5c}$$

In particular, $0.45 < n < 0.8$ for $r^* < r < R$ and $0.4\,R \leqslant r^* \leqslant 0.6\,R$ for cyclone separators of radius R. Sometimes it is also called the Rankine vortex.

This preliminary consideration covers the basic models of flow, which different authors use for the mathematical description of motion of gas in vortex tubes.

To better understand the processes and structures of flow in vortex tubes, the distinguishing features of rotational three-dimensional flows in tubular channels should be considered in addition to plane vortices. We shall consider a long tube (Fig. 2), at the closed end of which there is a swirler or a gas distribution device to spin the gas when it enters the tube. There are various designs of swirlers [11, 14]. However, in practice only tangential or snail swirlers are used in vortex

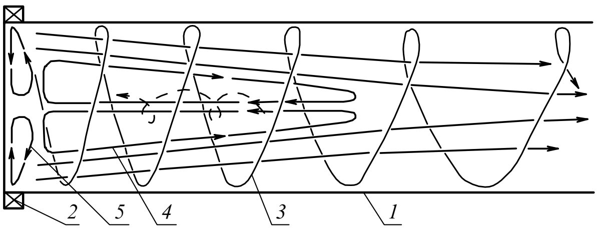

Figure 2. Typical flow pattern in tube near a tangential or helical swirler: *1* — smooth cylindrical tube with a closed end; *2* — swirler; *3* — peripheral vortex flow; *4* — central zone of reverse flows; *5* — face circulation flow.

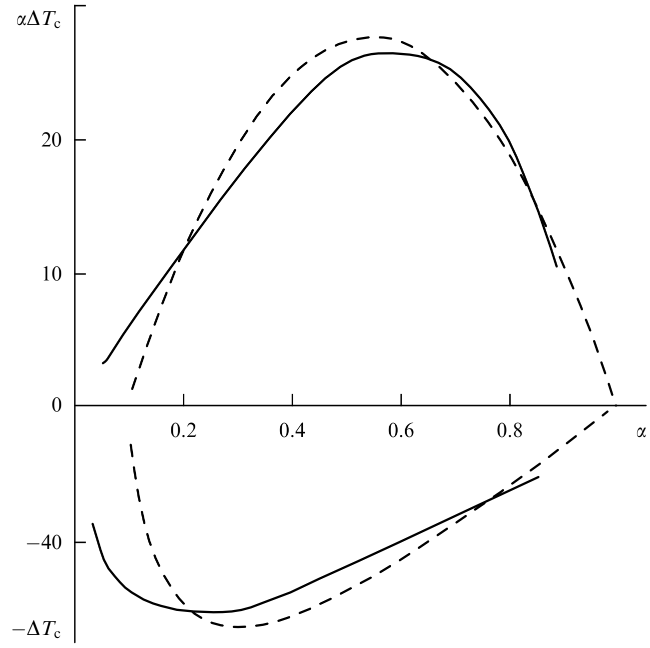

Figure 3. Typical experimental characteristics of a vortex tube (solid line) [3]: $P_0 = 0.6$ MPa; $P_c = 0.1$ MPa; $T_0 = 303$ K; $d_c/D = 0.48$; and calculations by means of formulae (61a) and (62a) (dotted line) for the same parameters of gas under the assumption that the 9% fraction of the total flow rate of gas flows through the boundary layer over the diaphragm.

tubes. For these designs, the formation of a reverse current in the central zone near the swirler is typical, with a diameter of about half the tube diameter and a length of some tens of the tube diameter [11]. Reverse currents appear because the circumferential velocity of the intensively rotating gas drops as a result of the wall friction as the gas moves along the tube, and consequently, the radial pressure differential also drops. If the velocity of the translational motion of the rotating gas along the tube is relatively small, i.e., the rate at which the pressure drops in the longitudinal direction is insufficient at the periphery of the tube, then the rapid decrease of the radial pressure differential along the tube leads to a negative pressure gradient on the tube axis and this gradient brings about the reverse current. This phenomenon is widely used, for example, to stabilize the flame in gas vortex burners [14].

Vortex tubes, which are used usually as cheap unattended refrigerators, can be of counter flow (Fig. 1a) as well as of parallel flow (Fig. 1b) types. Compressed air is fed into the Ranque vortex tube at a high rate (usually about the speed of sound) through one or more tangential input channels (nozzles) at the end of the tube. Hot gas escapes, as a rule, through a peripheral circular outlet at the opposite end of the tube, while cold gas is removed through a central outlet. This orifice can be located at either end of the tube: near the input nozzles (Fig. 1a, design with the counter flow) or near the hot gas outlet (Fig. 1b, parallel flow design).

Until recently counter flow vortex tubes have predominantly been used because of their efficiency [3], though it is already known [10] that the direct flow vortex tube, with the tangential input nozzle at a height of one half of the diameter of 'the energy separation chamber' (i.e., the tube), is more efficient in terms of temperature.

Since R Hilsch's work [37], the dependence of the temperature difference of the incoming and cold flows $\Delta T_c = T_0 - T_c$ on the relative mass flow rate of the cold gas $\alpha = Q_c/Q$ (here Q is the mass flow rate of the initial gas) is considered to be the base characteristic of the vortex flow.

Another important practical characteristic of a vortex tube is the specific refrigerating capacity $q_c = \alpha \Delta T_c c_p$ (c_p is the specific heat capacity at constant pressure). It describes the refrigerating power of the tube. Figure 3 presents the typical characteristics of a vortex tube [3].

The efforts of many researchers and designers, working with vortex tubes, have been focused on improvement of their efficiency by changing the relative and absolute dimensions of both the individual components and the tube as a whole. At present the following tube parameters are considered to be optimal [14]: diameter $D = 94$ mm; length $L = 520$ mm ($L/D \approx 5.5$); two inlet tubular tangential nozzles of diameter $d_T = 25$ mm (their total relative area $4S_T/(\pi/D^2) = 2(d_T/D)^2 = 0.14$); the relative outlet area for hot air $4S_h/(\pi D^2) = 0.052$; the outlet diameter for cold air $d_c = 35$ mm ($d_c/D = 0.37$); Mach number of incoming flow $M_0 = 0.4-0.5$. In other literature different, though close, geometrical parameters for vortex tubes are given as optimal, for example, in Ref. [3]: A Merkulov advises setting up a four-blade spider inside the tube at the hot end to reduce the vortex tube length to nine tube diameters; the relative diameter of the outlet for cold air is $d_c/D = 0.45$ or, in general, $d_c/D = 0.350 + 0.313\,\alpha$, where the cold air fraction α is taken from the operational interval for the vortex tube $0.2 < \alpha < 0.8$; the relative area of the flow section of the inlet nozzle is $0.085 < 4S_T/(\pi D^2) < 0.1$ and in this design only one nozzle with rectangular cross-section is used of width b and height h, in the ratio $b/h = 2$; the nozzle itself has the snail-like design of an Archimedean spiral; and Merkulov notes that the performance of the vortex tube improves greatly with an increase in linear dimensions while $D < 33$ mm, but further increases have no effect.

To summarize the data on the performance of different vortex tubes it is convenient to use the dimensionless quantity η that Hilsch introduced in Ref. [7]. This quantity is called the temperature efficiency and represents the ratio of the actual cooling value ΔT_c to the cooling value ΔT_s for isoentropic gas expansion from the parameters at the tube inlet (P_0 is the total pressure, T_0 is the stagnation temperature) to the pressure of the cold outflow P_c:

$$\eta = \frac{\Delta T_c}{\Delta T_s} \,. \tag{6}$$

The value of ΔT_s is related to the initial temperature T_0, the pressure differential $n = P_0/P_c$, and the adiabatic exponent k ($k = c_p/c_v$) by the expression

$$\Delta T_s = T_0 \left[1 - n^{(1-k)/k}\right] . \tag{7}$$

It turns out that the temperature efficiency of a vortex tube changes only weakly when the cooling effect is close to maximum: η_{max} changes from 0.47 for $n = 2$ to 0.5 for $n = 6$ for a particular tube (see Ref. [3]) and from 0.4 to 0.63 for different vortex tubes with the same inlet parameters (see Ref. [10]). Note that this clear experimental relationship between the cooling effects ΔT_c and ΔT_s represents the dependence of ΔT_c on the parameters of gas at the tube inlet and is by no means accidental.

The flow structure in vortex tubes (mostly in counter-flow tubes) has been studied many times (see, for example, Refs [3, 10, 14, 26]) and its pattern is quite clear, though not very simple. Most of all it resembles the flow pattern in a cyclone dust separator [14]. If we compare the absolute values of the circumferential v, axial u and radial j velocities, the circumferential velocity clearly has the highest value. The maximum of its radial distribution, approximately equal to the velocity of the incoming tangential flow, is offset to the tube wall in the section near the nozzle inlet and it shifts, decreasing in magnitude, towards the center in sections which are closer to the hot air outlet. As for the description of the radial profile of the circumferential velocity, opinions differ greatly: Merkulov and Kuznetsov [3, 10] think that the motion of wall layers is governed by the equation of a free vortex (3), Kurosaka et al., and Alekseev [25, 36] believe that the forced vortex (1) occupies the whole inlet section except the boundary layer, which builds up as the hot gas outlet is approached. If we compare the data on the tangential velocity distribution in Fig. 4 [10] with the data on flows in cyclones [14] [formulae (5a – 5c)], they are evidently similar.

The peak values of the axial velocity u are about half an order of magnitude less than the peak values of the tangential velocity v. The region of counter flow occupies a significant part of the tube section, and naturally fills the whole section of the cold gas outlet in the nozzle plane of the counter-flow vortex tube. A remarkable point is that the value of the longitudinal velocity of the cold reverse flow reaches minimum at the tube axis and sometimes a region appears in the diaphragm plane in which the velocity is positive. This means that a leak of the outer air into the tube has appeared.

The value of the radial component of the velocity j is 2.5 – 3 orders in magnitude less than the tangential velocity. In the case of a tubular chamber for energy separation (sometimes conical tubes are used which slightly diverges to the 'hot' end) the radial velocity is negative in almost the whole section (it is directed to the center) and it peaks near the edge of the cold air outlet.

Local measurements of the total temperature of gas (stagnation temperature) in counter-flow vortex tubes show that the gas layers have minimal energy on the tube axis near the nozzle section and the maximal energy near the hot gas outlet and also that the radial differential of the total temperature peaks at the nozzle section.

For common designs of uni-flow vortex tubes the flow pattern near the entry section is similar to the flow in the tube with a swirler when the tube is closed at one end (Fig. 2), and the pattern is very close to the flow near the cold air outlet in counter-flow vortex tubes. For such uni-flow vortex tubes the minimal value of the stagnation temperature is also near the entry section and because of this they have low efficiency.

It is worth emphasizing yet another common feature of vortex tubes of various designs, i.e., that the flow is highly turbulent and that these tubes are very noisy as a consequence.

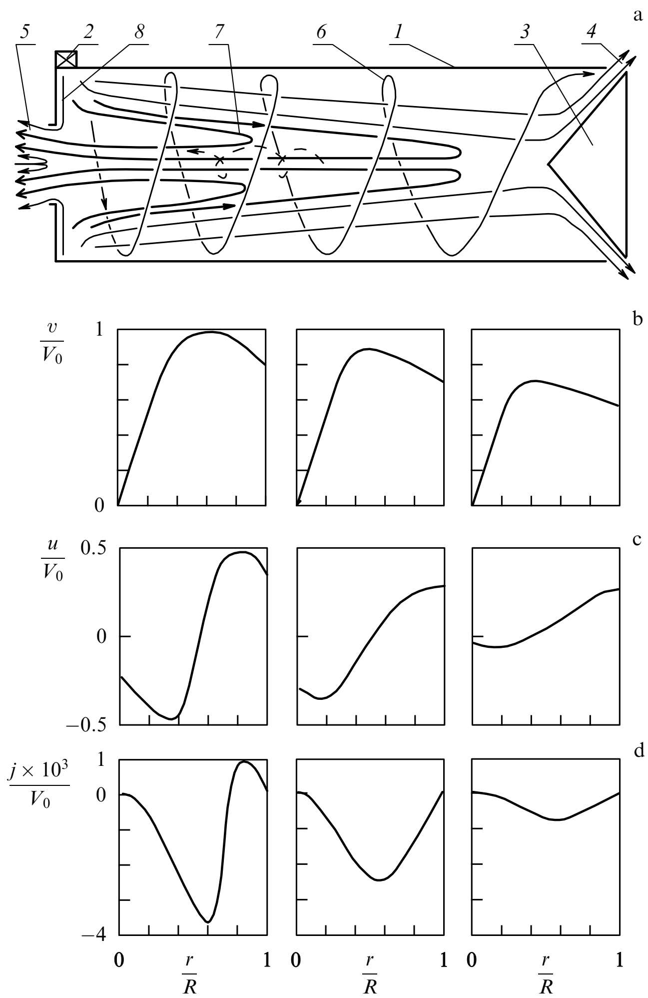

Figure 4. Flow pattern in a counter-flow vortex tube and typical radial profiles of tangential (v), axial (u) and radial (j) velocities in the pertinent cross-sections. *1* — tube with smooth wall; *2* — tangential or helical swirler; *3* — throttle (valve); *4* — exit of hot gas through ring slot; *5* — exit of cold gas through circular diaphragm; *6* — peripheral vortex flow; *7* — reverse vortex flow; *8* — radial flow in the boundary layer over the diaphragm.

In this section we shall not consider the features of particular vortex tubes, for which experimental data are not verified by multiple independent researchers. Such examples will be considered in subsequent sections because they require individual interpretation and because they often contradict existing theories.

3. Existing theories: inherent contradictions and contradictions with experimental data

Today the most advanced and popular theory among specialists [3, 4, 11, 14] is that the temperature separation of gas in a vortex tube is explained by intensive turbulent pulsation in the radial direction. According to this theory, turbulent elements adiabatically expanding and compressing in their motion in the high-gradient static pressure field (a pressure gradient appears because of high velocities of rotation of gas) 'get into cold cycles, transferring heat to the peripheral layers, while turbulence provides the source of mechanical energy for these cycles' [3]. The general flow pattern in a vortex tube is then made up of two vortices: an external vortex, which travels from the swirler to the valve, and an internal axial vortex, which travels from the valve to the diaphragm. In the external vortex the dependence of the velocity of rotation v on the radius r is described approximately by the potential vortex law (3). This law is derived from the law of conservation of angular momentum of a vortex flow. In the case of viscous flow, such a law of rotation means that tangential stresses should appear and which would decelerate inner layers and accelerate outer layers. Many researchers considered these stresses to be the major cause of the energy transfer from the inner layers to the outer, and consequently, explain in this way the temperature separation in vortex tubes. A more accurate consideration [3, 10] of the forces acting on a fluid element in a free vortex shows that the viscosity force accelerating the element on the side of the smaller radius is equal in size and opposite in direction to the viscosity force decelerating the element on the side of the larger radius. This means that the element will not change its velocity, i.e., there is no radial kinetic energy transfer in a potential vortex.

According to the theory of 'turbulent pulsations' a free 'vortex can start to break down on its radial boundaries, where the equilibrium of moments of forces is violated as a result of friction on wall and of interaction with axial elements,' and 'with the decrease of the circumferential velocity as the vortex travels along the tube, the radial gradient of the static pressure decreases as well and the vortex moves towards the axis. The decrease of the radial pressure gradient creates an axial pressure gradient. This last gradient forces the gas elements in the axial region to reverse their axial velocity and to move towards the nozzle section. The gas elements become intensively turbulent when they get into the axial region, and because of the high turbulent viscosity, they form a counter flow, rotating in accordance with the law of rotation of a solid body, i.e., this is a forced vortex with a constant angular velocity ω. The induced axial counter flow is swirled in its motion by a more and more intensive free vortex.' (Cited from Ref. [3]). In other words, the kinetic energy of rotation is transferred from the outer free vortex to the inner forced vortex and the thermal energy is transferred in the opposite direction, as noted above, due to radial pulsations in the presence of a high static pressure gradient. The principal conclusion of this theory is that 'turbulent transfer in circular flow will always take place when the radial temperature distribution diverges from the adiabatic law' [3]. (Here the adiabatic radial temperature distribution is understood to be the radial temperature distribution that would appear when a gas expands adiabatically in a pressure field with the same radial static pressure distribution as in the rotating gas.)

As for the flow pattern described by this model, it agrees with most of the experimental data (Fig. 4), though the explanation seems somewhat doubtful.

First, the assertion that the free vortex is internally stable, seems questionable if we recall that tangential stresses in a viscous fluid inevitably lead to the conversion of kinetic energy to heat. Let us consider a circular fluid element of unit length with density ρ and let it move between the radii R and $R + \Delta R$, having velocity V at radius R and moving in accordance with the law of a potential vortex (3). If we integrate the kinetic energy of the fluid over the radius, we arrive at the value of the kinetic energy for this element:

$$E_k = \pi \rho V^2 R^2 \ln \frac{R + \Delta R}{R} \,. \tag{8}$$

The power $\mathrm{d}W$ that is transformed into heat in an elementary ring of unit length with a fluid of dynamic viscosity μ and of radial gradient of circumferential velocity $\mathrm{d}v/\mathrm{d}r$, can easily be found from considerations of the dimensions:

$$\mathrm{d}W = 2\pi r \mu \left(\frac{\mathrm{d}v}{\mathrm{d}r} \right)^2 \mathrm{d}r \,. \tag{9}$$

If we integrate this power over all the ring volume, we obtain the value

$$W = 2\pi \mu V^2 R^2 \left[R^{-2} - (R + \Delta R)^{-2} \right] , \tag{10}$$

which characterizes the rate of conversion of kinetic energy of the potential vortex (3) into heat. If the ratio $\Delta R / R$ is small enough, then Eqns (8), (10) can be written as

$$E_k \approx \pi \rho V^2 R \Delta R \,, \tag{11}$$

$$W \approx 4\pi \mu V^2 \frac{\Delta R}{R} , \tag{12}$$

and the characteristic time

$$\tau \sim \frac{E_k}{W} = \rho \frac{R^2}{4\mu} = \frac{R^2}{4\nu} \tag{13}$$

can be obtained, in which the free vortex becomes a forced quasisolid vortex (1), i.e., one without slipping between adjacent rotating layers. Here ν is the kinematic viscosity. It follows directly from expression (13) that the free vortex becomes a forced vortex in a very short time when the radius of rotation is small. Note that for a typical radius of a vortex tube $R \approx 2$ cm and for the kinetic viscosity of air at the room temperature $\nu \approx 1.5 \times 10^{-5}$ m^2 s^{-1}, the time in which a potential vortex degrades because of internal viscosity forces is large: $\tau \approx 7$ s. Transition of the free vortex (3) to a small radius of rotation does not change the Reynolds number

$$\mathrm{Re} = \frac{\rho v r}{\mu} = \frac{v r}{\nu} , \tag{14}$$

so it should not bring about turbulent flow. The high level of turbulence observed in experiment probably has another cause.

The second inherent contradiction of this theory is more important and refers to the mechanism by which heat is carried away from the flow center. In fact, on one side it is known [39] that the heat propagation velocity in a turbulent jet exceeds the momentum propagation velocity. On the other side it is also known that the adiabatic motion of gaseous volumes in the presence of a pressure gradient causes an adequate temperature field to be established (recall, for example, the drop of temperature in the atmosphere with altitude). However, the simultaneous application of these two considerations to processes in vortex tubes — on which, in essence, the theory in question is based — seems unjustified, since the second consideration suggests that the process is adiabatic while the first, on the contrary, assumes that there is an intensive heat exchange. The Prandtl number, i.e. the ratio of the viscosity and diffusion coefficients, is less than unity for turbulent jets [39] but this only means that the spatial distribution of heat levels off more rapidly in the presence of turbulence than that of kinetic energy, but the distributions of either type of energy do level off, and these processes are much more intensive than in the absence of turbulence. In its primary sense turbulence is an intensive mixing by which system parameters are averaged. Turbulence cannot, therefore, produce spatial energy separation by itself, and if some unknown mechanism brings about this separation, turbulence must lower its efficiency. The proposition that turbulence can speed up the establishment of an equilibrium at which the temperature distribution in the pressure field of a vortex flow will match the adiabatic law

$$P^{k-1}T^{k} = \mathrm{const} \tag{15}$$

is approximately equivalent to the proposition that intensive mixing helps a faster stratification of two immiscible fluids or assists the separation of two gases with different molecular weights in the field of gravity.

The basic idea behind another theory is 'the hypothesis that excess energy is transferred by viscosity forces from the axial layers to the peripheral ones as a result of their different angular velocities' [10]. This theory is a modification of the view of energy transfer as the 'spin-up' of outer layers of gas by rapidly rotating inner layers. These approaches got a 'second wind' by introduction of a number of new, but not quite obvious assumptions. Kuznetsov [8 – 10, 40] supposes that in counter-flow vortex tubes the central vortex (Fig. 4) forms exclusively near the valve from the gas which the peripheral vortex carries and that it rotates approximately in accordance with the law

$$vr^{-2} = \mathrm{const}\,, \tag{16}$$

and that the gas does not get through the boundary between vortices. In its foundation this model contradicts the experimental data (including that of the author of the hypothesis [10]) on the actual flow pattern in vortex tubes. Firstly, over all the surface on which the longitudinal velocities are zero (i.e., on the interface between the central and peripheral vortices) the radial velocity has an essentially non-zero value, and as noted earlier, the radial velocity reaches its peak value just near the diaphragm edge. This makes it possible to conclude that the inner vortex forms through the whole length of the vortex tube, and primarily in the nozzle section. Secondly, the increase of the rotation velocity of the inner vortex in its motion to the outlet can only mean that the energy of rotation is transferred from the peripheral vortex to the central vortex, but not vice versa. This conclusion is supported by the increase of the angular velocity of the inner vortex with radius [a direct corollary from formula (16)].

An interesting and unexpected approach to the explanation of the Ranque effect was set forth in a series of works (Refs [22 – 26]) from the University of Tennessee. The basic idea probably came to mind when the undesirable temperature separation of a gas flow was successfully suppressed in a so-called 'annular cascade' using an acoustic suppressor. The mathematical model [22], which was constructed under certain assumptions, demonstrates that the loud 'whistle' specific to vortex tubes speeds up the peripheral layers of the vortex flow if it is caused by the basic circulation mode of acoustic vibrations inside the tube. Acoustic streaming provides a mechanism for this acceleration. Although no quantitative estimates have been derived explicitly from this model, its authors believe this consideration to be sufficient to explain the vortex effect. The experimental sections of Refs [22 – 25] show that the temperature increases significantly at the axis of a counter-flow vortex tube with a closed 'cold' exhaust for the resonant suppression of the basic mode of the vortex whistle by an acoustic suppressor, installed in the perforated energy separation chamber. According to the mathematical model [22] this phenomenon also has to be accompanied by a rearrangement of the forced vortex, which fills the vortex tube, into a free vortex and this fact is supposedly supported by an experimentally observed change in the gas flow pattern at the tube outlet for resonant noise suppression: the outlet flow rearranges itself from a radially divergent flow into a jet flow.

Close inspection of the main work [22], in which both a theoretical consideration and experimental data are presented, brings up a number of questions, and consequently, some doubt in the validity of the approach. Firstly, it seems strange that a counter-flow vortex tube with a closed cold air outlet (in fact, a uni-flow vortex tube) is selected as 'the simplest model' both for the theory and for experiment, yet the existence of a return flow is totally ignored. Secondly, the authors present no data to verify that the peripheral layers are accelerated by the action of sound; the rearrangement of the outlet flow from a divergent flow into a jet flow under resonant noise suppression can attest only that the total angular momentum of outlet gas decreases. This seems quite natural since the noise suppression resonance means a much closer association between the rotating gas and the gas in acoustic chambers, through the holes in the perforated tube, giving rise to stronger friction. The stronger friction should retard the flow rotation, also decreasing the Ranque effect temperature differential.

If the earlier works [22 – 25] could convince some reader that this approach is valid since their experimental data demonstrate a clear relationship between the sound level and temperature separation effect, the final work [26] (not, unfortunately, publicly available) can convince anyone except the authors that it is not. In Ref. [26] the following facts were established by radial measurements in an installation of increased diameter. Firstly, with resonant sound absorption the cold gas increases somewhat in temperature, but only near the axis of the system, while it lowers perceptibly in temperature in a substantial range between 0.42 R and 0.80 R (here R is the tube radius). This rearrangement of the temperature field is apparently related to the build-up of the return flow that the authors discovered. This build-up can be explained by the aforementioned steep growth of friction, and as a consequence, by an increased inverse pressure gradient on the system axis. Secondly, radial measurements have not revealed any increase of the circumferential velocity in the peripheral layers in the presence of a 'vortex whistle'. Thus, the energy separation of gas in vortex tube can hardly be explained by such fine effects as 'acoustic wind', though the results of Refs [22 – 26] are undoubtedly of interest because of the factual data they contain.

The recent mathematical model of a helical vortex [16, 17] has enabled an estimate of the precessional frequency of the vortex to be made, the return flow on the axis of an intensive vortex flow to be described [43], and interesting experimental data on the change of the helical symmetry in a specifically perturbed vortex fluid flow to be explained [44]. However, this model has not clarified the reasons for the energy separation.

The approach of Ref. [13] assumes that a polytropic process (with constant specific heat) proceeds in a vortex tube and that the polytropic index varies along the tube, but it has not clarified this issue any further.

In the recent paper [19] V Merkulov showed theoretically that a negative temperature gradient develops in the field of mass forces when there is convection and density stratification. However, there is a long way to go from the assertion that 'this makes it possible to give a rational explanation of the temperature stratification in the atmosphere and in Ranque-Hilsch tubes' to a clear description of the processes in the vortex tube, and it remains unclear whether this distance can be covered.

Though the parameters of common vortex tubes can be calculated in one way or another, for example, using the models [3, 10] above, some experimental results are in direct contradiction with established views. For example, the results of the experiment described in Ref. [45] defies not only all the familiar theories but also common sense (if we did not mind that the Ranque effect itself defies common sense to a certain extent). In a vortex cooler (VC), about which the author says only that it is 'an improved vortex tube' (VT), both outlet flows have been cooled down. In addition 'the band spectrum radiation was recorded by an IKS-29u infrared spectrophotometer in the range 5 – 12 μm at the cold gas outlet when the input pressure was larger than 2 MPa. 'The radiation intensity increases with input pressure. In certain modes of operation of the VC a blue radiation was observed to originate from the flow core (the intensity and spectral distribution were not measured). When a rod of diameter 2 – 4 mm, with one end fixed in a plane bearing, is inserted along the VC axis, the rod rotates in the opposite direction to the main flow and the velocity measured by strobotachometer is 3000 revolutions/min.' The author [45] set forth his own model of temperature 'separation', according to which the reason of separation is 'wave expansion and compression of gas. The tube itself can be a resonator of acoustic vibrations. The peculiar feature of its action is a very high noise level. Thus, a relatively high and rapid heating of part of the gas becomes possible at the expense of other parts and thus they do the work of adiabatic expansion and compression on themselves.' There is no explanation why the outer layers heat up as a result of acoustic vibrations. The author's opinion that the reason for gas cooling in a vortex tube is 'adiabatic expansion doing external work' since 'gas expanding in a VT or in a VC already does work inside the tube on the previously injected gas and on the surrounding atmosphere since the gas passes to the atmosphere' seems unconvincing. Even in the case of a supercritical pressure differential, gas can be adiabatically cooled down by a supersonic flow in a Laval nozzle, but any attempt to retard the supersonic flow will reestablish the initial gas temperature [38].

Before we come to the next experiments I want to note once again that specialists [20, 21] consider only the 'hypothesis of vortex interaction' [3] as a theory which ascertains that the cause of energy separation is 'adiabatic compression and expansion of turbulent vortices in the field of centrifugal forces with a non-adiabatic temperature distribution' [14]. Therefore, experimental results are usually, as in this case, first compared against this theory. As yet another argument against this theory, the results of [29] can be cited, according to which there is an energy separation in a serial air vortex tube when a nearly incompressible fluid (water) is used as a working medium. Although the energy separation was not as significant as in the gas (the temperature difference $\Delta T = T_h - T_c$, where T_h is the temperature of hot flow, varied from 10 K to 20 K when the input pressure varied from 20 to 50 MPa), it was noticeable. In this case the temperature of cold flow T_c was higher than the initial water temperature T_0 but lower than the water temperature T_g at the swirler outlet and lower than the 'hot' water temperature: $T_0 < T_c < T_g < T_h$.

Balmer [29] notes that the change of air density is about 700% when this vortex tube operates in normal conditions ($P_0 = 0.8$ MPa). At the same time the water density increases by 2% when the pressure is increased to 50 MPa. The author concludes that in vortex flow the temperature separation mechanism is independent of the compressibility of working medium. He shows that these results do not violate the second principle of thermodynamics and that the traditional theory [3, 14] cannot explain them.

A just criticism of the theory of 'turbulent migration' [14] can be found in Ref. [22], where Kurosaka gives examples of apparatus, in which there are turbulent vortex flows and in which gas is tangentially input as in vortex tubes but in which there is no temperature separation. The author considers the inability to explain these facts as the major drawback of the conventional theory.

We shall dwell on yet another result [33, 34] because it gave impetus to the development of the new hypothesis set forth in this paper. As noted above, the theory of 'turbulent migration' is directly followed by one principal conclusion that if there is vortex flow in the tube and all the gas or its part is output on the swirler side (as in a Ranque cyclone dust separator [2] or in water-cooled vortex tubes [3]), then heat is intensively transferred from the central zone. In part to verify this proposition an experiment was conducted at the 'Fialka' serial micro-wave (MW) plasma installation using gas-vortex heat insulation, stabilization of plasma fluid [1] on the plasmotron axis, and side supply of radiation. The goal of the experiment is to compare the efficiencies of vortex heat insulation of plasma in 'direct' vortex flow and in a cyclone-type flow typical for vortex tubes. In the first three series of experiments the plasmotron, which produced the plasma, was linked with a laboratory apparatus for plasmochemical processing of solutions [46, 47]. In the first scheme (Fig. 5a) the outlet of the plasmatron's smooth-wall quartz tube *1*, 45 mm in inner diameter, was joined with the massive tubular

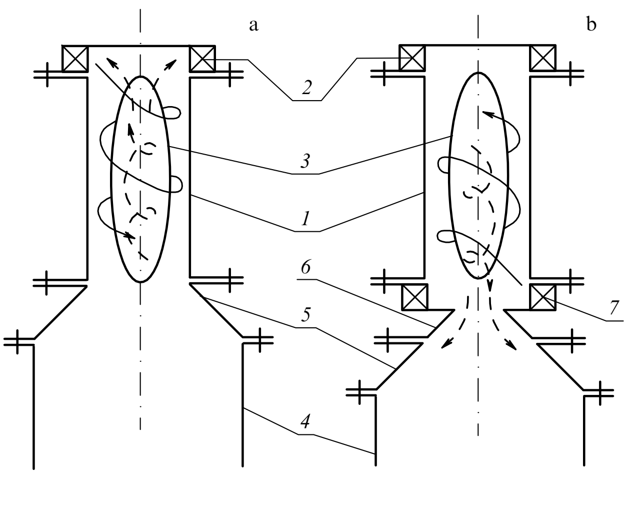

Figure 5. Pattern of vortex heat insulation of plasma [33, 34]: (a) in a 'direct' flow; (b) in a 'cyclone'-type flow. *1* — quartz tube with smooth wall; *2* — upper swirler; *3* — MW plasma formation; *4* — reactor in plasmochemical installation [46, 47]; *5* — connecting cone; *6* — water-cooled nozzle-diaphragm; *7* — lower swirler.

heat-insulated reactor *4*, made of temperature-resistant steel 80 mm in inner diameter, using the heat-insulated uncooled cone *5*, 80 mm in height. The vortex stabilization of plasma *3* of the MW discharge was achieved using the original swirler *2* mounted in the lid of the plasmatron. In two other schemes (Fig. 5b) the water-cooled nozzle-diaphragm *6* (opening 26 mm in diameter) and the lower swirler *7*, the total height of which was 43 mm, were placed between the quartz tube *1* and the cone *5*. The nozzle was included in the general coolant loop of the plasmatron. In the second scheme, air was supplied through the upper swirler *2* as in the first (Fig. 5a shows the flow pattern), while in the third scheme it was supplied through the lower swirler *7*, i.e., in the third scheme the plasma was stabilized in a 'cyclone-type' flow. Calorimetric and electrical measurements combined made it possible to determine the MW radiating power W_p absorbed by the discharge and the heat output W_T of the plasmotron's coolant loop. Figure 6 presents the results of these experiments (the numbers of curves *1 – 3* match the numbers of the schemes) as the relative heat losses W_T/W_p versus the specific energy supply to the discharge $J = W_p/Q$ (Q is the consumption of plasma gas).

Since in these experiments the heat losses in the plasmotron were too high because of the uncontrolled heat flux from reactor *4* (Fig. 5) we carried out additional experiments, in which the plasmotron was disconnected from the reactor and turned upside down. As a result, the nitrogen plasma emitted upwards escaped into the ambient air. Since scheme *1*, in which the plasmatron outlet had no diaphragm, was less efficient (curve *1* in Fig. 6), additional experiments were conducted for fixed geometry of the plasmatron with the second swirler *7* and nozzle-diaphragm *6* (Fig. 5b). In Fig. 6 the results of these experiments are presented by curves *2'* (stabilization in 'translational' vortex flow) and *3'* (stabilization in cyclone-type flow).

These results show that the change-over to a cyclone-type flow reduces heat losses by a factor of five and the thermal efficiency of an ordinary plasmotron becomes not less than that of industrial complex systems, in which gas is supplied through porous walls of the chamber [48]. The result seems quite natural if we compare the gas flow patterns in the plasmatron in Figs 5a, 5b. In fact, in the case of the 'uni-flow' geometry (Fig. 5a) the reverse vortex flow ejects a portion of the hot gas from discharge into wall layers. In the case of the cyclone-type flow, the discharge gas is instantly withdrawn from the plasmatron. In spite of the apparent efficiency of cyclone-type flows for heat insulation of plasma and other high-temperature and reacting systems, these flows have not been considered earlier in this context. Meanwhile scrupulous research has been conducted on the vortex stabilization of plasma in 'uni-flow' systems (see, for example, Refs. [49 – 51]). These facts can be explained only by the popularity of the theory of turbulent withdrawal of heat from the central part of a vortex tube [3].

It should be noted that experimental data close to the above results were obtained earlier, but were not properly examined. As an example, we can offer the work [52], in which Chigier et al. describe an experiment on flame stabilization by rotating gas flow. Unlike many studies of flames in rotating flows [53], in Ref. [52] a wire mesh screen was used to spin the air around the flame (Fig. 7). The same schlieren photographs of the flame jet as in Ref. [52] are also presented in Ref. [14]. Analysis of feasible flows of air around the flame in this experiment shows that the flow inside the wire mesh (Fig. 7) reproduces roughly the flow typical for cyclones and vortex tubes. In fact, rotation of the net screen creates a radial air

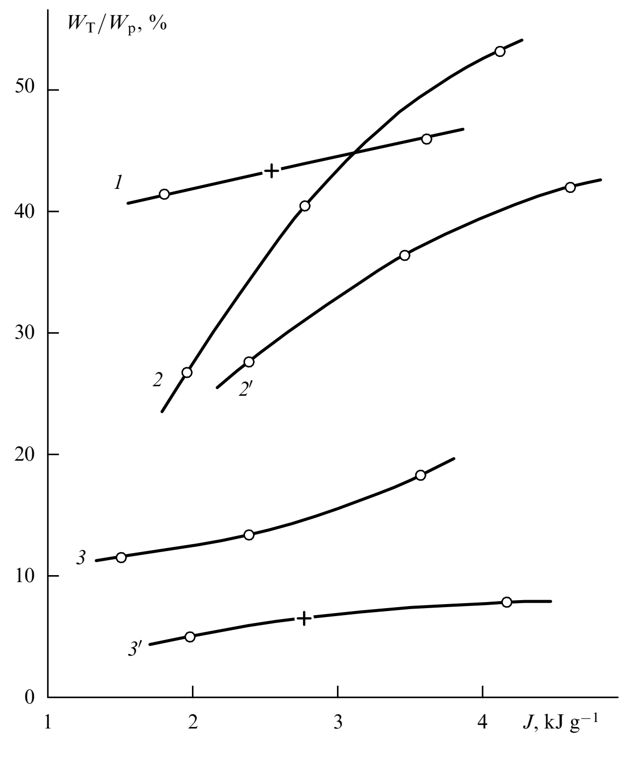

Figure 6. Dependence of thermal losses on the energy input into the discharge for different schemes of vortex stabilization [33] (see Fig. 5): *1* — scheme with 'direct' flow without outlet; *2* and *2'* — schemes with 'direct' flow with outlet (*2'* without reactor); *3* and *3'* — schemes with 'reverse' flow (*3'* without reactor).

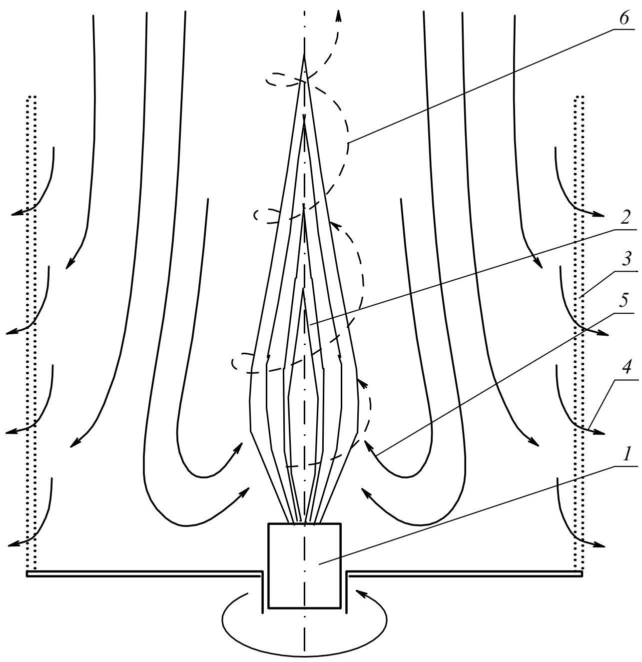

Figure 7. Air flow patterns in the experiment [51] on flame stabilization using a rotating net screen: *1* — burner; *2* — flame; *3* — rotating cylindrical metallic net screen; *4* — expelled air due to centrifugal forces; *5* — flow of air sustaining burning; *6* — flow of combustion products.

pressure gradient. Therefore, air will flow outside through the net. This flow through the net and the consumption of air required to maintain the flame must be compensated by the air inflow through the open end of the system contrary to the central flow of combustion products. In the experiment described, the flame temperature was higher, its length was longer by a factor of three, and its stabilization was many times better. Rotation of the net screen causes the turbulent flame to become a laminar one. The results of Ref. [52] make it possible to conclude that turbulence and intensive heat transfer from the central zone are not integral features of cyclone-type flows. These conclusions, which were not made in Ref. [52], are in radical conflict with the dominant theory of turbulent heat transfer from central zone of vortex tube.

Thus, analysis of the existing theories of the Ranque effect and their comparison with experimental data demonstrate principal contradictions and reveal a noticeable negative influence of these incorrect theories on the advancement of allied fields of science and technology.

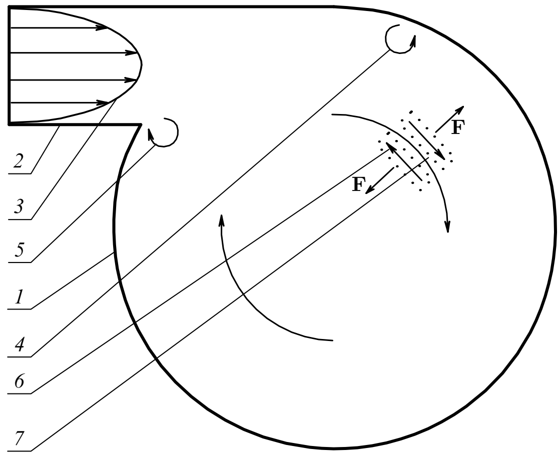

Figure 8. Formation of turbulent elements at the vortex tube inlet and their separation in the field of the centrifugal forces: *1* — tube wall; *2* — tangential inlet nozzle; *3* — velocity profile at the vortex tube inlet; *4* — microvortex which forms as a result of interaction between the tangential flow and cylindrical wall; *5* — microvortex which forms as a result of interaction between the tangential flow and vortex flow; *6* — gas element with a negative velocity pulsation; *7* — gas element with a positive velocity pulsation; **F** — resultant force.

4. New approach to the Ranque effect

Analysis of the bulk of experimental data shows that any hypothesis which claims that it can explain the Ranque effect should answer the following question: how a significant part of the incoming tangential flow reaches the vortex center without the initial kinetic energy and without the equivalent thermal energy. The fact that the kinetic energy of the central part of the vortex is close to zero follows immediately from the axial symmetry of the system, but how the stagnant gas near the axis has not been heated up in the course of its deceleration in circumstances where the central portions of gas are constantly refreshed is unclear. The hypothesis proposed gives a simple answer to this principal question: only the portions of incoming flow with low initial kinetic energy get into the vortex center and the mechanism, by which these portions get there, is the separation of flow elements with different circumferential velocity in the field of centrifugal forces.

Let us clarify the essence of the hypothesis. Because of friction and turbulence both at the outlet of the tangential nozzle and in the tube itself, there are flow elements with different velocities, i.e., they have different kinetic energies, all other factors being equal. We shall consider the shape of the distribution function of gas volumes in velocity later. However, we shall keep in mind that in this distribution the velocity varies in different layers from zero to $V_{\max}$ as a result of the gas flow 'sticking' to the fixed wall. Imagine that there are two microvolumes at the same radius in the rotating gas (Fig. 8) and that one of the volumes has a positive fluctuation of the circumferential velocity while the other has a negative one. The existence of different tangential velocities for the same centripetal acceleration will bring about separation of these elements: the faster element will move from the center while the slower element will shift towards the center. Thus, the periphery of the flow will gain fast gas, while the central core will gain slow gas. As a result, a stagnant gas will gather in the central region with reduced pressure and will experience, because of low heat conduction, nearly adiabatic cooling when it expands in circumstances where the pressure drops from the initial pressure at the tube inlet to atmospheric pressure, while on the periphery the fast gas will be partially slowed down and heated because of friction caused by the walls. In other words, the temperature separation of gas is equivalent to effects which anyone can observe when he stirs his tea: all particles that move slower than the surrounding liquid, i.e. sugar, tea-leaves, and bubbles, gather at the vortex center.

Thus, we can formulate that the cause of energy separation of gas in vortex tubes is the centrifugal separation of turbulent elements in tangential velocity.

This hypothesis allows a qualitative explanation to be given for the bulk of the reliable experimental data available and also to certain peculiar features of the design of vortex apparatus. Some of the relevant considerations together with numerical estimates are presented in the next section. Here we dwell only on certain fundamental conclusions and discuss qualitative results.

The first conclusion we can make if we accept this hypothesis is that since the cooling of the central layers results from two simultaneous processes, namely, the centrifugal separation of 'stagnant' elements and their adiabatic expansion, energy separation will take place, though to a much lesser extent, even if the second process is impossible. In this case a small part of the initial accumulated pressure energy, first converted into kinetic energy, will however, be transferred to the central layers of the vortex and inevitably transformed into heat. Therefore, the temperature of 'cold' water at the vortex tube outlet will be greater than the initial temperature, but of course, less than the stagnation temperature at the nozzle exit (where all the kinetic energy is transformed into heat and is divided equally between all parts of the flow) and especially less than the temperature of 'hot' water, which accounts for an disproportionally large quantity of kinetic energy, then transformed into heat. The results of Ref. [29] fit well into this picture.

Secondly, it should be noted that with this hypothesis the quantity η in Eqn. (6) acquires a natural meaning and that it should still not exceed unity.

The third fundamental conclusion refers to the cause and role of turbulence in the energy separation process. The forced vortex occupies the central part of the vortex tube and complies with the Raleigh stability criterion [11]:

$$\frac{\mathrm{d}(\rho v/r)}{\mathrm{d}r} > 0\,. \qquad (17)$$

This criterion means that the turbulent pulsations must be damped out, not amplified. The experimentally observed high level of turbulence in vortex tubes is caused by radial motion of 'slow' gas elements, i.e. turbulence is brought to the forced vortex from the outside by the inhomogeneous incoming tangential flow. Clearly, if the scale of this turbulence is small in comparison with the dimensions of the system, then the energy separation will be insignificant: 'slow' elements will be 'washed out' before they get into the vortex center. The characteristic sizes of microvolumes with essentially different forward velocities in the tangential nozzle depend on the transversal dimensions of the nozzle. These considerations suggest the design of the entry nozzle: its size should be as great as possible. Clearly, this conclusion explains why a single-thread spiral or tangential swirler, with a very impressively sized nozzle, is used in many designs of entry nozzle [3, 4, 10]. At the same time, for example, in swirlers used to stabilize discharges, the number of tangential gas inlets, as a rule, is not less than four, since studies [50, 51] have shown that a smaller number of tangential swirler slots does not provide a proper radial symmetry of flow.

Temperature separation is surely affected not only by the turbulence in the incoming jet, but also by the resultant turbulence in the temperature separation chamber, the source of which is the mixing zone of the incoming jet and the vortex flow (Fig. 8). Turbulent perturbations transfers the energy of the 'hot' peripheral gas from the tubular boundary layer to the cold central flow. Therefore, either the peripheral turbulence should be suppressed (by polishing the wall or rotating the chamber), or the tubular boundary layer should be additionally cooled (in vortex tubes with a water-cooled temperature separation chamber). All the above methods were employed in different designs of vortex tubes [4, 5, 10]. Ambiguous results of experiments of different authors on the efficiency of vortex tubes with a rotating tubular chamber [3, 10] are probably related to the twofold role of turbulence in the energy separation process. If rotation of the wall suppresses the initial turbulence (i.e., smooths out the velocity profile immediately after the nozzle inlet), then the energy separation is less efficient; if rotation of the wall suppresses only the turbulence in the boundary layer because of which the retarded high-temperature flow elements are brought from the wall into the center of the vortex tube, then the energy separation is more efficient.

It follows from the model in question that the vortex tube should not be a 'counter-flow' vortex tube (Fig. 1). In fact 'uni-flow' vortex tubes, rejected earlier because of their low efficiency, can be even more efficient than 'counter-flow' tubes provided that the return vortex is suppressed on the system axis. In Ref. [10] a tangential nozzle of height equal to the radius of the vortex tube was used for this purpose and, incidently, provided the maximum characteristic scale of turbulence. The flow patterns in a counter-flow vortex tube (Fig. 4) are such that suppression of the return vortex is extremely undesirable for efficient operation of the tube. Therefore, the height of the nozzle in a counter-flow vortex tube cannot be so large.

The initial turbulence is formed not only in the tangential nozzle itself, but also at the nozzle outlet (Fig. 8). As a result of the interaction between the flow and the wall of the vortex tube, local vortices are formed and rotate in the opposite direction to the main flow in the vortex tube. If now we recall the high stability of vortices in a low-viscosity medium [34] and take regard for the fact that the average velocity of gas in a resultant local vortex is less than the velocity of the main flow (i.e., centripetal forces shift this local vortex to the tube center), then the phenomenon of 'counter-rotation' of the rod, which was observed in Ref. [45], can be explained. So, if the design features of the vortex cooler were more favorable to the formation of 'counter-rotating' local vortices than to the formation of 'co-rotating' ones in the boundary region between the inward jet and developed vortex flow (Fig. 8), and if the scale of the former local vortices was sufficient for them to reach the system center without their disintegration, then a 'counter-rotating' vortex could form at the center of the vortex cooler. This central vortex would break down under extended contact with the outer vortex but as was mentioned in Ref. [54] the vortex cooler had a short length, probably, insufficient for such a contact to take effect.

If we follow the Prandtl theory of 'shift path' [35, 38] or later models of 'vortex filaments' [54], it is natural to expect that the radial and tangential pulsation velocities of vortex motion are of the same order in magnitude. According to the mechanism of formation of turbulence in vortex tubes (Fig. 8) the tangential pulsation velocity has the same order of magnitude as the tangential velocity. As noted above, the local values of the tangential velocity at the inlet to the vortex tube can reach and even exceed the velocity of sound [3, 10]. Remixing turbulent gas volumes with such high pulsation velocities in a variable pressure field will inevitably bring about high-intensity sound waves. Although the tangential velocity of a steady vortex can probably not exceed the velocity of sound because of inevitable development of a continuous system of shocks [34], the presence of regions in which the velocity is supersonic, is possible [3, 10], and thus shocks, or more precisely, low-intensity shock waves, will develop. An increase of the pressure differential at the tube inlet will be accompanied by an increase in the gas velocity in supersonic regions of the flow and by an increase in the intensity of shocks inside the vortex tube. (We recall that the vortex cooler [45] operated at an unusually high inlet pressure.) The propagation of shock waves in gases with slow excitation of some molecular degrees of freedom leads to a significant rise in temperature (sometimes, by a factor of 2 to 3) on the shock wave front in comparison with the final temperature behind the front [55]. In turn this rise can bring about a noticeable vibrational-rotational excitation of a fraction of the molecules, the radiative relaxation of which will result in a band spectrum. If we take regard for the fact that a low gas temperature is favorable for the appearance of highly excited molecules [56] when a part of the molecules are vibrationally excited, then the final result of this process may be not only the electronic excitation of molecules with a subsequent relaxation through radiation in visible light but also the dissociation of a part of the molecules. Probably, Fin'ko observed processes of this kind when he conducted experiments with a vortex cooler [45].

Energy release from acoustic tube through acoustic vibrations and electromagnetic modes must, of course, decrease the total energy of outlet flows in comparison with the initial energy of the gas, but probably not to the level at which both outward flows have temperatures sufficiently less than the initial one [45]. Since Fin'ko conveys in Ref. [45] that the 'alignment' of the outlet flow causes a rise in the temperature of 'hot' gas and then, as can be concluded from the data he presents, the energy balance is quite satisfactory, the reason for the imbalance should be sought in the rotation of the outward 'hot' flow. Taking into account the fact that a 'counter-rotating' vortex occupies a significant part of the 'cold' outward flow [45], in accordance with the law of conservation of angular momentum the intensity of rotation of the 'hot' flow should be very high which can bring about radial temperature separation of the gas in the output device itself, where the temperature of 'hot' flow is measured. Therefore, since it is in common practice to measure temperature at the flow axis, the results could be highly underrated.

In discussing the hypothesis, we began to doubt the mechanism by which the initial turbulence perturbations are shifted. This doubt arose because the theory of turbulence, based on the ideas [35, 38] of Prandtl, who considered the propagation of turbulent volumes to be similar to the motion of molecules of a gas, is much outdated. Today it is common practice to consider vortex perturbations of a certain scale and intensity rather than pulsations of velocity [54]. In addition, the lift (Zhukovskii) force

$$F = \rho v \Gamma \,, \tag{18}$$

where v is the flow velocity relative to the entity with circulation Γ, has to be taken into account when interaction of vortices with an encircling flow is considered. The comparison of the acceleration, for which the lift force is responsible, and the additional centripetal acceleration applied to a 'retarded' microvortex of radius r_{m}, the forward velocity of which is half the circumferential velocity in the vortex tube (Fig. 8), yields a ratio of the order R/r_{m}. In addition, the direction of the Zhukovskii force depends on the sense of rotation of the vortex past which the fluid is flowing, i.e., if microvortices, which have been formed upon interaction of the incoming tangential flow with the inner side of the wall of the vortex tube, accelerate towards the center, then microvortices, which have been formed upon interaction of the incoming flow with the developed 'main' vortex, must for the most part experience an acceleration away from the center. There are weighty arguments against consideration of the lift though it has a big relative value. Firstly, it is doubtful that the results obtained for a steady problem on the straight, uniform in height, flow past an adjacent vortex can be extended to the case of a flow past turbulent vortices, for which the typical time of disintegration is about the time of one-half revolution [39], especially, if we consider the fact that the flow has a highly variable velocity (at distances of the order of the diameter of the vortex around which the flow is taking place) as well as a sufficient pressure gradient. Secondly, the assumption that any perturbation of the rapid flow brings about vortices in a real gas seems somewhat unjustified, especially, if we recall laminar viscous flows of gas in channels.

A purely theoretical consideration requires various assumptions to be made. So, not to make an erroneous conclusion, we conducted a simple experiment to resolve this problem (Fig. 9). A deep circular vessel of 245 mm in diameter was filled with milk-colored water and fixed axisymmetrically on a grinder's wheel rotating horizontally at a speed of two revolutions per second. When the relative motion of the water with respect to the vessel stopped, an ink-colored water jet of 5 mm in diameter was poured into the rotating white water from a height of 0.2 m at a radius of 80 mm. The estimate of the Reynolds number (14) for the flow around the jet showed that turbulence should develop at the region where they merge. (Generally turbulence develops for a sufficiently small Reynolds number when jets merge [38].) The experiment was performed to check whether the mechanism proposed by this hypothesis plays a dominant role in radial shifts of turbulent entities. If so, the 'ink' would shift towards the center because it has a smaller velocity. If the acceleration is caused primarily by the lift, then the resultant turbulent entities would diverge in different directions depending on the sense of their rotation. After several seconds all the liquid inside the circle of the 'radius of injection' was ink-colored while the rest of the liquid in the vessel remained white. This experiment indicates directly that turbulent perturbations, which have a smaller tangential velocity than that of the encircling flow, shift towards the center and this is exactly the reason for the energy separation in vortex tubes according to the hypothesis we propose. The effect of lift seems to be less important.

a

b

Figure 9. Scheme of an experiment for the demonstration of the shift of retarded microvolumes to the axis of rotation: a) injection of ink into a rotating deep vessel filled with a milk-white fluid; b) in several seconds the inner region becomes colored.

To understand why such devices as the conical temperature separation chamber and the cross on the tube axis near the 'hot' outlet are widely used in designs of vortex tubes [3], we should call attention to the negative effect of the boundary layers which build up near the tubular wall, and more importantly, near the diaphragm and valve (Fig. 1). Through the face boundary layers hot or non-separated gas gets into the central reverse vortex or directly into the cold air outlet. While the trouble with the boundary layer on the diaphragm has not been resolved to any extent [3], the runoff of the hot gas into the return vortex by the valve surface can be suppressed in various ways (these methods are mostly empirical and their effectiveness can be checked by the efficiency of temperature separation). These methods are a longer temperature separation chamber, the length of which exceeds that of the return vortex; a cross on the axis near the hot outlet to retard rotation of the flow and to restrict the length of the return vortex; and a conical temperature separation chamber, expanding towards the hot gas outlet, to decrease the intensity of rotation and the radial pressure gradient near the valve. In addition, the conical flare of a vortex tube prohibits an efficient diaphragming of its throat because the boundary layer builds up lengthways. Such diaphragming leads to the shift of the entire incoming flow towards smaller radii of rotation, i.e., to the formation of a free vortex [2]. The formation of a free vortex in the inlet section is undesirable because in this case the kinetic energy transforms into heat by the action of viscous forces between layers; it is especially undesirable when the inward flow has a transonic velocity because a further increase of the circumferential velocity will be accompanied by a rise of the gas temperature in shocks that will appear. The expansion of the temperature separation chamber and the installation of a cross is probably required to increase the axial pressure gradient near the diaphragm†. An increase of the axial gradient leads to an increase of the velocity, with which the central, the most cold, portions of the gas flow out (Fig. 4).

The above discussion shows that the model enables us to explain qualitatively the most unexpected experimental data on vortex tubes. If we look at reverse vortex flows typical for counter-flow vortex tubes from a wider perspective, not only in terms of the energy separation of gases, then we should point out once again that they show promise for aero- and hydrodynamic insulation of a surrounding medium, and particularly for apparatus walls from a zone of reaction (from flame, plasma, etc.) [57]. To enhance the efficiency of such insulation the input turbulence should be suppressed in all ways, or at least, its scale should be decreased, though as operating experience with vortex tubes shows, return-vortex geometry provides a sufficiently high level of insulation of the walls from the central zone (but not vice versa) even if the initial turbulence is high. To some extent the effect of a return vortex flow is similar to that of a one-way membrane since it delivers the action from the outer layers to the central layers.

5. Estimates for processes in vortex tubes

As justly noted in the introductory chapter to the monograph [54], 'the only unquestionable proposition about turbulence is that this is the most complex motion of fluid'. However we shall try to evaluate the geometrical sizes of vortex tubes and their energy characteristics from general physical consideration and well-known experimental facts. We shall in principle try not to use any dimensionless criteria in order to retain the physical backgrounds of the estimates in their explicit form. The estimates will be established for the most studied counter-flow vortex tubes and they will be checked primarily against the experimental data presented in the monograph [3]. Note that the present hypothesis does not negate estimates of many other authors who have tried to determine optimal characteristics of vortex tubes on the basis of experimental data on flow patterns without a proper understanding of the nature of energy separation (see, for example, Ref. [36]).

† It is known [11, 14] that the expansion of rotational flows favors reverse flows.

5.1 Estimates for the geometrical sizes of vortex tubes and the parameters of gas inside them

First we shall assume that turbulent elements are adiabatically distributed on a radius in the main vortex according to their initial velocities and that they form a forced vortex (1) in the central part of the tube. If we want the circular outlet to release a substantially cooled gas, then only the low-velocity and low-temperature central portions should be removed from the inner vortex, and the pressure differential there should not be large (otherwise the surrounding air would flow into the tube along the axis). Let us consider the radial distributions of the main parameters of gas, i.e., the pressure, temperature, and density in the forced vortex. For this purpose we shall adopt the model of a plane quasisolid ideal vortex (1) of radius R_r, for which the Clapeyron equation

$$\frac{P}{\rho T} = \text{const} \tag{19}$$

holds.

We shall assume that the turbulent elements have already been separated and that this process was adiabatic, i.e., the Poisson equation

$$\frac{P}{\rho^k} = \text{const} \tag{20}$$

holds for the forced vortex.

Since the pressure distribution in a plane low-rate vortex (i.e. in a vortex with insignificant radial velocities) conforms to the law

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{\rho v^2}{r}\,, \tag{21}$$

integration of Eqn. (21) yields the dependencies

$$\frac{P}{P_1} = \left[1 + 0.5(k-1)\left(\frac{V_f}{c_1}\right)^2\left(\frac{r}{R}\right)^2\right]^{k/(k-1)}, \tag{22}$$

$$\frac{T}{T_1} = 1 + 0.5(k-1)\left(\frac{V_f}{c_1}\right)^2\left(\frac{r}{R}\right)^2, \tag{23}$$

$$\frac{\rho}{\rho_1} = \left[1 + 0.5(k-1)\left(\frac{V_f}{c_1}\right)^2\left(\frac{r}{R}\right)^2\right]^{1/(k-1)}. \tag{24}$$

Here P_1, T_1, and ρ_1 are the parameters of gas at the vortex center; V_f is the circumferential velocity on the outer boundary of the forced vortex; c_1 is the velocity of sound at the vortex center. To find how the effluent fraction of gas α is related to the radius of diaphragm r_0, Eqn. (24) should be integrated from 0 to r_0 and from 0 to R_f. The ratio of these integrals yields the desired dependence:

$$\alpha = \left\{\left[1 + \frac{k-1}{3-k}\left(\frac{r_0}{R_f}\right)^2\right]^{k/(k-1)} - 1\right\} \times \left[\left(1 + \frac{k-1}{3-k}\right)^{k/(k-1)} - 1\right]^{-1}. \tag{25}$$

Note that Eqns (22) and (23) were obtained in Ref. [36] but there the effluent fraction of gas was found without consideration of the radial variance of the density of gas, and as a consequence, the resultant dependence is quite different (Fig. 10). Given α, the relative radius of the diaphragm can be obtained from Eqn. (25). For example, $k = 1.4$ for air with $\alpha = 0.5$ when $r_0/R = 0.76$. The limiting parameters of gas on the periphery of the forced vortex are $P_f/P_1 = 2.184$, $T_f/T_1 = 1.25$, and $\rho_f/\rho_1 = 1.747$ when the circumferential velocity V_f is equal to the velocity of sound. In contrast to the work [36] we shall not assume that the forced vortex occupies the entire section of a vortex tube near the nozzle. Instead we shall recall that the designers are forced to use large nozzles to support the energy separation. To check our estimates against experimental data we shall take the same ratio of sizes of nozzle and tube as in Ref. [3] (Section 2):

$$h = \left(\frac{S_T}{2}\right)^{0.5} = \left(0.09\pi\frac{R_0^2}{2}\right)^{0.5} = 0.376\,R_0\,.$$

(Here R_0 is the tube radius). Assuming that in the inlet section of a counter-flow vortex tube where the cold air outlet is located, the distribution of the circumferential velocity is the combination of the forced vortex of radius R_f and the annular flow of approximately constant velocity between the vortex and wall, we derive from formula (25) the dependence

$$\frac{r_0}{R_0} = \frac{d_c}{D} = 1.248\left[(1.18\alpha+1)^{1/3.5} - 1\right]^{0.5}, \tag{26}$$

which is close to the experimental distribution in Ref. [3], especially, for large α (Fig. 10). Divergence for small α is probably related to the fact that in this estimate we do not consider the velocity profile of the outlet cold flow, which is very non-uniform, especially for small α (see Fig. 4). In addition, if we try to remove the coldest part of the gas, then the incompleteness of the adiabatic process (as a result of admixing the face boundary layer as well as a result of heat conduction in the course of 'slow' output of cold gas) will manifest itself in a rise of temperature at the vortex center, and in a smaller fraction of gas in the central part of the flow.

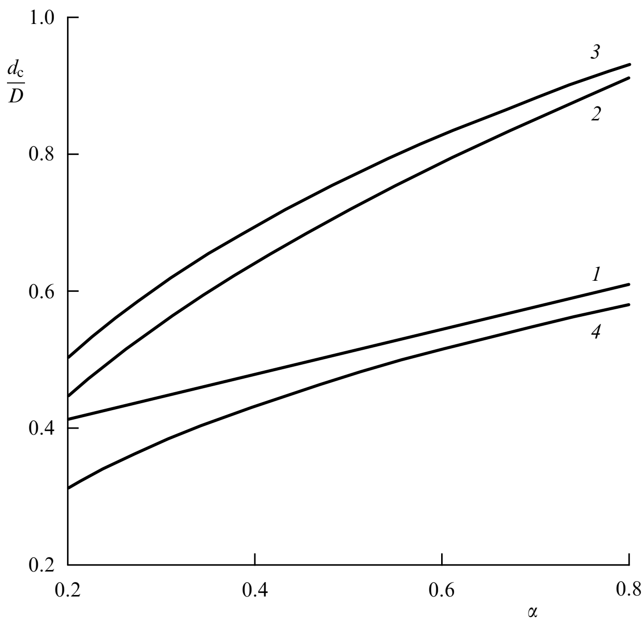

Figure 10. Dependence of the relative size of diaphragm on the effluent fraction of cold gas: *1* — calculation by means of empirical formula based on the experimental data from Ref. [3]; *2* — calculation by means of the formula from Ref. [36] on the basis of the forced vortex model for an incompressible gas; *3* — calculation by means of formula (25) on the basis of the forced vortex model for a compressible gas; *4* — calculation by means of formula (26) for a combined vortex.

The pressure and temperature distributions can be found in the peripheral region of the plane vortex with a radius-independent circumferential velocity. In this region integration of Eqn (21) yields

$$\frac{P}{P_f} = \left[1 + (k-1)\ln\frac{r}{R_f}\right]^{k/(k-1)}, \tag{27}$$

$$\frac{T}{T_f} = 1 + (k-1)\ln\frac{r}{R_f}\,. \tag{28}$$

These equations enable us to determine the pressure P_p and temperature T_p on the periphery of the vortex tube. For example, $P_p/P_f = 1.83$ and $T_p/T_f = 1.188$ for the given air vortex tube, for which $R_0/R_f = (R_f + h)/R_f = 1.6$. In optimal modes of operation the velocity of gas at the vortex tube inlet should not exceed the velocity of sound on the periphery of the forced vortex in order to preclude an additional heating of gas in shocks. Therefore, the peak velocity at the nozzle outlet is $V_0 = V_p$, and with regard for the above ratio of temperatures $T_p/T_f = 1.188$, it corresponds to the flow Mach number $M = (T_f/T_p)^{0.5} = 0.92$. The stagnation pressure P_0 of the initial air can be calculated from the equation

$$P\left[1 + 0.5(k-1)M^2\right]^{k/(k-1)} = \text{const} \tag{29}$$

for an adiabatically accelerated jet of gas [38]: $P_0/P_p = 1.724$. If we now find the expression for the pressure P_d on the boundary of the diaphragm using Eqn. (22) and put it equal to the atmospheric or cold outflow pressure P_c, then the peak pressure differential can be found, for which vortex tube is still working in optimal mode of operation. For example, for an air vortex tube with a diaphragm, the relative size $r_0/R_0 = 0.5$ of which matches approximately the effluent fraction of cold gas $\alpha = 0.5$, we have

$$n = \frac{P_0}{P_c} = \frac{P_0}{P_p}\,\frac{P_p}{P_f}\,\frac{P_f}{P_1}\,\frac{P_1}{P_d} = 5.57\,.$$

This value agrees well with the experimental data presented in Ref. [3], where the author indicates that the temperature effect ΔT_c rises significantly up to $n = 8$ while the temperature efficiency η remains approximately the same up to $n = 6$.

Thus, we can conclude that the hypothesis that formation of the central forced vortex is an adiabatic process leads to plausible estimates for important parameters of a vortex tube such as the relative size of diaphragm versus the cold air fraction and the optimal pressure range.

Now we shall evaluate the characteristic time of temperature separation since the size of a vortex tube may depend on it. Let the minimal size of the inlet nozzle be h. Then the characteristic size of a turbulent element will be about $h/2$ and the characteristic value of a velocity pulsation of the incoming flow will be about $V_0/2$. The force by which this element will be pushed towards the center, will be

$$F_c \sim \left(\frac{h}{2}\right)^3 \frac{\rho\left[V_0^2 - (V_0 - V_0/2)^2\right]}{R_0} \sim \left(\frac{h}{2}\right)^3 \frac{\rho V_0^2}{R_0}\,,$$

in the rotating coordinate system of the forced vortex. If we now assume that the velocity of separation of turbulent elements V_d can be calculated from the equality of this force and the viscous force, then we find that the value of the separation velocity is many times larger than the velocity of sound. We are therefore induced to suppose that turbulent elements are separated very rapidly so that the resistance to the motion is dictated by the developing pressure differential on the forward and rear edges of the moving element. In this case the resistance force will be of the order of $F_f \sim (h/2)^2 \rho V_d^2$. By equating F_c and F_f we can evaluate the characteristic velocity of the radial motion of separated turbulent elements: $V_d/V_0 \sim 0.5\,h/R_0$.

For the above tube this means a very high velocity $V_d \sim 0.2\,V_0$ and a very short characteristic time of separation $T_d \sim 5\,R_0/V_0$ approximately equal to the time of one revolution of the forced vortex. The distance L_d, at which the essential temperature separation takes place, can be evaluated in this case as follows:

$$L_d \sim t_d U_d \approx \frac{2V_0}{\pi(R_0^2 - r_0^2)} \frac{5R_0}{V_0} \sim R_0 \,.$$

In this estimate we have considered the fact that near the inlet almost all the gas $Q = \rho S V_0$ moves towards the valve, that it occupies the tube section from r_0 to R_0, that the density of the gas is approximately equal to the density of gas at the inlet, and the maximal forward velocity U_d is about two times larger than the mass-averaged velocity. In fact, the measurements of radial velocity showed [10] that there is still a flow region with positive radial velocity at a distance of two tube diameters (Fig. 4) (according to the hypothesis this means that accelerated microvolumes are in process of separation), though the radial velocity is negative along the remaining length of the tube in all the section of the tube (according to the hypothesis this is related to the migration of retarded turbulent elements from the boundary layer to the center). All this means that the length of a vortex tube should be determined from other considerations rather than by the time of energy separation.

Two considerations should be taken into account in the evaluation of the relative area of the inlet nozzle and the absolute sizes of the tube. First, the forward velocity of the cold flow should be much less than the velocity of sound for the subsequent deceleration not to cause an excessive heating of the flow. At the same time we shall keep in mind that the average velocity of gas at the outlet of a tangential intake nozzle is, as a rule, close to the velocity of sound (refrigeration will be insignificant for a smaller velocity). The gas has such an average circumferential velocity just after it gets into the vortex tube. Thus, we arrive at an estimate for the forward velocity of the cold gas at the diaphragm outlet:

$$\frac{\alpha Q}{\pi \rho_a r_0^2} \ll V_0 \,.$$

Here Q is the total flow rate. Given that $Q = m\rho_a S V_0$, where S is the total area of the inlet nozzles and m is the ratio of the density of gas in the section of the tangential nozzle to the density ρ_a of gas output from the vortex tube for atmospheric pressure, we arrive at the estimate

$$\pi r_0^2 \gg \alpha m S \,. \qquad (30)$$

The second consideration that we should keep in mind in examination of processes in vortex apparatus is the influence of the face boundary layer. The importance of the face boundary layer is illustrated in the work [58]. The boundary layer on the surface perpendicular to the axis of a vortex flow differs fundamentally from boundary layers in ordinary systems. The point is that the pressure gradient† which brings about the secondary flow in the boundary layer is perpendicular to the direction of the main flow. Therefore, the general flow pattern in the surface layer is very complex: the closer a given element of gas is to the face wall, the smaller the velocity of its circular motion and the stronger the effect of the radial pressure gradient, which brings about the motion towards the center of the system. If we add that at the time when the monograph [54] was written 'all the available information was on the boundary layer with a constant pressure', it becomes clear why it is so difficult to obtain correct estimates for this portion of the vortex flow.

However we shall try to find these estimates. For this purpose we shall mentally divide the region of flow near the face surface in two. The first region is directly adjacent to the surface and there the flow has primarily a radial direction and the static pressure is dominant; the second region is a relatively weakly perturbed part of the vortex flow, the turbulent elements of which are partially decelerated, and together with the retarded elements of the main flow, shift to 'lower orbits' where they are drawn into vortex turbulent motion‡. If in addition we consider the data from Ref. [38], according to which a laminar boundary layer on a plane surface becomes a turbulent one for a sufficiently large Reynolds number $\mathrm{Re}_x \geq 2 \times 10^6$, then we have grounds to assume that the radial boundary-layer flow is mostly laminar. In terms of the ordinary theory of turbulent boundary layers the radial flow can be called a 'laminar sublayer'. Generally, even without the assumption of the laminar nature of the flow it is clear that the resultant turbulent perturbations will inevitably be involved into vortex motion. The flow velocity V_δ in the laminar layer must be close to the velocity of sound because of the tremendous pressure difference between the wall and axis of the vortex tube. Then the flow rate of gas, which flows through this boundary layer to the tube center without energy separation and dilutes the cold flow, can be estimated as follows:

$$Q_d \approx 2\pi r_0 \delta_d V_0 \rho_a \,.$$

The thickness of the laminar boundary layer on the inner side of the diaphragm δ_d is specified by the length x_δ at which the boundary layer develops. Using data on the thickness of a boundary layer for ordinary flows [37], we shall assume that

$$\delta_d \sim \frac{4.64 x_\delta}{(\mathrm{Re}_x)^{1/2}} = 4.64 \left(\frac{x_\delta \nu}{V_\delta}\right)^{1/2} \approx 4.64 \left[\frac{(R_0 - r_0)\nu}{V_0}\right]^{1/2} .$$

Here, as earlier, ν is the kinematic viscosity. Then

$$Q_d \sim 2\pi r_0 \delta V_0 \rho_a \approx 25 r_0 \rho_a \left[(R_0 - r_0)\nu V_0\right]^{1/2} . \qquad (31)$$

For a vortex tube to be efficient it is required that the quantity of gas, which flows through the laminar boundary layer to the center of the system without energy separation, be much less than the total quantity of 'cold' gas, i.e., $Q_d \ll \alpha Q$. This

† In vortex tubes it is of the order of 10^7 Pa m^{-1}!

‡ These very 'vortex' flows over two surface faces are probably responsible for the efficient decrease of the chamber radius in Ref. [56].

proposition is equivalent to the inequality

$$\alpha m S \gg 25 r_0 \left[\frac{\nu(R_0 - r_0)}{V_0}\right]^{1/2} . \quad (32)$$

To evaluate the thickness of the perturbed surface turbulent layer we shall use the data on turbulent mixing of flows with nearly the same velocity from Ref. [38] and write down the formula for the rate of widening of the turbulent jet running down to the center over the surface of diaphragm:

$$0.22 V \frac{\mathrm{d}t}{\mathrm{d}b} \approx 2 \frac{V}{V/2} = 4 .$$

($V\,dt$ is the path that an element of the vortex tube travels in the time $\mathrm{d}t$; $\mathrm{d}b$ is the widening of the jet along this path; the factor 0.22 is known for the formula of turbulent jet widening in a co-current flow from numerous experiments [38]). The radial flow on the surface of the diaphragm passes smoothly into the main vortex flow, and therefore, the perturbation from the diaphragm will propagate with the same relative velocity as that from the co-current flow with a close velocity; in the co-current flow with a close velocity the jet widening is approximately the same as that in a flow with a double velocity [38] and this fact is taken into account in this formula. The rate of widening of a turbulent perturbation deep into the heart of the vortex flow is $V_\delta = \mathrm{d}b/\mathrm{d}t \sim V/20$. Given the characteristic time τ_0 of confinement of gas in a vortex tube,

$$\tau_0 \sim \frac{\pi R_0^2 L}{V_0 S} ,$$

(here L is the length of the vortex tube) we obtain an estimate for the thickness of the perturbed layer

$$\delta_{\mathrm{t}} \approx V_\delta \tau_0 \sim \frac{\pi R_0^2 L}{20 S} .$$

All the gas inside the tube will take part in the process of energy separation and there will be no decrease in the efficient radius [58] provided that the perturbation in question does not propagate to the opposite end of the tube, i.e., $\delta_{\mathrm{T}} \ll L$ or

$$20 S \gg \pi R_0^2 . \quad (33)$$

It follows directly from inequalities (30) and (32) for the tube in a typical operating mode that $\alpha \approx 0.5$ corresponds to the optimal relative radius of diaphragm $r_0/R_0 = 1/2$ (Fig. 10); $m \sim 3$ corresponds approximately to the velocity of sound at the tube inlet and the initial difference of pressure $n = 5$; the relative total area of the inlet nozzles,

$$S \approx 0.1 \pi R_0^2 , \quad (34)$$

fits the experimental data fairly well (Section 2). If the signs $\gg$ and $\ll$ imply differences no less than one-half order of magnitude, then it follows from inequality (31) that the radius of a vortex tube should be greater than $R_0 \gg 3000\nu/V_0$ or $R_0 \gg 1.5 \times 10^{-4}$ m for an air vortex tube in which the cold gas is exhausted into the atmosphere.

According to the above estimates the distance at which the main temperature separation takes place is small. Therefore, the length of a counter-flow vortex tube is probably specified by the length of the return vortex. If the tube is made shorter, then the return vortex will 'rest' on the valve and the retarded gas over the surface of the valve will get into the cold flow and increase its temperature. Prior to the establishment of the dependence of the length of the return vortex X_0 on the tube parameters we shall consider the limiting cases. As noted earlier (see Fig. 2 and Ref. [11]) X_0 can be some tens of the tube diameter when the diaphragm is closed. On increasing the fraction of gas withdrawn through the diaphragm, X_0 increases and when $\alpha = 1$ and the valve is closed $X_0 \to \infty$. This is related to the fact that there is a radial pressure gradient (which decreases with distance from the swirler) while there is a rotation of gas. Therefore, in the absence of a mass-averaged flow from the swirler deep into the tube (this corresponds to the condition $\alpha = 1$, or for example, to the flow in the electrode cup of an arc plasma generator [51]), the pressure gradient on the axis will be pointing towards the swirler over the entire length of the tube, and as experiments show, the counter-flow zone can be very lengthy and in some cases can be divided into several separate zones because of uncontrolled perturbations [51]. Moreover, for $\alpha = 1$ the length of the return flow zone should increase infinitely even for an insignificant rotation of gas provided that there is a peripheral flow deep into the tube. If $\alpha < 1$ and the mass-averaged velocity is directed away from the swirler, then at a certain distance the viscosity force, which acts on the central region of the flow from the peripheral flow directed initially away from the swirler, will exceed the pressure force, which acts on the central region towards the swirler because of the existing pressure gradient. Under these conditions X_0 should depend on the fraction of gas flowing out through the diaphragm and on the tube parameters characteristic of the rotation of the flow. Even from this qualitative consideration it is clear that varying α for the same tube can bring it out of the optimal operating conditions.

To evaluate the length of the reverse vortex we shall determine the coordinate X_0 for the point on the axis of the vortex flow, at which the oppositely directed forces acting on the central region of the gas flow, i.e. the frictional force from the outer vortex and the force, for which the radial pressure gradient is responsible, become equal in magnitude. We suppose that the profile of the axial velocity $u(r) = u(0) + (r/R)^2 U$ is quadratic in the main section of the tube up to the boundary layer. (Here R is the radial coordinate of the edge of the boundary layer at which the longitudinal velocity U is at a maximum. This maximum is shifted towards to the periphery because of the inverse pressure gradient at the vortex center.) Then the frictional force F_v, which acts on a tubular element of r in radius and of ΔL in length at the flow center near the 'point of reverse', can be expressed as follows:

$$F_v = 2\pi r \Delta L \mu \frac{\mathrm{d}u}{\mathrm{d}r} = 4\pi \Delta L \mu U \left(\frac{r}{R}\right)^2 . \quad (35)$$

To understand how the pressure gradient changes in the central region of the flow we must determine the law by which the flow parameters change along the flow. In deriving the estimates we shall assume that the medium is incompressible, i.e. $\rho = \mathrm{const}$. Taking into account the hypothesis on the drift of retarded turbulent elements towards the center of the flow we shall assume that the boundary layer is laminar over all the surface of the tube and that its thickness δ is specified by the critical Reynolds number Re_{k} for which the flow becomes turbulent at a given distance from the wall, i.e.

$$\mathrm{Re_k} = \delta\rho\mu^{-1}(U^2 + V^2)^{1/2} = \mathrm{const}\,. \tag{36}$$

Here V is the maximal tangential velocity and in this estimate it also is supposed to be shifted towards the periphery (the forced vortex model). We shall assume that $\delta \ll R$. Therefore, variation of the thickness of the boundary layer along the tube length only weakly affects the flow parameters. By considering the viscous friction force, which acts on a tubular flow element of short length Δ from this boundary layer and changes the momentum and angular momentum of this element, we obtain the following equations:

$$-\pi R^2 \Delta\rho \frac{\mathrm{d}U}{\mathrm{d}t} = 2\pi R\Delta\rho \frac{U^2}{\mathrm{Re_k}}\,, \tag{37}$$

$$-\frac{\pi R^2 \Delta\rho R^2}{2}\frac{\mathrm{d}(V/R)}{\mathrm{d}t} = \frac{2\pi R\Delta\rho V^2}{\mathrm{Re_k}}\,R\,. \tag{38}$$

The derivatives $\mathrm{d}U/\mathrm{d}t$ and $\mathrm{d}V/\mathrm{d}t$ can be replaced by $(\mathrm{d}U/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t)$ and $(\mathrm{d}V/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t)$, because of the translational motion of the element. Then, since $\mathrm{d}x/\mathrm{d}t = V$, we have

$$\frac{\mathrm{d}U}{\mathrm{d}x} = -\frac{2U}{R\,\mathrm{Re_k}}\,, \tag{39}$$

$$\frac{\mathrm{d}V}{\mathrm{d}x} = -\frac{4V^2}{UR\,\mathrm{Re_k}}\,. \tag{40}$$

The peripheral pressure P varies along the tube length because of the deceleration of the flow and because of frictional forces

$$-\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\rho}{2}\frac{\mathrm{d}(U^2)}{\mathrm{d}x} + \frac{2\pi R\rho U^2}{\mathrm{Re_k}\pi R^2}\,. \tag{41}$$

For a forced vortex with a fixed density the pressure variation P_1 at the center can be expressed as follows:

$$-\frac{\mathrm{d}P_1}{\mathrm{d}x} = \frac{\rho}{2}\frac{\mathrm{d}(V^2)}{\mathrm{d}x} - \frac{\mathrm{d}P}{\mathrm{d}x} = \frac{2U^2\rho}{R\,\mathrm{Re_k}} + \frac{\rho}{2}\frac{\mathrm{d}(U^2+V^2)}{\mathrm{d}x}\,.$$

Thus, the longitudinal pressure force on the central flow element is

$$F_\mathrm{p} = -\pi r^2 \frac{\mathrm{d}P_1}{\mathrm{d}x}\Delta L = \pi r^2 \Delta L\rho\left[\frac{2U^2}{R\,\mathrm{Re_k}} + \frac{1}{2}\frac{\mathrm{d}(U^2+V^2)}{\mathrm{d}x}\right]. \tag{42}$$

Given Eqns (39) and (40) it becomes

$$F_\mathrm{p} = -\frac{4\pi r^2 \Delta L\rho V^3}{UR\,\mathrm{Re_k}}\,. \tag{43}$$

At the 'point of reverse' $F_v + F_\mathrm{p} = 0$ and we arrive at the equation

$$V^3 R\rho = U^2\mu\mathrm{Re_k}\,. \tag{44}$$

By integrating Eqns (39) and (40) under the initial conditions $V(x=0) = V_0$ and $U(x=0) = U_0 \approx 2Q/\left[\rho\pi(R^2 - r_0^2)\right]$ ($Q = \rho S V_0$ is the total flow rate of fluid moving initially away from the swirler; r_0 is the radius of diaphragm) and by considering for the fact that $(r_0/R)^2 \approx \alpha$ for an incompressible fluid, we obtain

$$U = V_0\left\{\exp\left[\frac{2x}{R\,\mathrm{Re_k}}\right](1-\alpha)\frac{\pi R^2}{S}\right\}^{-1}, \tag{45}$$

$$V = V_0\left\{1 + 2\left[\exp\frac{2x}{R\,\mathrm{Re_k}} - 1\right](1-\alpha)\frac{\pi R^2}{S}\right\}^{-1}. \tag{46}$$

By substituting the resultant dependencies into Eqn. (44) we obtain a cubic equation in $\exp\left[2x/(R\,\mathrm{Re_k})\right] \equiv E$,

$$\left[2Z(E-1)+1\right]^3\frac{\mathrm{Re_k}}{\mathrm{Re_0}} = Z^2E^2\,, \tag{47}$$

where $(1-\alpha)\pi R^2/S \equiv Z$ and $\rho V_0 R/\mu \equiv \mathrm{Re_0}$. Analysis of this equation shows that for a flow with a large angular momentum, i.e., for $\mathrm{Re_k}/\mathrm{Re_0} \ll 1$ there is a solution $E \to \infty$ when $Z \to 0$ and the solution $E \approx 3 + (8Z\,\mathrm{Re_k}/\mathrm{Re_0})^{-1}$ when $Z \gg 1$. This means that the reverse vortex has infinite length when $\alpha \to 1$ and finite length when $\alpha = 0$ and that the characteristic scale of length is the quantity $R\,\mathrm{Re_k}/2 \gg R$. This probably explains the fact that in counter-flow vortex tubes the length significantly exceeds the diameter. The above estimate for the length of a reverse vortex fits the qualitative consideration well. However, this estimate is not very strict and is made for the case of an incompressible fluid. Thus, it is not quite appropriate for a quantitative consideration.

The above estimates for the optimal parameters of the gas and for sizes of the counter-flow vortex tubes fit the available experimental data quite well. The proposed mechanism of motion of turbulent elements has been used so far to evaluate how rapidly the energy separation proceeds. In all the other estimates the proposed hypothesis has been used primarily in an implicit manner. In the case of energy estimates the situation is fundamentally different.

5.2 Estimates for the energy characteristics

One of the fundamental differences of the proposed hypothesis from previous ones is that the flow is assumed to be basically inhomogeneous at the vortex tube inlet. This inhomogeneity should be described somehow if we want to obtain estimates for the energy efficiency of a vortex tube. As mentioned in Ref. [60], pulsations of velocity in a turbulent flow in tube are usually not very large, and as a rule, do not exceed† $\pm 10\%$. At the tangential nozzle outlet the flow pattern changes radically and becomes more complex (Fig. 8). On two sides of the incoming flow (through a rectangular nozzle) there are 'free' boundaries on which the jet intermixes with the vortex flow; on the third side the jet is bounded by the surface of the diaphragm; and on the fourth side it is bounded by the tubular wall, the radius of curvature of which is comparable with the transverse dimension of the jet. Therefore, the flow becomes primarily turbulent at the nozzle outlet. This turbulence producing effect is such that circulation zones can appear (especially in the case of a tubular inlet nozzle). We shall assume that all the flow breaks up into microvortices. Given the simplest model of velocity distribution in the initial turbulent elements (microvortices),

† Note that in real vortex tubes, in which the pressure difference is $n > 2$, the estimate for the Reynolds number (14) in the tangential channel shows that the flow should be turbulent there.

we shall also assume that in the flow with average velocity V_0 each microvortex has the following distribution of the projection of the forward velocity on the direction of averaged motion:

$$v = 2V_0 \frac{y}{y_0} \quad \left(0 \leqslant \frac{y}{y_0} \leqslant 1\right). \tag{48}$$

Here y_0 is the diameter of a microvortex and y is the distance along the axis, which is perpendicular to the direction of the averaged motion and to the axis of rotation of the microvortex†. Assuming that the pressure and density do not change over the diameter of the microvortex and taking into account the fact that the flow rate of gas $\mathrm{d}Q$ through the elementary section of a microvortex of $\mathrm{d}y$ in height with coordinate y is $\mathrm{d}Q = \rho v\,\mathrm{d}y$ (the 'thickness' of the microvortex is set to be unity) the dependence of the flow rate Q on y can be determined by integrating $\mathrm{d}Q$ from zero to y:

$$Q(y) = V_0 \rho \frac{y^2}{y_0}. \tag{49}$$

The flow rate $\mathrm{d}Q$ carries a kinetic energy $\mathrm{d}w = \mathrm{d}Q\, v^2/2 = \rho v^3\,\mathrm{d}y/2$. Integrating and accounting for Eqn. (48) we have:

$$w(y) = V_0^3 \rho \frac{y^4}{y_0^3}. \tag{50}$$

The specific kinetic energy is

$$W = \frac{w(y)}{Q(y)} = V_0^2 \left(\frac{y}{y_0}\right)^2. \tag{51}$$

The average specific kinetic energy W_0 that the flow carries is $W_0 = w(y_0)/Q(y_0) = V_0^2$. (The absence of the 1/2 factor of V_0^2 is related to the fact that the kinetic energy is stored in translational motion as well as in vortex motion.) Since according to formula (48) the layer with coordinate y carries specific energy $v^2/2 = 2V_0^2 (y/y_0)^2$, the layer with coordinate $y_s = y_0/\sqrt{2}$ will have the average specific kinetic energy. Gas elements in this layer will acquire the initial temperature upon expansion in the vortex tube and subsequent deceleration, while gas elements from a layer with a greater or lesser coordinate will acquire, respectively, a higher or lower temperature. To find a gas characteristic such as the fraction of gas $\alpha_{\max}$ when the cooling effect of a vortex tube $q_c = \alpha c_p \Delta T_c$ (here, as earlier, T_c is the temperature difference between the temperature of the initial gas and that of the 'cold' gas) is at maximum, it suffices to find the fraction of gas carried by layers whose coordinates are smaller than y_s:

$$\alpha_{\max} = \frac{Q(y_s)}{Q(y_0)} = \frac{1}{2}.$$

Note that this fraction of gas carries the average specific kinetic energy:

$$W_s = \frac{w(y_s)}{Q(y_s)} = \frac{W_0}{2}.$$

Now we can recall that gas elements acquire the specific kinetic energy W at the expense of a decrease in enthalpy I. Therefore, the specific kinetic energy is

$$W = I_0 - I = c_p T_0 - c_p T = c_p (T_0 - T).$$

Here $I_0 = c_p T_0$ is the specific enthalpy of the initial gas. As is known [38] the temperature T to which the gas cools upon expansion, is determined by the pressure difference and by the stagnation temperature T_0‡:

$$T = T_0 n^{(1-k)/k}.$$

Therefore, the average stored kinetic energy is

$$W_0 = c_p T_0 \left(1 - n^{(1-k)/k}\right). \tag{52}$$

The process is reversed when the gas is decelerated, for example, at the outlet from a vortex tube. Therefore, the cold gas temperature is

$$T_c = T + \frac{W_c}{c_p},$$

(here W_c is the kinetic energy stored by the 'cold' gas). In other words, the difference of temperatures in the initial and 'cold' flows depends on the difference of the stored kinetic energies:

$$\Delta T_c = T_0 - T_c = \frac{W_0 - W_c}{c_p} = \frac{1}{c_p} \frac{W_0 - w(y)}{Q(y)} = \frac{\left[1 - (y/y_0)^2\right] V_0^2}{c_p}.$$

In view of Eqn. (52) and $(y/y_0)^2 = Q(y)/Q(y_0) = \alpha$ we have the dependencies:

$$\Delta T_c(\alpha) = T_0 \left[1 - n^{(1-k)/k}\right](1 - \alpha), \tag{53}$$

$$\frac{q_c}{c_p} = \alpha \Delta T_c = T_0 \left[1 - n^{(1-k)/k}\right](\alpha - \alpha^2). \tag{54}$$

For the peak refrigerating capacity q_c

$$\frac{q_c}{c_p} = \alpha_{\max} \Delta T_c = \frac{T_0 \left[1 - n^{(1-k)/k}\right]}{4}. \tag{55}$$

Now we shall check how well the resultant dependencies fit the experimental curve (Fig. 3) in the case of $n = 6$, $T_0 = 303$ K, and $k = 1.4$. By means of formula (55) we have $q_c/c_p \approx 30$ K. This value only slightly exceeds the experimental value. According to formula (53) the possible peak cooling effect $\Delta T_{\max} \approx 121$ K corresponds to $\alpha = 0$ and $\eta = 1$. This result differs radically from the experimental data according to which $\Delta T_{\max} \to 0$ as $\alpha \to 0$.

Here we should recall the boundary layer, through which several percent of the gas (according to formula (31) for a vortex tube with typical size $R_0 = 2$ cm) reaches the outlet without energy separation. Let us denote the fraction by $\alpha_d \equiv Q_d/Q_0$, where Q_0 is the total flow rate. Since the conditions on the diaphragm surface depend only weakly on the fraction of gas exhausted through the valve (Fig. 1), we can assume that the gas from the boundary layer always gets out through the diaphragm and that in the case of $\alpha < \alpha_d$ the outlet of a fraction of this gas is compensated by the leak of the surrounding gas through the central part of the diaphragm (in this zone the pressure is minimal). According to formula (49) the total quantity of gas, which has

† This distribution corresponds to a 'rolling' quasisolid microvortex.

‡ This is simply another presentation of Eqn. (7).

experienced energy separation, is $V_0\rho y_0 = Q_0 - Q_d = (1-\alpha_d)Q_0$. Therefore, $Q_0 = V_0\rho y_0/(1-\alpha_d)$. Instead of formula (49) we have

$$Q(y) = \alpha Q_0 = \frac{V_0\rho y_0\alpha}{1-\alpha_d} \quad \text{when } 0 \leqslant \alpha \leqslant \alpha_d\,, \tag{56}$$

$$Q(y) = \frac{V_0\rho y_0\alpha_d}{1-\alpha_d} + \frac{V_0\rho y^2}{y_0} \quad \text{when } \alpha_d < \alpha \leqslant 1\,. \tag{56a}$$

Equation (50) is transformed into

$$w(y) = \frac{V_0^3\rho y_0\alpha}{1-\alpha_d} \quad \text{when } \ 0 \leqslant \alpha \leqslant \alpha_d\,, \tag{57}$$

$$w(y) = V_0^3\rho\left(\frac{y^4}{y_0^3} + \frac{y_0\alpha_d}{1-\alpha_d}\right) \quad \text{when } \ \alpha_d < \alpha \leqslant 1\,. \tag{57a}$$

With the aid of

$$\alpha \equiv \frac{Q(y)}{Q_0} = \left[\frac{\alpha_d}{1-\alpha_d} + \left(\frac{y}{y_0}\right)^2\right](1-\alpha_d) \tag{58}$$

we determine

$$\left(\frac{y}{y_0}\right)^2 = \frac{\alpha-\alpha_d}{1-\alpha_d} \quad \text{when } \ \alpha_d < \alpha \leqslant 1\,. \tag{59}$$

For the specific kinetic energy $W(y) \equiv w(y)/Q(y)$ accounting for Eqns. (56)–(58) we have

$$W(y) = V_0^2 \quad \text{when } \ 0 \leqslant \alpha \leqslant \alpha_d\,, \tag{60}$$

$$W(y) = V_0^2\,\frac{\alpha^2 - 2\alpha\alpha_d + \alpha_d}{\alpha - \alpha\alpha_d} \quad \text{when } \ \alpha_d < \alpha \leqslant 1\,. \tag{60a}$$

For the temperature difference we have

$$\Delta T_c(\alpha) = 0 \quad \text{when } \ 0 \leqslant \alpha \leqslant \alpha_d\,, \tag{61}$$

$$\Delta T_c(\alpha) = T_0\left[1 - n^{(1-k)/k}\right]\frac{(\alpha-\alpha_d)(1-\alpha)}{\alpha - \alpha\alpha_d} \quad \text{when } \ \alpha_d < \alpha \leqslant 1\,. \tag{61a}$$

For the refrigerating capacity we have instead of Eqn. (54) that

$$\alpha\Delta T_c = 0 \quad \text{when } \ 0 \leqslant \alpha \leqslant \alpha_d\,, \tag{62}$$

$$\alpha\Delta T_c = T_0\left[1 - n^{(1-k)/k}\right]\frac{(\alpha-\alpha_d)(1-\alpha)}{1-\alpha_d} \quad \text{when } \ \alpha_d < \alpha \leqslant 1\,. \tag{62a}$$

In this case for $\alpha_d = 0.09$ the calculated energy characteristics of the vortex tube in the main working area are close to the experimental ones (Fig. 3). As $\alpha \to 0$ effects of the next order should be considered, e.g. portions of gas with a higher temperature than the stagnation temperature T_0 can get into the face boundary flow from the tubular boundary layer and this can lead to a so-called 'reverse' of the vortex tube when the heated gas is exhausted through the diaphragm. The reason of the 'reverse' can also be an excessively large diameter of the diaphragm as a result of which high-velocity portions of the temperature-separated gas get into the 'cold' flow.

In the above energy estimates we have not considered one effect, on which the energy characteristics can depend essentially in the region where $\alpha \to 0$. The great difference between the translational velocities of specific elements of gas brings about an excessive centripetal acceleration of the 'retarded' microvolumes as well as a tangential acceleration. In other words in the course of radial drift microvolumes exchange kinetic energy and their energy distribution is no longer be uniform as follows from formula (48). However, since the mechanism of energy separation operates at any radius, it should not be expected that the resulting energy distribution be similar to that for a gas in equilibrium. The total energy of the 'low-energy' half of the flow should not change noticeably in this case because the separation of gas on the boundary $v = V_0$ occurs very early, prior to the redistribution of the kinetic energy of microvolumes. Therefore, the shapes of the energy dependencies remain the same near $\alpha = 1/2$ and noticeable distortions are possible only as $\alpha \to 0$ and as $\alpha \to 1$.

It is possible to compare the radial density distributions in the central vortex, obtained under the assumptions that it is 'quasisolid' and 'adiabatic', with the distribution based on the energy estimates (49) under the assumption that each microvolume will eventually be located on the radius, the value of which is proportional to the initial velocity of the microvolume (48) (i.e., it is possible to assume that the forced central vortex is formed from elements which have not taken part in a kinetic energy exchange). However, even in the simplest case the result of the comparison will depend essentially on the distribution of the axial velocity of the outflowing gas on radius of the diaphragm. A complete coincidence is possible only if the fluid is incompressible and the velocity of outflow is independent of radius†. If we take into account the facts that the real profile of the axial velocity is very complex, that gas is compressible, that the central elements tend to form a forced vortex, and that microvolumes exchange their kinetic energy, then we obtain slightly different energy characteristics as well as different radial dependencies of flow parameters.

We can make certain additional conclusions about the design of vortex tubes on the basis of the above energy estimates. The first change in design which suggests itself upon adoption of the proposed hypothesis is a reduction of the tube length to 2–3 diameters. This change is practical for a uni-flow vortex tube with a suppressed return vortex [10]. Kuznetsov studied two tubes of this type of 5 and 10 diameters in length [10]. However, because of the relatively large area of the inlet section $S/(\pi R_0^2) \approx 1/3$, where R_0 is the tube radius at the inlet, an essential temperature separation could only be obtained [according formula (34)] in the case of an essential increase in the tube diameter near the gas outlet (the conic temperature separation chamber allows condition (30) to be satisfied). According to the data in Ref. [10] the chambers in the two uni-flow tubes have the same taper of 3.7°. This corresponds to an increase in the cross-section of the longer chamber by a factor of 2.56, i.e., $S/(\pi R_0^2) \approx 1/8$, and to an increase in the cross-section of the shorter chamber by a factor of 1.69 (in Ref. [10] Kuznetsov erroneously cited the same factor of 2.56), i.e. $S/(\pi R_0^2) \approx 1/5$ and this ratio is well below the optimal. Accordingly, the author did not find any advantages in the shorter tube.

It follows from the hypothesis in question that the performance of a vortex tube can be improved fundamen-

† Note that the distribution (24) was obtained under the assumption that the vortex has a zero flow rate. This is equivalent to the condition that the velocity of outflow is independent of radius.

tally only by enhancing the inhomogeneity of the velocity at the vortex tube inlet.

Since the energy characteristics are impaired mainly by the gas flow over the face boundary layers, designers must apply their efforts to the elimination of these flows if they want to achieve a better cooling effect. For example, the total suppression of the diaphragm boundary layer effect of the cold outlet made it possible to achieve an unprecedented cooling effect on the axis in a self-evacuated vortex tube the design of which is symmetric about the nozzle cross-section [3]†. Other features of vortex tube designs which suggest themselves in consideration of energy characteristics, as noted earlier, were found by designers usually by empirical methods and were described in the previous section. General examination of the above estimates shows that a vortex tube cannot have optimal performance over a wide range of the various initial parameters and its design should be chosen according to its intended task.

6. Conclusion. Topical problems related to the Ranque effect

This paper has acquired the combined form of a critical review and the presentation of a new idea as the result of the attempt of the author — who himself came across the problem accidentally — to convey the essence of the idea to other 'accidental' amateurs. This attempt is based on the belief that the understanding of the simple nature of apparently paradoxical phenomena is extremely important to any scientist even if the field of his or her interests is far from that in question.

Here it is important to note that the approach taken in this paper and based on the treatment of gas (or fluid) flow as a collection of microvolumes with different forward velocities, and consequently, with different momenta (which is close to the Prandtl model of turbulence [34, 38]), can be fruitful also to solve other problems. An example of such a problem is the cause of the abnormally high heating of the closed end of a resonator tube in a gas-jet sound generator [60]. This phenomenon was revealed in 1954 [61]. It seems that in this case the process is quite similar to the phenomenon of energy separation in vortex tubes. In addition, this approach makes it possible to view from a different angle the reasons for the generation of sound in Hartmann generator type gas-jet acoustic radiators. Obviously, only high-speed microvolumes can reach the closed end of the cup-shaped resonator in such sound generators [62] and their quasi-corpuscular flow, on encountering with the bottom of the cup, brings about the initial acoustic vibrations, then amplified by resonator. I hope that this hypothesis will be fruitful for other problems, including those beyond my scope.

As for the Ranque effect itself, the absence of a plausible explanation of this phenomenon was, as noted earlier, the reason for scientific and technological errors in allied fields.

The study of the Ranque effect is by no means finished even if the proposed hypothesis is favored by most researchers. This hypothesis should be supported by both theoretical and experimental investigations to become a theory. The top priority theoretical task is to build a more rigorous model of energy separation accounting for such factors as kinetics of microvolumes and kinetic energy exchange; the actual distribution of microvolumes in forward velocity depending on the initial conditions; the arrival of turbulent entities to the central part of the flow from boundary layers; and the radial dependence of the forward velocity of the exhausted gas. Consideration of the kinetics of motion of microvolumes accounting for viscosity (since it affects the rate of separation as well as transformation of kinetic energy into heat) and heat exchange would yield a qualitatively understandable dependence of the efficient energy separation efficiency on the tube scale.

This model would be much closer to the real flow if researchers considered the fact that the main flow is three-dimensional. Advances in this field could probably be achieved by numerical modeling of laminar flows. The next step could be the consideration of compressible media and energy separation.

Since one of the important applications of vortex tubes is the separation of gaseous condensate [20, 21], a sensible improvement of equipment will require the introduction of condensation and separation kinetics, and droplet evaporation into the model.

As for experimental investigations of the Ranque effect, the hypothesis presents a fairly extensive area of research. Since the process of drift of retarded turbulent elements towards the center seems beyond question, it should be checked whether there are other mechanisms responsible for perceptible energy separation. The answer to this question can be found by studying the efficiency of energy separation in relation to the initial inhomogeneity of the flow. The role of rotation of turbulent elements for the process of energy separation is also open: the answer to this question can most probably be found in experiment, but does not seem simple.

The proposed hypothesis can probably give impetus to a new stage of activity by designers. This activity could achieve major advances by the intensification of the inhomogeneity of the velocity of the incoming flow and by further improvement of purely studied uni-flow vortex tubes with suppressed reverse flow.

It seems extremely important to intensify study of reverse vortex flows with the intent of using them in energy and chemical engineering apparatus for the insulation of surrounding media and chamber walls from the action of reactive zones.

† The self-evacuation effect consists in a decrease in pressure on the vortex axis by virtue of diffusers on opposite ends of the tube.

References

1. Gol'dshtik M A (Ed.) *Vikhrevaya Termoizolyatsiya Plasmy* (Vortex Heat Insulation of a Plasma) (Novosibirsk: 1979)
2. Ranque G J *J. Phys. Radium* **4** 112 (1933)
3. Merkulov A P *Vikhrevoĭ Effekt i Ego Primenenie v Tekhnike* (Vortex Effect and Its Application in Technique) (Moscow: Mashinostroenie, 1969)
4. Suslov A D et al. *Vikhrevye Apparaty* (Vortex Apparatuses) (Moscow: Mashinostroenie, 1985)
5. Merkulov A P (Ed.) *Vikhrevoĭ Effekt i Ego Primenenie v Tekhnike* (Vortex Effect and Its Application in Technique) (Kuĭbyshev: KuAI, 1988)
6. Gol'dshtik M A, Shtern V N, Yavorskiĭ N I *Vyazkie Techeniya s Paradoksal'nymi Svoĭstvami* (Viscous Flows with Paradoxial Properties) (Novosibirsk: Nauka, 1989)
7. Belostotskiĭ Yu G, Smirnov A P *Byulleten' Izobr.* (2) 68 (1996)
8. Kuznetsov V I, Thesis for Doctorate of Physicomathematical Sciences (Leningrad: Leningrad Technological Institute of Refrigiration Industry, 1991)
9. Barsukov S I, Kuznetsov V I *Vikhrevoĭ Effekt Ranka* (Vortex Ranque Effect) (Irkutsk: Irkutsk University, 1983)

10. Kuznetsov V I *Teoriya i Raschet Effekta Ranka* (Theory and Calculation of the Ranque Effect) (Omsk: Omsk State Technology University, 1995)
11. Khalatov A A *Teoriya i Praktika Zakruchennykh Potokov* (Theory and Practice of Rotational Flows (Kiev: Nauka Dumka, 1989)
12. Gol'dshtik M A *Vikhrevye Potoki* (Vortex Flows) (Novosibirsk: Nauka, 1981)
13. Shtym A N *Aerodinamika Tsiklonno-Vikhrevykh Kamer* (Aerodynamics of Vortex Chambers) (Vladivostok: Far East University, 1985)
14. Gupta K, Lilley D G, Syred N *Swirl flows* (England: Abacus Press,1984) [Translated into Russian (Moscow: Mir, 1987)]
15. Chernov A N, Thesis for Candidate of Physicomathematical Sciences (Moscow: Mosow Higher Technological School, 1982)
16. Borisov A A, Kuĭbin P A, Okulov V L *Dokl. Akad. Nauk* **331** 28 (1993) [*Phys. Dokl.* **38** 277 (1993)]
17. Borisov A A, Kuĭbin PA, Okulov V L *Sibirsk. Fiz. Tekh. Zh.* **V1** 30 (1993)
18. Dyskin L M *Izv. Vyssh. Uchebn. Zaved., Stroitel.* (11) 116 (1995)
19. Merkulov V I *Dokl. Akad. Nauk* **343** 57 (1995) [*Phys. Dokl* **40** 353 (1995)]
20. Zaporozhets E P, Kholpanov L P *Teor. Osnovy Khim. Tekhnol.* **30** 123 (1996)
21. Zaporozhets E P, Kholpanov L P *Khim. Prom.* 433 (1996)
22. Kurosaka M *J. Fluid Mech.* **124** 139 (1982)
23. Kurosaka M, Chu J Q, Goodman J R *AIAA Paper* 82-0592
24. Chu J Q, Ph D Thesis (Knoxville: The University of Tennessee, 1982)
25. Kurosaka M et al. *AIAA Paper* 83-0740
26. Kuroda H, Ph D Thesis (Knoxville: The University of Tennessee, 1983)
27. Stephan K et al. *Int. J. Heat Mass Transfer* **26** 341 (1983)
28. Amitani T, Adachi T, Kato T *T. JSME* **49** 877 (1983)
29. Balmer R T *Trans.ASME, J. Fluids Eng.* **110** 161 (1988)
30. Skornyakov G V *Pis'ma v Zh. Tekh. Fiz.* **15** 12 (1989) [*Sov. Tech. Phys. Lett.* **15** 877 (1989)]
31. Skornyakov G V *Zh. Tekh. Fiz.* **65** 35 (1995)
32. Skornyakov G V *Zh. Tekh. Fiz.* **66** 3 (1996)
33. Gutsol A F, in *Khimiya i Tekhnologiya Pererabotki Kompleksnogo Syr'ya Kol'skogo Poluostrova* (Chemistry and Technology of Processing of Complex Raw Materials on Kola Peninsula (Apatity: Kola Scientific Centre, Russian Academy of Science, 1996) p. 68
34. Kalinnikov V T, Gutsol A F *Dokl. Akad. Nauk* **353** 469 (1997)
35. Loitsyanskiĭ L G *Mekhanika Zhidkosti i Gaza* (Fluid and Gas Mechanics) (Moscow: Nauka, 1987)
36. Alekseev T S *Inzh.-Fiz. Zhurn.* **VII** (4) 121 (1964)
37. Hilsch R *Rev. Sci. Instrum.* **18** 108 (1947)
38. Abramovich G N *Prikladnaya Gazovaya Dinamika* (Applied Gas Dynamics) (Moscow: Nauka, 1976)
39. Abramovich G N, et al. *Teoriya Turbulentnykh Struĭ(Theory of Turbulent Jets) (Moscow: Nauka, 1984)*
40. Kuznetsov V I, Thesis for Candidate of Physicomathematical Sciences (Novosibirsk: 1975)
41. Rakowski W J, Ellis D H *Experimental analysis of blade instability* **1** R78 AEG 275, *General Electric Company Rep. for* F 33615-76-C-2035 *to Air Force Propulsion Lab.* WPAFB (1978) p. 67
42. Rakowski W J, Ellis D H, Bankhead H R *AIAA Paper* 78-1089
43. Borisov A A, Kuĭbin PA, Okulov V L *Fiz. Goreniya i Vzryva* (5) 107 (1993)
44. Alekseenko S V et al. *Dokl. Akad. Nauk* **345** 611 (1995) [*Phys. Dokl.* **40** 650 (1995)]
45. Fin'ko V E *Zh. Tekh. Fiz.* **53** 1770 (1983) [*Sov. Phys. Tech. Phys.* **28** 1089 (1983)
46. Gutsol A F *Khim. Vysokikh Energ.* **29** 303 (1995)
47. Gutsol A F, Kalinnikov V T *Zh. Prikl. Khim.* **68** 885 (1995)
48. Karabut A B et al. *Teplofiz. Vysokikh Temp.* **17** 618 (1979) [*High Temp.* **17** 514 (1979)
49. Sorokin L M *Fiz. i Khim. Obrab. Mater.* (4) 32 (1980)
50. Volchkov E P, Terekhov V I *Izv. Akad. Nauk SSSR, Sib. Otdel., Ser. Nauka i Tekh.* (11) Iss. 3, 14 (1967)
51. Zhukov M F, Koroteev A S, Uryukov B A *Prikladnaya Dinamika Termicheskoĭ Plasmy* (Applied Dynamics of Thermal Plasma) (Novosibirsk: Nauka, 1975)
52. Chigier N A et al. *Combustion and Flame* **14** 171 (1970)
53. Syred N, Beer J M *Combustion and Flame* **23** 143 (1974)
54. Bradshaw P (Ed.) *Turbulence* (Berlin, Heidelberg, New York: Springer-Verlag, 1978) [Translated into English (Moscow: Mashinostroenie, 1980
55. Zel'dovich Ya B, Raĭzer Yu P *Fizika Udarnykh Voln i Vysokotemperaturnykh Gidrodinamicheskikh Yavleniĭ* (Physics of Shocks and High-Temperature Hydrodynamic Phenomena (Moscow: Gos. Izd. Fiz. Mat. Lit., 1963)
56. Rusanov V D, Fridman A A *Fizika Khimicheski Aktivnoĭ Plasmy* (Physics of Chemically Active Plasma) (Moscow: Nauka, 1984)
57. Gutsol A F *Application for Patent of Russia* No. 951112323. Granted on 01.07.1996
58. Kostin N A et al. *Inzh. Fiz. Zh.* **67** 392 (1994)
59. Bradshaw P. *An Introduction to Turbulence and Its Measurement* (Pergamon Press, 1971) [Translated into Russian (Moscow: Mir, 1974)]
60. Dulov V G, Kustova E V *Vestnik St-Petersburg Univ.,* Ser. 1 (4) 61 (1995)
61. Sprenger M *Mitt. Inst. Aerodynamik* Zürich **21** 18 (1954)
62. Rozenberg L D (Ed.) *Fizika i Tekhnika Moshchnogo Ul'trazvuka* Kniga 1 *Istochniki Moshchnogo Ul'trazvuka* (Physics and Technique of Intensive Ultrasound, Book 1, Sources of Intensive Ultrasound) (Moscow: Nauka, 1967)