Physics – Uspekhi **40** (4) 433 – 440 (1997)

CONFERENCES AND SYMPOSIA

Jubilee scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences, celebrating the 80th anniversary of the birth of Academician V L Ginzburg (October 2, 1996)

A scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences was held at the P N Lebedev Physical Institute RAS on 2 October 1996. It marked an occasion of the 80th anniversary of the Academician V L Ginzburg's birth.

Nine reports were presented at this session:

(1) Opening address of **L V Keldysh** as the Academician-Secretary of the Division of General Physics and Astronomy of the Russian Academy of Sciences;

(2) **A F Andreev** (P L Kapitza Institute of Physical Problems RAS, Moscow) "Bose condensation and spontaneous distortion of the time homogeneity"

(3) **E L Feinberg** (P N Lebedev Physical Institute RAS, Moscow) "Special theory of relativity: how good-faith delusions come about";

(4) **R M Arutyunyan, V L Ginzburg, G F Zharkov** (P N Lebedev Physical Institute RAS, Moscow) "On the 'giant' thermoelectric effect in a hollow superconducting cylinder";

(5) **A V Gurevich, K P Zybin, V A Sirota** (P N Lebedev Physical Institute RAS, Moscow) "Cold dark matter and microlensing"

(6) **V V Zheleznyakov** (Institute of Applied Physics RAS, Nizhniĭ Novgorod) "Astrophysical plasma in extreme conditions";

(7) **B M Bolotovskiĭ, A V Serov** (P N Lebedev Physical Institute RAS, Moscow) "On the transient radiation interference with the proper field of an emitting charge";

(8) **E G Maksimov** (P N Lebedev Physical Institute RAS, Moscow) "Extreme evidences of lacking the strong change-correlation effects in HTSC systems";

(9) **M A Vasil'ev** (P N Lebedev Physical Institute RAS, Moscow) "Gauge theories of highest spins".

An abridged version of papers No 3, 4, and 6 is given below.

PACS number: **03.30.+p**

Special theory of relativity: how good-faith delusions come about

E L Feinberg

It is a fact of life in the field of the special theory of relativity that, while perfectly in terms of its technical aspects, many quite skilled and even very highly-skilled physicists, and among them Academician-level theorists are amazingly ignorant of its physical nature. To be specific (and leaving to the end of the article the discussion of the why's), our concern here is with the interpretation of the rod contraction and clock slowing down effects in going over from an inertial system 'at rest' to another one moving in a straight line x with velocity v. It is a misconception of the heart of the problem that this is a purely kinematic effect rather than a real physical change due to forces of some kind. Although most good physicists do know how things really stand, still already a quarter of century ago the present author found it necessary to clarify this question in a large Physics-Uspekhi 'methodological' note [1] full of quotations from both the 'introduction-to-relativity, manuals and not so serious 'relativity-for-millions' books (and poor millions those were!). Since then the number of such publications has grown and so has, understandably, the number of those in the dark. So what exactly are we talking about?

Let us recall here how Albert Einstein himself deduced the Lorentz transformations [2]. In the reference frame 'at rest' there are two identical rods and two identical sets of measuring yardsticks and synchronized clocks. Now let one of the rods and one of the sets be '*transferred*' to another inertial frame moving with the relative velocity v — which implies their being accelerated, of course. Once all acceleration effects come to a close, the rod — as the yardsticks and clocks of the initial frame will show — becomes $1/\beta$ ($\beta = \sqrt{1-v^2}$) times shorter (the speed of light $c = 1$), and the clocks are slow by the same factor. One can accelerate the rod in different ways, however — for example, by pushing it (say, 'to fire a gun') or by pulling its forward end or the middle. But, even though the elastic waves that emerge in the rod will be *different* depending on the particular way of acceleration chosen, once all calms down, the rod contraction will be the same. We are thus dealing with a *universality with respect to the acceleration regime*, the same being true of the slowing down of differently accelerated clocks. Now one may also apply nonmechanical forces by, say, electrically charging the rod and then passing it through an electrostatic, alternating magnetic, or some complex electromagnetic fields. Finally, present in the rod (clocks) themselves there also are strong and weak nuclear forces, gravitational forces, etc., and all these will also participate in the acceleration process. The result, however, will be as formerly at the same finite value of v and it is thus *universal with respect to the nature of forces at work*. The question raises, how is it possible?

The classics of the special theory of relativity knew how. Einstein did not say a word about any forces and acceleration regimes, and Pauli wrote that "the contraction of a scale is not

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a simple but ... rather a complex process.'' When all the laws ``governing the structure of the electron'' become known, Pauli went on, ``theory will be able to give an atomistic explanation'' [3]. Among those comprehended were also Laue [4] and Lorentz, of whom the latter wrote after he had finally accepted the special theory of relativity that the contraction effect was of the same type seen in the rod being cooled [5]. It is unfortunate that both considered the situation self-evident and so did not say more than a word or two on the subject.

Lorentz and Poincare wrote even before Einstein that these effects were possible if postulated to be the same for all types of forces, but having realized this for the case of gravitation, the latter of the two, the great Poincare was 'criminally' incautious in saying that ``this can only be explained in two ways: either all that exists in the world is of electromagnetic origin, or this property ... is nothing other than *external appearance* (italics mine, E F), something related to our measurement methods'' [6]. Neither is true, as we now know, and this is exactly where the disagreement between Poincare and Einstein stems from — for all that Lorentz and Poincare came seemingly very close to understanding and already knew much of the full truth.

So what is the essence of the problem then? The clue to this mystery is provided by Einstein's later comment [7] on the effect that the principle of relativity plays a role comparable to that of the law of conservation of energy in mechanics.

In fact, as the principle of relativity requires that the equations of motion for any set of particles and fields be covariant under the Lorentz transformations, so the Newtonian energy (and momentum) conservation law requires invariancy of the Lagrangian of the system under space-time translations. Covariant relativistic equations give, in principle, a full description of the acceleration process. Length contraction and clock slowing down represent a special but very general result (it was already obtained by Lorentz in his solution of Maxwell's equations for $v = \text{const}$, but this was of course something of a miracle because Maxwell, fully unaware of relativity, had nevertheless written his equations in a relativistically covariant form straight away, luckily not adding — which he could well have done! — any terms whatever insignificant for the low velocities v then known).

The conservation of energy also follows from equations of mechanics and is also a very general albeit special result. We will illustrate this point by the following example.

Suppose a stone is thrown upwards (along the z axis) with the initial velocity v_0 and one is asked to find the height h it will reach. One possibility is to solve Newton's equations with initial conditions $v_x(0) = v_y(0) = 0$, $v_z(0) = v_0$ and then to plot the stone trajectory thus finding the maximal height h required. Alternatively, however, one can simply write the energy conservation law, $mv_{0z}^2/2 = mgh$, for this purpose.

Now to complicate the problem, let $v_x(0) \neq 0$. Solving the appropriate equations yields the trajectory which is shown schematically in Fig. 1 and from which one finds h. But, again, we are able to write the same result, $h = v_{0z}^2/2g$.

Now suppose that a wind tunnel is directed along and has its orifice opened against the x axis. The air flow will blow the stone off to give the distorted trajectory shown in Fig. 2. But again no complicated equations need to be solved and the same result, $h = v_{0z}^2/2g$, is obtained.

The quantity $\beta = \sqrt{1 - v^2}$ in the special theory of relativity is universal in exactly the same way, whether we speak of the acceleration regime or the character of various fields involved. The analogy is complete and the origin of this universality lies in the fact that all the processes take place in Einstein's unified space-time.

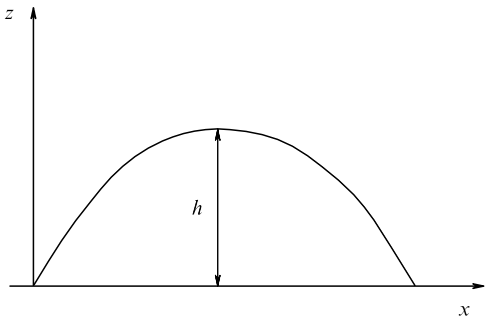

Figure 1.

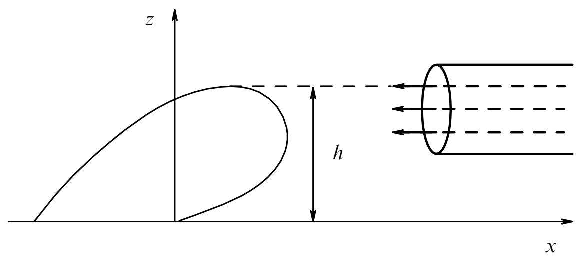

Figure 2.

And finally, the question is to be answered, why do so many people view scale contraction and clock slowing down as merely kinematic rather than dynamic effects? The blame lies with those numerous writers on the special theory of relativity who, while perfectly aware of the way things really are, still derive the Lorentz transformations 'in a simpler way' than Einstein did. It was Pauli [3] who first introduced this practice.

Suppose we have two frames of reference, one of which, K, with coordinates x, y, z, t, is 'at rest' and the other, $K'\,(x', y', z', t')$, moves (and is oriented) along the x axis. Suppose, further, that at the instant when the origins of the two coincide, a flash of light is emitted. According to the relativity postulate, the front of the light wave must be spherical in both frames, thus

$$x^2 + y^2 + z^2 - ct^2 = 0 = x'^2 + y'^2 + z'^2 - c^2t'^2 \,.$$

From this, by applying some group-theoretical arguments (the transition from K to K' and then to a third frame K'' is equivalent to the K–K'' transition, etc.) the Lorentz transformations follow. But, Pauli adds cautiously (and this hardly if at all out of his teens at the time of writing!): ``Group-theoretical considerations only yield the formulas of the transformations but not their physical content''.

Later authors ignored this important remark. The editor of the Russian publication [2] of Einstein's works wrote already in 1935 that ``transformation formulas are easier to obtain directly from the condition'' of the spherical shape of the wave as described above (see Russian translation of Ref. [2], p. 146). So they surely are — but in this case so some physical subtleties are left unaccounted-for. This practice has spread, however. Even L D Landau and E M Lifshitz, from whose excellent course several generations of physicists have grown, took this approach, and likewise many other authors have contented themselves with this simple derivation without giving much thought to the physical meaning of contraction and slowing down. These, they believe, are just kinematic effects ("we are just observing events from another frame of reference"), and dynamics has nothing to do with them.

To be sure, Landau had a clear understanding of the situation. This is perfectly evident — to cite but one example — from his remarkable hydrodynamical theory for multiple particle production in collisions of relativistically energetic nuclei: even though such nuclei, Lorentz-compressed into thin petals, do stop upon a collision in the centre-of-mass system, still they do not turn spherical instantaneously as nuclei at rest 'should' but rather, interacting with one another, they gradually expand to form a cylinder-shaped bidirectionally propagating continuous medium.

Which is a good lesson to all lecturers, popular physics authors and those writing texts on the special theory of relativity: do not keep silent about this complex dynamic process.

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On the 'giant' thermoelectric effect in a hollow superconducting cylinder

R M Arutyunyan, V L Ginzburg, G F Zharkov

It is known that a temperature gradient applied to a hollow superconducting bimetallic cylinder furnishes a small magnetic flux of order $10^{-2}\phi_0$, where $\phi_0 = 2 \times 10^{-7}$ G cm^2 is the magnetic flux quantum (for a treatment in detail, see the review in Ref. [1]). Indeed, the field in the cavity consists of a contribution due to the trapped flux $m\phi_0$ (m being the number of initially trapped quanta) plus the field H_{th} induced by the thermal current $\mathbf{j}_{\text{th}} = b\nabla T$ (b is the thermoelectric coefficient of the system). The latter contribution in the case of superconductors turns out to be strongly suppressed compared to its normal-metal value (because of the presence of two opposite currents in the bulk of the superconductor which mutually compensate one another so that $\mathbf{j}_{\text{s}} + \mathbf{j}_{\text{th}} = 0$ [1]), and it is for this reason that the thermal-current-related flux is expected to be of the order of $10^{-2}\phi_0$. In actual fact, however, much — indeed orders of magnitude — stronger fluxes of tens and hundreds of ϕ_0 have been measured [2]. There is at present no accepted explanation of this 'giant' thermoelectric effect.

A hypothesis was advanced [3] and later developed [4–8] that this effect is due to quantum transitions in which the number of the flux quanta 'trapped' in the cavity increases spontaneously under the action of the thermal current (thus producing a magnetic field in the cavity). There are serious arguments against this hypothesis, though. The quantum number m in a hollow superconductor is truly a topological invariant (see, e.g., [9]) and hence can change ($m \to m+1$) if only a vortex (carrying one magnetic flux quantum ϕ_0) enters through the outer surface of the sample. In the absence of an external field, however, there is at the outer surface a potential barrier [10] inhibiting the vortex motion into the superconductor. In the interior of the sample, topological arguments exclude vortex production. Because of this, the hypothesis has thus far remained just that because no specific transition mechanism has been proposed. It is the purpose of the present work to show that such a mechanism exists and thus to lend support to the hypothesis of Ref. [3].

The problem dealt with in [3] is a model one treating a homogeneous hollow cylinder of outer radius r_2 and internal radius r_1 in the presence of a specified normal current $\mathbf{j}_{\text{th}}$ circulating around the cavity (Fig. 1). The current $\mathbf{j}_{\text{th}}$ imitating a real thermoelectric current within this model is taken to be $\mathbf{j}_{\text{th}} = b\Delta T/\pi r$, where b is the thermoelectric coefficient of the system, and $\Delta T = T - T_1$, where T (T_1) is the hot (cold) junction temperature (for a treatment in detail, see Ref. [3]).

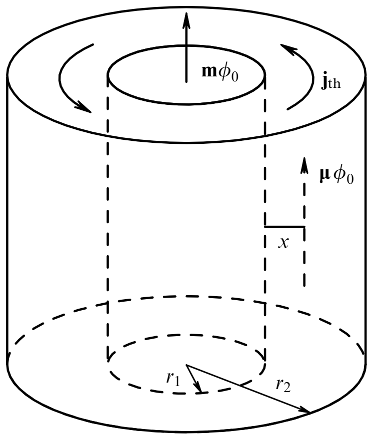

Figure 1. Cylinder of outer radius r_2 and internal radius r_1, $d = r_2 - r_1 \gg \lambda$, cavity field $\mathbf{H}_{\text{i}} = \mathbf{m}\phi_0/\pi r_1^2$; vortex $\boldsymbol{\mu}\phi_0$ is at a distance x from the cavity boundary. A fixed normal-state thermal current $\mathbf{j}_{\text{th}}$ flows round the cavity.

Suppose in the bulk of a type II superconductor of wall thickness $d = r_2 - r_1 \gg \lambda$ (λ is the superconducting penetration depth), at a distance x_1 from the cavity boundary (r_1), there occurs a vortex with the flux $\boldsymbol{\mu}_1\phi_0$ ($\boldsymbol{\mu}_1$ is a vector indicating whether the flow is along or against the z-axis of the cylinder). Another vortex, with flux $\boldsymbol{\mu}_2\phi_0$, let there is along the same radius at a distance x_2 from the cavity boundary. The thermodynamic potential (i.e. the Gibbs energy) of such a system is written as an integral over the sample volume and can be expressed in terms of surface integrals (see analogous transformations in Ref. [3]) as

$$
\begin{aligned}
G_{\rm s}(x_1,x_2) = \mathcal{F}_{\rm s0} + \frac{\phi_0}{8\pi}\big[&\mathbf{m}\cdot\mathbf{H}_{\rm i}(x_1,x_2) \\
&+ \boldsymbol{\mu}_1\cdot\mathbf{H}_{\mu 1}(0;x_1,x_2) + \boldsymbol{\mu}_2\cdot\mathbf{H}_{\mu 2}(0;x_1,x_2)\big] \\
&- \frac{\phi_0}{4\pi}\left[\mathbf{m}\cdot\mathbf{H}_{\rm th} + \boldsymbol{\mu}_1\cdot\mathbf{H}_{\rm th}\frac{\mathcal{L}(x_1)}{\mathcal{L}_0} + \boldsymbol{\mu}_2\cdot\mathbf{H}_{\rm th}\frac{\mathcal{L}(x_2)}{\mathcal{L}_0}\right] \\
&+ \frac{\lambda^2}{4\mathcal{L}_0}\mathbf{H}_{\rm th}\cdot\big(\mathbf{H}_{\rm i}(x_1,x_2) - \mathbf{H}_{\rm th}\big)\,. \qquad (1)
\end{aligned}
$$

Here $\mathcal{F}_{\rm s0}$ is the superconductor condensation energy; $\mathbf{m} = m\mathbf{e}_z$, m is the number of flux quanta initially trapped in the cavity; $\mathbf{e}_z$ is the unit vector along the z-axis; $\mathbf{H}_{\rm i}(x_1,x_2)$ is the field within the cavity (at $r = r_1$), whose value depends on the location of the vortices 1 and 2; $\mathbf{H}_{\mu 1}(0;x_1,x_2)$ is the field on the axis of the vortex 1 (and similarly for vortex 2); and

$$
\mathbf{H}_{\rm th} = \mathbf{e}_z H_{\rm th}\,, \qquad H_{\rm th} = \frac{4\pi}{c}\,b\mathcal{L}_0\frac{\Delta T}{\pi}\,,
$$
$$
\mathcal{L}_0 = \log\frac{r_2}{r_1}\,, \qquad \mathcal{L}(x) = \log\frac{r_2}{r_1+x}\,.
$$

Setting now $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 = 0$ in (1) yields an expression for $G_{\rm s0}$ appropriate for the vortex-free problem [3]. For $\boldsymbol{\mu}_1 = 0$ (or $\boldsymbol{\mu}_2 = 0$) we obtain the Gibbs energy $G_{\rm s}(x)$ for a hollow superconductor with one vortex in the presence of current $\mathbf{j}_{\rm th}$.

The expressions for the magnetic field strength on the axes of the vortices $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ take the form

$$
\begin{aligned}
\mathbf{H}_{\mu 1}(0;x_1x_2) &= \boldsymbol{\mu}_1 H_{\rm c0} f(x_1) + \boldsymbol{\mu}_2 H_{\rm c0} f(x_1,x_2) \\
&\quad + \mathbf{H}_{\rm i}(x_1,x_2)\exp\left(-\frac{x_1}{\lambda}\right), \\
\mathbf{H}_{\mu 2}(0;x_1,x_2) &= \boldsymbol{\mu}_2 H_{\rm c0} f(x_2) + \boldsymbol{\mu}_1 H_{\rm c0} f(x_1,x_2) \\
&\quad + \mathbf{H}_{\rm i}(x_1,x_2)\exp\left(-\frac{x_2}{\lambda}\right), \qquad (2)
\end{aligned}
$$

where

$$
H_{\rm c0} = \frac{\phi_0}{2\pi\lambda^2}\,,
$$

$$
\begin{aligned}
f(x) &= K_0(0) - K_0\left(\frac{2x}{\lambda}\right) - K_0\left(\frac{2d-2x}{\lambda}\right), \\
f(x_1,x_2) &= K_0\left(\frac{|x_1-x_2|}{\lambda}\right) - K_0\left(\frac{x_1+x_2}{\lambda}\right) \\
&\quad - K_0\left(\frac{2d-x_1-x_2}{\lambda}\right). \qquad (3)
\end{aligned}
$$

The functions $f(x_1)$, $f(x_2)$, and $f(x_1,x_2)$ account of both the own field of either vortex and that of the other vortex, as well as the contribution from the vortex images [11, 12] in the cylinder boundary mirror. (The surface may be considered plane since its curvature is negligible for $r_1 \gg \lambda$). The Bessel function of an imaginary argument $K_0(\rho)$ describes the magnetic field around the vortex axis at distances where one can neglect the effect of the vortex-induced field on the superconductor order parameter ($\rho > \xi$, ξ being the coherence length). On the axis of a vortex, numerical calculations of the type done in [13] show that we must set $K_0(0) = \log\varkappa$ and $K_1(0) = \varkappa$, where $\varkappa \gg 1$ is the familiar parameter of the Ginzburg – Landau theory.

The cavity field consists of two parts, $\mathbf{H}_{\rm i}(x_1,x_2) = \mathbf{H}_{\rm i0} + \delta\mathbf{H}_{\rm i}(x_1,x_2)$, where $\mathbf{H}_{\rm i0} = \mathbf{m}\phi_0/\pi r_1^2$ is the field in the cavity with m trapped flux quanta, and the field growth in the cavity due to the partial penetration into the cavity of fluxes from the vortices $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ is given by

$$
\delta\mathbf{H}_{\rm i}(x_1,x_2) = \boldsymbol{\mu}_1\frac{\phi_0}{\pi r_1^2}\exp\left(-\frac{x_1}{\lambda}\right) + \boldsymbol{\mu}_2\frac{\phi_0}{\pi r_1^2}\exp\left(-\frac{x_2}{\lambda}\right). \qquad (4)
$$

Writing the Gibbs energy $G_{\rm s}$ in the form $G_{\rm s} = G_{\rm s0} + \mathcal{G}(x_1,x_2)$, where $G_{\rm s0}$ is the Gibbs potential in the absence of vortices [3], the vortex-related term is found to be

$$
\mathcal{G}(x_1,x_2) = g(x_1,x_2)\,\frac{\phi_0 H_{\rm c0}}{8\pi}\,,
$$

where

$$
\begin{aligned}
g(x_1,x_2) &= \mu_1^2 f(x_1) + \mu_2^2 f(x_2) + 2\boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2 f(x_1,x_2) \\
&\quad + 2\frac{\lambda^2}{r_1^2}\left[\mu_1^2\exp\left(-\frac{2x_1}{\lambda}\right) + \mu_2^2\exp\left(-\frac{2x_2}{\lambda}\right)\right. \\
&\quad + 2\boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2\exp\left(-\frac{x_1+x_2}{\lambda}\right) + 2\boldsymbol{\mu}_1\cdot\mathbf{m}\exp\left(-\frac{x_1}{\lambda}\right) \\
&\quad \left. + 2\boldsymbol{\mu}_2\cdot\mathbf{m}\exp\left(-\frac{x_2}{\lambda}\right)\right] - a\big[\boldsymbol{\mu}_1\cdot\mathbf{e}_{\rm th}\mathcal{L}(x_1) \\
&\quad + \boldsymbol{\mu}_2\cdot\mathbf{e}_{\rm th}\mathcal{L}(x_2)\big]\,. \qquad (5)
\end{aligned}
$$

Here $\mathbf{e}_{\rm th} = \mathbf{H}_{\rm th}/H_{\rm th}$, $a = 2H_{\rm th}/\mathcal{L}_0 H_{\rm c0}$. (For $\boldsymbol{\mu}_2 = 0$, the function g describes a single vortex in the presence of a thermal current and is analogous to the Bean — Livingston function [10 – 12] used in describing a single vortex in the presence of an external field $H_{\rm e}$).

From (5) it follows that for $\boldsymbol{\mu}_2 = -\boldsymbol{\mu}_1$ the function $g(x_1,x_2)$ is zero at any x_1. In other words, the coexistence of a vortex and an antivortex (i.e. a vortex with an oppositely directed flux) at a certain point in a superconductor does not alter the energy of the system because the vortex-related field is fully compensated ($\boldsymbol{\mu}_1\phi_0 + \boldsymbol{\mu}_2\phi_0 = 0$) and hence does not affect the order parameter of the superconductor. This implies the possibility of fluctuation-assisted vortex — antivortex pair production at any point in the superconducting material, a process for which no energy is required. When the vortex and the antivortex move apart, however, two oppositely directed forces are operative: on the one hand, the vortex and antivortex attract each other (see [11, 12]), on the other hand, the thermal current tends to push them apart by displacing the vortex toward and the antivortex outward the cavity (as follows from the fact that the last two terms in (5) are opposite in sign at $\boldsymbol{\mu}_2 = -\boldsymbol{\mu}_1$). The function $g(x_1,x_2)$ (5) reflects the presence of various antagonistic factors in the system, among them the interaction of the vortices with one another and with the boundaries of the superconductor.

It can be seen that for $\boldsymbol{\mu}_2 = -\boldsymbol{\mu}_1$ the most favourable process is one in which a vortex — antivortex pair forms near the cavity ($x_1 = x_2 = 0$), where the field $H_{\rm th}(x)$ due to the current $j_{\rm th}$ is at its largest and acts to displace the vortex and the antivortex in opposite directions. While the antivortex will move outward by carrying away the flux $-\phi_0$ with itself, the vortex, having transferred its flux into the cavity, will turn into an additional current round the cavity in which the flux $(m+1)\phi_0$ will now be confined.

Setting $x_1 = 0$ in (5) we find the function $g(0,x_2)$ which gives the energy of the system for various positions of the antivortex x_2 relative to the cavity boundary. Formally, the vortex is on the inner boundary but, with its field having completely gone over to the cavity, it is not distinct from a current flowing round the cavity. The behaviour of $g(0, x_2)$ as a function of temperature T is shown schematically in Fig. 2.

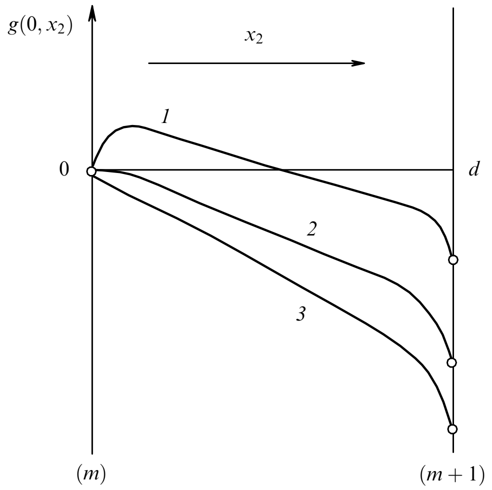

Figure 2. Function $g(0, x_2)$ for varied antivortex-cavity boundary distances at various temperatures: (*1*) $T < T_*$; (*2*) $T = T_*$; (*3*) $T > T_*$ (at the threshold temperature T_* the antivortex starts moving away from the cavity). Function $g(0, x_2)$ relates the states with m (for $x_2 = 0$) and $m + 1$ (for $x = d$) quanta in the system.

From Fig. 2 it is seen that near the cavity boundary ($x_2 = 0$) there exists a potential barrier inhibiting the antivortex detachment from the boundary (the force $F = -\partial g(0, x_2)/\partial x_2$ acting on the antivortex is directed into the cavity). As the temperature T increases, so does the coefficient $a \propto H_{\mathrm{th}} \propto j_{\mathrm{th}} \propto \Delta T = T - T_1$, and the barrier height lowers. The threshold temperature T_* at which the barrier disappears is found from the condition

$$g'_0 = \left.\frac{\partial g(0, x_2)}{\partial x_2}\right|_{x_2=0} = 0\,,$$

which can be written in the form

$$\varkappa + 2m\,\frac{\lambda_1^2}{r_1^2}\,\frac{1}{1-t} - \frac{a_0}{2}\,\frac{\lambda_1}{r_1}\,\frac{t}{(1-t)^{3/2}} = 0\,. \tag{6}$$

The reduced temperature $t = (T - T_1)/(T_c - T_1)$ takes values in the interval $0 \leqslant t \leqslant 1$. In deriving (6) the following formulas have been used:

$$a = a_0\,\frac{t}{1-t}\,, \qquad a_0 = \frac{16\pi}{c}\,\frac{bT_c\lambda^2(0)}{\phi_0}\,,$$

$$\lambda^2(T) = \frac{\lambda^2(0)}{1 - T_1/T_c}\,\frac{1}{1-t}\,, \qquad \lambda_1 = \frac{\lambda(0)}{\sqrt{1 - T_1/T_c}}\,.$$

The cubic equation (6) is solved using the Cardano formulas, but it is simpler to obtain from (6) the dependence $m(t)$ (taking the integral part of m):

$$[m] = \frac{r_1^2}{2\lambda_1^2}\left(\frac{a_0}{2}\,\frac{\lambda_1}{r_1}\,\frac{t}{\sqrt{1-t}} - \varkappa(1-t)\right), \tag{7}$$

i.e. the dependence of the total flux in the system, $\Phi = [m]\phi_0$, at points where it makes transitions from level m to $m + 1$. The derivative $\mathrm{d}\Phi/\mathrm{d}t$ is found to be

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \phi_0\,\frac{r_1^2}{2\lambda_1^2}\left[\frac{a_0}{2}\,\frac{\lambda_1}{r_1}\left(\frac{1}{\sqrt{1-t}} + \frac{t}{2(1-t)^{3/2}}\right) + \varkappa\right]. \tag{8}$$

Notice that for $t \to 1$ (i.e. for $T \to T_c$) we arrive at

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} \propto \frac{1}{(1-t)^{3/2}} \propto \frac{1}{(T_c - T)^{3/2}}\,.$$

The theory developed above is in principle adequate to explain the experimental data of Ref. [2]. Indeed, the quantum number m of the system (i.e. the total flux $\Phi_2 = m\phi_0$) remains unchanged when a vortex–antivortex pair is produced at any point in the superconductor bulk, nor topological laws are violated. If the vortex has its axis left at the cavity boundary ($x_1 = 0$), then its associated currents flow round the cavity and contribute to the field $\mathbf{H}_i$ which exists there. The field on the vortex axis therewith coincides with the weak cavity field and the order parameter Ψ shows no singularity at $x_1 = 0$. As the antivortex moves away from the boundary ($x_2 > 0$), a region with an oppositely directed field forms near its axis, the order parameter $\Psi (= 0)$ vanishing on the axis x_2 itself. (A detailed field-order parameter picture near the cavity boundary at $x_2 < \xi$ requires numerical vortex-structure calculations of the type discussed in Refs [11–13]). As the antivortex moves away from the boundary, the cavity field gradually increases, which means that additional round-the-cavity currents appear. The total flux in the system remains $\Phi_2 = m\phi_0$, however, and it is only when the antivortex comes within $\sim \lambda$ of the outer boundary and starts to give away its flux outward that the total flux gradually approaches $\Phi_2 = (m+1)\phi_0$. But the quantum number m of the system only jumps to $m + 1$ when the antivortex axis intersects the outer cavity boundary (in accordance with topology requirements), which is followed by the transition of the system to the $(m+1)\phi_0$ state. Thus, the proposed mechanism allows the system transition to a higher magnetic quantum level by first producing a vortex–antivortex pair and then pushing them apart by means of the thermal current. As a result, a clear physical picture emerges which seems to be a good basis for explaining the 'giant' thermoeffect observed.

To proceed to a more detailed discussion of the experiment of Ref. [2], notice that formula (7) yields the 'giant' effect outright (because every quantum produced in the system gives rise to a flux two orders of magnitude larger than the value $\sim 10^{-2}\phi_0$ expected from simple theoretical considerations [1]). According to [2], the total flux varies with temperature (near T_c) as $\mathrm{d}\Phi/\mathrm{d}t \propto (T_c - T)^{-3/2}$, which agrees with (8) for $t \to 1$. For lower t, temperature dependence (8) becomes weaker because of the large constant $\varkappa$ entering the formula. The same constant determines the large height of the barrier encountered by a single vortex entering the superconductor in the Bean–Livingston theory [10]. Note, however, that BL theory is only valid for mirror-smooth superconductor surfaces, for which the reflection method holds [10]. For rough surfaces, the measured threshold field [14] turns out to be much below the theoretical value [1], implying a smaller role of the last term in (9) and a wider range of validity for the $(T_c - T)^{-3/2}$ law. Also, it can be shown that increasing j_{th} (and hence the hot junction temperature, $T \to T_c$) lowers the barrier height for a vortex entering the sample through the outer boundary, where the presence of residual magnetic fields may be important.

Factors of this type must be taken into account when comparing the theory with experiment.

Notice that a quantitative comparison of formulas (5)–(7) and experimental results [2] is also complicated by the fact that the simple homogeneous model [3] we employ here is not entirely consistent with real experimental conditions and hence only a qualitative comparison is in fact possible. Let us first estimate the parameter a_0 determining the magnitude of the effect. Writing the coefficient b in the form α/ρ, where α is the thermoelectric coefficient and ρ the conductivity, and using tabulated values of α and ρ [15], it is found that $a_0 \sim 1-50$ for pure superconductors. Now note that we know nothing of the junction (alloy) characteristics of the pure In — pure Pb bimetallic samples used in [2]. This may be important because it is at the junction (which is the system's weakest point, with $\varkappa$ and λ at their greatest) where a vortex – antivortex pair is most likely to appear. On the other hand, the thermoelectric current j_{th}, and hence the parameter a_0, depend on the bulk characteristics of pure superconducting materials, and the bulk value of $\varkappa$ is not normally large. As a result, the parameters of the system may be chosen more or less at will. Taking $T_c = 5$ K, $1 - T_1/T_c = 10^{-2}$, $a_0 = 10$, $\varkappa = 10$, $r_1 = 0.1$ cm, $\xi_0 = 10^{-5}$ cm gives an estimate of $t_* \simeq 0.99$ for the temperature t_* at which flux jumps start to appear in the system. The onset of an anomalously large flux at lower values of t [2] can be due to a number of reasons. In particular, because the samples used in [2] were toroidal and had an internal cavity of rectangular cross section, geometrical factors affecting vortex formation differed considerably from those for the cylindrical case. In the discussion above, the effects of surface roughness and of junction properties have been mentioned. At the interface between two superconductors with widely different values of λ, it is known [16] that the height of the barrier to vortex motion is reduced significantly. Notice, further, that instead of pair appearing right away in the form of two extended antiparallel filaments, a vortex-antivortex pair may also be produced at less energy expenditure as a closed finite-size ring similar to vortex rings in superfluid helium [17, 18]. All the above factors can influence greatly the height of the pair-forming barrier.

Thus, the theory developed above can, in principle, account for the large effect observed in [2], although further investigation with inclusion of realistic experimental conditions is needed. This we hope will be the subject of future work.

Finally, we note that the flux-quantum production mechanism discussed above may be relevant to the problem of origin of very strong magnetic fields in rotating neutron stars, whose substance may be superconducting or superfluid at high densities [19].

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Astrophysical plasma in extreme conditions

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In this work, extreme conditions encountered in space and the corresponding unusual plasma properties are discussed. Our primary concern here is with the interaction of plasma and radiation in strong magnetic fields existing in degenerate stars, namely in white dwarfs and neutron stars. Special attention is given to the crucial role which cyclotron scattering, cyclotron-frequency radiation pressure, and vacuum magnetization play in forming the plasma shells and spectra of such stars.

Since astrophysical plasma covers a much wider range of conditions compared to the laboratory, situations occur in which it behaves in a qualitatively different way from its laboratory counterparts and from 'normal' space plasmas close generally to the laboratory plasma. We refer to such situations as extreme conditions. The scientific significance of plasmas under such conditions is not limited to astrophysics: the challenges and potentials of this field stimulate the progress of plasma physics as a whole.

The extreme properties of astrophysical plasma are exhibited in strong magnetic fields in white dwarfs and neutron stars, in the strong gravitational fields of black holes, and under high-density conditions existing in the interior of neutron stars and characteristic of the early stages of the Universe. Of primary value in this connection are plasma objects whose radiation carries information on the conditions existing in the radiation source, thus enabling a direct assessment of the nature of the radiation-plasma interaction under extreme conditions. On the contrary, high-density objects, such as matter at the initial stage of the Big Bang or in the neutron star interior, are much less directly informative and their study presents therefore a much more complicated problem.

There are certain specific features in plasmas on the surface or in the immediate neighbourhood of a degenerate star which influence strongly the nature of the electromagnetic radiation emitted. These features depend primarily on the magnitude of the magnetic field and are most pronounced at cyclotron frequencies. Since these frequencies ($\omega \simeq \omega_B$) lie in the optical range (fields $B \sim 10^7 - 10^9$ G; white dwarfs) or in the X-ray-gamma ranges ($B \sim 10^{11} - 10^{13}$ G; neutron stars), the role of cyclotron effects in these stars is quite evident. It should be noted also that the radiation-plasma interaction may be considerably influenced by the vacuum magnetization and electron-positron pair production effects occurring in strong magnetic fields characteristic of neutron stars. Pair production is also presumed to occur in the vicinity of black holes in double systems where, owing to the accretion from the star-companion, high densities of radiation and matter are produced.

The effect of the magnetized vacuum on propagation of radiation becomes appreciable, for example, for the radiation source of the X-ray pulsar Her X-1 with $kT \sim 10$ keV located at a neutron star with $B \simeq 4 \times 10^{12}$ G if the plasma concentration $N \ll 10^{23}$ cm^{-3}. For a plasma in a magnetized vacuum, the cyclotron absorption and scattering coefficients in both modes are comparable at frequencies $\omega \simeq \omega_B$, whereas in weaker magnetic fields, when the effect of the vacuum is insignificant, cyclotron absorption and scattering change their nature completely: for ordinary waves they are virtually of no importance compared to the corresponding effects for the unusual component (this is discussed more fully in Ref. [1]). However, for cyclotron frequencies $\omega \simeq \omega_B$ the interaction of a magnetized plasma with radiation becomes different already in weaker fields $B \sim 10^7 - 10^9$ G typical of white dwarfs. There are two time characteristics peculiar to this interaction, namely the neighbouring-level Landau transition time t_c and the electron intercollisional transit time $t_{\text{eff}} = 1/\nu_{\text{eff}}$ (where ν_{eff} is the effective collision frequency of the electrons). The strong magnetic fields of degenerate stars (white dwarfs and neutron stars) reduce considerably the time $t_c \propto B^{-2}$ thus realizing the inequality $t_c \ll t_{\text{eff}}$. The distribution function — and first and foremost that for velocities perpendicular to **B** — is then determined by the radiation intensity at cyclotron frequencies, the role of collisions being relatively small. The dominant mechanism for the interaction of radiation with such 'collisionless' plasma is cyclotron resonant scattering. The inclusion of collisions makes the transport equations more complicated [1, 2]. Radiation transfer solutions for $t_c \ll t_{\text{eff}}$ differ substantially from those for weaker magnetic fields typical of the Sun and nondegenerate stars. Therefore plasma of white dwarfs with $B \sim 10^8 - 10^9$ G, where the criterion $t_c \ll t_{\text{eff}}$ is fulfilled well, proves also to be in extreme conditions.

We consider below some specific examples of plasma-radiation interaction in extreme conditions, starting from the so-called radiation-driven diskons.

The small family of observable magnetic white dwarfs contains four objects — GD 229, PG 1031 + 234, GrW + 70°8247, and a recently examined one with coordinates 1408 + 3054 — whose spectra show wide absorption bands in the ultraviolet range. These spectral dips remained unaccounted for until it was noted that plasma can escape the upper photosphere of a hot magnetic white dwarf, filling a vast region around the star. The reason for this is the strong radiation pressure due to the cyclotron scattering at $\omega \simeq \omega_B$: as indicated by calculations [3], it is to this group of objects with unstable photosphere where white dwarfs with magnetic fields $B \sim 10^8 - 10^9$ G and surface temperature $T_{\text{ph}} \gtrsim 2 \times 10^4$ K may belong.

The presence of a strong magnetic field with a closed configuration of force lines prevents plasma from escaping the neighbourhood of the star: in the region which is occupied by the plasma the longitudinal component of the radiation pressure force f_{rad} acting on the rarefied plasma exceeds the corresponding projection of the gravitational force f_g. For white dwarfs, this region is typically about several star radii in size. Far away from the star $f_{\text{rad}} < f_g$ because of the sharp radiation pressure drop due to the cyclotron frequency ω_B being shifted towards low frequencies, where photosphere radiation decreases dramatically in intensity [4].

If the optical thickness τ of the plasma shell with respect to cyclotron scattering is less than unity, then the equilibrium configuration of the plasma is one in which it is accumulated along a closed 'equilibrium surface', where the longitudinal components of the forces f_{rad} and f_g mutually compensate, and also along that portion of the magnetic equator plane which is located between the star and the equilibrium surface. In the limit $\tau \gg 1$, when $f_{\text{rad}}(\tau) \propto \tau^{-1}$, a new equilibrium configuration sets in, in which the entire region within the equilibrium surface is filled by the plasma. The plasma density grows to a level at which at each point of the surface under study $f_g \simeq f_{\text{rad}}(\tau) \propto \tau^{-1} \propto N^{-1}$ (as projected on the line-of-force direction). This is a local relation obviously realizable in the plasma shell.

The cyclotron-scattering-thick plasma shell formed by radiation pressure at cyclotron frequencies influences strongly the observed spectra of white dwarfs as exemplified by the magnetic white dwarfs GD 229 and PG 1031 + 234 with their intense and deep absorption bands in the ultraviolet. Such depression occurs if the shell electron number density $N > 10^8$ cm^{-3} (provided $B \simeq 5 \times 10^8$ G and the inhomogeneity scale of the dipole magnetic field $L_B \simeq R_*/3 \sim 3 \times 10^8$ cm). In depression bands, a rather strong radiation polarization is to be expected, although for a certain orientation of the star's magnetic field relative to the observer, polarization contributions from various parts of the star may of course compensate each other considerably. Evidence for this is contained in the 1980 and 1994 IUE satellite data on the ultraviolet spectra of GD 229.

The formation of cyclotron lines in the spectra of X-ray pulsars and gamma-ray bursts is another example of plasma-radiation interaction in extreme conditions. The X-ray pulsar Her X-1 displays two cyclotron harmonics at ω_B and $2\omega_B$, from whose values the magnetic field of the neutron star is estimated to be $B \simeq 4 \times 10^{12}$ G. The pulsar 4U 0115 + 69 with a cyclotron feature in its X-ray spectrum is coupled to a neutron star with the magnetic field $B \simeq 10^{12}$ G.

Cyclotron lines from X-ray pulsars had been predicted before being discovered [5]. The observed lines are rather narrow in profile due to their formation in the hot polar spot with a quasi-uniform magnetic field. The source of heating here is the magnetic-field-controlled accretion of the star-companion matter. The spot plasma temperature, as estimated from the radiation level in the continuum spectrum, may be as high as $T \sim 10^8$ K.

The cyclotron radiation transfer in neutron star and white dwarf plasmas being dominated by resonance scattering, it was suggested [6] that cyclotron lines form in the same way as Fraunhofer lines in the spectra of ordinary stars do. Later, recognition came [7] that radiation transfer outside a cyclotron line is determined by nonresonant Thomson scattering on electrons as well as by bremsstrahlung and free-free absorption processes. The absorption line from an

X-ray pulsar then appears in the spectrum owing to a strong cyclotron scattering against the continuum background, which is also weakened by the Thomson scattering.

With the help of detectors onboard 'Venera' space stations, cyclotron lines from space gamma-ray bursts were also found, which have been detected in many later observations. The latest 'Phobos-2' and 'Ginga' data of this kind revealed features on two cyclotron harmonics, the corresponding burst-source fields being within an order of magnitude of those on X-ray pulsars.

The cyclotron lines from space gamma-ray bursts provide a most convincing evidence for the existence of a linkage between the bursts and neutron stars. We note, however, that observation of gamma-bursts from the GRO satellite showed no cyclotron lines, and that the sky distribution of burst sources turned out to be isotropic and gave no evidence for correlation with either the galactic centre or disk. These observations spotlighted alternative scenarios of this phenomenon, ones namely involving the extragalactic (and possibly cosmological) origin of the gamma-bursts — for example, through the formation of a hot plasma cloud following a collision of two neutron stars (see Ref. [8] for a careful discussion).

Turning now to specific theoretical interpretations of cyclotron lines from magnetic neutron stars, the narrow cyclotron features imply the involvement of a relatively cold plasma (with an average particle energy of about tens of keV) above the hot spot; the 'cold' layer may present an electron-positron plasma. The formation of cyclotron lines has been studied analytically by solving the scattering-type transport equations [9] and also by the Monte Carlo method [10]. In both studies the parameters of the cyclotron-line formation region were established, and in the former work also upper limits were found for the distance to gamma-burst sources with a second harmonic in addition to the fundamental one; these are 0.6 kps and 3 kps for the GB 880205 and GB 870303 bursts, respectively. At large distances the second harmonic seen in absorption disappears due to the high radiation density in the source. The existence of the limit indicates the galactic origin of bursts with such features.

Extreme conditions also realize near black holes, where at high levels of accretion plasma with high density of matter and radiation forms. In the vicinity of the Schwarzschild sphere, active electron-positron pair production (due to photon-photon interaction, for example) is possible [11]. On the other hand, electron-positron annihilation processes yielding narrow 0.5-MeV annihilation lines and wide-band boosts of hundreds MeV/quantum occur far enough from this sphere in X-ray sources presumably containing black holes. Aside from the high admixture of positrons, plasma in annihilation regions is very much classical. Thus, while a black hole serves as a direct source of particles (which may be accelerated to relativistic energies), observable radiation processes did not strictly occur in extreme conditions. A good case in point is the so-called Great Annihilator, the X-ray 1E1740.7 — 2942 source located near the centre of the Galaxy. An important point about the GA is that it was discovered in the microwave region using the VLA antenna [12]. In the radio frequency range this object was found to exhibit a complex structure consisting of two jets and a central radio source, the position of the latter coinciding with the X-ray source. This circumstance gave rise to a model capable to explain in a unified manner the origin of the 0.5-MeV line, the wide-band boosts, and microwave radiation from this source [13].

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