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Superconductivity and superfluidity (what was done and what was not)

V L Ginzburg

Contents

Abstract. **The author's theoretical work on superconductivity and superfluidity from 1943 to 1996 is described.**

1. Introduction. Early works

I, the author of the present paper, am 80 years old and I cannot hope for obtaining new important scientific results. At the same time, I feel I need to summarize my work of over fifty years. I do not mean my work in general (I have been engaged in solving quite a variety of physical and astrophysical problems, see Ref. [1] p. 312) but my activity in the field of superconductivity and superfluidity. In general, it is not traditional to write such papers. In my opinion, however, this comes of a certain prejudice. In any case, I decided to try and write such a paper, something like a scientific autobiography, but devoted only to two related problems — superconductivity and superfluidity. I may say that it is not associated with some priority or any other claims; it is only a desire to continue my work, though in an unusual form. I leave it to the reader to judge whether this attempt was pertinent and successful.

I began working, that is, obtaining some results in physics in 1938 – 1939 when I graduated from the physics faculty of Moscow State University. Before the War, that is, up to the mid-1941 I was engaged in classical and quantum electrodynamics, as well as the theory of higher spin particles. We somehow felt that the war would break out and were scared of it, but were unprepared and lived with the hope that the danger would pass by. I am not going to generalize, but this atmosphere reigned in the Department of Theoretical Physics of FIAN (P N Lebedev Physics Institute of the USSR Academy of Sciences). When it did not pass by, we began looking, while waiting for the call-up or some other changes in our lives, for an application of our abilities which might be of use for defense. I, for one, was engaged in problems of radio wave propagation in the ionosphere (see Refs [1, 2]). But these and similar subjects remained, at least in my case, far from finding application in defense. Therefore I went on working in various fields under these or other influences. The most important such influence, not to mention the continuation of research in the field of relativistic theory of spin particles, was exerted by L D Landau. In 1939, after a year's confinement in prison, Landau started working on the theory of superfluidity of helium II†. I was present, if I am not mistaken, in 1940, at Landau's talk devoted to this theory (the corresponding paper [4] was submitted for publication in May 15, 1941). At the end of the paper [4] he also considered superconductivity interpreted as superfluidity of an electron liquid in a metal. I do not know whether an assertion of the kind had ever been expressed before, but it is hardly probable. (Some hint in this respect was made in Ref. [5]). The point is that superfluidity in the proper sense of the word‡ was

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† As is well known, P L Kapitsa's plea for Landau's discharge from prison was motivated by the very wish to have his assistance in the field of superfluidity theory (see Ref. [3], p. 345).

‡ We mean a frictionless flow through capillaries and gaps. As to the anomalous behaviour of liquid helium (^{4}He) below the λ-point, i.e., at a temperature $T < T_\lambda = 2.17$ K, the study of this issue began, in effect, in 1911. Precisely in the year when superconductivity was discovered [7] (for more details see Refs [8], [9]; paper [7] is also included in [9] as an appendix), Kamerlingh Onnes reported a helium density maximum at T_λ [10], [11]. It was only in 1928 that the existence of two phases — helium I and helium II — became obvious, and in 1932 a clear λ-shaped curve for the temperature dependence of the specific heat near the λ-point was obtained. Superhigh thermal conductivity of helium II was discovered by W Keezom and A Keezom (see the references in [11], [12]) in 1936 and, finally, superfluidity was revealed [5], [6] in 1938. One can thus say that it took 27 years (from 1911 to 1938) to discover superfluidity. Such a long

discovered only in 1938 independently and simultaneously by Kapitsa [5] and Allen and Misener [6]. At the same time, the origin of superfluidity remained obscure. Landau believed [4] that the responsibility rested with the spectrum of 'elementary excitations' in a liquid while Bose statistics and Bose-Einstein condensation had nothing to do with it. F London and Tisza, on the contrary, associated [12] superfluidity with Bose-Einstein condensation. Validity of the latter opinion became obvious in 1948 after obtaining liquid ^{3}He with atoms obeying Fermi statistics, the properties differing radically from those of liquid ^{4}He. Theoretically, the same conclusion was drawn by Feynman (see Ref. [13]). But nothing could be derived from it in respect of superconductivity because electrons obey Fermi statistics. As we know today, the solution of the problem (or rather the puzzle) lies in the fact that electrons in a superconductor form 'pairs' with zero spin. Such pairs can undergo Bose-Einstein condensation with which the transition to a superconducting state is associated. My fairly modest contribution to this subject consists in pointing out that in the Bose-gas of charged particles the Meissner effect must be observed [14]. The idea of 'pairing' itself did not occur to me. To the best of my knowledge, Ogg was first to suggest it in 1946 [15]. This viewpoint was supported and further developed by Schafroth [16]. However the cause and mechanism of pairing remained absolutely vague; and it was only in 1956 that Cooper pointed out [17] a concrete mechanism of pairing in a Fermi-gas with attracting particles. This was the basis on which Bardeen, Cooper, and Schrieffer finally formulated the first consistent, though model type microtheory of superconductivity [18] in 1957. It is curious that paper [18] contains no indications of Bose-Einstein condensation, while it is in fact the crucial point.

However, I am running many years ahead as far as my own work is concerned. Concretely, in 1943 I tried [19], on the basis of the Landau theory [4] of superfluidity, to construct a quasi-microscopic theory of superconductivity. The paper postulated a spectrum of electrons (charged 'excitations') in a metal with a gap Δ. For such a spectrum, superconductivity (superfluidity of a charged liquid) must be observed. The introduction of a gap provided a temperature dependence of the critical field and of its penetration depth into a superconductor which approximately corresponded to the actual one. Comparison between the theory and experiment gave the value $\Delta/k_B T_c = 3.1$. As is well known, in the BCS theory $2\Delta_0/k_B T_c = 3.52$, but the most important point is that $\Delta_0 \equiv \Delta(0)$ is the value of the gap at $T = 0$, and with increasing temperature the gap decreases to yield $\Delta(T_c) = 0$. In my paper, the gap Δ was assumed to be constant, and a satisfactory agreement with experiment is possibly explained by the inaccuracy of the experimental data employed. I do not think that a more detailed analysis of this question is pertinent because model [19] is of no more than historical value now†.

process is in obvious contrast with the discovery of superconductivity which was practically a one-stroke occurrence [7] (for details see Refs [8], [9]). One can hardly doubt that the reason lies in different methods. Superconductivity was detected when the electric resistance of a wire (or, more precisely, a capillary filled with mercury) was measured. The character of a liquid flow (in this case, helium II) through gaps and capillaries is much more complicated and, moreover, one had to chance upon conducting such measurements.

† It should be noted that all the three papers [19, 21, 22] were submitted for publication on the same date (November 23, 1943). I do not remember why it was so. Most probably it was connected with some special conditions of the war time.

This explanation is all the more probable, for in Ref. [19], for example, the occurrence of resonance phenomena for incident radiation at a frequency $\nu = \Delta/h$ was mentioned. In any case, the fact is that in his well-known review [20], published in 1956, Bardeen covered the results of paper [19] rather extensively. Notice that paper [19] also presented a survey of the macrotheory of superconductivity. It was followed by note [21] considering gyromagnetic and electron inertia experiments with superconductors. Finally, in the same year of 1944 paper [22], devoted to thermoelectric phenomena in superconductors, was published. This latter paper remains topical even now, and we shall return to it in Section 5. The above-mentioned papers [19, 21, 22] were included in the monograph 'Superconductivity' [24] written in 1944. Before taking up superconductivity, I analysed [23], on the basis of the Landau theory, the problem of light scattering in helium II. In what follows I shall consider this and some other papers devoted to superfluidity (see Section 6).

2. Ψ-theory of superconductivity (the Ginzburg – Landau theory)

Within the first two decades after the discovery of superconductivity, its study went rather slowly compared to today's standards. This does not seem strange if we remember that liquid helium, which was first obtained in Leiden in 1908, became available elsewhere only after 15 years, i.e., in 1923. Without plunging into the history (see Refs [8, 9, 11]), I shall restrict myself to the remark that the Meissner effect was only discovered [25] in 1933, i.e., 22 years after the discovery of superconductivity. Only after that did it become clear that a metal in normal and superconducting states can be treated as two phases of a substance in the thermodynamic sense of this notion. As a result, in 1934 there appeared [26, 20] the so-called two-liquid approach to superconductors involving the relation

$$F_{n0}(T) - F_{s0}(T) = \frac{H_{cm}^2(T)}{8\pi}, \tag{1}$$

where F_{n0} and F_{s0} are free-energy densities (in the absence of a field) in the normal and superconducting phase, respectively and H_{cm} is the critical magnetic field destroying superconductivity. Differentiation of expression (1) with respect to T leads to expressions for the differences of entropy and specific heat.

According to the two-liquid picture, the total electric current density in a superconductor is

$$\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n, \tag{2}$$

where $\mathbf{j}_s$ and $\mathbf{j}_n$ are the densities of the superconducting and normal current.

The normal current in a superconductor does not, in fact, differ from the current in a normal metal, and in the local approximation we have

$$\mathbf{j}_n = \sigma_n(T)\,\mathbf{E}, \tag{3}$$

where $\mathbf{E}$ is the electric field strength and σ_n is conductivity of the 'normal part' of the electron liquid; for simplicity we henceforth take $\mathbf{j}_n = 0$, unless otherwise specified.

In 1935, F London and H London proposed [27] for $\mathbf{j}_s$ the equations (now referred to as Londons' equations)

$$\mathrm{rot}\,(\Lambda\,\mathbf{j}_s) = -\frac{1}{c}\,\mathbf{H}, \tag{4}$$

$$\frac{\partial(\Lambda \mathbf{j}_s)}{\partial t} = \mathbf{E}\,, \tag{5}$$

where Λ is a constant, and the magnetic field strength $\mathbf{H}$ here and below does not differ from the magnetic induction $\mathbf{B}$.

We arrive at such equations, for example, proceeding from the hydrodynamic equations for a conducting 'liquid' which consists of particles with charge e, mass m, and velocity $\mathbf{v}_s(\mathbf{r}, t)$:

$$\begin{aligned}\frac{\partial \mathbf{v}_s}{\partial t} &= -(\mathbf{v}_s \nabla)\mathbf{v}_s + \frac{e}{m}\,\mathbf{E} + \frac{e}{mc}\,[\mathbf{v}_s \mathbf{H}] \\ &= \frac{e}{m}\,\mathbf{E} - \nabla \frac{\mathbf{v}_s^2}{2} + \left[\mathbf{v}_s\left(\operatorname{rot}\mathbf{v}_s + \frac{e}{mc}\,\mathbf{H}\right)\right]. \end{aligned}\tag{6}$$

Such an equation corresponds to infinite (ideal) conductivity [28] and is not an obstruction to the presence of a constant magnetic field in a superconductor, which contradicts the existence of the Meissner effect. Therefore, the Londons imposed, so to say, an additional condition $\operatorname{rot}\mathbf{v}_s + e\mathbf{H}/mc = 0$ interpreted as the condition of a vortex-free motion for a charged liquid. If $\mathbf{j}_s$ is written in the form $\mathbf{j}_s = e n_s \mathbf{v}_s$, where n_s is the charge concentration, the additional condition for $n_s = \text{const}$ assumes the form (4), and

$$\Lambda = \frac{m}{e^2 n_s}\,. \tag{7}$$

Equation (6) transforms to (5) up to a small term proportional to $\nabla \mathbf{v}_s^2$ (see Section 5). Within such an approach, the principal Londons' equation (4) is, of course, merely postulated. This condition is an effect of quantum nature and follows from the Ψ-theory of superconductivity [29] considered below and from the microtheory of superconductivity [18, 30] which in turn transforms near T_c to the Ψ-theory [31]).

Londons' equation (4), along with the Maxwell equation

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\,\mathbf{j}_s \tag{8}$$

at $\Lambda = \text{const}$ (we are obviously dealing with the quasi-stationary case) leads to the equations

$$\Delta\mathbf{H} - \frac{1}{\delta^2}\,\mathbf{H} = 0\,, \quad \Delta\mathbf{j}_s - \frac{1}{\delta^2}\,\mathbf{j}_s = 0\,, \tag{9}$$

$$\delta^2 = \frac{\Lambda c^2}{4\pi} = \frac{mc^2}{4\pi e^2 n_s}\,. \tag{10}$$

From Eqn (9) it follows that the magnetic field $\mathbf{H}$ and the current density $\mathbf{j}_s$ attenuate through the superconductor depth according to the exponential law (for example, in the field parallel to and near a flat boundary, we have $H = H_0 \exp(-z/\delta)$, where z is the distance from the boundary), that is, the Meissner effect arises. Londons' equations still hold true, but only in the case of a weak field

$$H \ll H_c\,, \tag{11}$$

where H_c is the critical magnetic field that destroys superconductivity (under conditions of non-local connection between the current and the field, Londons' equations do not hold either [20, 30], but we do not consider such cases here)†. If the field is strong, i.e., comparable with H_c, Londons' theory may be invalid or otherwise insufficient. So, from Londons' theory it follows that the critical magnetic field H_c, in which the superconductivity of a flat film of thickness $2d$ is destroyed (in the field parallel to it), is

$$H_c = \left(1 - \frac{\delta}{d}\,\operatorname{th}\frac{d}{\delta}\right)^{-1/2} H_{cm}\,,$$

where H_{cm} is the critical field for a massive specimen (see Refs [32, 33, 24] and the references therein). This expression for H_c, however, contradicts experimental data. The situation can be improved by introducing different surface tensions σ_n and σ_s on the boundaries of the normal and the superconducting phases with a vacuum [32]. It turns out, however, that

$$\frac{\sigma_n - \sigma_s}{H_{cm}^2/8\pi} \sim \delta \sim 10^{-5}\,\text{cm}\,.$$

At the same time, it might be expected that $(\sigma_n - \sigma_s)$ equals about $(10^{-7} - 10^{-8})\,H_{cm}^2/8\pi$, i.e., is of the order of the volume energy $H_{cm}^2/8\pi$ multiplied by a length comparable with atomic scale. Moreover, in the theory based on Londons' equations, on the boundary between the normal and the superconducting phases, the surface tension (surface energy) connected with the field and the current, is $\sigma_{ns}^{(0)} = -\delta\,H_{cm}^2/8\pi$. Consequently, to obtain a positive surface tension $\sigma_{ns} = \sigma_{ns}^{(0)} + \sigma_{ns}^{(l)}$ observed for a stable boundary it is necessary to introduce a certain surface energy $\sigma_{ns}^{(l)} > \delta\,H_{cm}^2/8\pi$ of non-magnetic origin. The introduction of such a comparatively high energy is totally ungrounded. On the contrary, one can think that a rational theory of superconductivity must automatically lead to the possibility of expressing the energy σ_{ns} in terms of parameters characterizing the superconductor.

Such a theory that generalized the Londons' theory, eliminated the indicated difficulties, and suggested some new conclusions, was the Ψ-theory [29] formulated in 1950‡. In the same year I wrote a review [33] devoted to the macrotheory of superconductivity including the Ψ-theory.

In the absence of a magnetic field, the superconducting transition is a second-order transition. The general theory of such transitions always includes [34] a certain parameter (the order parameter) η which, when in equilibrium, differs from zero in the ordered phase and equals zero in the disordered phase. For example, in the case of ferroelectrics, the role of η is played by the spontaneous electric polarization P_s and in the case of magnetics, by the spontaneous magnetization M_s (not long before the appearance of our paper [29], both these cases were discussed in the review [35]). In superconductors, where the ordered phase is superconducting, for the order parameter we chose a complex function Ψ which plays the part of an 'effective wave function of superconducting electrons'. This function can be so normalised that $|\Psi|^2$ is the concentration n_s of 'superconducting electrons'.

The free energy density of a superconductor and the field was written in the form

$$\begin{aligned} F_{sH} &= F_{s0} + \frac{H^2}{8\pi} + \frac{1}{2m}\left| -\mathrm{i}\hbar\nabla\Psi - \frac{e}{c}\,\mathbf{A}\Psi\right|^2, \\ F_{s0} &= F_{n0} + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4\,, \end{aligned}\tag{12}$$

† We mean here type I superconductors. For type II superconductors the Londons' theory has a wider limit of applicability, including the vortex phase for $H \ll H_{c2}$ at any temperature.

‡ This theory is usually called the Ginzburg – Landau theory. I try however to avoid this term, not out of false modesty, but rather because in such cases the use of one's own name is not conventional in Russian. Furthermore, in its application to superfluidity (not superconductivity) the Ψ-theory was developed not with L D Landau, but with L P Pitaevskiĭ and A A Sobyanin (see Section 4).

where $\mathbf{A}$ is vector potential of the field $\mathbf{H} = \mathrm{rot}\,\mathbf{A}$. Without the field, in the state of thermodynamic equilibrium $\partial F_{s0}/\partial|\Psi|^2 = 0$, $\partial^2 F_{s0}/\partial^2|\Psi|^2 > 0$ and we must have $|\Psi|^2 = 0$ for $T > T_c$ and $|\Psi|^2 > 0$ for $T < T_c$. This implies that $\alpha_c \equiv \alpha(T_c) = 0$ and $\beta_c \equiv \beta(T_c) > 0$, and $\alpha < 0$ for $T < T_c$. Within the validity limits of expansion (12) in $|\Psi|^2$ one can put $\alpha = \alpha'_c(T - T_c)$ and $\beta(T) = \beta_{T_c} \equiv \beta_c$. From this, at $T < T_c$ [see also Eqn (1)] we have

$$|\Psi|^2 \equiv |\Psi_\infty|^2 = -\frac{\alpha}{\beta} = \frac{\alpha'_c(T_c - T)}{\beta_c}\,, \tag{13}$$

$$F_{s0} = F_{n0} - \frac{\alpha^2}{2\beta} = F_{n0} - \frac{(\alpha'_c)^2(T_c - T)^2}{2\beta_c} = F_{n0} - \frac{H_{cm}^2}{8\pi}\,.$$

In the presence of the field, the equation for Ψ is derived upon varying the free energy $\int F_{sH}\,\mathrm{d}V$ with respect to Ψ^* and, obviously, has the form

$$\frac{1}{2m}\left(-\mathrm{i}\hbar\nabla - \frac{e}{c}\,\mathbf{A}\right)^2\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0\,. \tag{14}$$

If on the superconductor boundary the variation $\delta\Psi^*$ is arbitrary, i.e., no additional condition is imposed on Ψ and no additional term corresponding to the surface energy is introduced in Eqn (12), then the condition of minimal free energy is the so-called natural boundary condition on the superconductor boundary,

$$\mathbf{n}\left(-\mathrm{i}\hbar\nabla\Psi - \frac{e}{c}\,\mathbf{A}\Psi\right) = 0\,, \tag{15}$$

where $\mathbf{n}$ is the normal to the boundary (for more details see Ref. [29] and Section 3). Condition (15) refers to the case of a boundary between a superconductor and a vacuum or a dielectric. As regards the equation for $\mathbf{A}$, under the condition $\mathrm{div}\,\mathbf{A} = 0$ and after variation of the integral $\int F_{sH}\,\mathrm{d}V$ over $\mathbf{A}$ it becomes

$$\Delta\mathbf{A} = -\frac{4\pi}{c}\,\mathbf{j}_s\,, \quad \mathbf{j}_s = -\frac{\mathrm{i}e\hbar}{2m}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - \frac{e^2}{mc}\,|\Psi|^2\mathbf{A}\,. \tag{16}$$

An expression similar to (14) is, of course, also obtained for Ψ^*, and as expected, we have $\mathbf{j}_s\mathbf{n} = 0$ on the boundary [see Eqn (15)]. The solution of the problem of the distribution of the field, current, and function Ψ in a superconductor is reduced to the integration of the system of equations (14) and (16). If we assume that $\Psi = \Psi_\infty = \mathrm{const}$, the superconducting current density is $\mathbf{j}_s = -e^2|\Psi_\infty|^2\mathbf{A}/mc = -e^2 n_s\mathbf{A}/mc$ (with normalization $|\Psi_\infty|^2 = n_s$). Applying the operation rot to this expression, we obtain Londons' equation (4) [see also Eqn (7)]. Thus, the Ψ-theory generalizes the Londons theory and passes over into it in the limiting case $\Psi = \Psi_\infty = \mathrm{const}$.

Paper [29] is rather long (19 pages) and solves several problems to which we shall return in what follows. After that, I myself, sometimes with co-authors, devoted a number of papers to the development of the Ψ-theory of superconductivity. These papers are mentioned below. Moreover, this theory was further promoted and accounted for in a lot of papers and books (see, for example, Refs [20, 30, 33, 36–41]). I do not follow the corresponding literature now, the more so as equation (14) and its extensions are widely used outside superconductivity or only in application to superconductors (see, for example, Refs [42–44]). This equation is also being investigated by mathematicians whose works are incomprehensible to me (see, for example, Ref. [45]). The relativistic generalization of the equations of the Ψ-theory and some of the concepts associated with this theory also enjoy wide application in quantum field theory (spontaneous symmetry breaking, etc.; see, for example, Ref. [46]). In such a situation, it seems absolutely impossible to elucidate here the present-day state of the Ψ-theory or even focus in detail on the original paper [29] and my subsequent papers.

But what I think necessary is to tell the story of the appearance of paper [29] and to speak about the role of Landau and myself. Nobody else can do this because regretfully Lev Davidovich Landau passed away long ago (he stopped working in 1962 and died in 1968). At the same time, this is, of course, a very delicate question. That is why, when 20–25 years ago I was addressed by the bibliographical magazine 'Current Contents' with a request to elucidate the history of the appearance of paper [29], I refused. My refusal was motivated by the fact that my story might be interpreted as an attempt to exaggerate my role. And in general I had no desire to prove that I was indeed a full co-author and not a student or a post-graduate to whom Landau 'had set a task', whilst actually doing everything himself. Without such a premise it is difficult to explain why our paper has been frequently cited as Landau and Ginzburg, although it is known to have Ginzburg and Landau in the title. Of course, I have never made protestations concerning this point and in general consider it to be a trifle, but still I believe that such a citation with a wrong order of authors is incorrect. It would certainly still be incorrect even if my role had indeed been a secondary one. But I do not think so, neither did Landau, and this fact was well known to his circle and generally in the USSR. As to foreigners, they really did not know much of scientific research in the USSR at that time, for in 1950 the 'cold war' was at its height. As far back as 1947, the USSR Journal of Physics, which was a good journal, stopped being published and paper [29] appeared only in Russian. We could not go abroad at that time. Perhaps we sent a reprint to D Shoenberg or he himself came across this article in ZhETF. In any event, Shoenberg translated the paper into English on his own initiative, distributed it among some people and it became available at least to some colleagues. Landau's name played, of course, a positive role and stimulated a lively interest in the paper.

One way or another, I decided to dwell on the history of the appearance of the work [29] because the present paper would be incomplete if I did not.

I regard the already mentioned paper [32], being accomplished as far back as 1944 (it was submitted for publication on December 21, 1944), as initial. From Ref. [32] it is quite clear that the Londons theory is invalid for the description of the behavior of superconductors in strong enough fields and, in particular, for the calculation of the critical field in the case of films. The introduction of the surface energies σ_n and σ_s was an artificial technique, and these quantities were absurdly large new constants whose values were not predicted by the theory. The same applies to the surface energy σ_{ns} on the boundary between the normal and superconducting phases. It was also absolutely unclear how the critical current should be calculated in the case of small-sized superconductors. Therefore, it was necessary somehow to generalize the Londons theory, to overcome its limits. Unfortunately, advancement in this direction was slow. One of the possible explanations is that like many theoretical physicists of my generation and the previous, I was simultaneously engaged in the solution of various problems and did not concentrate on anything definite (it can be seen, for instance, from the bibliographical index [47]). But there was gradual progress. So, on the basis of the conception of the Landau theory [4] I came to the conclusion [48] that electromagnetic processes in superconductors must be nonlinear and, incidentally, suggested a possible experiment for revealing such nonlinearity. The main point is that in note [48] I made the following remark: 'The indication of a possible inadequacy of the classical description of superconducting currents consists in the fact that the zero energy of excitation in a superconductor is equal in order of magnitude to $\hbar^2 n/m\delta \sim 1$ erg cm^{-2} (for $\delta \sim 10^{-5}$ cm and $n \sim 10^{22}$ cm^{-3}) and is thus higher than the magnetic energy $\delta H^2/8\pi \sim 0.1$ erg cm^{-2} (for $H \sim 500$ Oe)'. The feeling that the theory of superconductivity should take into account quantum effects was also reflected in note [49] devoted for the most part to critical velocity in helium II. At the same time, in that paper I also tried to apply the theory of second-order phase transitions to the λ-transition in liquid helium.

It seems surprising, and unfortunately, it did not occur to me at that time to ask, why Landau, the author of the theory of phase transitions [34] and the theory of superfluidity [4], had never posed the question of the order parameter η for liquid helium. In Ref. [49], I chose such a parameter ρ_s, i.e., the density of the superfluid phase of helium II. However, this choice raises doubts because the expansion of the free energy (thermodynamic potential) begins with the term $\alpha\rho_s$, whereas in the general theory the first term of the expansion has the form $\alpha\eta^2$. Hence, $\sqrt{\rho_s}$ is a more pertinent choice as the order parameter. But $\sqrt{\rho_s}$ is proportional to a certain wave function Ψ so far as it is precisely the quantity $|\Psi|^2$ that is proportional to the particle concentration. Unfortunately, I do not remember exactly whether it was these arguments alone that prompted me to introduce the order parameter $\eta = \Psi$, and nothing is said about it in Ref. [49]. More important for me was the desire to explain the surface tension σ_{ns} by the gradient term $|\nabla\Psi|^2$. In quantum mechanics this term has the form of kinetic energy $\hbar^2|\nabla\Psi|^2/2m$. It was precisely this idea that I suggested to Landau, probably in late 1949 (paper [29] was submitted on April 20, 1950, but it had taken much time to prepare it). I was on good terms with Landau, I attended his seminars and often asked his advice on various problems. Landau supported my idea of introducing the 'effective wave function Ψ of superconducting electrons' as the order parameter, and so we were immediately led to the free energy (12). The thing I do not remember exactly (and certainly do not want to contrive) is whether I came to him with the ready expression

$$\frac{1}{2m}\left| -\mathrm{i}\hbar\nabla\Psi - \frac{e}{c}\,\mathbf{A}\Psi\right|^2$$

or with an expression without the vector potential. The introduction of the latter is obvious by analogy with quantum mechanics, but perhaps this was only made during a conversation with Landau. I feel I should present my apologies to the reader for such reservations and uncertainty, but since that time nearly 50 years have passed (!), no notes have remained, and I never thought that I would have to recall those remote days.

After the basic equations (12), (14) and (16) of the Ψ-theory were derived, one had to solve various problems on their basis and compare the theory with experiment. Naturally, it was I who was mostly concerned with this, but I regularly met with Landau to discuss the results. What has been said above may produce the impression that my role in the creation of the Ψ-theory was even greater than that of Landau. But this is not so. One should not forget that the fundamental basis was the theory [50, 34] of second-order phase transitions developed by Landau in 1937 which I had employed in a number of cases [35, 49] and applied to the theory of superconductivity in paper [29]. Moreover, I find it necessary to note that the important remark made in [29] concerning the meaning of the Ψ-function used as a order parameter was due to Landau himself. I shall cite the relevant passage from [29]: 'Our function $\Psi(\mathbf{r})$ may be thought of as immediately related to the density matrix $\rho(\mathbf{r},\mathbf{r}') = \int \Psi^*(\mathbf{r},\mathbf{r}'_i)\,\Psi(\mathbf{r}',\mathbf{r}'_i)\,\mathrm{d}\mathbf{r}'_i$, where $\Psi(\mathbf{r},\mathbf{r}'_i)$ is the true Ψ-function of electrons in a metal which depends on the co-ordinates of all the electrons $\mathbf{r}_i$ ($i = 1,2,\ldots,N$) and $\mathbf{r}'_i$ are the co-ordinates of all the electrons except a distinguished one (its co-ordinates are $\mathbf{r}$ and at another point $\mathbf{r}'$). One may think that for a non-superconducting body, where the long range order is absent, as $|\mathbf{r}-\mathbf{r}'| \to \infty$ we have $\rho \to 0$, while in the superconducting state $\rho(|\mathbf{r}-\mathbf{r}'| \to \infty) \to \rho_0 \neq 0$. In this case it is natural to assume the density matrix to be related to the introduced Ψ-function as $\rho(\mathbf{r},\mathbf{r}') = \Psi^*(\mathbf{r})\Psi(\mathbf{r}')$.' Accordingly, the superconducting (or superfluid) phase is characterized by a certain long range order which is absent in ordinary liquids (see also Refs [30], § 26; [51, 51a] and [52], Section 9.7). This result is usually ascribed to Yang [51] and is referred to as ODLRO (off-diagonal long range order) [52]. However, as we can see, Landau realized the possibility of the existence of this long range order 12 years before Yang. I mentioned this fact in Ref. [53].

In expression (12) and those subsequent appear the coefficients e and m. These designations were of course chosen by analogy with the quantum-mechanical expression for the Hamiltonian of a particle with charge e and mass m. Our Ψ-function is however not the wave function of electrons. The coefficient m can be taken arbitrarily [29] because the Ψ-function is not an observed quantity; an observed quantity is the penetration depth δ_0 of a weak magnetic field [see Eqns (12), (13), (16)]:

$$\delta_0^2 = \frac{mc^2\beta_c}{4\pi e^2|\alpha|} = \frac{mc^2}{4\pi e^2|\Psi_\infty|^2}\,. \tag{17}$$

Since the Ψ-theory in a weak field (11) transforms to the Londons theory (though, a number of problems cannot be stated even in this case), the penetration depth δ_0 is frequently called the London penetration depth and is denoted by δ_L or λ_L.

If we assume [29] e and m to correspond to a free electron ($e_0 = 4.8 \times 10^{-10}$ CGS, $m_0 = 9.1 \times 10^{-28}$ g) then $|\Psi_\infty|^2 = n_s$, where n_s is the 'superconducting electron' concentration thus defined. In fact, one can choose any arbitrary value of m [29, 37] which will only affect the normalization of the observed quantity $|\Psi_\infty|^2$. In the literature $m = 2m_0$ occasionally occurs, which corresponds to the mass of a 'pair' of two electrons. As to the charge e in Eqn (12) and subsequent expressions, it is an observed quantity (see below). It seemed to me from the very beginning that one should regard the charge e in Eqn (12) as a certain 'effective charge' e_{eff} and take it as a free parameter. But Landau objected, and in paper [29] it is stated as a compromise that 'there is no reason to assume the charge e to be other than the electron charge'. Running ahead I shall note that I still went on thinking of the question of the role of the charge $e \equiv e_{\rm eff}$ as open and pointed out the possibility of clarifying the situation by comparing the theory with experiment (see Ref. [14], p. 107). The point is that the essential parameter involved in the Ψ-theory is the quantity

$$\varkappa = \frac{mc}{e\hbar}\sqrt{\frac{\beta_{\rm c}}{2\pi}} = \frac{\sqrt{2}e}{\hbar c}\,H_{\rm cm}\delta_0^2\,. \qquad (18)$$

In paper [29] we set $e = e_0$ and could therefore determine $\varkappa$ from experimental data on $H_{\rm cm}$ and δ_0. At the same time, the parameter $\varkappa$ enters the expressions for the surface energy $\sigma_{\rm ns}$, for the penetration depth in a strong field ($H \gtrsim H_{\rm cm}$) and the expressions for superheating and supercooling limits. Using the approximate data of measurements available at the time, I came to the conclusion [54] (this paper was submitted for publication on August 12, 1954) that the charge $e \equiv e_{\rm eff}$ in Eqn (18) is two-three times greater than e_0. When I discussed this result with Landau, he put forward a serious objection to the possibility of introducing an effective charge (he had apparently had this argument in mind before, when we discussed paper [29], but did not then advance it). Specifically, the effective charge might depend on the composition of a substance, its temperature and pressure and, therefore, might appear to be a function of coordinates. But in that case, the gradient invariance of the theory would be broken, which is inadmissible. I could not find arguments against this remark, and with the consent of Landau I included it in paper [54]. The explanation seems now to be quite simple. No, an effective charge $e_{\rm eff}$, which might appear to be coordinate-dependent, should not have been introduced. But it might well be supposed that, say, $e_{\rm eff} = 2e_0$. And this was exactly the case, but it became obvious only after the creation of BCS theory [18] in 1957 and the appearance of the paper by Gor'kov [31] who showed that the Ψ-theory near $T_{\rm c}$ follows from the BCS theory. More precisely, the Ψ-theory near $T_{\rm c}$ is certainly wider than the BCS theory in the sense that it is independent of some particular assumptions used in the BCS theory. But this is a different subject. The formation of pairs with charge $2e_0$ is a very general phenomenon, too. I have already emphasized above that the idea of pairing, and what is important, the realistic character of such pairing, was far from trivial.

So, in the Ψ-theory we have $e = 2e_0$, and consequently [see Eqn (18)]

$$\varkappa = \frac{2\sqrt{2}e_0}{\hbar c}\,H_{\rm cm}\delta_0^2\,. \qquad (19)$$

As can be seen from the calculations, the surface tension $\sigma_{\rm ns} > 0$ is positive only for $\varkappa < 1/\sqrt{2}$. An analytical calculation of $\sigma_{\rm ns}$ encounters difficulties. In paper [29] this was only done for sufficiently small $\varkappa$:

$$\sigma_{\rm ns} = \frac{\delta_0 H_{\rm cm}^2}{\sqrt{2}\cdot 3\pi\varkappa}\,, \quad \Delta = \frac{\sigma_{\rm ns}}{H_{\rm cm}^2/8\pi} = \frac{1.89\delta_0}{\varkappa}\,, \quad \sqrt{\varkappa} \ll 1\,. \quad (20)$$

From this it is already seen that the Ψ-theory leads to $\sigma_{\rm ns}$ values of the required order of magnitude. It is only in the recently received preprint [55] that the energy $\sigma_{\rm ns}$ is calculated analytically up to terms of the order of $\varkappa\sqrt{\varkappa}$. The result is as follows [the value $\Gamma = 2\sqrt{2}/3$ corresponds to expression (20)]:

$$\sigma_{\rm ns} = \frac{\delta_0 H_{\rm cm}^2}{4\pi\varkappa}\,\Gamma\,,$$
$$\Gamma = \frac{2\sqrt{2}}{3} - 1.02817\sqrt{\varkappa} - 0.13307\varkappa\sqrt{\varkappa} + \ldots \qquad (21)$$

As $\varkappa$ increases, the energy $\sigma_{\rm ns}$ decreases, and in Ref. [29] it was pointed out that according to numerical integration

$$\sigma_{\rm ns} = 0\,, \quad \varkappa = \frac{1}{\sqrt{2}}\,. \qquad (22)$$

But it was also shown that for $\varkappa > 1/\sqrt{2}$ there occurs some specific instability of the normal phase, namely, nuclei of the superconducting phase are formed in it. Concretely, this instability arises in the field

$$H_{\rm c2} = \sqrt{2}\varkappa H_{\rm cm}\,. \qquad (23)$$

(It should be noted that formula (23) is present in Ref. [29] in an implicit form; it was written explicitly in Ref. [56]). In case $\varkappa < 1/\sqrt{2}$, the field $H_{\rm c2}$ corresponds to the limit of a possible supercooling of the normal phase (for $H < H_{\rm c2}$ this phase becomes metastable; see also Ref. [56], where, as in some of my other papers, the field $H_{\rm c2}$ is denoted by $H_{\rm k1}$). When $\varkappa > 1/\sqrt{2}$, it is clear from Eqn (23) that superconductivity is preserved in some form in the field $H > H_{\rm cm}$ too and vanishes only in the field $H_{\rm c2}$. Generally, it is just for $\varkappa = 1/\sqrt{2}$ that the change in the behavior of a superconductor becomes pronounced. Hence, there were no doubts in the validity of the result (22). Analytically this is proved, for example, in Refs [30, 37, 38]. It turns out that for pure superconducting metals we typically have $\varkappa < 1/\sqrt{2}$ or even $\varkappa \ll 1/\sqrt{2}$ (for instance, according to Ref. [30] $\varkappa$ is equal to 0.01 for Al, 0.13 for Sn, 0.16 for Hg, and 0.23 for Pb). Such superconductors are called type I superconductors. If $\varkappa > 1/\sqrt{2}$, the surface tension $\sigma_{\rm ns}$ is negative, and we then deal with type II superconductors (for the most part alloys) whose behavior was first investigated thoroughly in experimental studies by L V Shubnikov† and co-authors as far back as 1935 – 1936 (for references and explanations see Refs [57, 24]). In Ref. [29] we considered only type I superconductors, and there is such a phrase there: 'For sufficiently large $\varkappa$, on the contrary, $\sigma_{\rm ns} < 0$, which is indicative of the fact that such large $\varkappa$ do not correspond to the typically observed picture'. So, we in fact overlooked the possibility of the existence of type II superconductors. Neither was I engaged in the study of type II superconductors later on. In this respect I only made a remark in Ref. [56]. The theory of the behavior of type II superconductors based on the Ψ-theory was constructed in 1957 by Abrikosov [58] (see also Refs [30], [41]). As indicated in Refs [58] and [30], p. 227, Landau was the first to suggest that in alloys $\varkappa > 1/\sqrt{2}$.

Allowing for Eqns (13) and (17), one can write

$$H_{\rm cm} = \left[\frac{4\pi(\alpha_{\rm c}')^2}{\beta_{\rm c}}\right]^{1/2}(T_{\rm c} - T)\,,$$
$$\delta_0 = \left(\frac{m_0 c^2\beta_{\rm c}}{16\pi e_0^2\alpha_{\rm c}'}\right)^{1/2}(T_{\rm c} - T)^{-1/2}\,. \qquad (24)$$

† L V Shubnikov, a prominent physicist experimenter, was guiltlessly executed in 1937.

These expressions, the same as the whole Ψ-theory, are strictly speaking valid only in the vicinity of T_c, i.e., the condition $(T_c - T) \ll T_c$ is needed. However, the condition of applicability of the theory for small $\varkappa$ is in fact more rigorous, because to satisfy the local approximation, the penetration depth δ_0 must significantly exceed the size ξ_0 of the Cooper pair (the corresponding condition written in Refs [30], §45 has the form $(T_c - T) \ll \varkappa^2 T_c$, but in Ref. [29] this, of course, could not yet be discussed). Along with the penetration depth δ_0, the Ψ-theory involves one more parameter which has the dimension of length — the so-called coherence length or the correlation radius (length)†

$$\xi = \frac{\hbar}{\sqrt{2m_0|\alpha|}} = \frac{\hbar}{\sqrt{2m_0\alpha'_c(T_c - T)}} = \frac{\hbar\tau^{-1/2}}{\sqrt{2m_0\alpha'_c T_c}} = \xi(0)\tau^{-1/2}, \qquad (25)$$

where $\tau = (T_c - T)/T_c$ and $\xi(0) = \hbar/\sqrt{2m_0\alpha'_c T_c}$ is a conditional correlation radius for $T = 0$ (we call it conditional because the Ψ-theory is strictly speaking applicable only in the vicinity of T_c).

As is readily seen [see Eqns (18), (19), (24)],

$$\varkappa = \frac{m_0 c}{2e_0\hbar}\sqrt{\frac{\beta_c}{2\pi}} = \frac{\delta_0(T)}{\xi(T)} \,. \qquad (26)$$

In addition to the above-mentioned problems, some more points were considered in Ref. [29], namely, the field in a superconducting half space and critical fields for plates (films) in the case where superconductivity is destroyed by the field and current. The penetration depth of the field in a superconducting half space adjoining a vacuum has the form

$$\delta = \delta_0\left[1 + f(\varkappa)\left(\frac{H_0}{H_{cm}}\right)^2\right], \quad f(\varkappa) = \frac{\varkappa(\varkappa + 2\sqrt{2})}{8(\varkappa + \sqrt{2})^2}, \qquad (27)$$

where H_0 is the external field (the field for $z = 0$), and by definition $\delta = \int_0^\infty H(z)\,dz/H_0$. The nonlinearity of the electrodynamics of superconductors, which was assumed already in Ref. [48] and is reflected in the dependence of δ on H_0, is fairly small. So, even for $\varkappa = 1/\sqrt{2}$ and $H_0 = H_{cm}$, the depth is $\delta = 1.07\,\delta_0$. In 1950 there were no accurate enough experimental measurements of $\delta(H)$. I am not sure that they have yet been carried out, though it is probable.

Now I should make, or rather repeat, one general remark. I was never long engaged in studying superconductivity, but researched various fields (see Refs [1], p. 312 and [47]). As to the macroscopic theory of superconductivity (the Ψ-theory and its development), it was generally beyond the scope of my interest from a certain time (see Section 3). As a result, I am ignorant of the current state of the problem as a whole. Unfortunately, neither am I aware of the existence of a monograph compiling all the material (I am afraid there is no such book). Moreover, I forgot much of what I had done myself and now recollect the old facts, sometimes with surprise, when reading my own papers. That is why I cannot be convinced that my old calculations were unerring, I do not know the subsequent calculations and the results of their comparison with experiment. But the present paper does not even claim to make a current review, it is only an attempt to elucidate some problems of the history of studies of superconductivity and superfluidity in an autobiographical context. Those uninterested will just not read it, and in this I find some consolation.

The concluding part of paper [29] is devoted to a consideration of superconducting plates (films) of thickness $2d$ in an external magnetic field H_0 parallel to the film and also in the presence of a current $J = \int_{-d}^{+d} j(z)\,dz$ (where $j(z)$ is the current density) flowing through the film. Instead of J, it is convenient to work in terms of the field $H_J = 2\pi J/c$ created by the current outside the film.

In the absence of current, the critical field H_c destroying superconductivity for thick films with $d \gg \delta_0$ is [see Eqn (27)]

$$\frac{H_c}{H_{cm}} = 1 + \frac{\delta_0}{2d}\left(1 + \frac{f(\varkappa)}{2}\right), \quad d \gg \delta_0 \,. \qquad (28)$$

For sufficiently thin films a transition to the normal state is a second-order transition (i.e., for $H_0 = H_c$ the function Ψ is equal to zero) and for small $\varkappa$ we have

$$\left(\frac{H_c}{H_{cm}}\right)^2 = 6\left(\frac{\delta_0}{d}\right)^2 - \frac{7}{10}\varkappa^2 + \frac{11}{1400}\varkappa^4\left(\frac{d}{\delta_0}\right)^2 + \ldots, \quad d \ll \delta_0 \,. \qquad (29)$$

For films with half thickness $d > d_c$, where

$$d_c^2 = \frac{5}{4}\left(1 - \frac{7}{24}\varkappa^2 + \ldots\right)\delta_0^2 \,, \qquad (30)$$

we are already dealing with first order transitions with a release of latent transition heat (in other words, d_c is a tricritical point or, as it was termed before, a critical Curie point).

In the presence of a current and field (for $\varkappa = 0$)

$$\frac{H_{J_c}}{H_{cm}} = \frac{2\sqrt{2}}{3\sqrt{3}}\frac{d}{\delta_0}\left[1 - \left(\frac{H_0}{H_c}\right)^2\right]^{3/2}, \quad d \ll \delta_0 \qquad (31)$$

where H_c is the critical field for a given film in the absence of a current [see Eqn (29)], H_0 is the external field and J_c is the critical current destroying superconductivity ($H_{J_c} = 2\pi J_c/c$).

The field H_c for such films is much larger than the critical field H_{cm} for bulk samples, and $H_{J_c} \ll H_{cm}$. It is interesting, however, that according to Eqns (29) and (31) (for $\varkappa = 0$ and $H_0 = 0$) we are led to

$$H_c H_{J_c} = \frac{4}{3} H_{cm}^2 \,. \qquad (32)$$

In Ref. [29] we certainly tried to compare the theory with the then available experimental data. But the latter were not numerous and, particularly importantly, their accuracy was low. To the best of my knowledge, all the results of the theory were later confirmed by experiment.

3. The development of the Ψ-theory of superconductivity

In paper [29] we did not of course solve all the problems, not even those which it was easy to formulate. Therefore, I naturally continued, although with some intervals, to

† To compare the formulas written here with those of Ref. [30], one should bear in mind that in expression (12) in Ref. [30] $m = 2m_0$ and, of course, $e = 2e_0$.

develop the Ψ-theory for several years. For example, in paper [59] (see also Ref. [14]) I considered in more detail than in Ref. [29] the destruction of superconductivity of thin films having half thickness $d > d_c$ [see Eqn (30); the condition $(\varkappa d/\delta_0)^2 \ll 1$ was used]. Critical fields were found for supercooling and superheating. I note that not for films, but for cylinders and balls, critical fields were calculated (on the basis of the Ψ-theory) by Silin in Ref. [60] and myself in Ref. [61]. The critical current for superconducting films deposited onto a cylindrical surface was found in paper [62]. The question of normal phase supercooling [see Eqn (23)] was discussed in paper [56], already mentioned above, and the critical field for superheating of the superconducting phase in bulk superconductors was calculated in paper [61]. So, for small $\varkappa$ the critical field for superheating (denoted as the field H_{k2} in Ref. [61]) is

$$\frac{H_{c1}}{H_{cm}} = \frac{0.89}{\sqrt{\varkappa}}\,, \quad \sqrt{\varkappa} \ll 1\,, \tag{33}$$

where the coefficient is obtained from numerical integration.

In several papers (see Refs [14, 32, 54, 63]) I discussed, in particular, the behavior of superconductors in a high-frequency field, but later on showed no interest in this issue and am now unaware whether these papers were of interest and importance for experiments (in respect of the behavior in a high-frequency field).

As I have already emphasized, the Ψ-theory can be immediately applied only in the vicinity of T_c. Naturally, I wished to extend the theory to the case of any temperature. In the framework of the phenomenological approach this goal can be achieved in different ways. So, Bardeen suggested [64] replacement of the expression for the free energy F_{s0} from Eqn (12) by another expression involving a more complicated dependence of $F_{s0}\left(|\Psi|^2\right)$ on $|\Psi|^2$. The same object can, however, be attained [65] without changing expression (12) but assuming a certain dependence of the coefficients α and β on temperature or, more precisely, on the ratio T/T_c. A somewhat different approach to the problem consists [66] not in assuming the dependence $F_{s0}\left(|\Psi|^2\right)$ in advance, but rather in finding it from comparison with experiment.

After creation of the BCS theory in 1957 and the papers [31] by Gor'kov, I almost lost interest in the theory of superconductivity. Superconductivity was no longer an enigma (it had been an enigma for a long 46 years after its discovery in 1911). There existed quite a lot of other attractive problems and I thought that I would drop superconductivity for ever. It was merely by inertia that in 1959, when it became finally clear that the effective charge in the Ψ-theory was $e_{\text{eff}} \equiv e = 2e_0$, I compared [67] the Ψ-theory with the available experimental data and made sure that everything was all right. I will also mention the note [68] devoted to the allowance for pressure in the theory of second-order phase transitions as applied to a superconducting transition.

It was F London [69] who had already pointed out that a magnetic flux through a hollow massive superconducting cylinder or a ring must be quantized, and that the flux quantum must be $\Phi_0 = hc/e$ and the flux $\Phi = k\Phi_0$, where k is an integer and e is the charge of the particles carrying the current. Naturally, London assumed $e = e_0$ to be a free electron charge. It was only in 1961 that the corresponding experiments were carried out (for references and the description of the experiments see, for example, Ref. [70]) demonstrating that, in fact, $e = 2e_0$. The latter is quite clear from the point of view of the BCS theory according to which it is pairs of electrons that are carried over. Thus,

$$\Phi = \frac{hck}{2e_0} = \frac{\pi\hbar ck}{e_0} = \Phi_0 k\,,$$

$$\Phi_0 = 2\times 10^{-7}\ \text{G cm}^2\ (k = 0, 1, 2, \ldots)\,. \tag{34}$$

This result (34) refers, however, only to the case of doubly connected bulk samples, for instance, hollow cylinders with wall thickness substantially exceeding the magnetic field penetration depth δ in a superconductor. And yet, samples of any size, as well as those located in an external magnetic field, etc., are also of interest. Within the framework of the Ψ-theory, I solved this problem in paper [71]. A similar, but less thorough and comprehensive analysis appeared nearly simultaneously in papers [72, 73] (all the papers [71 – 73] were submitted for publication in mid-1961).

I have not yet mentioned my papers [74] and [75] which were written before the creation of the BCS theory which however fell out of the scope of direct application of the Ψ-theory [29]. So, in Ref. [74] the Ψ-theory was extended to the case of anisotropic superconductors. In the 'low-temperature' (conventional) superconductors known at that time, anisotropy is either absent altogether (isotropic and cubic materials) or is fairly small. It was apparently for this reason that in Ref. [29] we assumed, even without reservations, that metals are isotropic. But already in paper [22], when I considered thermoelectric phenomena, I had to examine an anisotropic (i.e., non-cubic) crystal, and in view of this I generalized the Londons theory (4), (5) by introducing a symmetric tensor of rank two, Λ_{ik}, instead of the scalar Λ so that $\operatorname{rot}\boldsymbol{\Lambda}(\mathbf{j}) = -\mathbf{H}/c$, $\Lambda_i(\mathbf{j}) = \Lambda_{ik} j_k$, (here $\mathbf{j} = \mathbf{j}_s$ is the superconducting current density). Such a generalization is, of course, obvious enough, but I mention it here because in the extensive review [20] Bardeen refers in this connection only to papers [76, 77] by Laue which appeared later.

In Ref. [74], the complex scalar function $\Psi(\mathbf{r})$ for anisotropic material is introduced as before, but the free energy is written not in the form (12) but as

$$F_{sH} = F_{s0} + \frac{H^2}{8\pi} + \frac{1}{2m_k}\left|-i\hbar\frac{\partial\Psi}{\partial x_k} - \frac{2e_0}{c}\,A_k\Psi\right|^2, \tag{35}$$

where doubly occurring indices are of course summed up, and in Ref. [74] the charge e is taken instead of $2e_0$; for an isotropic or cubic material $m_1 = m_2 = m_3 = m$, and we obtain Eqn (12).

As mentioned above, the anisotropy in 'conventional' superconductors is not large, i.e., the 'effective masses' m_k differ little from one another. But in the majority of high-temperature superconductors, on the contrary, the anisotropy is very large, and it is expression (35) and the corollaries to it, partially mentioned already in Ref. [74], that are widely used.

Among the superconductors known in the 1950s there was not a single ferromagnetic. This is, of course, not accidental. The point is that even digressing from microscopic reasons, the presence of ferromagnetism hampers the occurrence of superconductivity [75]. Indeed, one can see that in the depth of a ferromagnetic superconductor the magnetic induction $\mathbf{B}$ must also be zero. But spontaneous magnetization $\mathbf{M}_s$ causes induction $\mathbf{B} = 4\pi\mathbf{M}_s$. Consequently, in a ferromagnetic superconductor, even in the absence of an external magnetic field, there must flow a surface superconducting current compensating for the 'molecular' current responsible for magnetization. From this it follows that a thermodynamic critical magnetic field for a ferromagnetic superconductor is

$$H_{\mathrm{cm}}(T) = \frac{H_{\mathrm{cm}}^{(0)}(T)}{\sqrt{\mu}} - \frac{4\pi M_{\mathrm{s}}}{\mu}\,, \qquad (36)$$
$$H_{\mathrm{cm}}^{(0)} = \sqrt{8\pi(F_{\mathrm{n}0} - F_{\mathrm{s}0})}\,,$$

where the ferromagnetic is assumed to be 'ideal', i.e., for it $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} = \mu\mathbf{H} + 4\pi\mathbf{M}_{\mathrm{s}}$ (μ is magnetic permittivity) and $F_{\mathrm{n}0}$ and $F_{\mathrm{s}0}$ are free energies for the normal and superconducting phases of a given metal in the absence of magnetization and a magnetic field. Obviously, superconductivity is only possible under the condition $H_{\mathrm{cm}}^{(0)}(0) > 4\pi M_{\mathrm{s}}/\sqrt{\mu}$ which can hold, in fact, only for ferromagnetics with a not very large spontaneous magnetization M_{s}. With the appearance of the BCS theory it became clear that superconductivity and ferromagnetism obstruct each other even irrespective of the above-mentioned so-called electromagnetic factor. Indeed, conventional superconductivity is associated with the pairing of electrons with oppositely directed spins, while ferromagnetism corresponds to parallel spin orientation. Thus, the exchange forces that lead to ferromagnetism obstruct the appearance of superconductivity. Nevertheless, ferromagnetic superconductors were discovered, but naturally with fairly low values of T_{c} and the Curie temperature T_{M} (see Refs [75a], [213]). Unfortunately, I am unacquainted with corresponding experiments and wish to emphasize here that the 'electromagnetic factor' was allowed for above in only the simplest, trivial case of an equilibrium uniform magnetization of bulk metal. However, there exist alternative possibilities [75]. So, let us assume that a ferromagnetic metal possesses a large coercive force and that in the external field $H_{\mathrm{c}} < H_{\mathrm{coer}}$ magnetization can remain directed opposite to the field (for simplicity we consider cylindrical samples in a parallel field). Then for $M_{\mathrm{s}} < 0$ (the magnetization is directed oppositely to the field) superconductivity may exist under the condition $H_{\mathrm{cm}}^{(0)}(0) > 4\pi|M_{\mathrm{s}}|/\sqrt{\mu} - \sqrt{\mu}H_{\mathrm{coer}}$, i.e., in principle, the 'electromagnetic factor' may be absolutely insignificant. Of even greater interest are possibilities arising in the case of thin films and generally small-size samples. For them, the critical field $H_{\mathrm{c}}^{(0)}$, as is well known and has already been mentioned above, may substantially exceed the field $H_{\mathrm{cm}}^{(0)}$ for bulk metal. At the same time, a critical field for a ferromagnetic superconducting film, even for $M_{\mathrm{s}} > 0$ (when the magnetization is directed along the field) has, as before, the form (36) but with $H_{\mathrm{cm}}^{(0)}$ replaced by $H_{\mathrm{c}}^{(0)}$. Now, the presence of magnetization M_{s} may already be of no importance. Thus, additional possibilities arise for investigating ferromagnetic superconductors. I do not know if these possibilities have ever been considered.

We have up to now discussed only equilibrium or metastable (superheated or supercooled) states of superconductors, fluctuations being totally ignored. Meanwhile, fluctuations near phase transition points, especially for second-order transitions, generally speaking play an important role (see, for example, Refs [34], §146). In the case of superconductors one should expect fluctuations of the order parameter Ψ both below and above T_{c}. I can tell the reader about my activity in this field. In 1952, at the end of paper [78] it was noted that fluctuations of the 'concentration of superconducting electrons' n_{s} must also be present above T_{c}, and this must affect first of all the complex dielectric constant of a metal. At the end of review [14] this remark was made again with emphasis on the fact that as $T \to T_{\mathrm{c}}$ the fluctuations must be large. However, I never elaborated upon this observation later. 14 years had passed before V V Schmidt [79] (untimely demised in 1958) went farther and considered (with a reference to the paper [78]) the question of the fluctuational specific heat of small balls above T_{c} and also mentioned the possibility of observing the fluctuational diamagnetic moment of such balls. It is curious that another two physicists with this name investigated [80, 81] the same issue and, moreover, considered fluctuational conductivity above T_{c} (for the fluctuation effects see also Refs [30, 82, 83]).

Let us now turn to a very important question of the applicability limits of Landau's phase transition theory both in the general context and in its application to superconductors [84].

Landau's phase transition theory [50, 34] is well-known to be the mean field theory (or, as it is sometimes referred to, molecular or self-consistent field theory). This means that the free energy (or a corresponding thermodynamic potential) of the type

$$F = F_0 + \alpha\eta^2 + \frac{\beta}{2}\,\eta^4 + \frac{\gamma}{6}\,\eta^6 + g(\nabla\eta)^2 \qquad (37)$$

does not allow for the contribution from the fluctuations of η.

As we have seen in the example of a superconductor, when $\eta = \Psi$ [see Eqns (12), (13)], below the second-order transition point (we set $\gamma = 0$) the equilibrium value is

$$\eta_0^2 = -\frac{\alpha}{\beta} = \frac{\alpha_{\mathrm{c}}'(T_{\mathrm{c}} - T)}{\beta_{\mathrm{c}}}\,. \qquad (38)$$

Taking the Landau theory as the first approximation and using it as a basis, one can find the fluctuations of various quantities, in particular, the parameter η itself. Naturally, the Landau theory holds true and the fluctuations calculated on its basis hold true only as long as they are small compared to the mean values obtained within the Landau theory. In application to η this means that the condition

$$\overline{(\Delta\eta)^2} \ll \eta_0^2\,, \qquad (39)$$

must hold, where obviously $\overline{(\Delta\eta)^2}$ is the statistical mean of the fluctuation of the quantity η (the fluctuation $\overline{(\Delta\eta)}$ is zero because we calculate the deviations from the value η_0 corresponding to the minimum free energy).

The use of criterion (39) leads to the following condition of applicability of the Landau theory [see Eqns (37), (38)]

$$\tau \equiv \frac{T_{\mathrm{c}} - T}{T_{\mathrm{c}}} \gg \frac{k_{\mathrm{B}}^2 T_{\mathrm{c}}\beta_{\mathrm{c}}^2}{32\pi^2\alpha_{\mathrm{c}}' g^3}\,, \qquad (40)$$

where k_{B} is the Boltzmann constant. This means that the Landau theory can be exploited within the temperature range in the vicinity of the transition point T_{c} satisfying the inequality (40). A condition of type (40) or similar was derived in different but close ways in Refs [84–86, 34]. So, in Ref. [86] the condition of applicability of the Landau theory is written in the form (in our notation; moreover, in Refs [86] and [34] k_{B} was assumed to be equal to unity)

$$\mathrm{Gi} = \frac{T_{\mathrm{c}}\beta_{\mathrm{c}}^2}{\alpha_{\mathrm{c}}' g^3} \ll \tau \ll 1\,, \quad \tau = \frac{T_{\mathrm{c}} - T}{T_{\mathrm{c}}}\,. \qquad (41)$$

The number Gi in Ref. [86] was called the Ginzburg number, but I never employ this terminology for the reason mentioned above in respect of the Ψ-theory. In my opinion it is more appropriate to employ a criterion of the form (40) because the coefficient $1/32\pi^2$ is fairly small, and this extends, in fact, the limits of applicability of the Landau theory (note that in Ref. [84] the coefficient $1/32\pi^2$ in the final expression (5b) is omitted, but it is clear from formula (4) for $\overline{(\Delta\eta)^2}$).

Obviously, the smaller the number Gi, the closer to the transition point the Landau theory can be used, in which, in particular, the specific heat simply undergoes a jump (without λ-singularity) and $\eta_0^2 \sim T_c - T$. This immediately implies, for example, that in liquid helium (^{4}He) the parameter Gi is large, and this results in the existence of the λ-singularity. In Ref. [84], various transitions are discussed, the most detailed consideration being given to ferroelectrics to which the Landau theory is generally well applicable, as to other structure phase transitions. This subject was discussed many years later in paper [87], but we shall not touch upon it here. In the present paper we are concerned with superconducting transitions and the λ-transition in liquid helium. The latter is dealt with in Section 4. As far as superconductors are concerned, from comparison of the expressions (12) with $e = 2e_0$ and $m = m_0$, (25), (26), (37) and (40) it follows that condition (40) takes on the form

$$\tau \equiv \frac{T_c - T}{T_c} \gg \tau_G \equiv \frac{(k_B\beta_c)^2}{32\pi^2(\alpha_c')^4 T_c^2 \left[\xi(0)\right]^6} . \tag{42}$$

This expression, however, bears no specific features for superconductors and refers to any second-order transition described by the Landau theory. In the framework of this theory, as is clear from Ref. [34] and, for example, from Eqns (13) or (37), the jump ΔC of specific heat $C = T\,\mathrm{d}S/\mathrm{d}T$, where $S = -\partial F/\partial T$ is entropy, at transition is

$$\Delta C = \frac{(\alpha_c')^2 T_c}{\beta_c} . \tag{43}$$

From Eqn (43) it is clear that condition (42) involves, in particular, the directly measurable quantity ΔC. Next, for superconductors† [see Eqns (13), (23), (25), (26) and (34)]

$$\begin{aligned}
&H_{cm}^2 = \frac{4\pi(\alpha_c')^2}{\beta_c}(T_c - T)^2 = \frac{4\pi(\alpha_c')^2 T_c^2}{\beta_c}\tau^2 = H_{cm}^2(0)\tau^2 ,\\
&H_{c2}^2 = 2\varkappa^2 H_{cm}^2 , \quad \xi^2 = \frac{\hbar^2}{2m_0\alpha_c' T_c}\tau^{-1} = \xi^2(0)\tau^{-1} ,\\
&\varkappa^2 = \frac{m_0^2 c^2 \beta_c}{8\pi e_0^2 \hbar^2} , \quad \xi^{-2}(0) = \frac{2e_0}{\hbar c} H_{c2}(0) = \frac{2\pi H_{c2}(0)}{\Phi_0} ,\\
&H_{c2}^2(0) = 2\varkappa^2 H_{cm}^2(0) .
\end{aligned} \tag{44}$$

† To avoid misunderstanding, we shall stress that all our consideration, as well as the Ψ-theory itself, refers directly to the region in the vicinity of T_c only. Consequently, the quantities $H_{cm}(0)$ and $H_{c2}(0)$ are somewhat formal and are not at all the true values of the fields $H_{cm}(T)$ and $H_{c2}(T)$ at $T = 0$. In view of this, it would be more correct to employ the derivatives

$$\left(\frac{\mathrm{d}H_{cm}}{\mathrm{d}T}\right)_{T=T_c} = -\frac{H_{cm}(0)}{T_c} \quad \text{and} \quad \left(\frac{\mathrm{d}H_{c2}}{\mathrm{d}T}\right)_{T=T_c} = -\frac{H_{c2}(0)}{T_c}$$

which can be measured in experiment.

Allowing for Eqns (43) and (44), one can rewrite condition (42) in the form

$$\tau \gg \tau_G = \left(\frac{2\pi}{\Phi_0}\right)^3 \frac{H_{c2}^3(0)}{32\pi^2(\Delta C)^2} , \quad \Phi_0 = \frac{\pi\hbar c}{e_0} . \tag{45}$$

For type I superconductors, the substitution in Eqns (42) and (45) of the values of $\xi(0)$ (or $H_{c2}(0)$) and ΔC known from experiment, even without account of the factor $1/32\pi^2 \sim 3\times10^{-3}$, yields the estimate $\tau_G \sim 10^{-15}$ (see Ref. [84] for $T_c \sim 1$) or, on the basis of the BCS model, the estimate $\tau_G \sim (k_B T_c/E_F)^4 \sim 10^{-12} - 10^{-16}$ (here E_F the is Fermi energy; see Refs [86], [30] §45). Physically it is obvious that the smallness of the value τ_G for superconductors is due to the high value of the correlation radius $\xi(0)$ in type I superconductors. In this case, the characteristic value $\xi(0) \sim \xi_0 \sim 10^{-4} - 10^{-5}$ cm is of the order of the size of a Cooper pair. For structure phase transitions $\xi(0) \sim d \sim 3\times10^{-8}$ cm an is of the order of interatomic length, and the fluctuation region must be seemingly large. But in this case (in particular, in ferroelectrics) the relative smallness of τ_G is caused by other factors (see Refs [84], [87]).

Thus, the Ψ-theory is generally speaking well applied to superconductors. The words 'generally speaking' refer to several circumstances. Firstly, we have considered here the three-dimensional case. For quasi-two-dimensional (thin films), quasi-one-dimensional (thin wires, etc.), and quasi-zero-dimensional (small seeds, say, balls) superconductors the conditions of applicability of the theory are different; the fluctuation region is wider than for a three-dimensional system. Unfortunately, I do not know all aspects of the problem (see, however, Ref. [88]). Secondly, as has already been emphasized, good applicability of the mean field approximation (the Landau theory and, in particular, the Ψ-theory) is in no way an obstruction to the calculation of various fluctuation effects as long as they are sufficiently small (see, for example, Refs [79–83, 88, 89]). It is of importance, especially in application to high-temperature superconductors, that paper [88] analyses, on the basis of the expression (35), the anisotropic case. Third, in a number of superconductors (dirty alloys, high-temperature superconductors — HTSC), the parameter $\varkappa$ is large or even very large (reaching hundreds) while the correlation length is small. Then the fluctuation region, i.e., the temperature range in which inequality (42), (45) is violated, is not so small. So, in Ref. [88] we present the values $\tau_G = (0.2-2)\times10^{-4}$ for HTSC. Somewhat lower values are reported in Ref. [90]. For $\tau_G \sim 10^{-4}$ and $T_c \sim 100$ K, the width of the fluctuation region is $\Delta T \sim 10^{-2}$ K (in this region the fluctuations are already high and are therefore not a small correction). This region does not seem to be so very large, but in experiments the variation of the specific heat of some HTSC near T_c has a clearly pronounced λ-shaped form similar to the one we observe in helium II (see Ref. [91], p. 2 and p. 129, where the original literature is cited).

In view of the latter circumstance, it seems interesting to extend the Ψ-theory to the fluctuation region. We shall touch upon this issue in Section 4 because this extension was proposed in application to liquid helium. But after the discovery of HTSC in 1986–1987, such a 'generalized Ψ-theory' was suggested in application to superconductors as well [92, 88, 53].

Underlying the 'generalized' Ψ-theory of superconductivity is the following expression

$$\widetilde{F} = \widetilde{F}_{n0} + \frac{C_0 T_c}{2}\tau^2 \ln\tau + \int\Big[-a_0\tau^{4/3}|\Psi|^2 + \frac{b_0}{2}\tau^{2/3}|\Psi|^4 + \frac{g_0}{3}|\Psi|^6 + \frac{\hbar}{4m_k}\left|\left(\nabla_k - \mathrm{i}\frac{2e_0}{\hbar c}A_k\right)\Psi\right|^2\Big]\,\mathrm{d}V\,, \qquad (46)$$

for the free energy which leads to the equation for Ψ:

$$-\frac{\hbar^2}{4m_k}\left(\nabla_k - \mathrm{i}\frac{2e_0}{c\hbar}A_k\right)^2\Psi + \left(-a_0\tau^{4/3} + b_0\tau^{2/3}|\Psi|^2 + g_0|\Psi|^4\right)\Psi = 0\,. \qquad (47)$$

If one neglects anisotropy and sets $m_k = m_0/2$, then equation (47) will differ from (14) by a transformed temperature dependence of the coefficients and by the presence of the term proportional to $|\Psi|^4\Psi$. Taking the example of helium II we shall see in Section 4 that the 'generalized' Ψ-theory entails a number of consequences near T_c which correspond to reality in the case of liquid helium. One might think that this could also be extended to superconductors with a very small correlation length. Such a case corresponds in a certain measure to the Schafroth model [16] which involves small-sized pairs. One of the directions of HTSC theory is based precisely on this model [91].

An important point in the 'generalized' Ψ-theory is the problem of boundary conditions. Condition (15) is, generally speaking, already insufficient here and should be replaced [37, 88, 93] by a more general condition

$$n_k\Lambda_k\left[\frac{\partial\Psi}{\partial x_k} - \mathrm{i}\frac{2e_0}{\hbar c}A_k\Psi\right] = -\Psi \qquad (48)$$

on the boundary with a vacuum or a dielectric, where all the quantities are, of course, taken on the boundary, n_k are the components of the unit vector $\mathbf{n}$ perpendicular to the boundary and Λ_k are some coefficients having dimensions of length, sometimes referred to as extrapolation lengths. For the isotropic case, when $\Lambda_k = \Lambda$, Eqn (48) takes on the form

$$\mathbf{n}\left(\nabla\Psi - \mathrm{i}\frac{2e_0}{\hbar c}\mathbf{A}\Psi\right) = -\frac{1}{\Lambda}\Psi \qquad (49)$$

[this Λ should not be confused with the coefficient (17) involved in the Londons theory (4), (5)].

In case $\Lambda_k \gg \xi_k(T)$, condition (49) becomes condition (15) because, generally speaking, $\partial\Psi/\partial x_k \sim \Psi/\xi_k$. In the case $\Lambda_k \ll \xi_k(T)$, however, we arrive at the boundary condition

$$\Psi = 0\,. \qquad (50)$$

This condition on a rigid wall was chosen in the initial Ψ-theory of superfluidity [94]. As far as I know, the 'generalized' Ψ-theory of superconductivity was never used after paper [88]. Two reasons for this are possible. On the one hand, the 'generalized' Ψ-theory has no reliable microscopic grounds (as distinct from the conventional Ψ-theory of superconductivity considered above). On the other hand, the investigations of HTSC are obviously at such a stage now that it has probably not yet become necessary to solve problems requiring application of the 'generalized' Ψ-theory. As far as the conventional Ψ-theory is concerned, its application to HTSC is also now only rather small-scale.

I have dwelt above on the development of the initial Ψ-theory [29] in three directions: allowing for anisotropy [74], for ferromagnetic superconductors [75] and in a fluctuation region [88]. Of importance are also extensions in another two directions, namely, to the nonstationary case, when the function Ψ is time dependent, and to superconductors with order parameter not reduced to the scalar complex function $\Psi(\mathbf{r})$. I obtained no results in either of these two directions. True, in what concerns the non-stationary generalization of the Ψ-theory, I already understood [63] in 1950 that this task did exist, but restricted myself to the remark that equation (14) might be supplemented with the term $\mathrm{i}\hbar\partial\Psi/\partial t$. Meanwhile, an allowance for relaxation is more significant. The corresponding equations for $\Psi(\mathbf{r}, t)$ are discussed in reviews [83, 95]. As to the so-called unconventional superconductors in which Cooper (or analogous) pairs are not in the s-state, I did not only fail to contribute to this field, but have a poor knowledge of it. By the way, the possibility of 'unconventional' pairing was first pointed out [96] for superfluid ^{3}He, and this fact was later confirmed. In the case of superconductivity, the 'unconventional' pairing takes place for at least several superconductors with heavy fermions (UB_{13}, $CeCu_2Si_2$, UPt_3) and, apparently, several high-temperature superconductors — cuprates. I shall restrict myself only to pointing to one of the pioneering papers in this field [97] and the reviews [98–101]. It is a pleasure to me to note also that 'unconventional' superconductors are now the subject of successful research by Yu S Barash [102] — my immediate colleague (our joint research was however conducted in quite a different field — the theory of Van der Waals forces [103]). It is noteworthy that an appropriately extended Ψ-theory is extensively used for 'unconventional' superconductors as well [97–100].

4. Ψ-theory of superfluidity

As I have mentioned above, the behavior of liquid helium near the λ-point was beyond the scope of Landau's interests. He also remained indifferent to the behavior of superfluid helium near a rigid wall. As for me, I was for some reason interested in both these questions from the very beginning of my work in the field of superfluidity, i.e., from 1943 [19]. I have already mentioned the attempt [49] to introduce the order parameter ρ_s near the λ-point. As regards the behavior of helium near the wall, it looks like this. Helium atoms stick to the wall (they wet it, so-to-say). How can it be combined with a flow along the wall of the superfluid part of the liquid with a density ρ_s and a velocity $\mathbf{v}_s$? We know that in the Landau theory of superfluidity [4] the velocity $\mathbf{v}_s$ along the wall (as distinct from the velocity $\mathbf{v}_n$ of a normal liquid) does not become zero on the wall. This means that on the wall the velocity $\mathbf{v}_s$ must become discontinuous (the velocity $\mathbf{v}_s$ cannot tend gradually to zero because of the condition $\mathrm{rot}\,\mathbf{v}_s = 0$). This velocity discontinuity must be associated with a certain surface energy σ_s [104]. Estimates show that the energy σ_s is rather high ($\sigma_s \sim 3\times10^{-2}$ erg cm^{-2}) and its existence must have led to a pronounced effect. Specifically, something like dry friction must have been observed — to move a rigid body placed in helium II, the energy $\sigma_s S$ must have been expended, where S is the body (say, plate) surface area. However, specially conducted experiments showed [105] that no energy $\sigma_s S$ is actually needed and a possible value of σ_s is at least by many orders of magnitude smaller than the above-mentioned estimates [104]. How can this contradiction be eliminated? The solution of the problem I saw in the assumption that the density ρ_s decreases approaching the wall, and on the wall itself $\rho_s(0) = 0$. Thus the discontinuity of the velocity $\mathbf{v}_s$ on the wall is of no importance because the flow $\mathbf{j}_s = \rho_s \mathbf{v}_s$ tends gradually to zero on the wall itself even without a change of velocity $\mathbf{v}_s$. By that time (1957) the Ψ-theory of superconductivity [29] had long since been constructed and there was no problem in its extension to the case of superfluidity, and with the boundary condition $\Psi(0) = 0$ on the wall [see Eqn (50)], which provided the condition $\rho_s(0) = 0$, as well.

Unfortunately, I do not at all remember how far I had advanced in constructing the Ψ-theory of superfluidity before I learnt that L P Pitayevsky was engaged in solving the same problem. We, naturally, joined our efforts, and the outcome was our paper [94] which we submitted for publication on December 10, 1957.

The Ψ-theory of superfluidity constructed in [94] will henceforth be referred to as the initial Ψ-theory of superfluidity. The point is that this theory was later found to be inapplicable to helium II in the quantitative respect, and we had to generalize it. Such a generalized Ψ-theory of superfluidity, developed by A A Sobyanin and myself [106–109], is far from being so well grounded as the Ψ-theory of superconductivity. In this connection and, I think, in view of an insufficient awareness of the distinction between the generalized theory and the initial one [94], the Ψ-theory of superfluidity has not drawn much attention, and at the present time remains undeveloped† and not systematically verified. Meanwhile, the microtheory of superfluidity is not nearly so well developed as the microtheory of superconductivity, and the role of the macrotheory of superfluidity is particularly high. This has led Sobyanin and me to the conviction that the development of the Ψ-theory of superfluidity and its comparison with experiment would be highly appropriate.

The most comprehensive of the cited reviews devoted to the generalized Ψ- theory of superfluidity [108] amounts to 78 pages. This alone makes it clear that in this paper I have no way of giving an in-depth consideration to the Ψ-theory of superfluidity. Below we shall restrict ourselves to brief remarks.

We shall begin with the initial theory [94]. It is constructed in much the same manner as the Ψ-theory of superconductivity [29]. As the order parameter we took the function $\Psi = |\Psi| \exp \mathrm{i}\varphi$ acting as an 'effective wave function of the superfluid part of a liquid', and so the density ρ_s and the velocity $\mathbf{v}_s$ are expressed as

$$\rho_s = m|\Psi|^2\,, \quad \mathbf{v}_s = \frac{\hbar}{m}\nabla\varphi\,,$$
$$\mathbf{j}_s = \rho_s \mathbf{v}_s = -\frac{\mathrm{i}\hbar}{2}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) = \hbar|\Psi|^2\nabla\varphi\,, \tag{51}$$

where $m = m_{\mathrm{He}}$ is the mass of a helium atom, and a convenient normalization of Ψ is chosen; in Ref. [94] it is shown (see also below) that in the expression for $\mathbf{v}_s$ we have $m = m_{\mathrm{He}}$ irrespective of the manner in which Ψ is normalised. Then there come expressions

$$F = F_0 + \frac{\hbar^2}{2m}|\nabla\Psi|^2\,,$$
$$F_0 = F_{\mathrm{I}} + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4\,, \ \alpha = \alpha'_\lambda(T - T_\lambda)\,, \ \beta = \beta_\lambda \tag{52}$$

usual for the mean field theory (the Landau phase transitions theory), where $F_{\mathrm{I}}(\rho, T)$ is free energy of helium I and T_λ is the temperature of the λ-point. In equilibrium homogeneous helium II

$$|\Psi_0|^2 = \frac{\rho_s}{m} = \frac{|\alpha|}{\beta_\lambda} = \frac{\alpha'_\lambda(T_\lambda - T)}{\beta_\lambda}\,,$$
$$\Delta C_p = C_{p,\mathrm{II}} - C_{p,\mathrm{I}} = T_\lambda \frac{(\alpha'_\lambda)^2}{\beta_\lambda}\,. \tag{53}$$

In inhomogeneous helium II, the function Ψ obeys the equation

$$-\frac{\hbar^2}{2m}\nabla\Psi + \alpha\Psi + \beta_\lambda|\Psi|^2\Psi = 0\,, \tag{54}$$

which should be solved with the boundary condition (50) on a rigid wall. Like in Eqn (25), we introduce the correlation length (in Ref. [94] it is denoted by l)

$$\xi(T) = \frac{\hbar}{\sqrt{2m|\alpha|}} = \frac{\hbar\tau^{-1/2}}{\sqrt{2m\alpha'_\lambda T_\lambda}} = \xi(0)\tau^{-1/2}\,,$$
$$\tau = \frac{T_\lambda - T}{T_\lambda} = \frac{t}{T_\lambda}\,. \tag{55}$$

The estimate presented in Ref. [94] and based on the data of ΔC_p and ρ_s measurements [see Eqn (54)] gives approximately $\xi(0) \sim 3 \times 10^{-8}$ cm. At the same time, the Ψ-theory is applicable only provided the macroscopic Ψ-function changes little on atomic scales. This implies the condition $\xi(T) \gg a \sim 3 \times 10^{-8}$ cm (here a is the mean interatomic distance in liquid helium). Consequently, the Ψ-theory can only hold near the λ-point for $\tau \ll 1$, say, for $(T_\lambda - T) < (0.1 - 0.2)$ K. Of course, proximity to T_λ is also the condition of applicability of expansion (52) in $|\Psi|^2$. The small magnitude of the length $\xi(0)$ in helium leads at the same time to considerable dimensions of the fluctuation region [84]. Indeed, applying criterion (42), we arrive at the value $\tau_{\mathrm{G}} \sim 10^{-3}$ for helium [see Ref. [106], formula (2.46)]. Thus, it turns out that the initial Ψ-theory of superfluidity can only hold under the condition 10^{-3} K $\ll (T_\lambda - T) \lesssim 0.1$ K, i.e., it is practically inapplicable because in the studies of liquid helium, of particular interest is exactly the range of values $(T_\lambda - T) \ll 10^{-3}$ K. The fact that the mean field theory leading to the jump in specific heat (53) does not hold for liquid helium (we certainly mean ^{4}He) is attested by the existence of a λ-singularity in the specific heat as well as the circumstance that the density ρ_s near T_λ does not behave at all like $(T_\lambda - T)$ according to (53), but rather changes by the law

$$\rho_s(\tau) = \rho_{s0}\tau^\zeta\,, \quad \zeta = 0.6705 \pm 0.0006\,, \tag{56}$$

where the value of ζ is borrowed from the most recently reported data [110]. Note that in Ref. [106] we gave the value $\zeta = 0.67 \pm 0.01$ and in Ref. [108] the values $\zeta = 0.672 \pm 0.001$ and $\rho_s = 0.35\tau^\zeta$ g cm^{-3}. Hence, to a high accuracy we have

$$\zeta = \frac{2}{3}\,. \tag{57}$$

I cannot judge whether ζ actually differs from 2/3, but if it does, the difference does not exceed 1 percent. It is noteworthy that in 1957, when paper [94] was accomplished, the variation of ρ_s by the law (56) was not yet known. We therefore did not raise an alarm immediately (the λ-type

† One of the reasons, and perhaps even the main one, is that Sobyanin has gone into politics and for several years now has practically been not working as physicist. [A A Sobyanin, a talented physicist theoretician, died on 10 June, 1997.]

behavior of specific heat is less crucial in this respect because it may not be associated with variations of Ψ, whereas the density ρ_s is proportional to $|\Psi|^2$).

Thus, the initial Ψ-theory of superfluidity [94] is inapplicable to liquid helium (^{4}He). However, owing to its simplicity it has a qualitative and occasionally even quantitative significance for ^{4}He as well. The main thing is that liquid ^{4}He is not the only existing superfluid liquid, suffice it to mention liquid ^{3}He at very low temperatures, ^{3}He – ^{4}He solutions, non-dense ^{4}He films and neutron liquid in neutron stars, as well as possible superfluidity in an exciton liquid in crystals, in supercooled liquid hydrogen [111], and in the Bose-Einstein condensate of the gas of various atoms (it is this very question that is presently commanding the attention of physicists; see, for example, Ref. [112] and the literature cited there). In some of these cases, the fluctuation region may appear to be small enough, so that the initial Ψ-theory of superfluidity may prove sufficient. This is apparently the situation in the particularly important case of superfluidity in ^{3}He. We shall therefore dwell briefly on the results obtained in paper [94].

We found the distribution $\rho_s(z)$ near a rigid wall and in a liquid helium film of thickness d. The function $\Psi(z)$ and, of course, $\rho_s = m|\Psi|^2$, where z is the co-ordinate perpendicular to the film, has a dome-like shape because on the boundaries of the film we have $\Psi(0) = \Psi(d) = 0$ [see Eqn (50)]. Naturally, for a sufficiently small thickness d the equilibrium value is $\Psi = 0$, i.e., the superfluidity vanishes. The corresponding critical value d_c (for $d < d_c$ a film is not superfluid) is equal to

$$d_c = \pi\xi(T) = \frac{\pi\hbar\tau^{-1/2}}{\sqrt{2m\alpha'_\lambda T_\lambda}}\,, \quad \tau = \frac{T_\lambda - T}{T_\lambda}\,. \tag{58}$$

This result implies that for a film the λ-transition temperature is lower than that for 'bulk' helium. Concretely, from Eqn (58) it follows that for a film the λ-transition takes place at a temperature ($T_\lambda \equiv T_\lambda(\infty)$)

$$T_\lambda(d) = T_\lambda - \frac{\pi^2\hbar^2}{2m\alpha'_\lambda d^2} = T_\lambda - \frac{\pi^2 T_\lambda \xi^2(0)}{d^2}\,. \tag{59}$$

The specific heat of the film changes with varying d, too. Such effects in small samples are observed experimentally. In Ref. [94] we also solved the problem of the vortex line, the value of Ψ on its axis being equal to zero and the velocity circulation around the line being

$$\oint \mathbf{v}_s\,\mathrm{d}\mathbf{s} = \frac{2\pi\hbar k}{m}\,, \quad k = 0, 1, 2, \ldots \tag{60}$$

In this formula, the ^{4}He atom mass $m = m_{\mathrm{He}}$ should be used considering that the circulation cannot change with temperature, and as was shown by Feynman [113], at $T = 0$ it is the mass m_{He} that enters in Eqn (60). Finally, in Ref. [94] we found the surface energy on the boundary between helium II and a rigid body and the vortex line energy.

The fact that for liquid helium and a number of other transitions the mean field (Landau) theory does not hold led to the appearance of the generalized theory in which the free energy is written in the form (37), but with a different temperature dependence of the coefficients. Specifically, for the order parameter Ψ we immediately write

$$F_{\mathrm{II}} = F_{\mathrm{I}} - a_0\tau|\tau|^{1/3}|\Psi|^2 + \frac{b_0}{2}\,|\tau|^{2/3}|\Psi|^4 + \frac{g_0}{3}\,|\Psi|^6\,. \tag{61}$$

Since for small $|\Psi|^2$ in equilibrium [see Eqn (53)] $|\Psi_0|^2 = |\alpha|/\beta = a_0\tau^{2/3}/b_0$, this result is in agreement with (56), (57). Expression (61) is, naturally, so chosen as to correspond to experiment. Parenthetically, the same technique in application to the Ψ-theory of superconductivity was employed in paper [66], only not near but far from T_c. As far as I know, expression (61) was first applied by Yu G Mamaladze [114]. Some other authors also discussed a generalization of the phase transition theory in the spirit of involving an equation of the type (61) (see references in [106]). Sobyanin and I developed the generalized Ψ-theory of superfluidity [106 – 109] on the basis of expression (61) which in turn underlay the 'generalized' Ψ-theory of superconductivity (see Ref. [88] and Section 3 above). But while the latter is of limited significance, the generalized Ψ-theory of superfluidity is a unique scheme capable of describing the behavior of liquid helium near the λ-point, not counting the incomparably more sophisticated approach based on the renormalization group theory (see Ref. [115] and the literature cited therein). In addition, this approach [115] is either of no or limited validity for the inhomogeneous and nonstationary cases.

Without going into details, we shall immediately present the expression for the involved free energy density in some reduced units (instead of free energy, other thermodynamic potentials were used in Refs [106 – 109], but this is of no importance):

$$F_{\mathrm{II}} = F_{\mathrm{I}} + \frac{3\Delta C_p}{(3+M)T_\lambda}\left[-t|t|^{1/3}|\Psi|^2 + \frac{(1-M)|t|^{2/3}}{2}|\Psi|^4 + \frac{M}{3}|\Psi|^6 + \frac{\hbar^2}{2m}|\nabla\Psi|^2\right]. \tag{62}$$

Here $t = T_\lambda - T$, ΔC_p is the jump of specific heat determined by expression (53), M is the constant introduced in the theory, $\Psi = \Psi/\Psi_{00}$, $\Psi_{00} = \sqrt{1.43\rho_\lambda}/m$, $\rho_s = 1.43\rho_\lambda \times (T_\lambda - T)^{2/3}$.

In the simplest version of the theory we have $M = 0$, and irrespective of this fact the reduced order parameter Ψ is sometimes (for instance, in the vicinity of the axis of a vortex line) rather small, and the term $|\Psi|^6$ in Eqn (62) can be ignored. Comparison with experiment for helium II leads to the estimate $M = 0.5 \pm 0.3$ (see Ref. [109]). The transition is second-order for $M < 1$ and first-order for $M > 1$.

For a shift of the λ-transition temperature in a film (for $M < 1$) we have

$$\Delta T_\lambda = T_\lambda - T_\lambda(d) = 2.53 \times 10^{-11}\left(\frac{3+M}{3}\right)d^{-3/2}\,\mathrm{K}\,, \tag{63}$$

which generalizes expression (59) and corresponds to experimental data; for a capillary with diameter d, the coefficient 2.53 in Eqn (63) is replaced by 4.76. Expressions for a number of other quantities (density, specific heat, etc.) are obtained, and the effect of the external (gravitational, electric) fields as well as Van der Waals forces are taken into account. The behavior of ions in helium II, the dependence of the density ρ_s on velocity v_s, and the vortex line structure are considered [116]. Furthermore, the theory is extended to the case of the presence of a flow of the normal part of a liquid (density ρ_n, velocity $\mathbf{v}_n$) and the presence of dissipation and relaxation (for a non-stationary flow; for the initial Ψ-theory this was done partially in paper [117]). The problem of vortex creation in a superfluid liquid (see Ref. [108] where the corresponding literature is cited) is very interesting. We note that somewhat unexpectedly this question proved to be of interest for simulating the process of creation of so-called topological defects in cosmology [118]. I believe that in an analysis of corresponding experiments the Ψ-theory of superfluidity may turn out to provide quite suitable methods.

The generalized Ψ-theory of superfluidity was not developed 'from first principles' or on the basis of a certain reliable microtheory (as in the situation with the Ψ-theory of superconductivity). This is a phenomenological theory that rests on the general theory of second-order phase transitions (Landau theory and scaling theory) and on experimental data. Such data are unfortunately quite insufficient for drawing a vivid conclusion concerning the region of applicability of the Ψ-theory. In the papers [119, 120] we find rather pessimistic judgements in this respect, but Sobyanin was of the opinion that such a criticism is groundless. I do not hold any particular viewpoint here, but my intuition suggests a great positive role of both the initial [94] and the generalized [106–108] Ψ-theories of superfluidity. In any case, clarification of the precision and the role of the Ψ-theory is currently pressing because experimental studies of superfluidity in helium II are in full swing (see, for example, Refs [121, 122]).

5. Thermoelectric phenomena in superconductors

Different papers have their own fate. My first paper [19] on superconductivity now seems dull to me, and this is all bygone times. And what concerns the second paper [22] accomplished in the same year, 1943, remains topical up to the present date. It was devoted to thermoelectric phenomena in superconductors. Before that, thermoelectric effects had been considered (see, for example, Refs [57], [123]) to disappear completely in the superconducting state. Specifically, when a superconducting current passes through a seal of two superconductors, the Peltier effect is absent, the same as a noticeable thermoelectric current is absent upon heating one of the seals of a circuit consisting of two superconductors. But as a matter of fact, thermoelectric phenomena in superconductors do not vanish completely, although they can manifest themselves only under special conditions [22, 24]. The point is that in a superconductor one should take into account the possibility of the appearance of two currents — superconducting (the density $\mathbf{j}_s$) and normal (the density $\mathbf{j}_n$). In a non-superconducting (normal) state in a metal there may flow only one current $\mathbf{j}$, Ohm's law $\mathbf{j} = \sigma\mathbf{E}$ holding in the simplest case. If there exists a gradient of chemical potential μ of electrons in a metal and a temperature gradient, then

$$\mathbf{j} = \sigma\left(\mathbf{E} - \frac{\nabla\mu}{e_0}\right) + b\nabla T. \qquad (64)$$

In the superconducting state, as can readily be seen (see, for example, Ref. [125]), for the normal current we have

$$\mathbf{j}_n = \sigma_n\left(\mathbf{E} - \frac{\nabla\mu}{e_0}\right) + b_n\nabla T \qquad (65)$$

instead of Eqn (3), and in the Londons approximation equation (4) is preserved; instead of Eqn (5) we obtain

$$\frac{\partial(\Lambda\mathbf{j}_s)}{\partial t} = \mathbf{E} - \frac{\nabla\mu}{e_0} + \nabla\frac{\Lambda j_s^2}{2\rho_e}\,, \qquad (66)$$

where μ is the chemical potential of electrons and $\rho_e = e_0 n_s$, n_s is the concentration of 'superconducting electrons' ($\mathbf{j}_s = e_0 n_s \mathbf{v}_s$). Here we omit the detail connected with the necessity of introducing different chemical potentials μ_n and μ_s in non-equilibrium conditions for a normal and superconducting electron subsystems (see Ref. [125]). Note that the last term on the right-hand side of equation (66) is of hydrodynamic character [see Eqn (6)] and in Eqn (5) it was omitted because of its small magnitude. However, the contribution of this term can be observed experimentally (see Ref. [125] and the references therein). Forgetting again about the last term in Eqn (66) in the stationary case for a superconductor we have

$$\mathbf{E} - \frac{\nabla\mu}{e_0} = 0\,, \qquad (67)$$

from which it follows that [see Eqn (65)]

$$\mathbf{j}_n = b_n(T)\nabla T\,. \qquad (68)$$

Thus, in a superconductor the thermoelectric current $\mathbf{j}_n$ does not vanish completely. Why then is it not observed? As has already been mentioned, under particularly simple conditions a normal current is totally compensated for by a superconducting current, that is,

$$\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = 0\,, \quad \mathbf{j}_s = -\mathbf{j}_n\,. \qquad (69)$$

By 'particularly simple conditions' we understand a homogeneous and isotropic superconductor, say, a non-closed small cylinder (a wire) on one end of which the temperature is T_1 and on the other end T_2 (we assume that $T_{1,2}$ is less than T_c)†. In such a specimen, in the normal state (for $T_{1,2} > T_c$) we certainly have $\mathbf{j} = 0$ and $\mathbf{E} = \nabla\mu/e_0 - b\nabla T/\sigma$ [see Eqn (64)]; in the superconducting state we of course also have $\mathbf{j} = 0$, but [see Eqns (68), (69)]

$$\mathbf{j}_s = -\mathbf{j}_n = -b_n\nabla T\,, \quad \mathbf{E} - \frac{\nabla\mu}{e_0} = 0\,. \qquad (70)$$

If a superconductor is inhomogeneous and (or) anisotropic then, generally speaking, the total compensation (69) does not occur and a certain, although weak, thermoelectric current must be [22] and is, in fact, observed [125, 126]. But one should not think that in the simple case considered above, when $\mathbf{j} = 0$, all thermoelectric effects disappear. Indeed, the thermoelectric current $\mathbf{j}_n$ must be associated with some heat transfer, i.e., in superconductors there must occur an additional (say, circulational or convective) heat transfer mechanism similar to the one that exists in a superfluid liquid‡. This analogy was, properly speaking, the starting point for me in paper [22]. However, in Ref. [22] I made no estimate of the additional (circulational) thermal conductivity. Later I decomposed [63] the total heat conductivity $\varkappa$ involved into the heat transfer equation $\mathbf{q} = -\varkappa\nabla T$ into three parts: $\varkappa = \varkappa_{ph} + \varkappa_e + \varkappa_c$. Here $\varkappa_{ph}$ stands for the contribution due to phonons (the lattice), $\varkappa_e$ is due to electron motion such, that there is no circulation (i.e., under the condition $\mathbf{j}_n = 0$), and $\varkappa_c$ is due to circulation (convection). The estimates done

† I did not want to place figures in the paper, although perhaps they would not be out of place here. But all the necessary illustrations concerning thermoeffects can be found in the readily available papers [125, 126].

‡ Such heat transfer is also possible in semiconductors that possess simultaneously the corresponding electron and hole conductivities (see Ref. [128]).

in Ref. [63] indicated that $\varkappa_c$ must be negligibly small compared to $\varkappa_e$, but now, unfortunately, I do not understand these estimates.

After th BCS theory was created, it became possible to carry out a microscopic evaluation of $\varkappa_e$ and $\varkappa_c$. According to Ref. [127], at $T \sim T_c$

$$\frac{\varkappa_c}{\varkappa_e} \sim \frac{k_B T_c}{E_F}, \tag{71}$$

where E_F is the Fermi energy of electrons in a metal. However, the role of a superconducting sample boundaries where the superconducting current turns into normal and vice versa was not taken into account neither in this, nor in other papers of which I am aware, although it seems worth to do it. Thus, I shall give an estimate [129, 126] based on the assumption that the convective heat $\mathbf{q}_c = -\varkappa_c \nabla T$ is mainly due to breakdown of superconducting pairs at a higher temperature T_2 and to creation of such pairs out of normal electrons at a temperature T_1 at which the current $\mathbf{j}_n$ becomes $\mathbf{j}_s = -\mathbf{j}_n$. Upon pair creation or breaking, the energy $2\Delta(T)$, where $\Delta(T)$ is the width of the superconducting gap per electron, is released or absorbed. The normal current density is $\mathbf{j}_n = e_0 n_n \mathbf{v}_n = b_n \nabla T$ (recall that the electron charge is denoted by e_0; n_n is the concentration of 'normal' electrons and $\mathbf{v}_n$ is their velocity). Hence, on the end of a superconductor an energy of the order of $\Delta(T) n_n v_n = j_n \Delta / e_0 = b_n \Delta |\nabla T| / e_0$ is released (or absorbed) per unit time; and this is just the heat flow $q_c = \varkappa_c |\nabla T|$. From this we have $\varkappa_c \sim b_n(T)\Delta(T)/e_0$. Next, making use of the Wiedemann–Franz law

$$\varkappa_e = \frac{\pi^2 k_B^2}{3e_0^2} T\sigma_n,$$

we obtain

$$\frac{\varkappa_c}{\varkappa_e} \sim \frac{3e_0 S_n \Delta}{\pi^2 k_B^2 T} \sim \frac{\Delta(T)}{E_F} \sim \frac{k_B T_c}{E_F}, \tag{72}$$

where we have used the formula $S_n \equiv b_n/\sigma_n = \pi^2 k_B^2 T / 3e_0 E_F$ ($S = b/\sigma \equiv \mathrm{d}\mathcal{E}/\mathrm{d}T$ is the Seebeck coefficient or alternatively, the differential thermoelectric power, $\mathcal{E}$ is the thermoelectric power) valid for free electrons. In passing over to the last expression (72) it was obviously assumed that $\Delta(T) \sim k_B T_c$.

I have to say that I am unsatisfied with this estimate (72), coincident with (71), to say nothing of the fact that it only refers to free electrons (their attraction, which leads to the BCS results, being neglected). At the same time, for metals with impurities, allowing for anisotropy and 'unconventional' pairing [130, 131] the thermoeffect in superconductors increases significantly and, possibly, $\varkappa_c/\varkappa_e \sim 1$ or the convective (circulational) heat transfer may be even higher, but this problem has not been investigated at length.

For 'conventional' superconductors, which are in general well described by the BCS model, the estimate (71), (72) apparently holds true. In these cases, for $T_c \sim (1-10)$ K and $E_F \sim (3-10)$ eV, according to Eqn (71) we have $\varkappa_c/\varkappa_e \sim 10^{-4}$, and the convective heat transfer is negligibly small. But for HTSC (high-temperature superconductors), even according to Eqn (71), for example, for $T_c \sim 100$ K and $E_F \sim 0.1$ eV we already have $\varkappa_c/\varkappa_e \sim 0.1$. Taking into consideration what has been said above, it is quite likely that for HTSC we even have $\varkappa_c/\varkappa_e \gtrsim 1$. At the same time, for HTSC there exists a clearly pronounced non-monotonic dependence of the heat conductivity $\varkappa$ on T when the temperature falls below T_c [132, 133]. It is natural to believe [129] that such a behavior of $\varkappa$ is due to the appearance of a convective contribution $\varkappa_c$ into $\varkappa$ in the superconducting state; this contribution is certainly absent when $T > T_c$. Unfortunately, this is not the only possible explanation because the growth of $\varkappa$ with lowering T may, in principle, be caused by an increase of $\varkappa_{ph}$ or $\varkappa_e$. Furthermore, a certain increase of $\varkappa_{ph}$ is even natural because for $T < T_c$ the 'normal electrons' seem to 'freeze', for their concentration tends to zero as $T \to 0$. That is why the phonon scattering on electrons decreases, and therefore within some temperature range $\varkappa_{ph}$ increases. This is just the explanation that prevails in the literature (see, for example, Ref. [134]). According to some authors [134a], however, governing is the non-monotonic dependence of $\varkappa_e$ on the temperature. As far as I understand, it is the quantity $\varkappa_e + \varkappa_c$ that is directly calculated in Ref. [134a]. Indeed, as is clear from what we have said above, the total volume electron contribution to the heat conductivity of a superconductor is $\varkappa_e + \varkappa_c$, and $\varkappa_e$ is a conditional quantity corresponding to the case where $\mathbf{j}_n = 0$. Nevertheless, even if one calculates $\varkappa_e + \varkappa_c$, the convective heat conductivity (or, more precisely, the heat transfer) is allowed for only in part in case it is necessary to take into consideration the breakdown and creation of pairs at the ends of a superconductor or, more generally, in regions (say, at temperatures T_2 and T_1) where a normal current becomes superconducting. I am not aware of whether such a breaking and creation of pairs at the end of a superconductor should be taken into account or whether everything is already involved automatically in the calculations [131, 134a].†

I have repeatedly paid attention [126, 129, 135] to a possible role of the convective mechanism of heat transfer in superconductors and cannot understand why this question is ignored. Maybe, this is a matter of fashion, maybe someone has weighty arguments against the possibility of a convective mechanism or thinks of it as already involved but does not wish to publish his considerations or critical notes, for example, out of politeness. The latter hypothesis is, however, almost unbelievable. In any case, the question of the role of convective mechanisms of heat transfer in HTSC seems to be very interesting and deserving of investigation both in theory and experiment (for single crystals, depending on the orientation of ∇T relative to the crystal axes, etc.; some additional literature is cited in Refs [126, 134, 134a]).

I went at such length into convective heat conductivity (heat transfer) in superconductors because I feel some permanent dissatisfaction with the state of this problem. I have never been specially engaged in microtheory or, as it is more frequently referred to, the electron theory of metals, including superconductors. Therefore I could not (and have never even tried to) construct a microtheory of convective heat transfer. And now I am afraid it is already late for me. But I hope that someone will finally take up this interesting subject.†

If a superconductor is not homogeneous and isotropic, then, as has already been mentioned, a total compensation for the currents $\mathbf{j}_n$ and $\mathbf{j}_s$ does not take place and, generally speaking, some thermoelectric currents must flow. Particularly simple cases are as follows: an isotropic, but inhomogeneous superconductor and a homogeneous but anisotropic superconductor (a single crystal). Fifty three years ago (!), when paper [22] was finished, alloys and generally inhomo-

† (See note before the list of references)

geneous superconductors were thought of as something 'dirty' and it was not even clear whether the Londons equations can be employed under such conditions. Therefore, the case of an inhomogeneous superconductor was only touched upon in Ref. [22]. Specifically, it was pointed out that for a bimetallic plate (different superconductors, say, sealed (welded) to one another) in the presence of a temperature gradient along the seal line, an uncompensated current **j** arises and flows around the seal; this leads to the appearance of a magnetic field perpendicular to the plate and to the seal line (see Fig. 3a in Ref. [125] and Fig. 3 in Ref. [126]). As I have said, such a situation did not seem very interesting. That is why attention was concentrated on single crystals with a non-cubic symmetry, the case where the tensor Λ_{ik} is not degenerated into a scalar (for cubic and isotropic superconductors we have $\Lambda_{ik} = \Lambda\delta_{ik}$). If in such a plate-shaped crystal the temperature gradient ∇T is not directed along the symmetry axis, a current **j** flows around the plate, and across the plate appears a magnetic field H_T proportional to $|\nabla T|^2$. This field can easily be measured using modern methods. For details see Refs [22, 125, 126, 136]. Unfortunately, attempts to observe the thermoelectric effect in question were undertaken only in paper [137], the results of which remain ambiguous [125, 136].

As it turned out, the thermoeffect for inhomogeneous isotropic superconductors is easier to analyse and easier to observe. For this purpose, it is most convenient to consider not a bimetallic plate but rather a superconducting ring (a circuit) consisting of two superconductors (with one seal at a temperature T_2 and the other at a temperature $T_1 < T_2$; see Fig. 3b in Ref. [125] or Fig. 7 in Ref. [126]). The pertinence of the choice of this particular version was indicated in papers [138, 139]. In paper [138] it was stated, however, that the indicated effect was other than the one considered in Ref. [22], but this was a misunderstanding [125, 140]. Indeed, a bimetallic plate and a circuit of two superconductors differ topologically by the presence of a hole in the latter case, which leads to the possibility of the appearance of a quantized magnetic field flow through the hole (see Fig. 3 in Ref. [125]). A simple calculation (see Refs [125, 126, 138 – 141]) shows that the flow through the indicated hole is equal to

$$\Phi = k\Phi_0 + \Phi_T\,, \quad \Phi_T = \frac{4\pi}{c}\int_{T_1}^{T_2} (b_{\mathrm{n,II}}\,\delta_{\mathrm{II}}^2 - b_{\mathrm{n,I}}\,\delta_{\mathrm{I}}^2)\,\mathrm{d}T\,, \quad (73)$$

$$\Phi_0 = \frac{hc}{2e_0} = 2\times10^{-7}\ \mathrm{G\ cm^2}\,, \quad k = 0, 1, 2, \ldots,$$

where the indices I and II refer to metals I and II that form the superconducting circuit, $\delta \equiv \delta_0$ is the penetration depth; when $k = 0$, we obtain the result for a bimetallic plate. If we assume for simplicity that $(b_{\mathrm{n}}\delta^2)_{\mathrm{II}} \gg (b_{\mathrm{n}}\delta^2)_{\mathrm{I}}$ and $\delta_{\mathrm{II}}^2 = \delta_{\mathrm{II}}^2(0)(1 - T/T_{\mathrm{c,II}})^{-1}$, then from Eqn (73) $T_{\mathrm{c}} = T_{\mathrm{c,II}}$ we obtain

$$\Phi_T = \frac{4\pi}{c}\, b_{\mathrm{n,II}}\delta_{\mathrm{II}}^2(0)\, T_{\mathrm{c}} \ln\left(\frac{T_{\mathrm{c}} - T_1}{T_{\mathrm{c}} - T_2}\right). \quad (74)$$

The estimates for tin ($b_{\mathrm{n}}(T_{\mathrm{c}}) \sim 10^{11} - 10^{12}$ CGSE, $\delta(0) \approx 2.5\times10^{-6}$ cm) when $(T_{\mathrm{c}} - T_2) \sim 10^{-2}$ K, $(T_{\mathrm{c}} - T_1) \sim 0.1$ K and generally $\ln[(T_{\mathrm{c}} - T_1)/(T_{\mathrm{c}} - T_2)] \sim 1$ lead to the value $\Phi_T \sim 10^{-2}\Phi_0$. Such a flow can readily be measured, and this was done in a number of papers as far back as 20 years ago (for the references see [125, 141]). Here I will only refer explicitly to the new paper [142] which also confirmed the result (74).

As far as the thermoelectric current in a superconducting circuit is concerned, everything seems to be clear in principle, but this is not so. The point is that for a sufficiently massive and closed toroidal type circuit (a hollow cylinder made of two superconductors) the measured flow $\Phi(T)$ appeared [141] to be several orders of magnitude higher than the flow (74) and, moreover, to possess a different temperature dependence. The origin of such an 'enormous' thermoeffect in superconductors has not yet been completely clarified. The most probable explanation was suggested by R M Arutyunyan and G F Zharkov [143] (as for me, I totally support it, although it has not yet been confirmed by experiment). In this case, the measured flow through the hole is equal to $\Phi_T + k\Phi_0$ rather than Φ_T. When the critical temperature T_2 of the hottest seal approaches the temperature T_{c} of one of the superconductors, an increase of thermoelectric current makes an increase of the entrapped flow $k\Phi_0$, i.e., a growth of k, energetically advantageous. This question was discussed in a number of papers [143 – 146], but the mechanism responsible for the increase of the flow $\Phi(T)$ still remained unclear and corresponding experiments were not carried out. It is only quite recently that the mechanism of vortex formation in the walls of a superconducting cylinder that leads to an increase of an entrapped flow with increasing thermoelectric current has been proposed [147]. I hope, although not very much, that thermoeffects in superconductors (in a superconducting state) will no longer be ignored and there will finally appear experiments involving, in particular, HTSC.

Concluding this section, I would like to emphasize that in accordance with the general context of this paper I only concentrated on those thermoelectric phenomena in superconductors which I investigated myself. Nevertheless, there exist some other related aspects of the problem. In this respect, I shall restrict myself to references to the reviews [125, 126, 141] and the literature cited there, as well as the book [40] and the papers [148 – 150].

6. Miscellanea (superfluidity, astrophysics and other things)

As mentioned in Section I, my first work [23] in the field of low-temperature physics, which was accomplished at the beginning of 1943 was devoted to light scattering in helium II. This question was rather topical at that time because when comparing the transition in helium and the Bose-Einstein gas condensation one might expect a very strong scattering near the λ-point. At the same time, the Landau theory [4] suggested no anomaly. But this was, so-to-say, a trivial result. The most interesting thing is that the scattering spectrum must consist not of the central line and a Mandelstam – Brillouin doublet as in usual liquids, but of two doublets. Indeed, the Mandelstam – Brillouin doublet is associated with scattering on sound (or, more precisely, hypersonic) waves, while the central line is associated with scattering on entropy (isobaric) fluctuations. In the case of helium II and generally superfluid liquids, entropy fluctuations propagate (or, more precisely, are resorbed) in the form of a second sound. This is the reason why instead of a central peak a doublet must be observed that corresponds to scattering on second sound waves. In paper [23] I noted, however, that 'the inner anomalous doublet cannot be actually observed because on the one hand the corresponding splitting is too small ($\Delta\omega_2/\omega_2 \sim u_2/c \lesssim 10^{-7}$)

and on the other hand, and this is particularly important, the intensity of this doublet is quite moderate'. Indeed, the inner-to-outer doublet intensity ratio is $I_2/I_1 \approx C_p/C_V - 1$ ($C_{p,V}$ is specific heat at a constant pressure or for a constant volume). Even near the lambda point, at low pressure in helium II we have $C_p/C_V = 1.008$. However, as in many other cases in physics, the pessimistic prediction did not prove to be correct. Firstly, the intensity of the inner doublet increases greatly with pressure and, secondary, and this is especially significant, the use of lasers promoted great progress in the study of light scattering. As a result, the inner doublet could be observed and investigated (see Ref. [151] and the review [152], p. 907).

I have already mentioned the papers [49, 104] devoted to superfluidity, to say nothing of the papers [94, 106–109, 116] on the Ψ-theory of superfluidity. I would like also to mention the notes [153, 154] whose titles cast light on their contents. Finally, I shall dwell on the thermomechanical circulation effect in a superfluid liquid [140, 155]. In a ring-shaped vessel filled with a superfluid liquid (concretely, helium II was discussed) and having two 'weak links' (for example, narrow capillaries), in the presence of a temperature gradient there must occur a superfluid flow spreading to the entire vessel. Curiously, the conclusion concerning the existence of such an effect was suggested [140] by analogy with the thermoelectric effect in a superconducting circuit. At the same time, the conclusion was drawn concerning the existence of thermoelectric effects in superconductors [22], in turn, by analogy with the 'inner convection' occurring in helium II in the presence of temperature gradient.

The effect under discussion was observed [156], but the accuracy of measurements of the velocity v_s was not enough to fix the jumps of circulation in superfluid helium (the circulation quantum is $2\pi\hbar/m_{He} \approx 10^{-3}$ cm^2 s^{-1}) which had been predicted by the theory [155]. Meanwhile, there exist interesting possibilities of observing not only jumps of circulation of a superfluid flow, but also peculiar quantum interference phenomena (to this end, 'Josephson contacts' must be present in the 'circuit', for example, narrow-slit diaphragms). In my opinion, the circulation effect in a nonuniformly heated ring-shaped vessel is fairly interesting, and not only for ^{4}He or solutions of ^{4}He with ^{3}He, but perhaps also in the case of superfluidity of pure ^{3}He. Considering a very extensive front of research in the field of superfluidity all over the world, I cannot understand why this effect is totally neglected. I do not know whether this is a matter of fashion, a lack of information, or something else†.

To save space in the other sections of the present paper, I shall mention here the works [111, 157–159]. The first of them [111] stresses the fairly obvious fact that molecular hydrogen H_2 does not become superfluid only for the reason that at a temperature T_m exceeding the λ-transition temperature T_λ it solidifies. As is well known, for H_2 the temperature T_m is 14 K, whereas by estimation T_λ should be nearly 6 K. Perhaps liquid hydrogen may be supercooled, for example, by way of expansion (a negative pressure) and the use of films on different substrates.

The possibility of observing the secondary sound and convective heat transfer in superconductors, in the first place accounting for exciton type excitations (we mean bosons) was considered in paper [157]. I should say that paper [157] was written in 1961, and I am unaware of the present state of the questions discussed in it.

In 1978, there appeared reports on the observation of a very strong diamagnetism (superdiamagnetism) in CuCl, when the magnetic susceptibility χ is negative, and $|\chi| \sim 1/4\pi$ (of course, $|\chi| < 1/4\pi$ because $\chi = -1/4\pi$ corresponds to an ideal diamagnetism). After that (in 1980) there appeared indications of the existence of superdiamagnetism in CdS too. What it was that was actually observed in the corresponding experiments (for references see [158]) remains unclear, and this question was somehow 'drawn in the sand'. Many physicists believe that the measurements were merely erroneous. In any case, attempts were made to associate the observations with the possibility of the existence of superdiamagnetics other than superconductors‡. The last study in this direction in which I took part was reported in paper [158]. Further on, the question of superdiamagnetism somehow 'faded away' (see, however, Ref. [160]), and I am unacquainted with the progress in this field. When seeking ways of explaining superdiamagnetism, I made an attempt to generalize the Ψ-theory of superconductivity [159]. It is unknown to me whether this paper is of any value now.

Concluding this section, I shall dwell on an astrophysical problem, namely, the possibility of the existence of superconductivity and superfluidity in space.

It seems to me that a small digression will not lead us beyond the scope of the general context of the paper. When I was young and then middle-aged, I used to entertain myself or, maybe, to do an exercise which I called then a 'brain attack' (I wrote about it in my book [1], p. 309). The procedure of the 'attack' was as follows: looking at my watch, I set myself a task to think up some effect within a certain time interval, say, within 15–20 minutes. Here is a concrete example. If I am not mistaken, it was 1962, I was traveling by train from Kislovodsk to Moscow. I was alone in the compartment with no book to read and so decided to conceive of something. I had been engaged in low-temperature physics and astrophysics for a number of years, and therefore, a natural question for me was where and under what conditions superfluidity and superconductivity could be observed in space. To formulate a question is frequently equivalent to doing half the work. It actually took me no more than the prescribed time to think that the existence of superfluidity is possible in neutron stars and superconductivity in the atmosphere of white dwarfs and that there may exist superfluidity of the neutrino 'sea'. On returning to Moscow, I took up all three problems — the first two together with D A Kirzhnits [161, 162] and the third in collaboration with G F Zharkov [163].

The interaction among neutrons with antiparallel spins in the s-state corresponds to attraction, and therefore in a degenerate neutron gas there will appear pairing in the spirit of BCS theory. For the gap width $\Delta(0) \sim k_B T_c$ we obtained the estimate $\Delta(0) \sim (1-20)$ MeV, i.e., in the center of a neutron star (for a density $\rho \sim 10^{14}-10^{15}$ g cm^{-3}) we obtained $T_c \sim 10^{10}-10^{11}$ K, while on the neutron phase

† Sobyanin recently pointed out an interesting possibility of 'untwisting' the normal component of helium II inside a vessel by means of electric and magnetic fields acting on the helium ions [209].

‡ In these experiments, a very strong diamagnetism was observed, but the conductivity of the samples was not at all anomalously large. Such a situation is also possible for superconductors in case where the superconducting seeds (granules) are separated by non-superconducting layers. The question, however, arose whether or not superdiamagnetism can be observed in dielectrics and generally in non-superconductors.

boundary (for $\rho \sim 10^{11}$ g cm^{-3}) we had $T_c \sim 10^7$ K. It was also indicated that the rotation of a neutron star results in the formation of vortex lines. The fact that in nuclear matter there may occur superfluidity had actually been known before, but applied to neutron stars (at that time, in 1964, they had not yet been discovered), as far as I know, our paper was pioneering. Incidentally, in paper [164], in which I summarized my activity in the field of superfluidity and superconductivity in space, I also pointed to a possible superconductivity of nuclei-bosons (for example, α-particles) in the interior of white dwarfs and to superconductivity of protons which are present in a certain amount in neutron stars.

The possibility of the existence of superconductivity in a certain surface layer of the cold stars — white dwarfs was discussed in papers [162, 164]. The estimates give little hope. For example, for a density $\rho \sim 1$ g cm^{-3} the temperature is $T_c \sim 200$ K, and as the density increases, T_c falls rapidly. Somewhat more interesting is the possibility of superconductivity of metallic hydrogen in the depths of large planets — Jupiter and Saturn [164]. The estimates of the critical temperature T_c for metallic hydrogen, which are known from the literature, reach 100 – 300 K, but the temperature in the depth of the planets is unknown. I am unacquainted with the present-day state of the problem, but it seems to me that the existence of superconductivity in stars and large planets is hardly probable. The possibility of the appearance of superfluidity in the degenerate neutrino 'sea', whose existence at the early stages of cosmological evolution was discussed in some papers, was considered in the note [163] (see also Ref. [164]). Such a possibility, as applied to neutrinos or some hypothetical particles now involved in the astrophysical arsenal, is currently of no particular interest, but nevertheless it is reasonable to bear in mind.

7. High-temperature superconductivity

Beginning in 1964 I started investigating high-temperature superconductivity (HTSC) and from that time this problem remained, and remains, at the center of my attention although I was interested in many other things as well. My story about this work should however begin with quite a different question that concerns surface superconductivity. This question is as follows: Can there exist two-dimensional superconductors in which the electrons (or holes) participating in superconductivity are concentrated near the boundary of, say, a metal or a dielectric with a vacuum, on the boundary between, e.g., twins (i.e., on the boundary of twinning), etc. It seems to me that surface superconductivity might be particularly well pronounced for electrons on surface levels which were first considered by I E Tamm as far back as 1932 [165]. The possibility of this particular superconductivity was discussed in paper [166]. The answer was affirmative — the Cooper pairing and the whole BCS scheme works in the two-dimensional case as well. The following possibility was also pointed out: electrons are located at volume type levels, but their attraction, which leads to superconductivity, takes place only near the body surface (or on the twinning boundary). Note that surface ordering, although absent in the volume, may certainly take place not only in the case of superconductivity, it is also possible, for example, for ferro- and antiferromagnetics [167]. I subsequently saw experimental research testifying to realistic character of such situations. But I did not follow the appearance of the corresponding literature and cannot therefore give any references. Besides, this is not the subject of the present paper. As to surface superconductivity, it was emphasized in 1967 that long-range superconducting order is impossible in two dimensions [168]. At the same time, as distinguished from the one-dimensional case, in two dimensions (the case of a surface) the fluctuations that destroy the order increase with the surface size L only logarithmically. Accordingly, even for surfaces of macroscopic size ($L \gg a$, where a is atomic size) the fluctuations may be not so large [169]. An even more important circumstance is that in a two-dimensional system there may occur a quasi-long-range order under which superfluidity and superconductivity are preserved. This is an extensive issue, and we therefore restrict ourselves to mentioning paper [170] and the monograph [171] (Chap. 1, Sec. 5 and Chap. 6, Sec. 5), where one can find the corresponding citations. Briefly speaking, superconductivity may well exist in two-dimensional systems. From an electrodynamic point of view, surface superconductors must behave as very thin superconducting films [172, 173]. In a certain sense, surface superconductivity is realized. For instance, superconductivity is observed in a $NbSe_2$ film with a thickness of only two atomic layers [174]. It would be more interesting to obtain surface superconductors on the Tamm (surface) levels [166]. It is obvious how interesting and probably important from the point of view of applications would be a dielectric possessing surface superconductivity. I am not however definitely sure that such a version may be thought of as radically different from a dielectric covered itself by a superthin superconducting film. But after all, the difference does exist. The problem of surface superconductivity seems to be demanding and significant irrespective of the corresponding value of the critical temperature T_c.

The fates decreed, however, that surface superconductivity was to be associated with the problem of high-temperature superconductivity (HTSC). To be more precise, the association appeared in my own work.

Before clarifying the matter, I shall make several remarks (henceforth, I shall sometimes use the text of my paper [175] which may prove to be unavailable to the reader).

For a full 65 years, the science of superconductivity was a part of low-temperature physics, i.e., temperatures of liquid helium (and in some cases liquid hydrogen). Thus, for example, the critical temperature of the first known superconductor, mercury, discovered in 1911, is $T_c = 4.1$ K; the critical temperature of lead, whose superconductivity was discovered in 1913, is $T_c = 7.2$ K. If I am not mistaken, higher T_c values were not achieved until 1930, although it was definitely understood that higher T_c were desirable. The next important step on this way was the synthesis of the compound Nb_3Sn with $T_c = 18.1$ K in 1954. Despite a great effort, it was not until 1973 that the compound Nb_3Ge with $T = 23.2 - 24$ K was synthesized. Subsequent attempts to raise T_c were unsuccessful until 1986, which saw the first indications (soon confirmed) of superconductivity in the La–Ba–Cu–O system with $T_c \sim 35$ K [176]. Finally, in early 1987, a truly high-temperature superconductor† $YBa_2Cu_3O_7$ with $T_c = 80 - 90$ K was created.

† This statement reflects my opinion that the term 'high-temperature' is appropriate only for superconductors with $T_c > T_{b,N_2} = 77.4$ K, where, obviously, T_{b,N_2} is the boiling nitrogen temperature at atmospheric pressure.

The discovery of high-temperature superconductors (HTSC) became a sensation and gave rise to a real boom. One of the indicators of this boom is the number of publications. For example, in the period of 1989 – 1991, about 15 000 papers devoted to HTSC appeared, that is, on average, approximately 15 papers a day. For comparison, one of the reference books states that in the 60 years from 1911 to 1970, about 7 000 papers in total were devoted to superconductivity. Another indicator is the scale of conferences devoted to HTSC. Thus, at the conference M^{2}HTSC III in Kanazawa (Japan, July 1991) there were approximately 1 500 presentations, and the Conference proceedings occupied four volumes with a total size of over 2 700 pages (see Ref. [178]). Undoubtedly, such a scale of research is to a large extent explained by the high expectations for HTSC applications in technology. These expectations, by the way, from the very beginning appeared to me to be somewhat exaggerated, and this was later confirmed in practice. But, of course, the potential importance of HTSC for technology, medicine (NMR-tomograph), and physics itself leaves no doubts. Nevertheless, I still do not completely understand such a hyperactive reaction of the scientific community and of the general public to the discovery of HTSC: it is some sort of social phenomenon.

Another phenomenon that may be attributed either to sociology or to psychology is the complete oblivion to which the researchers of high-temperature superconductors, who began working successfully in 1986, consigned their predecessors. Indeed, the problem of HTSC was born not in 1986, but at least 22 years earlier — in its current form this problem was first stated by Little in 1964 [179]. First of all, Little posed the question: Why was the critical temperature of the superconductors known at the time not so high? Secondly, he pointed out a possible way of raising T_c to the level of room temperature or even higher. Specifically Little proposed replacing the electron-phonon interaction, responsible for superconductivity in the model of Bardeen, Cooper, and Schrieffer (BCS) [18], by the interaction of conduction electrons with bound electrons, or in a different terminology which Little did not use, with excitons. In terms of the well-known BCS formula for the critical temperature

$$T_c = \theta \exp\left(-\frac{1}{\lambda_{\text{eff}}}\right) \qquad (75)$$

the meaning of the exciton mechanism is that the region of attraction between conduction electrons θ is set to be $\theta \sim \theta_{\text{ex}}$, where $k_B\theta_{\text{ex}}$ is the characteristic exciton energy. On the other hand, for the electron-phonon mechanism of attraction in Eqn (75) we have $\theta \sim \theta_D$, where θ_D is the Debye temperature of the metal. Since the situation in which $\theta_{\text{ex}} \gg \theta_D$ is quite possible and even typical, it follows that for the same value of the effective dimensionless interaction parameter λ_{eff}, for the exciton mechanism T_c is $\theta_{\text{ex}}/\theta_D$ times higher than for phonons. Concretely, Little proposed to create an 'excitonic superconductor' on the basis of organic compounds by designing a long conducting (metallic) organic molecule (a 'spine') surrounded by side polarizers — other organic molecules [179].

It is not appropriate to go into details here. Let me just point out that Little's work did not remain unnoticed. Quite the opposite: it attracted a lot of attention. In particular, I also followed up Little's work by suggesting a somewhat different version: roughly speaking, replacing the quasi-one-dimensional conducting thread in Little's model with a quasi-two-dimensional structure ('sandwich'), i.e., with a conducting thin film placed between two polarizers (dielectric plates) [180]. More precisely, in paper [180], with a reference to the paper [166] on surface superconductivity it was assumed that T_c may be raised with the help of some dielectric coverings of metallic surfaces. It was emphasized that quasi-two-dimensional structures are much more advantageous than quasi-one-dimensional structures [179] because of an considerably smaller role of fluctuations (this argument was worked out in Ref. [169]). Later on I was engaged in earnest in the HTSC problem and concentrated on 'sandwiches', i.e., thin metallic films in dielectric and semiconducting 'coatings' and on layered superconducting compounds — these kind of 'files' of sandwiches [181 – 185, 171].

I should say that I write rather easily and, moreover, I even feel the necessity of expressing my thoughts in written form. As a result, during the 32 years in which I have been interested in the HTSC problem, I wrote many (probably, too many) papers on the subject, particularly, popular papers. I do not think I need to refer to many of them here. Among the published works, special attention is deserved by the monograph [171]. This book was the outcome of the joint efforts undertaken by L N Bulaevskiĭ, V L Ginzburg, D I Kholmskiĭ, D A Kirzhnits, Yu V Kopaev, E G Maksimov, and G F Zharkov (I E Tamm Department of Theoretical Physics of P N Lebedev Physical Institute of the USSR Academy of Sciences, Moscow) who had been 'attacking' the HTSC problem for several years. This monograph was published in Russian in 1977 and in English translation in 1982 and was the first, and up to 1987 the only one devoted to this issue. In Ref. [171], a whole spectrum of possible ways of obtaining HTSC was considered.

I shall dwell on some of the results of our work.

A very important question is whether or not there are some limitations on admissible T_c values in metals, say, due to the requirement of crystal lattice stability. Such limitations are possible in principle, and moreover in the 1972 paper [186] it was stated that it was the requirement of lattice stability that fully obstructs the possibility of the existence of HTSC. The point is that the dimensionless parameter of the interaction force λ_{eff} in the BCS formula (75) can be written in the form

$$\lambda_{\text{eff}} = \lambda - \mu^* = \lambda - \frac{\mu}{1 + \mu \ln(\theta_F/\theta)} \,. \qquad (76)$$

Here λ and μ are respectively the dimensionless coupling constants for phonon or exciton attraction and Coulomb repulsion and $k_B\theta_F = E_F$ is the Fermi energy. At the same time, in the simplest approximation (homogeneity and isotropy of material, and weak coupling) we have

$$\mu - \lambda = \frac{4\pi e^2 N(0)}{q^2 \varepsilon(0, q)} \,, \qquad (77)$$

where $\varepsilon(\omega, q)$ is the longitudinal permittivity for the frequency ω and for the wave number q and the factor $1/q^2\varepsilon(0, q)$ should be understood as a certain mean value in $\mathbf{q}$; $N(0)$ is the density of states on the Fermi boundary for a metal in the normal state. If, as was assumed in Ref. [186], the stability condition has the form

$$\varepsilon(0, q) > 0 \,, \qquad (78)$$

then from Eqn (77) it follows that

$$\mu > \lambda \,. \qquad (79)$$

This inequality and Eqn (76) imply that superconductivity (for which, certainly, $\lambda_{\text{eff}} > 0$) is generally possible only due to the difference between μ^* and μ, the T_c value being not large. It was, however, already known empirically that $\mu < 0.5$ and sometimes $\lambda > 1$, and thus that inequality (79) is violated. Apart from such and some other arguments already expressed in the early stages [184], it was later shown strictly (see Refs [187, 171, 188] and the literature cited there) that the stability condition (78) is invalid and, in fact, the stability condition has the form (for $q \neq 0$)

$$\frac{1}{\varepsilon(0,q)} \leqslant 1\,, \tag{80}$$

i.e., is satisfied if one of the inequalities

$$\varepsilon(0,q) \geqslant 1\,, \quad \varepsilon(0,q) < 0 \tag{81}$$

holds. It is interesting that the values $\varepsilon(0,q) < 0$ for large q, important in the theory of superconductivity, are realized in many metals [189, 190]. From the second inequality (81) and expression (77) it is obvious that the parameter λ may exceed μ. On the basis of this fact, our group came to the conclusion even before 1977 (I mean the Russian edition of the book [171]) that the general requirement of stability does not restrict T_c and it is quite possible, for example, that $T_c \lesssim 300$ K.

As has already been mentioned above, the idea of the exciton mechanism is connected with the possibility of raising T_c by increasing the temperature θ in Eqn (75) which determines the energy range $k_B\theta$ where the electrons attract one another near the Fermi surface and, thus, form pairs. It is assumed that weak coupling takes place here, when

$$\lambda_{\text{eff}} \ll 1\,. \tag{82}$$

It is only under this condition that formula (75) and the BCS model are applicable. But the BCS theory is on the whole more extensive and admits consideration of the case of strong coupling [191], when

$$\lambda_{\text{eff}} \gtrsim 1\,. \tag{83}$$

Under the conditions (83) of strong coupling formula (75) is, already, of course, not valid although it is clear from it that the temperature T_c rises with increasing λ_{eff}. In the literature, a large number of expressions for T_c are proposed for the case of strong coupling (see Refs [171, 188, 192, 193] and some references therein). The simplest of these expressions is as follows:

$$T_c = \theta \exp\left(-\frac{1+\lambda}{\lambda-\mu^*}\right)\,. \tag{84}$$

Exactly as it should be under the conditions (82) of weak coupling or, more precisely, under the condition $\lambda \ll 1$, formula (84), of course, becomes (75). If in Eqn (84) we set $\mu^* = 0.1$ then, for example, for $\lambda = 3$ the temperature is $T_c = 0.25\,\theta$. Therefore, for the value $\theta = \theta_D = 400$ K, which is readily admissible for the phonon mechanism, we already have $T_c = 100$ K. More accurate formulae also suggest that for the strong coupling (83) the phonon mechanism can already allow temperatures $T_c \sim 100$ K and even $T_c \sim 200$ K. But the analysis carried out in the book [171] and later, showed that for 'conventional' superconductors with strong coupling, the temperature T_c is rather small. For example, for lead we have $\theta_D = 96$ K and, therefore, in spite of the high value $\lambda = 1.55$, the critical temperature is $T_c = 7.2$ K. For such a conclusion, i.e., that θ_D falls with increasing λ, there also exist theoretical arguments (see Ref. [171], Chap. 4). That was the reason why we (or, at least, I) did not hope for the creation of high-temperature superconductors at the expense of strong coupling but possessing the phonon mechanism. In any case, as I have mentioned above, in Ref. [171] there prevailed a versatile and unprejudiced approach to the HTSC problem. Below I cite the last part of Chapter 1 written by me for the book [171]:

'On the basis of general theoretical considerations, we believe at present that the most reasonable estimate is $T_c \lesssim 300$ K, this estimate being, of course, for materials and systems under more or less normal conditions (equilibrium or quasi-equilibrium metallic systems in the absence of pressure or under relatively low pressures, etc.). In this case, if we exclude from consideration metallic hydrogen and, perhaps, organic metals, as well as semimetals in states near the region of electronic phase transitions, then it is suggested that we should use the exciton mechanism of attraction between the conduction electrons.

In this scheme, the most promising materials' from the point of view of the possibility of raising T_c, are apparently layered compounds and dielectric – metal – dielectric sandwiches. However, the state of the theory, let alone the experiment, is still far from being such as to allow us to regard as closed other possible directions, in particular, the use of filamentary compounds. Furthermore, for the present state of the problem of high-temperature superconductivity, the most sound and fruitful approach will be one that is not preconceived, in which attempts are made to move forward in the most diverse directions.

The investigation of the problem of high-temperature superconductivity is entering into the second decade of its history (if we are talking about the conscious search for materials with $T_c \gtrsim 90$ K using exciton and other mechanisms). Supposedly, there begins at the same time a new phase of these investigations, which is characterized not only by greater scope and diversity, but also by a significantly deeper understanding of the problems that arise. There is still no guarantee whatsoever that the efforts being made will lead to significant success, but a number of new superconducting materials have already been produced and are being investigated. Therefore, it is in any case difficult to doubt that further investigations of the problem of high-temperature superconductivity will yield many interesting results for physics and technology, even if materials that will remain superconducting at room (or even liquid-nitrogen) temperatures will not be produced. However, as has been emphasized, this ultimate aim does not seem to us to have been discredited in any way. As may be inferred, the next decade will be crucial for the problem of high-temperature superconductivity'.

This was written in 1976. Time passed, but the multiple attempts to find a reliable and reproducible way for creating HTSC have been unsuccessful. As a result, after the burst of activity came a slackening which gave cause for me to characterize the situation in a popular paper [194] published in 1984 as follows:

'It somehow happened that research into high-temperature superconductivity became unfashionable (there is good reason to speak of fashion in this context since fashion sometimes plays a significant part in research work and in the scientific community). It is hard to achieve anything by making admonitions. Typically it is some obvious success (or reports of success, even if erroneous) that can radically and rapidly reverse attitudes. When they sense a 'rich strike' the former doubters, and even dedicated critics, are capable of turning coat and become ardent supporters of the new work. But this subject belongs to the psychology and sociology of science and technology.

In short, the search for high-temperature superconductivity can readily lead to unexpected results and discoveries, especially since the predictions of the existing theory are rather vague'.

I did not expect, of course, that this 'prediction' would come true in two years [176, 177]. It came true not only in the sense that HTSC with $T_c > T_{b,N_2} = 77.4$ K were obtained, but also, so-to-say, in the social aspect: as I have mentioned above, a real boom began and an 'HTSC psychosis' started. One of the manifestations of the boom and psychosis was an almost total oblivion of everything that had been done before 1986, as if the discussion of HTSC problem had not begun 22 years before [179, 180]. I have already dwelt on this subject above and in the papers [175, 192] and would not like to return to it here. I will only note that J Bardeen, whom I always respected, treated the HTSC problem with understanding both before 1986 and after it (see [195]).

The present situation in solid state theory and, in particular, the theory of superconductivity, does not allow us to calculate the temperature T_c or indicate, with sufficient accuracy and certainly, especially in the case of compound materials, what particular compound should be investigated. Therefore I am of the opinion that theoreticians could not have given experimenters better and more reliable advice as to how and where HTSC could be sought than was done in the book [171]. An exception is perhaps only an insufficient attention to the superconductivity of the $BaPb_{1-x}Bi_xO_3$ (BPBO) oxide discovered in 1974. When $x = 0.25$, for this oxide we have $T_c = 13$ K which is a high value of T_c when it is estimated in a way similar to that used for conventional superconductors. In the related oxide $Ba_{0.6}K_{0.4}BiO_3$ (BKBO), superconductivity with $T_c \sim 30$ K was discovered in 1988. Most importantly, the compound $La_{2-x}Ba_xCuO_4$ (LBCO) in which superconductivity with $T_c \sim 30-40$ K was discovered in 1986 [176] and is thought of as the discovery of HTSC belongs to the oxides. However even now, 10 years later, one cannot predict, even roughly, the values of T_c for a particular material, and moreover, even the very mechanism of superconductivity in cuprates and, in particular, in the most thoroughly investigated cuprate $YBa_2Cu_3O_{7-x}$ (YBCO) with $T_c \sim 90$ K is not yet clear.

It is inappropriate to dwell here extensively on the current state of the HTSC problem. I shall restrict myself to several remarks.

At first glance, HTSC cuprates differ strongly from 'conventional' superconductors (see, for example, Refs [52, 178, 196, 210]). This circumstance gave rise to the opinion that HTSC cuprates are something special — either the BCS theory is inapplicable to them or, in any case, a non-phonon mechanism of pairing is acting in them. This tendency was very clearly expressed at the 1991 M^{2}HTSC III Conference [178].

Indeed, the phonon mechanism has no exclusive rights. In principle, may exist the exciton (electronic) mechanism, the Schafroth mechanism (creation of pairs at $T > T_c$ with a subsequent Bose-Einstein condensation), the spin mechanism (pairing due to exchange of spin waves or, as it is sometimes called, spin fluctuations), and some other mechanisms (for some more details and references see, for example, Refs [193, 210]). Since I have always been a supporter of the exciton mechanism, I would be only glad if this very mechanism proves to act in HTSC. But there is not yet any grounded basis for such a statement. In the BKBO oxide and in doped fullerenes (fullerites) of K_3Cu_{60} and Rb_3Cu_{60} type (they all possess cubic structure) with $T_c \sim 30-40$ K the phonon mechanism obviously prevails. The situation is more complicated with oxides-cuprates which are highly anisotropic layered compounds. However, E G Maksimov, O V Dolgov and their colleagues indicate, I believe, convincingly that the phonon mechanism may quite possibly also dominate in HTSC cuprates. In any case, HTSC cuprates in the normal state differ from ordinary metals in only a quantitative respect. Formally, a standard electron-phonon interaction with a coupling constant $\lambda \approx 2$ accounts well for the high values $T_c \sim 100-125$ K as being due to the high Debye temperature $\theta_D \sim 600$ K (see Refs [193, 197, 198–201] and the literature cited there)†. The properties of the superconducting state of HTSC cuprates are a more complicated entity. To explain them, it is already insufficient to use a standard isotropic approximation in the model of a strong electron-phonon interaction. However, allowing for the anisotropy of the electron spectra and interelectron interaction, the electron-phonon interaction all the same may play a decisive role in the formation of a superconducting state. As has been shown [211, 212] (see also Refs [202–204]), in the framework of multi-zone models allowing for standard electron-phonon and Coulomb interactions, one can obtain a strongly anisotropic superconducting gap including its sign reversal in the Brillouin zone, which imitates d-pairing. It is also possible that the electron-exciton interaction and peculiarities of the electron spectrum, which are almost insignificant for understanding the properties of the normal state, make their contribution to the formation of the superconducting state. I do not regard myself competent enough to think of such statements as proved. But it is beyond doubt that a general denial of the crucial role of the phonon mechanism of HTSC (in cuprates) typical of the recent past (see Ref. [178]) is already behind us.

Suppose, for the sake of argument, that in the already known HTSC the exciton mechanism does not play any role. This is, of course, important and interesting, but in no way discredits the very possibility of a manifestation of the exciton mechanism. As has already been mentioned, we are not aware of any evidence contradicting the action of the exciton mechanism. But it is actually not easy for the exciton mechanism to manifest itself. This will require some special conditions which are not yet clear (see, in particular, Ref. [201]).

The highest critical temperature fixed today (for $HgBa_2Ca_2Cu_3O_{8+x}$ under pressure) reaches 164 K. Such a value can be attained with the phonon mechanism. But if one succeeds in reaching a temperature $T_c > 200$ K , the phonon mechanism will hardly be sufficient (when $\lambda = 2$, the temperature $T_c = 200$ K is obtained for $\theta_D \approx 1000$ K). As to the exciton mechanism, even room temperature is not a limit

† I find it necessary to note that the report [197] was, in fact, prepared by E G Maksimov alone. My name appeared in [197] only because there was a difficulty with including this report on the agenda, and I had, by E G Maksimov's consent, to include my name which provided the possibility of his participation in the 1994 M^{2}HTSC IV Conference. It is not a pleasure to speak about such morals and manners, but this is a truth.

for T_c. A search for HTSC with the highest possible critical temperatures is now being and will, of course, be undertaken. It seems to me, as before, that the most promising in this respect are layered compounds and dielectric – metal – dielectric 'sandwiches'†. It would be natural to use the atomic layer-by-layer synthesis here [205, 214]. The role of a dielectric in such sandwiches can be played by organic compounds in particular. By the way, the possibilities that may open on the way are virtually boundless. It is therefore especially reasonable to be guided by some qualitative consideration (see, for example, Ref. [171], Chap. 1).

For 22 years (from 1964 to 1986), which however flew by very quickly, high-temperature superconductivity was a dream for me, and to think of it was something like a gamble. Now it is an extensive field of research, tens of thousand papers are devoted to it, hundreds or even thousands of researchers are engaged in the study of one or another of its aspects. Much has already been done but much remains to do. Even the mechanism of superconductivity in HTSC cuprates is rather obscure, to say nothing of the myriad particular questions. I think that among these questions the first place belongs to the question of the maximum attainable value of the critical temperature T_c under not very exotic conditions, say, at atmospheric pressure and for a stable material. More concretely, one can pose a question concerning the possibility of creating superconductors with T_c values lying within the range of room temperatures (the problem of RTSC — room temperature superconductivity). RTSC is, in principle, possible, but there is no guarantee in this respect. The problem of RTSC took, generally, the place that had been occupied by HTSC before 1986 – 1987. I am afraid that I do not see any possibility for myself to do something positive in this direction, and it only remains to wait impatiently for coming events.

8. Concluding remarks

By 1943, when I began studying the theory of superconductivity, 32 years had already passed since the discovery of the phenomenon. None the less, at the microscopic level superconductivity had not yet been understood and had actually been a 'white spot' in the theory of metals and, perhaps, in the whole physics of condensed media. Superfluidity of helium II had been discovered in its explicit form no more than 5 years before that time, and its connection with superconductivity had only been outlined. The world was at terrible war, and I myself hardly understand now why the enigmas of low-temperature physics seemed so tempting to me when I was cold and semi-starving in evacuation in Kazan'. But it was so. Poor command of mathematical apparatus, an inability to concentrate on one particular task (I was simultaneously engaged in several problems), difficulties in exchange of scientific information, especially with experimenters, in the war and post-war years obstructed a rapid advance, and it was only in 1950 that something appeared completed (I mean the Ψ-theory of superconductivity). But this completeness is, of course, rather conditional because new questions and problems constantly arose.

† Besides being intuitive, (see Refs [171, 182, 184, 185]), there are also some concrete arguments [197, 201] in favor of such quasi-two-dimensional structures. It has already been proved that in HTSC cuprates (or, at least, in some of them) superconductivity is of a quasi-two-dimensional character [214].

At the same time, the character of studies in the field of low-temperature physics, as well as the whole physics was changing radically. It is even hard to imagine now that it was only one laboratory that succeeded in obtaining liquid helium between 1908 and 1923. It is hard to imagine that applications of superconductivity in physics, to say nothing of technology, were fairly modest for three decades. And it was not until the 1960s that strong superconducting magnets were created and extensively used. At the present time superconductivity finds numerous applications (see, for example, Refs [70, 206]). Even the small book [207] intended for schoolchildren presents various applications of superconductivity, including giant superconducting magnets in tokamaks and tomographs. Creation of high-temperature superconductors (HTSC, 1986 – 1987) gave rise to great expectations of the possibility of new applications of superconductivity. These expectations were partly exaggerated, but nevertheless now, after 10 years, much has already been done in this direction, even in respect of electric power lines and strong magnets [208], not to mention some other applications. I wrote in Section 7 about the boom provoked by the creation of HTSC. Many thousands of papers and hundreds or even thousands of researchers — what a contrast with what was observed in, say, 1943 or as recently as 10 years ago!

In the light of the present state of the theory of superconductivity and superfluidity, much of what has been said in this paper is only of historical interest and in other cases is somewhere far from the forefront of the current research. At the same time, and it is now very important, I have mentioned a large number of questions and problems which still remain unclear. This lack of clarity concerns the development of the Ψ-theory of superconductivity and its application to HTSC, the application of the Ψ-theory of superfluidity, the problem of surface (two-dimensional) superconductivity, the question of thermoeffects in superconductors (and especially their connection with heat transfer), the circulation effect in a non-uniformly heated vessel filled with a superfluid liquid and some other things, to say nothing of HTSC theory. The aim of the paper will have been attained if it at least helps to draw attention of both theoreticians and experimenters to these problems.

Taking the opportunity, I express my gratitude to Yu S Barash, E G Maksimov, L P Pitaevskiĭ, A A Sobyanin, and G F Zharkov for their interest in the manuscript and fruitful remarks.

Note added to the current edition

When this paper had already been published in Russian, I understood that calculations of the heat conductivity with the help of kinetic equation [127] and with account of creation and breakdown of pairs at the ends of a specimen [126, 129] are equivalent (provided the mean-free path of 'normal' electrons is small in comparison with the length of a specimen). Therefore, coincidence of estimates (71) and (72) is quite natural. In this connection note that calculation of the total electron contribution $\varkappa_e^{\text{tot}} = \varkappa_e + \varkappa_c$ to the heat conductivity with the aid of kinetic equation and under the condition that $\mathbf{j}_n = b_n \nabla T$ is in principle correct and takes into account the convective heat transfer. In isotropic ordinary superconductors the convective corrections are small (see Ref. [216], §98) and this agrees with estimates (71), (72) since they result in $\varkappa_c / \varkappa_e \ll 1$. But in an anisotropic case (in unusual superconductors, see Ref. [131]) the convective heat flow caused by $\mathbf{j}_n \neq 0$ may be large. It is not clear to me if calculations [134a] are consistent with the allowance for the convective flow. In any case now it is absolutely clear that the maximum of $\varkappa(T) = \varkappa_e^{\text{tot}} + \varkappa_{\text{ph}}$ observed in HTSC materials is connected with $\varkappa_e^{\text{tot}}$ but not with $\varkappa_{\text{ph}}$ [134a, 217]. This is clearly demonstrated by observations of the Righi-Leduc effect in HTSC superconductors [217]. The only question of principal character which is not clear to me now is whether the maximum of the function $\varkappa_e^{\text{tot}}(T)$ in HTSC (at $T < T_c$) is connected with nonzero normal current or whether the fact that $\mathbf{j}_n \neq 0$ is not important. In the first case we may say that the above maximum of $\varkappa_e^{\text{tot}}$ is due to the convective heat flow [22, 126, 129].

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