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Photons and leptons in external fields at finite temperature and density

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Contents

Abstract. Theoretical aspects of the propagation of photons, electrons, and neutrinos in external electromagnetic fields at finite temperature and density are considered. The photon polarization operator and the radiative mass shifts and anomalous magnetic moments of the electron and the massive neutrino are investigated based on the finite-temperature quantum field theory with the use of exact solutions to the relativistic equations of motion for particles in external fields of various configurations. The present approach permits using model results as reference ones for experimental data as well as presenting a wider choice of interpretations for the results obtained.

1. Introduction

In recent years a new varied line of investigations in the quantum field theory (QFT) has been formed that is related to consideration of the influence of external conditions, such as classical fields, finite temperature and density of medium, as well as boundary conditions for the system in a finite volume. When the factors mentioned are taken into account then, on the one hand, it makes the results of analysis of quantum-field systems more informative, at the same time eliminating some principal difficulties that are inherent in the system of free fields, and, on the other hand, such an approach enables the physical effects that are observed in real laboratory experiments and in astrophysical conditions to be predicted and explained with a high degree of reliability.

Historically this line can be traced back to the works devoted to construction of the Heisenberg – Euler effective Lagrangian [1] (see also Refs [2, 3]), the quantum theory of synchrotron radiation [4 – 6] (see also Refs [7, 8]) and that of quantum processes in strong external fields [9 – 11], to papers [12, 13], where phase transitions in a system of interacting fields have been predicted, as well as to the Casimir's calculation of the energy of electromagnetic vacuum between plates (see, for instance, Refs [3, 14]) that paved the way for modern investigation of the effects of this kind.

Further development of this trend of investigation has led to the creation of modern theoretical methods that enabled a number of rather delicate physical effects to be predicted, as embodied in an enormous amount of papers published for the past twenty years. In the present review we consider only a part of these works, and concentrate primarily on rigorous mathematical approaches to the description of the influence of external fields, finite temperatures and substance density on the radiative and vacuum effects in QFT. However, in the analysis of the physical essence of these effects and in the introduction of their theoretical derivation we did not aim at giving a comprehensive presentation of formal mathematical justifications for particular methods of calculations.

Due to limitations of the article size, we unfortunately had to leave outside the scope of our consideration some questions that being very interesting ones are nevertheless rather special in our opinion to be presented here. Among them are the Casimir effect and its modern manifestation, as well as cosmological aspects of the phase transition phenomenon in the gauge theories of interactions between elementary particles. Moreover, such questions as the effects related to the

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nontrivial structure of the vacuum state in the quantum chromodynamics (QCD), phase transitions in QFT, and new particles creation in the framework of models extending the Standard Model of elementary particles interaction are to be discussed in a separate publication.

The plan of the review is as follows. Section 2 is devoted to a rather concise presentation of basic approaches to theoretical description of quantum fields interaction in the presence of an external field, at finite temperature and density of medium.

In Section 3, radiative effects in an electron-positron plasma and, in particular, the polarization operator and propagation of a photon across a constant magnetic field are considered in the framework of the Standard Model.

In the last Section 4, the results of the study concerning the properties of fermions moving in external fields at finite temperature and nonzero chemical potential are given. The mass operator and radiative corrections to the fermion masses are considered, and a detailed study on the dynamic nature of the anomalous magnetic moment of fermions as a function of the magnetic field strength, the fermion energy and properties of the medium (temperature and density) is presented as well.

2. QFT formalism in the presence of an external field, at finite temperature and density of medium

2.1 Application of an external field

In solving fundamental problems of QFT in the presence of external electromagnetic fields, particularly when the external field is fairly strong and expansion of equations in terms of the coupling constant inherent to the charged particle and the field is no more valid, or in the case where a bound state of a particle in the external field may be of interest, the method of finding the exact solutions to relativistic wave equations has proved to be of considerable significance. This method has been widely used in constructing the quantum theory of synchrotron radiation (SR) [4, 6–8], and later it was developed in studies of other quantum processes in an external field (see, for example, Refs [9–11, 14, 15]). The method of exact solutions should be also used when working in the Furry's representation [16], where the Feynman–Dyson formalism of QED is extended from the case of a free electron to that of an electron bound state.

The essence of the method is as follows [9, 10, 15]. The QED Hamiltonian with an external field unlike that without the external field includes a term describing interaction with an external classical field A^{ext}:

$$H = H_{\text{e}} + H_{\gamma} + H_{\text{int}} + \int j^{\mu}(x) A_{\mu}^{\text{ext}}(x)\, \mathrm{d}^3 x\,, \tag{2.1}$$

where H_{e}, H_{γ} stand for free quantized electron-positron and electromagnetic fields, respectively, and H_{int} describes their interaction. In constructing the interaction representation in Furry's approach, an unperturbed Hamiltonian H_0 is taken as the following one

$$H_0 = H_{\text{e}} + H_{\gamma} + \int j^{\mu}(x) A_{\mu}^{\text{ext}}(x)\, \mathrm{d}^3 x\,, \tag{2.2}$$

which includes an interaction of a spinor field with an external field, and the perturbation theory is constructed with respect to the radiation interaction H_{int} only. Then, to generalize the secondary quantization scheme to the case of nonvanishing external field one should expand the secondary-quantized operators of the electron-positron field in terms of the complete system of the Dirac equation solutions in an external electromagnetic field, i.e.

$$\hat{\psi}(x) = \sum_s \left[a_s \psi_s^{(+)}(\mathbf{r}) \exp(-\mathrm{i}\varepsilon_s^{(+)} t) + b_s^{+} \psi_s^{(-)}(\mathbf{r}) \exp(\mathrm{i}\varepsilon_s^{(-)} t) \right], \tag{2.3}$$

where $\psi_s^{(+)}$, $\psi_s^{(-)}$ are the solutions of the Dirac equation in an external field, which belong to positive $(\varepsilon_s^{(+)})$ or negative $(-\varepsilon_s^{(-)})$ frequency states, respectively. In order to evaluate any particular physical effect in the Furry's picture one may use the Feynman diagram technique. In this case all electron lines correspond to the Dirac equation solutions. An electron in the initial and final states is described by the solution of a homogeneous Dirac equation for a given external field A_{μ}^{ext}:

$$\begin{aligned} &(\hat{p} - e\hat{A}^{\text{ext}} - m)\psi(x) = 0\,, \\ &\hat{p} = \mathrm{i}\gamma^{\mu}\partial_{\mu}\,, \end{aligned} \tag{2.4}$$

and an internal line corresponds to the Feynman propagator $S^c(x, x')$ of an electron in the external field:

$$S^c(x, x') = -\mathrm{i}\langle 0 | T\psi(x)\bar{\psi}(x') | 0 \rangle\,, \tag{2.5}$$

which is a solution of the inhomogeneous equation

$$(\hat{p} - e\hat{A}^{\text{ext}} - m) S^c(x, x') = \delta(x - x')\,. \tag{2.6}$$

It should be emphasized that the above-mentioned approach can be immediately applied only to a limited class of external fields, which allow a one-particle interpretation for the Dirac equation, and one can introduce a unique system of in- and out- states in the interaction representation and a unique vacuum state [15]. Moreover, it is essential that a quantum number exists such that electrons propagating forward and backward in time may be distinguished by an invariant way, and hence the same may be done with an electron and positron. We note that this quantum number can also exist in the case of fields that vary with time and when the energy is not conserved. Such a possibility exists for external fields such that the vacuum stability is not disturbed and hence electron-positron (and also other) pair production is impossible. In particular, the vacuum in a constant magnetic field, in the field of a plane electromagnetic wave, and also in constant crossed fields ($\mathbf{E} \perp \mathbf{H}, E = H$) is stable with respect to spontaneous particle pair production. In terms of the Dirac equation solutions this means that for the case of external fields which do not disturb the vacuum stability there exists a complete system of the Dirac equation solutions $\{\psi_s^{(\pm)}\}$ distinguished at any instant of time by the particle–antiparticle criterion.

A systematic consideration of all those alterations that are introduced into the *S*-matrix QFT formalism by the presence of an external field capable of pair production has been carried out in papers of Schwinger [2], Nikishov and Ritus [9], Fradkin with coworkers [15], Grib and Mostepanenko with coworkers [14], and Bagrov [17], and some other authors as well.

Here we shall turn our attention to a certain important point. As known, in formulating the problem of pair production by an external electromagnetic field it is assumed that the latter is switched off at $t \to \mp\infty$. Then a classification of the Dirac equation solutions by the frequency sign ($\pm$) can be made in the past (future), and in Refs [9, 18] it was shown that a complete set of solutions with a definite frequency sign in the past ${}_{\pm}\psi_n(x)$ and in the future ${}^{\pm}\psi_n(x)$ can be chosen so that conservation of the set of quantum numbers $\{n\}$ characterizing the solutions be ensured. In this case at any instant of time ${}_{+}\psi_n(x)$ can be represented as a superposition of wave functions ${}^{+}\psi_n(x)$ and ${}^{-}\psi_n(x)$ with the same $\{n\}$, and moreover probabilities of pair production and scattering processes induced by an external field can be found via the coefficients of this superposition. As a result, the S-matrix in Ref. [9] was expressed in terms of creation out-operators and annihilation in-operators, but in contrast to Ref. [15] the use of the Dirac equation solutions with conserved quantum numbers did not lead to any difference from the conventional S-matrix formalism and there was no need for propagators other than the Feynman ones.

In the non-Abelian gauge theory, an external non-Abelian field is introduced in completely the same way as it is done via the Furry's representation in QED [11]. The total gluon field A_μ^a is represented as the sum of an external (classic) field $\overline{A}_\mu^a$ and small quantum fluctuations Q_μ^a around it:

$$A_\mu^a = \overline{A}_\mu^a + Q_\mu^a \,. \tag{2.7}$$

Furthermore, similar to QED, interaction of quarks (gluons) with an external field $\overline{A}_\mu^a$ is taken into account exactly, and that with a quantized gluon field Q_μ^a is considered in the perturbation theory.

In the unified theory of electromagnetic and weak interactions of Weinberg, Salam and Glashow (WSG), the equations for propagators $D_{\mu\nu}(x, x')$ of the W-boson and the charged scalar $D(x, x')$ in an external field have the form

$$\left[(D^2 + m_{\mathrm{W}}^2)g^{\mu\nu} + 2\mathrm{i}eF^{\mu\nu} + \left(\frac{1}{\xi} - 1\right)D^\mu D^\nu\right]D_{\nu\lambda}(x, x') = \delta_\lambda^\mu \delta(x - x') \,, \tag{2.8}$$

$$(D^2 + \xi m_{\mathrm{W}}^2)D(x, x') = \delta(x - x') \,, \tag{2.9}$$

where $D_\mu = \partial_\mu + \mathrm{i}e\overline{A}_\mu$ is a covariant derivative (e is the electron charge), $\overline{A}_\mu$ is an external electromagnetic field potential, $F_{\mu\nu} = \partial_\mu \overline{A}_\nu - \partial_\nu \overline{A}_\mu$, and ξ is the gauge parameter.

We note that charged scalars in the WSG model are nonphysical particles and they appear only in virtual states. They can be excluded by choosing the unitary gauge ($\xi = \infty$), though in particular calculations the Feynman gauge ($\xi = 1$) is usually employed. There are few exact solutions known for the Dirac equation, as well as for the corresponding relativistic wave equations governing the W-boson and the charged scalar in an external electromagnetic field. Most important of them are the solutions of the wave equations in the Coulomb field, in the homogeneous magnetic field, in the electromagnetic plane wave field, in the homogeneous electric field, and in some cases of combinations involving the fields mentioned. Solutions of various wave equations in the above-mentioned external fields, as well as the methods of obtaining explicit expressions for the Feynman propagators of particles are described, for instance, in Refs [9–11, 15, 19].

One point has to be discussed in more detail, since it proves to be of particular importance in computations of quantum processes with charged particles in arbitrary stationary electromagnetic fields. As it is well known, the latter are either crossed fields or such that in a certain Lorentz reference system the electric and magnetic fields are parallel [20]. The total probability of the process with only one participating charged particle, calculated per unit time and volume (along with other physical quantities such as radiative mass shift), and being an invariant quantity, depends on two dimensionless field invariants [21, 9]

$$\begin{pmatrix} \varepsilon \\ \eta \end{pmatrix} = \frac{1}{B_0}\left[(f^2 + g^2)^{1/2} \mp f\right]^{1/2} , \tag{2.10}$$

where

$$f = \frac{1}{4} F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(H^2 - E^2) \,, \qquad g = \frac{1}{4} F_{\alpha\beta}\widetilde{F}^{\alpha\beta} = \mathbf{EH} \,,$$

$$\widetilde{F}^{\alpha\beta} = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu}F_{\mu\nu} \,, \qquad \text{and} \qquad B_0 = \frac{m^2}{e} = 4.41 \times 10^{13}\ \mathrm{G}$$

is the critical field, and also on the dynamic parameter

$$\varkappa = \frac{1}{mB_0}\left[-(F_{\mu\nu}p^\nu)^2\right]^{1/2} , \tag{2.11}$$

where p_ν is a constant 4-momentum of the particle in the external field, which upon switching off the field goes over into the 4-momentum of a free particle. Thus, the physical quantity of interest W is the function of three invariant parameters:

$$W = W(\varepsilon, \eta, \varkappa) \,. \tag{2.12}$$

In the crossed fields $\varepsilon = \eta = 0$. Therefore, as mentioned in Refs [21, 9], when the conditions

$$\varepsilon, \eta \ll 1 \,, \qquad \varepsilon, \eta \ll \varkappa \tag{2.13}$$

are fulfilled, the results of calculations of physical quantities in an arbitrary stationary field upon expressing them in an invariant form coincide in the first approximation [the level of approximation is determined by conditions (2.13)] with the exact result for the case of the crossed fields $\mathbf{E} \perp \mathbf{H}$, $E = H$ ($\varepsilon = \eta = 0$). The physical conditions $\varepsilon, \eta \ll 1$ imply that the constant field is weak as compared with the critical field B_0, and the condition $\varepsilon, \eta \ll \varkappa$ imply that any constant field looks almost as the crossed one from almost all directions for an ultrarelativistic particle in its rest frame. The homogeneous field approximation has recently also found extensive application in calculations of various processes proceeded in crystals. Here, for prompt particles propagating at small angles with crystalline axis or planes the dimensions of the regions of process formation are small as compared to the scales of inhomogeneities in crystalline fields [22], which can be effectively described with the use of the homogeneous field model.

2.2 Inclusion of finite temperature and density of medium

The method of Green's functions whose fundamentals have been developed in Refs [23–32] is the most efficient and universal technique in QFT at finite temperatures. Here methods of solution of the QFT problems both at vanishing and finite temperatures demonstrate a remarkable similarity — all problems are reduced to determination of corresponding Green's functions. In particular, evaluation of the energy spectrum of a macroscopic system at finite temperature T and nonzero chemical potential μ amounts to finding out the time-dependent Green's functions. For definiteness, let us consider a Fermi-system in an external stationary and homogeneous magnetic field, and assume that there is no condensation in all cases where generalization to Bose-systems is made. The time-dependent single-particle Green's function $G^c(x,x')$ is defined as follows [23–25]:

$$G^c(x,x') = -\mathrm{i}\,\frac{\mathrm{Sp}\{\exp[-\beta(\hat{H}-\mu\hat{N})]\,T\psi(x)\bar{\psi}(x')\}}{\mathrm{Sp}\{\exp[-\beta(\hat{H}-\mu\hat{N})]\}}\,, \tag{2.14}$$

where $\beta = T^{-1}$ is an inverse temperature, $\hat{H}$ is the system Hamiltonian that includes interaction with an external field, $\hat{N}$ is the particle number operator, μ is the chemical potential, and the trace operation (Sp) is carried out in the Fock space. Thus, definition of the time Green's functions at $T \neq 0$ and $\mu \neq 0$ differs from that of the causal Green's functions in the ordinary QFT ($T=\mu=0$) by the averaging procedure established for the chronological product of Heisenberg field operators, which is carried out not over the vacuum state but rather over the grand canonical Hibbs distribution. Formula (2.14), upon substituting the secondary-quantized ψ-operators (in an explicit form) of the fermionic field in the Furry's representation, provides the following representation for the time Green's function of an ideal electron-positron gas in a constant magnetic field [33]:

$$G^c(H,T,\mu) = S_c(H, T=\mu=0) + S_\beta(H,T,\mu)\,, \tag{2.15}$$

where

$$S_c(H, T=\mu=0) = -\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{+\infty}\mathrm{d}\omega\exp[\mathrm{i}\omega(t-t')] \times \sum_{s,\,\varepsilon=\pm1}\frac{\psi_s^{(\varepsilon)}(\mathbf{x})\bar{\psi}_s^{(\varepsilon)}(\mathbf{x}')}{\omega+\varepsilon E_s(1-\mathrm{i}\delta)} \tag{2.16}$$

is an ordinary Green's function of an electron in a constant magnetic field, and the temperature-dependent part of the time Green's function equals

$$S_\beta(H,T,\mu) = \mathrm{i}\sum_{s,\,\varepsilon=\pm1}\frac{\varepsilon\psi_s^{(\varepsilon)}(\mathbf{x})\bar{\psi}_s^{(\varepsilon)}(\mathbf{x}')}{\exp[\beta(E_s-\varepsilon\mu)]+1}\exp[-\mathrm{i}\varepsilon E_s(t-t')]\,. \tag{2.17}$$

It should be noted that the time Green's function is the sum of the Feynman propagator at zero temperature and the pure temperature-dependent part, with the representation for the latter function $S_\beta(H,T,\mu)$ obtained in paper [32] also in the form of a double integral, one of which being the integral taken around proper time. In (2.16), (2.17) summation is carried out over all quantum numbers $\{s\}$ of the positive ($\varepsilon=+1$) and negative ($\varepsilon=-1$) frequency states, $\psi_s^{(\varepsilon)}(\mathbf{x})$ is the coordinate part of the Dirac equation solution in a constant magnetic field, and the electron energy levels in a stationary magnetic field are determined by the formula [8]

$$E_n = \sqrt{2eHn+m^2+p_z^2}\,, \tag{2.18}$$

where $n=0,1,2,\ldots$ is the principle quantum number, p_z ($-\infty<p_z<+\infty$) is the electron momentum projection on the magnetic field direction $\mathbf{H}$.

In the momentum representation with respect to the variable $\tau = t-t'$, Eqns (2.15)–(2.17) yield

$$G^c(p_0,\mathbf{r},\mathbf{r}') = \sum_{s,\,\varepsilon=\pm1}\Bigg[\mathrm{V.p.}\,\frac{\psi_s^{(\varepsilon)}(\mathbf{x})\bar{\psi}_s^{(\varepsilon)}(\mathbf{x}')}{p_0-\varepsilon E_n} - \mathrm{i}\pi\delta(p_0-\varepsilon E_n)\psi_s^{(\varepsilon)}(\mathbf{x})\bar{\psi}_s^{(\varepsilon)}(\mathbf{x}')\tanh\frac{p_0-\mu}{2T}\Bigg]\,. \tag{2.19}$$

Along with the time Green's function (2.14), the retarded $G^R(x,x')$ and advanced $G^A(x,x')$ Green's functions are used also in QFT:

$$\begin{pmatrix} G^R(x,x') \\ G^A(x,x') \end{pmatrix} = -\mathrm{i}\begin{pmatrix} \theta(x_0-x_0') \\ -\theta(x_0-x_0') \end{pmatrix} \times \frac{\mathrm{Sp}\{\exp[-\beta(\hat{H}-\mu\hat{N})]\,[\psi(x),\bar{\psi}(x')]_+\}}{\mathrm{Sp}\{\exp[-\beta(\hat{H}-\mu\hat{N})]\}}\,. \tag{2.20}$$

In Ref. [25], the following relation between $G^A(p_0,\mathbf{r},\mathbf{r}')$, $G^R(p_0,\mathbf{r},\mathbf{r}')$ and the time Green's function $G^c(p_0,\mathbf{r},\mathbf{r}')$ has been established in the general case by means of direct comparison of spectral representations:

$$\begin{pmatrix} G^R(\mathbf{x},\mathbf{x}') \\ G^A(\mathbf{x},\mathbf{x}') \end{pmatrix} = \mathrm{Re}\,G^c(p_0,\mathbf{x},\mathbf{x}') \pm \mathrm{i}\coth\frac{p_0-\mu}{2T}\,\mathrm{Im}\,G^c(p_0,\mathbf{x},\mathbf{x}') \tag{2.21}$$

for Fermi-systems, and

$$\begin{pmatrix} G^R(\mathbf{x},\mathbf{x}') \\ G^A(\mathbf{x},\mathbf{x}') \end{pmatrix} = \mathrm{Re}\,G^c(p_0,\mathbf{x},\mathbf{x}') \pm \mathrm{i}\tanh\frac{p_0-\mu}{2T}\,\mathrm{Im}\,G^c(p_0,\mathbf{x},\mathbf{x}') \tag{2.22}$$

for Bose-systems.

Relations (2.21), (2.22) follow directly from definitions (2.14) and (2.20), thus solving the problem of construction G^A (G^R) in accordance with the time Green's function G^c. Hence, there follows a relation between the mass (polarization) operator of the retarded Green's function and the mass operator of the time Green's function [23, 25]:

$$\mathrm{Re}\,\Sigma^R = \mathrm{Re}\,\Sigma^c\,, \qquad \mathrm{Im}\,\Sigma^R = \coth\frac{p_0-\mu}{2T}\,\mathrm{Im}\,\Sigma^c \tag{2.23}$$

for the Fermi-system, and

$$\mathrm{Re}\,\Sigma^R = \mathrm{Re}\,\Sigma^c\,, \qquad \mathrm{Im}\,\Sigma^R = \tanh\frac{p_0-\mu}{2T}\,\mathrm{Im}\,\Sigma^c \tag{2.24}$$

for the Bose-system.

We considered it necessary to discuss these fundamental relations since in recent papers (see, for instance, [34–36]) the correspondence between the real and imaginary parts of the self-energy diagrams [(2.23) and (2.24)] had not been only derived anew but also verified in the one-loop approximation by the direct calculation. In order to compute the time Green's function for the system of interacting particles in the real time representation, both the Keldysh diagram technique [37, 38] and the equivalent method of the thermo-field dynamics [31] can be employed. In so doing, the equations for the Green's functions obtained by this way are equivalent in their sense to the kinetic equations. We remind [25] that the retarded Green's function $G^R(p_0, \mathbf{r}, \mathbf{r}')$ is analytic in the upper half-plane of the complex variable p_0, and the poles of its continuation to the lower half-plane define the dispersion law (the energy and decay) of quasi-particles. Whereas the advanced Green's function $G^A(p_0, \mathbf{r}, \mathbf{r}')$ is analytic in the lower half of the p_0-plane, and the poles of its continuation to the upper half-plane determine the energy spectrum of 'holes'. To calculate the spectrum of elementary excitations, the method of temperature (Matsubara) Green's functions is also used, which is known as the imaginary time representation [26, 39]. From the geometric point of view, QFT at finite temperature and nonzero chemical potential in the imaginary time representation corresponds to the quantum field theory on the hypercylinder $R^3 \times S^1$ with the base radius $r = \beta/2\pi$, and when the fields $\Phi(\mathbf{x}, \tau)$ depending on the imaginary time $\tau = \mathrm{i}t$ satisfy the periodicity (antiperiodicity) conditions in the boson (fermion) cases:

$$\Phi(\mathbf{x}, \tau) = \pm \exp(-\beta\mu)\Phi(\mathbf{x}, \tau - \beta) . \qquad (2.25)$$

For further presentation it will suffice to mention that the diagram technique in the framework of the imaginary time formalism is analogous to the Feynman rules in the ordinary QFT. The diagram technique in the imaginary time representation is developed according to the following replacements [25, 30, 39, 40]:

$$p_0 \to \mathrm{i}\omega_l + \mu , \qquad (2.26)$$

$$\int \frac{\mathrm{d}p_0}{2\pi} \to \mathrm{i}T \sum_{l=-\infty}^{+\infty} , \qquad (2.27)$$

$$\mathrm{i}2\pi\delta(p_0) \to \beta\delta_{l,0} , \qquad (2.28)$$

where $\omega_l = 2\pi T(l + 1/2)$, $l = 0, \pm 1, \pm 2, \ldots$ for fermions, and $\omega_l = 2\pi T l$ for bosons. To illustrate the procedure, we present below analytical expressions for the temperature Green's function of an ideal electron-positron gas in a constant magnetic field. The electron Green's function in a homogeneous and stationary magnetic field in the proper-time representation is given by the formula [11]

$$G(x'', x') = \Phi(x'', x') \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \exp(-\mathrm{i}px) G(p) , \qquad (2.29)$$

where

$$G(p) = \mathrm{i} \int_0^\infty \mathrm{d}s \, \exp\left[-\mathrm{i}s\left(m^2 - p_0^2 + p_z^2 + p_\perp^2 \, \frac{\tan z}{z}\right)\right] \times \left(\frac{m + \gamma^0 p^0 - \gamma^3 p_z}{\cos z} \exp(\mathrm{i}z\Sigma_3) - \frac{\boldsymbol{\gamma}\mathbf{p}_\perp}{\cos^2 z}\right) ,$$

$$\Phi(x'', x') = \exp\left[-\mathrm{i}e \int_{x'}^{x''} A_\mu^{\text{ext}}(x) \, \mathrm{d}x^\mu\right] ,$$

$$z = eHs , \qquad x = x'' - x' , \qquad \boldsymbol{\gamma}\mathbf{p}_\perp = \gamma^1 p_x + \gamma^2 p_y . \qquad (2.30)$$

The temperature Green's function is obtained from formulas (2.29) and (2.30) by the replacement

$$\int \frac{\mathrm{d}p_0}{2\pi} \exp\left[-\mathrm{i}(p_0 x_0 - \mathbf{px})\right] G(p_0, \mathbf{p}) \to \mathrm{i}T \sum_l \exp\left[-\mathrm{i}(\tau\omega_l - \mathrm{i}\mu\tau) + \mathrm{i}\mathbf{px}\right] G(p_0 = \mathrm{i}\omega_l + \mu, \mathbf{p}) \qquad (2.31)$$

with $\tau = \tau'' - \tau' \in [-1/T, 1/T]$.

The application of the proper time method to the temperature Green's function technique can be examined in more detail in papers [32, 41].

The simple correlation (2.26) – (2.28) between the diagram technique rules in the imaginary time formalism and in the ordinary QFT may be considered in a certain sense as a convenient peculiarity of the temperature Green's function method that emerges in calculating the thermodynamic quantities of the system. However, in order to find the spectrum of elementary excitations and the kinetic coefficients of a nonequilibrium system as well, one has to know the time Green's functions. Hence, a relation between the temperature Green's functions and the time Green's functions has to be discussed. Let $G_M(p_0 = \mathrm{i}\omega_l + \mu; \mathbf{r}, \mathbf{r}')$ be a component of the Fourier-series expansion of the temperature Green's function in terms of the variable $\tau = t - t'$ on the interval $[-1/T, 1/T]$:

$$G_M(\tau; \mathbf{r}, \mathbf{r}') = \mathrm{i}T \sum_l \exp(-\mathrm{i}\omega_l\tau - \mu\tau) G_M(p_0; \mathbf{r}, \mathbf{r}) . \qquad (2.32)$$

Then, as it has been shown in Ref. [25], one gets

$$G^R(p_0 = \mathrm{i}\omega_l + \mu; \mathbf{r}, \mathbf{r}') = G_M(\mathrm{i}\omega_l + \mu; \mathbf{r}, \mathbf{r}') , \qquad \omega_l > 0 , \qquad (2.33)$$

$$G^A(p_0 = \mathrm{i}\omega_l + \mu; \mathbf{r}, \mathbf{r}') = G_M(\mathrm{i}\omega_l + \mu; \mathbf{r}, \mathbf{r}') , \qquad \omega_l < 0 . \qquad (2.34)$$

Thus, the problem of constructing G^R (G^A) with the help of the temperature Green's function is reduced to the problem of analytic continuation of the temperature Green's function from the discrete set of points in the upper (lower) half-plane of the complex variable p_0 to the whole upper (lower) half-plane, where the retarded (advanced) function is analytic. This analytic continuation is possible, in principle, though generally speaking it is not a trivial problem. Here, it is essential that the temperature and time Green's functions are defined in terms of the same spectral density [25]. Knowledge of the dispersion law for quasi-particles is sufficient for the description of thermodynamic properties of the system as well.

Another even simpler way of describing the thermodynamic properties of a system consists in computation of the thermodynamic potential $\Omega = \Omega(T, V, \mu)$:

$$\Omega = -T \ln \mathrm{Sp}\left\{\exp\left[-\beta(\hat{H} - \mu\hat{N})\right]\right\} , \qquad (2.35)$$

where $\hat{H}$ and $\hat{N}$ are the system Hamiltonian and the particle number operators, respectively. A consistent procedure of computing the thermodynamic potential with consideration for the interaction between particles in the system, which is based upon the Green's function method, was explained in Refs [42, 43]. As for the particular calculations, the physically transparent results with the effects of an external field, finite temperature and nonzero chemical potential being simultaneously taken into account, were obtained until now only in the framework of a one-loop approximation. In the two-loop approximation, as far as we know, only the effective Lagrangian of a constant electromagnetic field in QED at $T=\mu=0$ [44–46] and the two-loop thermodynamic potential of QED in a constant magnetic field [47] have been calculated. Papers [48–50] should also be mentioned, where in the framework of QCD at $T\neq 0$ a multiloop contribution of quarks and gluons to the effective potential of the gluon field in a special form $A^a_\mu=\delta_{\mu 0}(\delta^{a3}A^3_0+\delta^{a8}A^8_0)=\text{const}$ has been calculated. The one-loop Euclidean action W_E in QFT is determined by the expression [51]

$$W_\mathrm{E}=k\tau\int\frac{\mathrm{d}p_4}{2\pi}\sum_s\ln(p_4^2+\varepsilon_s^2)-\text{c.t.}\,, \tag{2.36}$$

where ε_s is the energy spectrum of particles in an external field, $k=+1$ for fermions, and $k=-1$ for charged bosons, and the contact term, c.t., corresponds to a free particle contribution. Formula (2.36) determines the one-loop polarization contribution to the vacuum energy

$$E^{(1)}=-\frac{W_\mathrm{E}}{\tau}\,. \tag{2.37}$$

The simplest way of calculating the one-loop thermodynamic potential in the imaginary time representation consists in making a substitution in the expression for $E^{(1)}$ similar to (2.26), (2.27) [27]:

$$\int\frac{\mathrm{d}p_4}{2\pi}\to T\sum_{l=-\infty}^{+\infty}\,,\qquad p_4\to\omega_l-\mathrm{i}\mu\,. \tag{2.38}$$

Another approach suitable for calculating Ω-potential in the presence of external fields, which is especially convenient when the Schwinger–Fock proper time method is employed, implies that firstly we evaluate the corresponding temperature Green's function, and then perform integration over the coupling constant or some other appropriate parameter in order to find the Ω-potential of the system. In Ref. [25], for example, the Ω-potential in QED with the external field specified by the potential A_μ in the one-loop approximation was represented in the form

$$\Omega=T\int_0^\beta\mathrm{d}x_4\int\mathrm{d}^3x\,\mathrm{tr}^s\left(\int_0^e\gamma_\mu G(x,x)A_\mu(x)\,\mathrm{d}e'\right)+\Omega\Big|_{e=0}\,, \tag{2.39}$$

where e is the electron charge, $G(x,x)$ is the temperature Green's function in an external field, and the trace operation tr^s is carried out with respect to the spinor indices. A more popular method of evaluating the one-loop effective Lagrangian of QED in an external field in the one-loop approximation $L^{(1)}$ consists in the use of the known relation [14]

$$\frac{\partial L^{(1)}}{\partial m}=\mathrm{tr}^s\,G(x,x)\,. \tag{2.40}$$

As it was mentioned above, the Green's function in the real time representation is expressed in the form of a sum of the causal Feynman propagator at $T=\mu=0$ and the temperature-dependent part, which is the solution of the corresponding homogeneous equation. In the fermionic case we arrive at

$$G^c(x,x'|A)=S_c(x,x'|A)+S_{\beta,\mu}(x,x'|A)\,,$$
$$(\hat p-e\hat A-m)S_c(x,x'|A)=\delta(x-x')\,,$$
$$(\hat p-e\hat A-m)S_{\beta,\mu}(x,x'|A)=0\,. \tag{2.41}$$

As a consequence, direct generalization of the known formula for the one-loop effective action [30]

$$W_\mathrm{E}=\mathop{\mathrm{Tr}}_{x,s}\ln S^{-1}S_0 \tag{2.42}$$

for the case $T\neq 0$ and $\mu\neq 0$ proves to be impossible, and the second method is usually employed when evaluating the effective Lagrangian in the real time formalism [52, 53]. It should be emphasized that in the real time representation the contribution of the finite temperature and density effects due to (2.41) is automatically separated from the pure vacuum part of the corresponding physical quantity and does not contain any ultraviolet divergences.

Concluding this section we note that particular features of QED at finite temperatures and with an external field, disturbing the vacuum stability, were considered in Ref. [32].

3. Radiative effects in electron-positron plasma

3.1 Polarization operator in quantum electrodynamics

Before proceeding to the rigorous statement of the problem of photon propagation across external electromagnetic fields with simultaneous consideration for the nonlinear vacuum and plasma influence, it would be of interest to discuss briefly the physical meaning of the problem. Recall the classical problem of electromagnetic wave propagation through plasma being in a thermodynamic equilibrium. The external electromagnetic field induces charges and currents in plasma and they in turn produce additional electromagnetic fields, thus changing the total field in the system. In the simplest case of collisionless plasma, this self-consistent interaction of an external field and plasma particles is described on the basis of the system of equations incorporating an electromagnetic field in a plasma and taking into account both induced currents and charges, and of the Vlasov kinetic equation with a self-consistent field. It is in this way that an effect, called Landau's damping, was predicted in Ref. [54], i.e. dissipation of waves in isotropic plasma takes place even when there are no particle collisions at all, as a result of the Cherenkov wave absorption by the plasma particles. Without pretending to rigorous consideration one may draw the analogy between the plasma or insulator behaviour in an external field and the phenomenon of elementary particles vacuum polarization in QFT. Vacuum in elementary particle physics is defined as the state where there are no particles and which corresponds to the zero eigenvalue of the particle number operator. There appear no physical particles in it and physical states are brought about by acting the particle creation operator on the vacuum vector. The quantized field operator does not commute at the same time with the particle number operator. Physically this is demonstrated by the fact that in a local quantum field theory the vacuum state is not the state in which there is no field, i.e. the so-called zero vacuum field with infinite energy density is available in the vacuum state and, as Grib comments [55]: 'non-existence meaning the absence of field and particles is impossible'. Using the language of virtual particles one may conclude that virtual particle – antiparticle pairs are constantly created and annihilated in the vacuum, the distance between them being of the order of the particle's Compton wavelength. In an external electromagnetic field the 'vacuum plasma' of virtual pairs is polarized analogously to the classical plasma and it is in itself a source of an additional electromagnetic field. And certain properties of an external field enable the bound virtual pairs to become free and production of real pairs from vacuum by an external field takes place. For a stationary and homogeneous electric field, the probability of the effect for electrons becomes essential with the so-called critical field, which is determined from the condition that the work done by the field on an electron at the distance of the Compton wavelength is equal to the rest energy of the particle:

$$eE_{\mathrm{cr}}\,\frac{\hbar}{mc} = mc^2\,, \qquad E_{\mathrm{cr}} = \frac{m^2c^3}{e\hbar}\,. \tag{3.1}$$

Another approach to the vacuum polarization phenomenon is based upon the idea that the electron-positron vacuum is, according to Dirac, thought of as a system of electrons filling all the negative energy levels. The energy levels of vacuum electrons are shifted in an external field, and the relative change of the vacuum energy pertaining to the state without the field and that with the field is finite and it leads to the radiative correction to the classical Lagrange function of the external field, which in its turn breaks the linearity of Maxwell equations. We note that if the one-loop contribution [see Eqn (2.36)] to the effective Lagrangian is really determined by the energy spectrum of particles that do not interact with each other, then the two-loop contribution in a constant field takes account of the change of the Coulomb electron-positron interaction in the virtual pair by the external field. In the vacuum polarization phenomenon, the role of an external field may be played by boundary conditions imposed on the quantized field, which is usually considered in a free space. Boundaries or a nontrivial topology, as well as an external field alter the vacuum zero oscillations spectrum, and the change in the vacuum expectation value of the energy-momentum tensor operator turns out to be finite as compared to the 'empty' space and it leads to the physical effect (Casimir effect) examined in experiments [14]. It should be noted that scientific interest to the Casimir effect in various QFT models, and in the presence of an external field as well, was not lost in recent years [56].

In papers of Ritus [44 – 46], a deep analogy between asymptotic behaviour of the effective QED Lagrangian in strong fields and behaviour of the polarization function of a photon at large values of the virtual photon 4-momentum squared $t = -k^2$ has been drawn. As known [57], the photon polarization function at large values of $t \gg m^2$ determines the behaviour of QED over the region of small distances.

Let $D = (1/k^2) d_{\mathrm{R}}(-k^2/m^2, \alpha)$ be an exact photon propagator with regard to all the radiative corrections. Then in the region of large values of $t \gg m^2$, the renorminvariant quantity $\alpha d_{\mathrm{R}}(t/m^2, \alpha)$ tends to a certain function $\Phi_1\big(\varphi(\alpha)t/m^2\big)$, i.e. contribution of all the radiative corrections reduces to the scale factor $\varphi(\alpha)$ of the dynamic variable t/m^2. In Refs [44 – 46] it was shown that in the asymptotic region of strong fields, when $eF \gg m^2$ and a substantial contribution to the Lagrangian function is provided by the small distances $x \sim (eF)^{-1/2}$, which is related to the localization of vacuum electrons in the regions small in size as compared with the electron Compton wavelength, the invariant charge $\alpha l_{\mathrm{R}}^{-1}$, where $l_{\mathrm{R}} = L_{\mathrm{R}}/L_0$ is the ratio of the exact Lagrangian L_{R} to the classical one L_0, tends to a certain function $\Phi_2\big(\varphi(\alpha)eF/m^2\big)$ with the same scale factor $\varphi(\alpha)$, as in the case of the photon propagator. This circumstance demonstrates that investigation of the QFT peculiarities in the region of small distances can be carried out basing upon the study of the effective Lagrangian in a strong external field. Moreover, not only the charge renormalization can be made using the effective electromagnetic Lagrangian and the knowledge about its real part behaviour in a weak field, but also renormalization of the electron mass can be performed without traditionally exploiting the mass operator for this purpose. In the latter case, the behaviour of the imaginary part of the effective Lagrangian in a weak field becomes essential [44 – 46].

In the present section we consider the photon propagation across an external field (Section 3.2) and the photon (plasmon) propagation through a relativistic electron-positron plasma placed in a constant magnetic field (Section 3.3). In both cases we effectively deal with the photon propagation through an anisotropic medium, and these problems will be considered in a uniform way, basing upon the Green's function method. In the Feynman gauge, the Dyson – Schwinger equations for the exact electron $G(x, y)$ and photon $D_{\mu\nu}(x, y)$ Green's functions in the presence of an external field A_μ^{ext} take the form [15]:

$$\partial^2 D_{\mu\nu}(x, y) - \int \Pi_\mu^{\alpha}(x, z) D_{\alpha\nu}(z, y)\,\mathrm{d}^4 z = g_{\mu\nu}\delta\,(x - y)\,, \tag{3.2}$$

$$(\hat{p} - e\hat{A}^{\,\mathrm{ext}} - e\hat{a} - m)G(x, y) - \int \Sigma(x, z)\,G(z, y)\,\mathrm{d}^4 z = -\delta\,(x - y)\,, \tag{3.3}$$

where $\partial^2 = -p_\mu p^\mu = \partial_0^2 - \nabla^2$, $\Sigma(x, y)$ and $\Pi_{\mu\nu}(x, y)$ are the mass and polarization operators:

$$\Sigma(x, y) = -\mathrm{i}e^2\gamma^\mu \int S(x, z)\,\Gamma^\nu(z, y, z')\,D_{\nu\mu}(z', x)\,\mathrm{d}^4 z\,\mathrm{d}^4 z'\,, \tag{3.4}$$

$$\Pi^{\mu\nu}(x, y) = -\mathrm{i}e^2\,\mathrm{tr}\,\gamma^\mu \int S(x, y)\,\Gamma^\nu(z, z', y)\,S(z', x)\,\mathrm{d}^4 z\,\mathrm{d}^4 z'\,. \tag{3.5}$$

We note that in formulas (3.4), (3.5) $\Gamma^\mu(x, y, z)$ is an exact vertex function, and in the case of an external field that does not break vacuum stability, the sum of the initial external field A_μ^{ext} and the radiative correction to it $a^\mu(x)$ determines an exact average electromagnetic field in the system. Propagation of a photon across an external electromagnetic field is described by the polarization operator in modified Maxwell equations that are obtained from (3.2) by discarding its right-hand side:

$$\partial^2 A_\mu(x) - \int \Pi_\mu^{\alpha}(x, y) A_\alpha(y)\,\mathrm{d}^4 y = 0\,. \tag{3.6}$$

In a constant external field, the polarization operator is diagonal in the momentum space, and its general structure according to the requirements of relativistic, gauge and charge invariances has been studied in Refs [9, 58 – 63]:

$$\Pi_{\mu\nu}(k, k'|A) = \int \mathrm{d}^4 x\,\mathrm{d}^4 x'\,\exp\big[-\mathrm{i}(kx - k'x')\big]\,\Pi_{\mu\nu}(x, x', A) = (2\pi)^4\delta\,(k - k')\,P_{\mu\nu}(k)\,. \tag{3.7}$$

The polarization operator $P_{\mu\nu}(k)$ in this case possesses four mutually orthogonal eigenvectors $b_\mu^{(j)}$:

$$P_{\mu\nu} b^{\nu}_{(j)} = \varkappa_j b_\mu^{(j)} , \quad j = 1, 2, 3 , \tag{3.8}$$

with the eigenvalues $\varkappa_j$ ($\varkappa_0 = 0$ due to gauge invariance) and is defined by the formula

$$P_{\mu\nu}(x) = \sum_{j=1}^{3} \varkappa_j \frac{b_\mu^{(j)} b_\nu^{+(j)}}{|b_\rho^{(j)}|^2} , \tag{3.9}$$

while the solution of Eqn (3.6) in the momentum space has the form

$$A_\mu(k) = \sum_{j=1}^{3} b_\mu^{(j)} \delta\,(k^2 + \varkappa_j)\, \varphi_j(k) . \tag{3.10}$$

Thus, the eigenvectors $b_\mu^{(j)}$ of the polarization operator play a part of the polarization 4-vectors of photons propagating across a constant external field, and its own dispersion law is evidenced for each polarization, being determined from the equation

$$\mathbf{k}^2 - k_0^2 = \varkappa_j , \quad j = 1, 2, 3 . \tag{3.11}$$

Similar to classical electrodynamics one may introduce an index of refraction for each of the eigenmodes, which may traverse the given external field:

$$n_j = \frac{|\mathbf{k}|}{k_0} = \left[1 + \frac{\varkappa_j(k_0, \mathbf{k})}{k_0^2} \right]^{1/2} . \tag{3.12}$$

Another essential result obtained in Refs [60, 63] deals with the explicit form of the polarization 4-vectors of the eigenmodes. In an arbitrary stationary field the vector $b_\mu^{(1)}$ shows a purely kinematic origin:

$$b_\mu^{(1)} = (F^2 k)_\mu k^2 - k_\mu (k F^2 k) . \tag{3.13}$$

In the particular case of a crossed field ($f = g = 0$), as well as at $g = 0$ but with $f \neq 0$, the vectors $b_\mu^{(2)}$ and $b_\mu^{(3)}$ are as follows

$$b_\mu^{(2)} = \tilde{F}_{\mu\nu} k^\nu , \quad b_\mu^{(3)} = F_{\mu\nu} k^\nu . \tag{3.14}$$

In the one-loop approximation, the photon polarization operator in QED is written in the form

$$\Pi_{\mu\nu}(x, y) = -\mathrm{i} e^2 \,\mathrm{tr} \left[\gamma^\mu S_c^{(\mathrm{e})}(x, y|F)\, \gamma^\nu S_c^{(\mathrm{e})}(y, x|F) \right] , \tag{3.15}$$

where $S_c^{(\mathrm{e})}(x, x'|F)$ is the electron causal Green's function in the given field, which is separated from the set of all the solutions of Eqn (2.6) according to the Feynman prescription $m^2 \to m^2 - \mathrm{i}\varepsilon$. To renormalize the polarization operator one should subtract from (3.15) its value in the zero external field and add the value of the renormalized polarization operator calculated in the case of a vanishing external field. The photon polarization operator, as it will be seen later, possesses much physical information even on the mass shell ($k^2 = 0$). Let us consider now the elastic scattering amplitude of a real ($k^2 = 0$) photon:

$$T_\alpha = \frac{1}{2\omega} e_\mu^{(\alpha)} P^{\mu\nu}(k|F) e_\nu^{(\alpha)} , \tag{3.16}$$

where $\omega = k_0 = |\mathbf{k}|$ is the photon energy, $e_\mu^{(\alpha)}$ is the polarization 4-vector ($\alpha = 1, 2$). Amplitude (3.16) determines the radiative shift of the photon mass squared [64, 46]:

$$\Delta m^2_{\gamma_{1,2}} = 2\omega \,\mathrm{Re}\, T_{1,2} \tag{3.17}$$

and the rate of the photon decay into the $\mathrm{e}^+\mathrm{e}^-$-pair in the given external field [6] equals

$$\omega_{1,2} = 2 \,\mathrm{Im}\, T_{1,2} . \tag{3.18}$$

The photon (plasmon) propagation problem applied to an electron-positron plasma in external fields is solved in the manner completely similar to the solution of the photon propagation problem in the case of vacuum in the presence of an external field. To solve the problem, the following has to be done [63]:

(1) calculate the advanced or the retarded polarization operator of an electron-positron gas in the corresponding order of the Furry's picture;

(2) diagonalize the polarization operator obtained and find its eigenfunctions and eigenvalues. The eigenfunctions describe independent polarization properties of the three plasmon eigenmodes, and the eigenvalues define a dispersion law for each eigenmode.

We remind that the photon polarization operator in an external homogeneous and stationary magnetic field is defined in the one-loop approximation as

$$\Pi_{\mu\nu}(k_0, \mathbf{k}) = -\mathrm{i} e^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \,\mathrm{Sp} \left[\gamma_\mu G(p)\, \gamma_\nu G(p + k) \right] , \tag{3.19}$$

where $G(p)$ is the electron propagator determined by Eqn (2.29).

3.2 Photon propagation across an electromagnetic field

3.2.1 Photon dispersion in a constant magnetic field. Here we shall follow paper [63], where a comprehensive bibliography on the problem discussed can also be found. Polarizations of the three types of electromagnetic waves that may propagate across a constant magnetic field are determined by Eqns (3.13), (3.14). The directions of the vectors of the electric and magnetic field strengths for each of the eigenmodes are depicted in Fig. 1. In the general case only the mode 3 is completely transversal. In the modes 1 and 2, the vectors $\mathbf{E}$, $\mathbf{B}$, $\mathbf{k}$ lie in the same plane, $\mathbf{H}$ is always perpendicular to them,

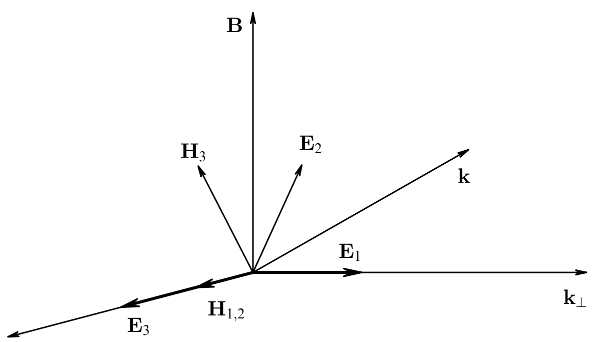

Figure 1. The senses of electric $\mathbf{E}$ and magnetic $\mathbf{H}$ vectors relating to eigenmodes in the magnetic field $\mathbf{B}$. Vectors $\mathbf{B}$, $\mathbf{k}$, $\mathbf{k}_\perp$, $\mathbf{H}_3$, and $\mathbf{E}_{1,2}$ lie in the plane of the drawing, while $\mathbf{E}_3$ and $\mathbf{H}_{1,2}$ vectors are orthogonal to the latter, and, finally, $\mathbf{H}_3 \perp \mathbf{k}$.

and $\mathbf{E}_1 \perp \mathbf{B}$ in the mode 1. In the mode 3, the vectors $\mathbf{H}, \mathbf{B}, \mathbf{k}$ lie in the same plane and $\mathbf{E}$ is perpendicular to this plane. In the case of a parallel propagation ($\mathbf{k} \uparrow\uparrow \mathbf{B}$), the mode 2 represents a longitudinal electric wave ($\mathbf{H}_2 = 0,\ \mathbf{E}_2 \uparrow\uparrow \mathbf{k}$) and, due to the isotropy of the polarization operator (PO) in the plane perpendicular to the direction of the external field $\mathbf{B}$, the equality $\varkappa_1 = \varkappa_3$ holds. On the contrary, if we have $\mathbf{k} \perp \mathbf{B}$, then the mode 1 becomes a longitudinal electric wave ($\mathbf{H}_1 = 0, \mathbf{E}_1 \uparrow\uparrow \mathbf{k}$). From the general properties of the photon polarization operator in a constant external field it follows that its eigenvalue $\varkappa_1 = k^2$. Therefore, one might expect that the mode 1 at $k^2 = 0$ describes the real electromagnetic wave. However, the polarization 4-vector $b_\mu^{(1)} \propto k_\mu$ at $k^2 = 0$, i.e. it becomes purely longitudinal and such a wave is lacking. Massless solutions corresponding to real photons can have dispersion equations for the modes 2 and 3. Analytic properties of PO and its eigenvalues with respect to complex variables $z_1 = k_0^2 - k_3^2$ and $z_2 = k_\perp^2$ have been studied in detail. They are analytic in z_1 everywhere except the infinite number of branching points

$$z_{nn'} = \left[(2eBn + m^2)^{1/2} + (2eBn' + m^2)^{1/2}\right]^2 , \tag{3.20}$$

where n and n' take on integral nonnegative values, m is the electron mass, and in addition they are entire functions of z_2. We remark that the spectral representations for $\varkappa_j$ within the one-loop approximation in the form of sums over n and n' and their singular behaviour of the $(z_1 - z_{nn'})^{1/2}$ type have been for the first time mentioned in Ref. [65], while the branching point $z_1 = z_{nn'}$ physically corresponds to the photoproduction threshold for an electron-positron pair with the principle quantum numbers n and n' in a constant magnetic field. The above-mentioned singularities $\varkappa_j$ $(j = 1, 2, 3)$ have been referred to as cyclotron resonances in the scientific literature. At this point, consider two physical effects whose origin lies in the substantial deviation of the dispersion law from the vacuum one $k^2 = 0$. They have been predicted by Shabad in his analysis of the dispersion equations solutions near the resonance thresholds. For our purposes it is enough to study, for example, the dispersion equation for the mode 2 in the vicinity of the first resonance $(n = n' = 0)$:

$$z_1 - z_2 = 2\alpha eBm \exp\left(-\frac{z_2}{2eB}\right)(4m^2 - z_1)^{-1/2} . \tag{3.21}$$

It can be brought to the cubic equation with respect to $X = z_1/m^2$ and is solved explicitly. Let $X_1 = X_1(z_2)$ be a solution of (3.21) which describes propagation of an electromagnetic wave without absorption. Using the explicit form of $X_1 = X_1(z_2)$, where z_2 is real, one can check that the component of the wave group velocity transversal to the vector $\mathbf{B}$ satisfies the inequality

$$\left(\frac{\mathrm{d}k_0}{\mathrm{d}k_\perp}\right)_{k_\parallel = \mathrm{const}} < 1 \tag{3.22}$$

near the threshold and vanishes at $z = 4m^2 - \mathrm{i}0$. The longitudinal component of the group velocity is equal in this case to

$$\left(\frac{\mathrm{d}k_0}{\mathrm{d}k_\parallel}\right)_{k_\perp = \mathrm{const}} = \frac{k_\parallel}{(4m^2 + k_\parallel^2)^{1/2}} . \tag{3.23}$$

Thus, for $k_\parallel \neq 0$ the wave group velocity near the cyclotron resonance is directed along the external magnetic field. This is the so-called effect of a photon channelling along the magnetic field.

Another phenomenon, which is characteristic of crystal optics with regard to the spatial dispersion, implies that the dispersion equations have infinite number of solutions near the production threshold [see, for instance, Eqn (3.21)], for which $z_1 = k_0^2 - k_3^2$ is real, while the transversal momentum squared z_2 is determined by a complex expression. Such solutions have been identified as new waves [66].

Conditions for possible manifesting the effect of the photon deflection from its trajectory towards the magnetic field in the pulsar magnetosphere, as well as in a semiconductor placed in a magnetic field, were studied in Refs [67, 68]. We note that since the forbidden band width in a semiconductor (of the order of 1 eV) is much less than the energy gap between positive- and negative-frequency solutions to the Dirac equation in a constant magnetic field ($2mc^2 \approx 1$ MeV), the channelling effect in a semiconductor can be observed already in fields of much lower strength as compared with the vacuum case.

Finally we shall discuss two results relating to the dispersion equation solutions in a superstrong magnetic field, when the following conditions are fulfilled [10, 63, 69]:

$$B \gg B_0 , \qquad z_1 \frac{B_0}{B} \ll 1 . \tag{3.24}$$

(1) For the mode 3, the dispersion equation has the solution $k^2 = 0$ which corresponds to a transverse electromagnetic wave propagating with the speed of light through the vacuum in any direction.

(2) For the mode 2 far from the threshold, when $z_1 \ll 4m^2$ and $z_2 \ll 2eB$, there exists a solution, which describes the wave propagation with the index of refraction, close to the vacuum one, namely

$$n \cong 1 + \frac{\alpha}{6\pi} \frac{eB}{m^2} \sin\theta , \tag{3.25}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{B}$.

3.2.2 Photon in the field of a plane electromagnetic wave. The polarization operator in an arbitrary plane-wave field, as well as in the particular case of the plane electromagnetic wave, which fairly well models an electromagnetic field of real laser beams, was studied in Refs [70, 71, 10]. Omitting calculation details, we shall qualitatively describe the following two effects: rotation of the polarization plane (the Faraday effect), and circular birefringence (the Kerr effect). To this end, we shall consider the problem of a linearly polarized photon of the 4-momentum q propagating across the field of a circularly polarized wave. From modified Maxwell equations (3.6) in the physically interesting case

$$\lambda = \frac{\omega(q^0 - q^3)}{2m^2} \ll 1 , \qquad \xi = \frac{eE}{m\omega} \ll 1 , \tag{3.26}$$

where E and ω are the strength amplitude and the frequency of an external wave field, m is the electron mass, ξ is the classical parameter of the wave intensity, it follows that near the vacuum dispersion law ($q^2 = 0$) only two eigenmodes have physical meaning. They describe states of transversely polarized photons with right $(+)$ and left $(-)$ circular polarization, and the refraction indices corresponding to these modes are equal to

$$n_{\pm}^2 = 1 + \frac{22}{45}\,\frac{\alpha}{\pi}\left(\frac{m}{q^0}\right)^2 \xi^2\lambda^2\left(1 \pm \frac{16\lambda}{77}\right). \qquad (3.27)$$

In this case the polarization plane of a linearly polarized photon propagating across the field of a circularly polarized wave in the path of length L, will rotate through the angle equal to

$$\varphi = \frac{1}{2}\,\omega L(n_+ - n_-) = \frac{176}{630}\,\frac{\alpha}{\pi}\,\xi^2\lambda^3\,\frac{\omega}{c}\left(\frac{mc^2}{\hbar\omega}\right)^2 L\,. \qquad (3.28)$$

We note that the phenomenon considered provides an integral effect, i.e. the angle of rotation of the polarization plane increases proportionally to L and may prove to be substantial even if $\xi \ll 1$, which is the condition for the perturbation theory in terms of the wave intensity to be valid.

3.2.3 Polarization operator and photon propagation across the stationary crossed field. The polarization operator and the Green's function of a photon were considered within the framework of QED theory in Refs [64, 9, 60, 61]. Here we shall restrict ourselves to consideration of the dispersion equations for the modes 3 and 2, which take the form

$$k^2 + P_{1,2}(\lambda, \varkappa) + P_3(\lambda, \varkappa) = 0\,, \qquad (3.29)$$

where $P_i(\lambda, \varkappa)$ $(i = 1, 2, 3)$ are the functions of the 4-momentum squared k_μ of a photon

$$\lambda = \frac{k^2}{m^2} \qquad (3.30)$$

and of the dynamic variable

$$\varkappa = \frac{1}{m^3}\sqrt{-(F_{\mu\nu}k^\nu)^2}\,. \qquad (3.31)$$

We remark that in a special reference system, where a photon moves in the direction opposite to the Poynting vector $\mathbf{E}\times\mathbf{H}$, the modes 3 and 2 are transversely polarized with respect to $\mathbf{E}$ and $-\mathbf{H}$, correspondingly. In the one-loop approximation, the functions $P_i(\lambda, \varkappa)$ are determined by the integrals

$$P_{1,2} = -\frac{2\alpha m^2}{3\pi}\int_4^\infty \mathrm{d}v\,\frac{2v + 1 \mp 3}{v\sqrt{v(v-4)}}\left(\frac{\varkappa}{v}\right)^{2/3} f'(z)\,,$$
$$P_3 = \frac{4\alpha m^2\lambda}{\pi}\int_4^\infty \frac{\mathrm{d}v}{v^2\sqrt{v(v-4)}}\left[f_1(z) - \ln\left(1 - \frac{\lambda}{v}\right)\right],$$
$$z = \left(\frac{v}{\varkappa}\right)^{2/3}\left(1 - \frac{\lambda}{v}\right), \qquad (3.32)$$

where $f'(z)$ is the derivative of the Airy–Hardy function [64, 9]:

$$f(z) = \Upsilon(z) + \mathrm{i}\sqrt{\pi}\,\Phi(z) = \mathrm{i}\int_0^\infty \mathrm{d}t\,\exp\left[-\mathrm{i}\left(zt + \frac{t^3}{3}\right)\right], \qquad (3.33)$$

and the following function was introduced:

$$f_1(z) = \int_z^\infty \mathrm{d}x\left[f(x) - \frac{1}{x}\right]. \qquad (3.34)$$

Near the vacuum dispersion law, from Eqns (3.29), (3.32)–(3.34) it follows that

$$k^2 = k_0^2 - \mathbf{k}^2 = k_0^2(1 - n_{3,2}^2) = \mu_{3,2}^2$$
$$= \frac{2\alpha m^2}{3\pi}\int_4^\infty \mathrm{d}v\,\frac{2v + 1 \mp 3}{v\sqrt{v(v-4)}}\left(\frac{\varkappa}{v}\right)^{2/3} f'(z)\,, \qquad (3.35)$$

where $\mu_{3,2}^2$ is the complex square of the photon mass for modes 3 and 2, and an index of refraction of the medium $n_{3,2}$ is introduced. Thus, since the process of electron-positron pair photoproduction is possible in an external field, the latter plays the role of an anisotropic medium with dispersion and absorption for the photon propagating through it. The probability of absorbing the i-th eigenmode is determined in the general case by the dispersion equation

$$\Gamma_i = \mathrm{Im}\,k_0 = \frac{1}{2\,\mathrm{Re}\,k_0}\,\mathrm{Im}\,\mu_i^2\,, \qquad (3.36)$$

where the complex frequency of a normal wave constitutes the solution of the dispersion equation at fixed real value of $\mathbf{k}$. It should be emphasized that only near the vacuum dispersion law ($k^2 = 0$) Eqn (3.36) takes the form of the above-mentioned optical theorem, where the eigenvectors $b_\mu^{(3)}$ and $b_\mu^{(2)}$ of the polarization operator have to be borrowed for the polarization 4-vectors, and the following relation

$$\Gamma_i = \frac{w_i}{2} = \mathrm{Im}\,T_i\,, \qquad i = 3, 2 \qquad (3.37)$$

holds, where w_i is the rate of photon decay into e^+e^--pair, and T_i is the corresponding amplitude of photon elastic scattering. In the limiting cases of large and small values of the dynamic parameter $\varkappa$, it follows from Eqn (3.35) [64, 9] that

$$\mu_{3,2}^2 = \alpha m^2\begin{cases} -\dfrac{11 \mp 3}{90\pi}\,\varkappa^2 - \mathrm{i}\sqrt{\dfrac{3}{2}}\,\dfrac{3 \mp 1}{16}\,\varkappa\exp\left(-\dfrac{8}{3\varkappa}\right), & \varkappa \ll 1\,, \\ \dfrac{5 \mp 1}{28\pi^2}\sqrt{3}\,\Gamma^4\left(\dfrac{2}{3}\right)(1 - \mathrm{i}\sqrt{3})(3\varkappa)^{2/3}, & \varkappa \gg 1\,. \end{cases} \qquad (3.38)$$

We note that the real part of μ^2 is negative at small $\varkappa$, and with growing $\varkappa$ it changes the sign at $\varkappa \approx 15$ and increases proportional to $\varkappa$ for $\varkappa \gg 1$. In the framework of the Weinberg–Salam–Glashow (WSG) unified theory of electroweak interactions, the W-boson contribution to the photon PO should also be taken into account. In a constant magnetic field, this has been done for the first time in Refs [72, 73], and the behaviour of the W-boson contribution to the photon PO in the superstrong magnetic field $H \to H_0^{\mathrm{W}} = M_{\mathrm{W}}^2/e \approx 10^{24}$ G (M_{W} is the mass of the W-boson) was studied in Ref. [74]. In the case of comparatively weak fields and large transverse momenta of a photon, viz.

$$H \ll H_0^{\mathrm{W}}\,, \qquad k_\perp \gg M_{\mathrm{W}}\,, \qquad (3.39)$$

the one-loop W-boson contribution to the asymptotics of the shift of the real part of the photon mass squared is determined by the expression [73]

$$\mathrm{Re}\,\mu_{3,2}^2 = \frac{\alpha}{\pi}\,M_{\mathrm{W}}^2\begin{cases} \dfrac{42 \pm 1}{30}\,\varkappa^2\,, & \varkappa \ll 1\,, \\ 2\varkappa\,, & \varkappa \gg 1\,, \end{cases} \qquad (3.40)$$

while the rate of photon decay into the W^+W^--pair has the asymptotics

$$w_{3,2} = \frac{\alpha M_W^2}{k_0} \begin{cases} \frac{35 \pm 2}{32} \left(\frac{3}{2}\right)^{1/2} \varkappa \exp\left(-\frac{8}{3\varkappa}\right), & \varkappa \ll 1\,, \\ 2\sqrt{3}\,\varkappa\,, & \varkappa \gg 1\,, \end{cases} \tag{3.41}$$

where the invariant parameter

$$\varkappa = \frac{e}{M_W^3} \sqrt{-(F_{\mu\nu}k^\nu)^2} = \frac{k_\perp}{M_W} \frac{H}{H_0^W}\,. \tag{3.42}$$

has been introduced.

Comparison of the results (3.40), (3.41) with the analogous findings for the spinor (3.38) and scalar [9] particles contribution to the photon elastic scattering amplitude shows that:

(1) the contribution increases with growing particle spin;

(2) the forms of the T-amplitude dependence on the parameter $\varkappa$ at $\varkappa \ll 1$ coincide in all cases ($T \sim \varkappa^2$);

(3) the situation is essentially altered for $\varkappa \gg 1$: in the case of scalars and electrons $T \sim \varkappa^{2/3}$, and for W-bosons $T \sim \varkappa$.

Thus, at $\varkappa \gg 1$ the W-boson contribution T_W to the amplitude of photon elastic scattering considerably exceeds that of electrons T_e, and a superhigh-energy photon in a magnetic field will decay most probably into the W^+W^--pair. We note that in a quasi-classical region for $\varkappa \gg 1$, where $\varkappa$ is the parameter determined by Eqn (2.11), the synchrotron radiation power of a vector particle also increases faster with growing $\varkappa$, i.e. proportional to $\varkappa^{4/3}$ [75], than does the radiation power of scalar particles and electrons, which is proportional to $\varkappa^{2/3}$.

3.2.4 Polarization operator and the photon propagation across the electromagnetic field of the Redmond configuration. A superposition of a constant magnetic field and a field of a plane electromagnetic wave propagating along the magnetic field direction is called the Redmond configuration (RC). A potential of this field for the case of a circular polarized wave propagating along the Z-axis can be chosen in the form

$$A^\mu(x) = A_1^\mu(H) + A_2^\mu(\varphi)\,, \qquad \varphi = kx = \omega(t - z)\,,$$
$$A_1^\mu = (0, 0, xH, 0)\,, \qquad A_2^\mu = \frac{m}{e}\,\xi(0, -\sin\varphi, g\cos\varphi, 0)\,, \tag{3.43}$$

where $\xi = eE/m\omega$ is the above-introduced parameter of the wave intensity, $g = +1$ ($g = -1$) for the right (left) circular polarization of the wave. The explicit form of PO in the RC field and solutions of the corresponding modified Maxwell equations were obtained in Ref. [70] (see also Ref. [10]). We note that in the variable RC electromagnetic field the polarization operator $\Pi_{\mu\nu}(p, q|F)$ contains, as in the case of the plane electromagnetic wave field, terms that are off-diagonal with respect to the photon momenta outside the mass shell. This provides a drastic distinction of this case from that with a constant external field.

Let us analyze in more detail the one-loop elastic scattering amplitude for a circularly polarized photon moving in the direction opposite to that of the wave propagation (in the case of a photon moving in the direction of wave propagation its interaction with an external field is impossible). Here we shall limit ourselves by mostly quantitative justification of some nontrivial physical effects taking place just in this field (3.43).

We remind that the Dirac equation solutions in the field (3.43) satisfy the conditions of the time and spatial in z periodicity [21, 76, 77, 9]:

$$\psi(t + T) = \exp(-\mathrm{i}q_0 T)\psi(t)\,,$$
$$\psi(z + \lambda) = \exp(-\mathrm{i}q_z\lambda)\psi(z)\,, \tag{3.44}$$

where $T = 2\pi/\omega$ is the oscillation period, λ is the wavelength. Therefore, the quasi-energy q_0 and the third component of the quasi-momentum q_z can be introduced for the particle in the RC field, as in the case of an electron motion in the spatially periodical field of a crystal lattice:

$$q_0 = p_0 + \xi^2 m\omega\,(2\Delta)^{-1}\,, \qquad q_z = p_z + \xi^2 m\omega\,(2\Delta)^{-1}\,,$$
$$\Delta = \alpha\omega - g\omega_H\,, \qquad \alpha = \frac{q_0 - q_z}{m} = \frac{p_0 - p_z}{m}\,, \qquad \omega_H = \frac{eH}{m}\,, \tag{3.45}$$

where p_0 and p_z are the electron energy and the z-component of the electron momentum in a constant magnetic field. The quasi-energy q_0 is defined to an accuracy of the modulus of ω [see (3.44)] and can play in the RC field the same role as an ordinary energy plays in a stationary problem. The laws of conserving the quasi-energy and quasi-momentum hold for a partial process of pair creation by an external photon with the 4-momentum $\varkappa^\mu = (\varkappa^0, 0, 0, -\varkappa^0)$:

$$\varkappa^0 + l\omega - q_0^- - q_0^+ = 0\,, \qquad -\varkappa^0 + l\omega - q_z^- - q_z^+ = 0\,. \tag{3.46}$$

Due to conservation of the total moment projection onto the magnetic field direction, the following relation holds as well:

$$n - n' - gl + \sigma = 0\,, \tag{3.47}$$

where $\sigma = \pm 1$ for the cases of right and left circular polarization of an external photon, and n and n' are the principle quantum numbers of an electron and a positron in a constant magnetic field. Hence, there follows an intimate connection between n, n', g and σ, which, by the way, does not take place for the wave of linear polarization or for other direction of an external photon propagation. We note, for instance, that as it follows from (3.47) creation of a pair with $n = n' = 0$ at $\sigma g = -1$ is available only under the condition that a quantum of radiation is emitted from the wave ($l = -1$). At the same time, for $\sigma g = +1$ the process can proceed if $l = +1$, i.e. as a result of absorption of a quantum from the external wave. The above-mentioned sensitivity of the pair creation process in the field (3.43) to the polarization directions of the wave and of the external photon certainly reveals itself also in the study on the problem of threshold heights in the pair production on fixed Landau levels n and n', i.e. of singularities in the photon elastic scattering amplitude, which also depends on the parameters

$$\mu = \frac{H}{H_0}\,, \qquad \eta = \frac{k\varkappa}{eH} = \frac{2\omega\varkappa^0}{eH}\,, \qquad \xi = \frac{eE}{m\omega}\,. \tag{3.48}$$

From conservation laws (3.46), (3.47) it truly follows that the pair production process with fixed n and n' can occur only for those values of parameters (3.48) that satisfy the equality

$$\frac{\xi^2\eta^2\beta_+\beta_-}{(1-g\beta_+\eta)(1+g\beta_-\eta)} = (1+2n\mu)\beta_- + (1+2n'\mu)\beta_+ - 2g\mu\eta\beta_+\beta_-(n-n'+\sigma)\,, \quad (3.49)$$

where $\beta_\pm$ are correlated to the variable $\beta \in [-1, 1]$ by the condition

$$\beta_\pm = \frac{\alpha_\mp}{\alpha_+ + \alpha_-} = \frac{1}{2}(1 \pm \beta)\,. \quad (3.50)$$

We deduce from Eqn (3.49) and rigorous computations of the process probability, in particular, that:

(1) In the weak magnetic field ($H \ll H_0$) at $\eta \gg 1$ the condition for the process to proceed is as follows

$$\lambda g(n-n'+\sigma) \geqslant 4(1+\xi^2)\,,$$
$$\lambda = 2\mu\eta = \frac{4\omega\varkappa^0}{m^2}\,. \quad (3.51)$$

We note that the opening condition for the partial channel of the pair photoproduction in the plane electromagnetic wave [21, 9] takes the form analogous to (3.51), i.e.

$$\lambda l \geqslant 4(1+\xi^2)\,, \quad (3.52)$$

where l is the number of quanta absorbed from the wave. As for the probability of the process, with $\sigma g = +1$ ($\sigma g = -1$) it increases (decreases) as compared to the case when an external photon is linearly polarized.

(2) In the limiting case $\eta \ll 1$, the condition for pair creation is determined by the inequality

$$\lambda \geqslant \frac{2\mu}{\xi}\left\{\left[\left(\frac{\mu g(n-n'+\sigma)}{\xi}\right)^2 + \left[(1+2\mu n)^{1/2} + (1+2\mu n')^{1/2}\right]^2\right]^{1/2} - \frac{\mu g(n-n'+\sigma)}{\xi}\right\}. \quad (3.53)$$

It is seen that pair production can occur even with very weak restriction on the external photon energy, which in this case may be considerably lower than the threshold energy for the corresponding reaction in a pure magnetic field. The lowest possible value of the parameter λ is reached at $n = n' = 0$:

$$\lambda \geqslant \frac{2\mu}{\xi}\left[\left(\frac{\mu^2}{\xi^2}+4\right)^{1/2} - \frac{\mu(\sigma g)}{\xi}\right]. \quad (3.54)$$

Eqn (3.54) also states that there exists the photon energy region wherein an external photon is absorbed as a result of pair production ($\sigma g = 1$), while for other photons ($\sigma g = -1$) the medium in question is transparent.

(3) In the case of a strong magnetic field ($H \gg H_0$), which is interesting from the physical point of view, when the conditions

$$\eta \ll 1\,, \qquad \frac{\xi^2\eta^2}{\mu} < 1\,, \quad (3.55)$$

are fulfilled, an electron and a positron may be produced only in the ground state ($n = n' = 0$) with the probability

$$W_\pm = \frac{4\alpha\omega\xi^2\eta}{(\xi^2\eta^2 \pm \lambda)^{3/2}(\xi^2\lambda^2 \pm \lambda - 4)^{1/2}} \exp\left(-\frac{\xi^2\eta^2}{2\mu}\right). \quad (3.56)$$

A threshold singularity should be noticed in Eqn (3.56), which is characteristic of the processes in a magnetic field and appears under the condition that longitudinal momenta of particles produced are equal to zero.

Finally we dwell on one more property characteristic for the processes in variable fields, which deals with the real part of the photon mass squared. We limit ourselves by the case of a weak ($\xi^2 \ll 1$) plane electromagnetic wave, when upon averaging over photon polarizations the following asymptotics can be obtained [78, 10]:

$$\operatorname{Re}\mu^2 = -\xi^2\frac{\alpha m^2}{2\pi}(\ln^2\lambda - 2\ln\lambda - \pi^2 + 6)\,,$$
$$\lambda = \frac{2k\varkappa}{m^2} \gg 1\,. \quad (3.57)$$

If we have a power-like ($\sim \varkappa^{2/3}$) growth of both the real and imaginary parts of μ^2 in a stationary external field at large values of the dynamic parameter $\varkappa$, then in a variable field the real part of the photon mass squared is, on the one hand, dominating and, on the other hand, it increases proportional to the logarithm squared of λ at high energies, which is also characteristic of radiative corrections in vacuum [79].

3.2.5 Photon propagation through the electron-positron medium at finite temperature, found in an external electromagnetic field. As it was already mentioned, the study of problems related to propagation and absorption of electromagnetic waves in an electron-positron plasma is carried out basing upon examination of the polarization operator at finite temperature and nonzero chemical potential. For a wealth of basic researches in the field of interest (see, e.g., Refs [80, 81]), the characteristic feature is that the influence of the external field on the photon propagation through a plasma is allowed for in the framework of perturbation theory. At the same time, as Ginzburg emphasized in Ref. [81], consideration of a relativistic electron-positron plasma in a strong electromagnetic field, when the perturbation theory in terms of the external field intensity is no more valid, should be of much interest from the point of view of particular physical applications. Evaluation of particular physical effects even in the case of a collisionless plasma in equilibrium in the presence of a strong external magnetic field turns out to be a nontrivial problem and it has been initiated recently by Shabad and others, whose papers [63, 82, 83] we shall follow below. In Ref. [63], a relativistically covariant structure of the photon PO at finite temperature and nonzero chemical potential in a constant magnetic field $\mathbf{B}$ has been demonstrated. It should be emphasized that this has been done without any reference to the one-loop approximation, i.e. for an exact PO in an arbitrary reference frame, wherein plasma as a whole is at rest or moves along the magnetic field direction. It follows from the analysis of the properties of the plasmon eigenmodes conducted in Ref. [63] that in a general case of the three eigenmodes propagating at an arbitrary angle to the external field direction, they are elliptically polarized, with one of the semiaxes of the polarization ellipse lying in the plane $(\mathbf{H}, \mathbf{B})$, and the other being perpendicular to this plane. As to the analytic properties of the eigenvalues $\varkappa_j$ ($j = 1, 2, 3$) of the PO in the plane of the complex variable $z_1 = k_0^2 - k_3^2$ at fixed real values of k_3 and $z_2 = k_\perp^2$, they are (at fixed values of the pair of the principle quantum numbers n and n') analytic functions on the whole complex plane of z_1 with the exception of the real axis, where

$\varkappa_j$ are analytic over the region

$$z_1'' < z_1 < z_1', \tag{3.58}$$

as well as for $-\infty < z_1 < -k_3^2$, when k_0 takes on imaginary values. The points z_1' and z_2' are determined with the help of the expressions

$$\begin{aligned} z_1' &= \left[(2eBn + m^2)^{1/2} + (2eBn' + m^2)^{1/2}\right]^2, \\ z_1'' &= \left[(2eBn + m^2)^{1/2} - (2eBn' + m^2)^{1/2}\right]^2 \end{aligned} \tag{3.59}$$

and have the following physical meaning.

The point z_1' was already mentioned as the branching point coinciding with the photoproduction threshold for an electron-positron pair with the principle quantum numbers n and n' in a stationary magnetic field. In order to interpret the second branching point z_1'', we note that as a result of the absorption of an external photon an electron (or a positron) of the plasma can make a transition to another quantum state. In this case the following conservation laws for the energy and the momentum projection onto the magnetic field direction

$$k_0 + E_n = E_{n'}, \qquad k_z + p_z = p_z', \tag{3.60}$$

hold, where the primed quantities refer to the final state. The values of the electron momentum p_z, at which a photon with the energy k_0 can initiate transitions $n \to n'$, may be found from (3.60), while the value of z_1 whereat the imaginary part of p_z comes to nought for the first time is just the branching point z_1''.

Thus, there appear two different mechanisms of photon absorption in a collisionless plasma: via the photoproduction process for e^+e^--pair on fixed levels n and n', which corresponds to the right cut $-k_3^2 < z_1 < z_1'$, $z > z_1$, and via photon absorption by an electron (positron) of the plasma relevant to the left cut $-k_3^2 < z_1 < z_1''$. In the latter case at $n \neq n'$ we shall deal with the inverse magnetobremsstrahlung, and at $n = n'$ (when $z_1'' = 0$) absorption of a photon takes place via a process inverse to the quantum Cherenkov radiation. We shall not present here a detailed discussion of the solutions to dispersion equations near the magnetobremsstrahlung absorption thresholds or near the pair production threshold, rather we shall refer the reader to Ref. [63]. Only one result is to be demonstrated, which emphasizes the essentially dynamic character of the photon propagation process in an electron-positron plasma in the presence of an external magnetic field. It corresponds to the solution near the first threshold of magnetobremsstrahlung ($n = 0$, $n' = 1$ or $n = 1$, $n' = 0$) to the dispersion equation for the mode 2 at $k_\perp^2 = 0$, when it turns out to be a longitudinally polarized wave.

We confine ourselves to the case of zero temperature and the range of the external field strength satisfying

$$2eB > \mu^2 - m^2. \tag{3.61}$$

In so doing the electrons fill only the ground Landau level. Then the plasmon frequencies at $k_3 = 0$ are found from the equation [63]

$$\omega_p^2 = \frac{8\alpha eBm^2}{\pi}\,\frac{1}{\omega_p\sqrt{4m^2 - \omega_p^2}} \times \left\{\arctan\left[\frac{4m^2 - \omega^2}{\omega^2}\left(1 + \frac{eBm}{\pi^2 n}\right)\right]^{1/2} - \frac{\pi}{2}\right\}, \tag{3.62}$$

where the magnetic field B is explicitly present, and $\omega_p^2 = 4\pi\alpha n/2m$ is an ordinary Langmuir frequency (n is the electron concentration).

Notice that only under the condition

$$n \ll \frac{eBm}{\pi^2} \tag{3.63}$$

equation (3.62) goes over to the known relation $\omega^2 = \omega_p^2$.

In paper [82], the method of approximate calculation of PO eigenvalues in the case of a comparatively weak magnetic field $B \ll B_0$ has been developed. The polarization operator was represented in the form of the sum of two matrices

$$P_{\mu\nu}(k) = \Pi_{\mu\nu}(k) + \widetilde{\Pi}_{\mu\nu}(k), \tag{3.64}$$

where $\Pi_{\mu\nu}(k)$ is the diagonal matrix, whose elements coincide with the diagonal elements of $P_{\mu\nu}(k)$, while the matrix $\widetilde{\Pi}_{\mu\nu}(k)$ is constructed from the off-diagonal matrix elements of $P_{\mu\nu}(k)$. The matrix elements of $\widetilde{\Pi}_{\mu\nu}(k)$ offer the higher-order infinitesimal in terms of the parameter B/B_0 as compared to $\Pi_{\mu\nu}(k)$ and are proportional to the components of the photon momentum k_μ. Subject to the condition that $B \ll B_0$ and/or $|k_\mu| \to 0$, the eigenvalues $\varkappa_j$ of the matrix $P_{\mu\nu}(k)$ can be represented in the form of a convergent series

$$\varkappa_j = \varkappa_j^{(0)} + \varepsilon\varkappa_j^{(1)} + \varepsilon^2\varkappa_j^{(2)} + \ldots, \tag{3.65}$$

where $\varkappa_j^{(0)}$ are the eigenvalues of $\Pi_{\mu\nu}(k)$, i.e. the diagonal matrix elements of $P_{\mu\nu}(k)$, $\varkappa_j^{(n)}$ are the corrections determined by the structure of $\widetilde{\Pi}_{\mu\nu}(k)$, and the parameter ε is a power-like function depending on B/B_0 and $|k_\mu|/m$. Thus, the behaviour of PO in the limiting cases of high and low temperatures has been studied in Ref. [82]. In the limiting case of a superstrong magnetic field, when

$$eB \gg T^2,\, m^2,\, k^2, \tag{3.66}$$

the temperature correction to a photon PO and its asymptotics in the limits of high and low temperatures were investigated in Ref. [83]. We emphasize that the important plasma characteristics such as the Debye screening radius r_D:

$$r_D^{-2} = -\lim_{|\mathbf{k}|\to 0} \Pi_{00}(\omega, \mathbf{k}), \tag{3.67}$$

in the limit of high temperatures demonstrates essentially different behaviour depending on the field strength [83]:

$$r_D^{-2} = \begin{cases} \dfrac{2\alpha}{\pi}(eB), & eB \gg T^2 \gg m^2, \\[2ex] \dfrac{e^2T^2}{3}\left[1 - \dfrac{3}{2\pi^2}\left(\dfrac{m}{T}\right)^2 + \dfrac{7}{8}\,\dfrac{\zeta(3)}{\pi^3}\left(\dfrac{B}{B_0}\right)^2\left(\dfrac{m}{T}\right)^4 + \ldots\right], & \\ & eB \ll m^2, \quad T \gg m. \end{cases} \tag{3.68}$$

The above result (3.67) can be obtained from the Ward identities. Considering the higher orders of the perturbation theory leads to a corrected expression for r_D^{-2} [83]:

$$r_D^{-2} = -\lim \left\{\Pi_{00}(k_0^2, \mathbf{k}^2)\left[1 + \frac{\partial^2}{\partial\mathbf{k}^2}\,\Pi_{00}(k_0^2, \mathbf{k}^2)\right]^{-1}\right\}. \tag{3.69}$$

4. Radiative effects and propagation of fermions in the Standard Model

4.1 Mass operator and propagation of fermions in the Standard Model

As known from QED, consideration of higher orders of the perturbation theory in terms of the interaction with the radiation quantized field will result, for instance, in the electron mass renormalization and in appearance of an additional magnetic moment of an electron, which is referred to as an anomalous magnetic moment (AMM). The correction of order α to the proper electron magnetic moment followed from the Dirac equation and called the Bohr magneton, was computed by Schwinger as far back as in 1948. It proved to be in perfect agreement with Breit's assumption (made to explain the experiments of Nafe, Nelson and Rabi) that the true value of the electron magnetic moment deviates from the Bohr magneton μ_B [84]:

$$\mu_\mathrm{e} = -\mu_\mathrm{B}\left(1 + \frac{\alpha}{2\pi}\right), \qquad \mu_\mathrm{B} = \frac{e_0\hbar}{2mc}\,. \tag{4.1}$$

Presence of an external electromagnetic field alters the interaction between an electron and the radiation field, thus leading to the radiation shift of the mass and changing AMM of an electron. In papers [85–87, 64], an interesting physical effect has been predicted and studied. It implies that the electron AMM exhibits a complex dynamic nature in a constant magnetic field, i.e. this quantity depends on the field strength and the electron energy and at sufficiently strong fields and high energies it can essentially differ from Schwinger's value. There appeared an abundance of publications that deal with the study on the dynamic nature of the mass shifts both of charged and neutral particles in various configurations of intensive external fields, and their torrent is still growing [88–92]. Motion of an electron (charged lepton) in an external electromagnetic field is governed by the Dirac–Schwinger equation taking radiative corrections into account [2, 3, 9]:

$$(\hat{p} - e\hat{A} - m)\psi(x) = \int \Sigma(x, x')\psi(x')\,\mathrm{d}^4x'\,, \tag{4.2}$$

where $\Sigma(x, x')$ is the electron mass operator. In papers [93, 9], a complete set of operators was found which commute with the electron mass operator (exact in an external and radiative fields) for the case of an arbitrary stationary electromagnetic field. And an effective method based upon diagonalization of the mass operator and the use of its eigenfunctions (the QED eigenfunctions method in constant fields) was proposed for calculating the radiative effects. We note that if there exists a time-dependent part in the external field configuration, e.g., the plane electromagnetic wave field, then the processes outside the mass shell (accompanied by absorption and radiation of arbitrary number of photons of the wave) are possible, and the mass operator in this field is off-diagonal in the momentum space.

The electron mass operator in a constant electromagnetic field [9] shows an essential singularity and branching at the zero point in the dynamic variable $\varkappa$ proportional to the field strength and the charge, and the branching leads to appearance of two analyticity domains, where the mass operator turns out to be retarded or advanced.

To second order in the radiation field strength, the electron mass operator in QED is defined by the expression [9]

$$\Sigma^{(\gamma)}(x, x') = -\mathrm{i}e^2\gamma^\mu S_c^{(\mathrm{e})}(x, x')\gamma^\nu D_{\mu\nu}^{(\gamma)}(x, x')\,, \tag{4.3}$$

where $S_c^{(\mathrm{e})}(x, x')$ is the causal electron Green's function in an external field, and $D_{\mu\nu}^{(\gamma)}$ is the photon propagation function, for which, to the lowest order in α, the photon propagator in the absence of an external field should be taken.

In the WSG standard model of electroweak interactions besides contributions of the virtual process $\mathrm{e} \to \mathrm{e} + \gamma \to \mathrm{e}$ to the one-loop electron (charged lepton) mass operator, other contributions such as those of the charged W-boson, and the neutral Z-boson and the Higgs boson H, i.e. processes of the $\mathrm{e} \to \mathrm{W} + \nu \to \mathrm{e}$, $\mathrm{e} \to \mathrm{e} + \mathrm{Z} \to \mathrm{e}$, $\mathrm{e} \to \mathrm{e} + \mathrm{H} \to \mathrm{e}$ type, should be considered. As a result, the electron mass operator in the electroweak model and in the unitary gauge (to second order in the coupling constants) is represented in the form [92, 94–96]:

$$\begin{aligned}\Sigma(x, x') &= \Sigma^{(\gamma)}(x, x') + \Sigma^{(\mathrm{Z})}(x, x')\\ &\quad + \Sigma^{(\mathrm{W})}(x, x') + \Sigma^{(\mathrm{H})}(x, x')\,,\end{aligned} \tag{4.4}$$

where the electrodynamic (photon) contribution $\Sigma^{(\gamma)}(x, x')$ is determined by Eqn (4.3), and those of the neutral weak current (Z-boson), of the charged weak currents (W-boson) and of the Higgs boson (not yet discovered) are respectively equal to

$$\begin{aligned}\Sigma^{(\mathrm{Z})}(x, x') &= -\mathrm{i}G_\mathrm{Z}^2\gamma^\mu(1 + \beta\gamma^5)S_c^{(\mathrm{e})}(x, x')\\ &\quad\times \gamma^\nu(1 + \beta\gamma^5)D_{\mu\nu}^{(\mathrm{Z})}(x, x')\,,\end{aligned} \tag{4.5}$$

$$\begin{aligned}\Sigma^{(\mathrm{W})}(x, x') &= -\mathrm{i}G_\mathrm{W}^2\gamma^\mu(1 + \gamma^5)S_c^{(\nu)}(x, x')\\ &\quad\times \gamma^\nu(1 + \gamma^5)D_{\mu\nu}^{(\mathrm{W})}(x, x')\,,\end{aligned} \tag{4.6}$$

$$\Sigma^{(\mathrm{H})}(x, x') = -\mathrm{i}G_\mathrm{H}^2 S_c^{(\mathrm{e})}(x, x')D^{(\mathrm{H})}(x, x')\,. \tag{4.7}$$

Here the following notations have been adopted:

$$\begin{aligned}&\beta = \frac{g^2 + g'^2}{g^2 - 3g'^2}\,, \qquad G_\mathrm{Z} = \frac{g^2 - 3g'^2}{4(g^2 + g'^2)^{1/2}}\,,\\ &G_\mathrm{W} = \frac{g}{2\sqrt{2}}\,, \qquad G_\mathrm{H} = \frac{g}{2}\,\frac{m_\mathrm{e}}{M_\mathrm{W}}\,.\end{aligned} \tag{4.8}$$

In Eqns (4.5)–(4.8), $S_c^{(\mathrm{e})}(x, x')$ and $D_{\mu\nu}^{(\mathrm{W})}(x, x')$ are the electron and W-boson propagators in an external field; $S_c^{(\nu)}$, $D_{\mu\nu}^{(\mathrm{Z})}$ and $D^{(\mathrm{H})}(x, x')$ are the neutrino, Z-boson and Higgs boson propagators, respectively, and, finally, g and g' are the gauge coupling constants of the WSG theory.

The most direct method of calculating the electron mass radiation shift calls for the computation of the electron elastic scattering amplitude $T(p, q, A)$ in an external field:

$$\begin{aligned}T(p, q, A) &= \int \bar{\psi}_q(x)\Sigma(x, x')\psi_p(x')\,\mathrm{d}^4x\,\mathrm{d}^4x'\\ &= (2\pi)^4 VT\delta(p - q)T(q, A)\,,\end{aligned} \tag{4.9}$$

where $\psi_q(x)$ is the Dirac equation solution for an electron in an external field in the state with quantum numbers q.

The amplitude $T(q, A)$ should be renormalized according to the general prescription [9]:

$$T_\mathrm{R}(q, A) = T(q, A) - T(q, 0) + T_\mathrm{R}(q, 0)\,, \tag{4.10}$$

where $T_R(q, A)$ is the renormalized amplitude, $T(q, 0)$ is the value of $T(q, A)$ in the absence of the field, and $T_R(q, 0)$ is its renormalized value.

The real part of the elastic scattering amplitude (4.10) is related to the electron mass shift Δm by the formula

$$\mathrm{Re}\, T_R(q, A) = \frac{m}{p_0}\, \mathrm{Re}\, \Delta m\,, \tag{4.11}$$

and its imaginary part according to the optical theorem is proportional to the total probability of the electron radiative transition from the state with quantum numbers q in the given external field:

$$-\mathrm{Im}\, T_R(q, A) = \frac{w_q}{2}\,. \tag{4.12}$$

Another method of calculating the value of Δm is based upon using the dispersion relations; it can be acquainted by Refs [64, 97]. We only note that in the presence of the strong external field the validity condition for this method reduces to analyticity of $T_R(q, A)$ in the upper half-plane of the complex variable $\varkappa$ and its vanishing at $\varkappa = 0$ at least linearly.

The Dirac–Schwinger equation describing radiative corrections for the motion of a massive neutrino in an external field has the form similar to (4.2):

$$(\hat{p} - m_\nu)\psi_\nu(x) = \int \mathrm{d}^4x'\, \Sigma(x, x')\psi_\nu(x')\,. \tag{4.13}$$

In the one-loop approximation, the mass operator $\Sigma(x, x')$ determines the radiative correction to the neutrino energy in the form

$$\Delta E = \frac{1}{T}\int \mathrm{d}^4x\, \mathrm{d}^4x'\, \bar{\nu}(x)\Sigma(x, x')\nu(x')\,, \tag{4.14}$$

where T is the 'interaction time' ($T \to \infty$), and the neutrino bispinor $\nu(x)$ is taken in the zero approximation. The explicit expression for $\Sigma(x, x')$ in an external field in the framework of the one-loop approximation follows from the Lagrangian of the neutrino interaction with the charged particles [98, 11]:

$$\Sigma(x, x') = \Sigma_W(x, x') + \Sigma_\varphi(x, x')\,, \tag{4.15}$$

where

$$\Sigma_W(x, x') = -\frac{\mathrm{i}g^2}{2}\,\frac{1-\gamma^5}{2}\,\gamma^\mu S_c^{(\mathrm{e})}(x, x')\gamma^\nu\,\frac{1+\gamma^5}{2}\,D_{\mu\nu}^{(\mathrm{W})}(x, x') \tag{4.16}$$

is the W-boson contribution, while the mass operator $\Sigma_\varphi(x, x')$ contribution of the charged scalar is negligibly small $\left(\propto (m_e/m_W)^2\right)$ as compared to the W-contribution, and we omit it here (see Refs [98, 11] for more details).

In contrast to ΔE, the mass radiative correction

$$\Delta m = \frac{p^0}{m}\, \Delta E \tag{4.17}$$

is the Lorentz invariant, and similar to what has been done above it is necessary to renormalize routinely the neutrino mass by subtracting its value in the vanishing field from the nonrenormalized quantity Δm_ν.

We remark that since the Majorana neutrino (MN) is identical to its antiparticle, the main contribution to its mass operator, in contrast to the case of the Dirac neutrino (DN), is provided not only by the virtual process $\nu \to e^- W^+ \to \nu$, but also by the process $\nu \to e^+ W^- \to \nu$ being charge-conjugate to the previous one.

AMM of a lepton and its anomalous electric moment (AEM) induced by an external field are determined by those terms in the radiative mass shift that are proportional to $(s_\mu \widetilde{F}^{\mu\nu} p_\nu)$ and to $(s_\mu F^{\mu\nu} p_\nu)$, respectively [97]:

$$\mathrm{Re}\, \Delta m_H = \frac{\mu(s^\mu \widetilde{F}_{\mu\nu} p^\nu)}{m}\,, \tag{4.18}$$

$$\mathrm{Re}\, \Delta m_E = \frac{d(s^\mu F_{\mu\nu} p^\nu)}{m}\,, \tag{4.19}$$

where s^μ is the particle polarization 4-vector.

It should be emphasized that AEM is induced by an external field if the pseudoscalar of this field $\mathbf{EH} \neq 0$, and that its existence does not contradict the T-invariance of the WSG theory, while the third spin term in Δm, proportional to $s^\mu F_{\mu\alpha} F^{\alpha\beta} p_\beta$, is governed by the spatial parity nonconservation when weak-interacted. That the charged lepton [97] and the massive DN [98, 11] both exhibit AEM in an electromagnetic field constitutes a qualitatively new physical effect. As long ago as in papers of Landau and Zel'dovich [99, 100], the existence of a nonvanishing electric dipole moment of particles in the CPT-symmetric theory, which is determined by the well-known electromagnetic form factor $F_3(0)$ [101]

$$d = -\frac{F_3(0)}{2m}\,, \tag{4.20}$$

was proved to be possible only under the condition that the combined CP- (or T-) invariance is broken. An electron (DN) in vacuum has neither normal nor anomalous electric moments as a result of the T-invariance of the interaction in the Standard Model. However, modification of the electron radiative interaction in an external field leads to the appearance of an electron (massive DN) AEM.

The review of various models of the CP-invariance breaking from the standpoint of predicting the existence of the electron electric dipole moment is presented in Ref. [101]. As to the experimental situation, the achieved accuracy of indirect measurements of a possible nonvanishing electron electric dipole moment was estimated as [101]

$$\delta(d_e) \simeq 10^{-27}\, e \times \mathrm{cm}\,, \tag{4.21}$$

which is much higher than the accuracy of the electron AMM measurement reported in papers [102–104]:

$$\delta\left[\frac{1}{2}(g-2)\right] \simeq 1 \times 10^{-11}\,. \tag{4.22}$$

Much interest has also been paid recently to the study of finite-temperature fermion Green's functions in various realistic QFT models.

In the framework of QED, the one-loop electron mass operator and vertex function at finite temperature and nonzero chemical potential were calculated by a number of research groups. In papers [105–108], a charge-symmetric case ($\mu = 0$) was considered, when equal numbers of electrons and positrons are excited from vacuum at finite temperature. The electron self-energy at zero temperature with consideration for the contribution of the finite density effects was studied in Refs [109, 110]. In the charge-asymmetric case

($\mu \neq 0$), when electrons are in excess over positrons, the vertex function and the electron AMM at finite temperature were calculated in Ref. [111]. Along with QED, similar investigations are under way in the Standard Model of electroweak interactions. Today one may speak about new important trends in the theoretical physics such as, for instance, physics of neutrino at finite temperature and density of medium. Most complete account of the modern state of investigations in the physics of neutrino propagating through dispersive media (plasma in metals and stars, ferromagnetics, lepton plasma of the early Universe, etc.) can be found in Ref. [112], where references are exhibited to the original papers with the discussion of the dispersion law and electromagnetic properties of neutrino in an electron-positron plasma in a free case, i.e. in zero external field (see also Refs [113 – 121]).

In the one-loop approximation, the spectrum of quark-like excitations together with the gluon excitation spectrum in a hot quark-gluon plasma were calculated in papers (that are now considered as classical) written by Kalashnikov and Klimov [122, 123] and somewhat later in Ref. [124].

Along with the above-mentioned problems much attention is paid to the study of the radiative energy shift and AMM of fermions at finite temperature and nonzero chemical potential in the presence of strong external fields [125 – 130, 33].

4.2 Mass shifts and anomalous magnetic moments of an electron and neutrino in an external electromagnetic field due to the Standard Model

The most comprehensive study of electrodynamic contribution to the electron mass shift and its AMM in an arbitrary stationary electromagnetic field, as well as in the RC field, was given in Refs [64, 9, 10, 75]. We shall confine ourselves to the analysis and comparison of a number of characteristic physical effects in some particular cases of intensive external field configurations.

For ultrarelativistic electron energies and comparatively weak magnetic fields, when the following conditions

$$H \ll H_0 = \frac{m^2}{e} \simeq 4.41 \times 10^{13}\ \mathrm{G}\,, \qquad p_\perp = \sqrt{2eHn} \gg m\,, \tag{4.23}$$

are fulfilled, the electron AMM is determined by the formula [64]

$$\Delta\mu = \mu_{\mathrm{B}} \frac{\alpha}{2\pi} \int_0^\infty \frac{\mathrm{d}u}{(u+1)^3} \left(\frac{u}{\varkappa}\right)^{2/3} \Upsilon\left[\left(\frac{u}{\varkappa}\right)^{2/3}\right], \tag{4.24}$$

where $\mu_{\mathrm{B}} = e/2m$ is the Bohr magneton, Υ is the upsilon function (3.33), and the dynamic parameter of synchrotron radiation $\varkappa$ is equal to

$$\varkappa = \frac{1}{m^3}\sqrt{-(eF^{\mu\nu}p_\nu)^2} = \frac{H}{H_0}\frac{p_\perp}{m}\,. \tag{4.25}$$

We note that Eqn (4.24) is defined as an exact expression for the electron AMM in a constant crossed field [9], where the field dynamic parameter equals

$$\varkappa = \frac{F}{F_0}\frac{(p^0 - p^3)}{m}\,, \tag{4.26}$$

and $p^0 - p^3$ is the difference between the kinetic energy of the particle and its kinetic momentum component along the axis $\mathbf{E} \times \mathbf{H}$.

Eqn (4.24) allows the following asymptotic expansions to be derived [64]:

$$\frac{\Delta\mu}{\mu_{\mathrm{B}}} = \begin{cases} \dfrac{\alpha}{2\pi}\left[1 - 12\varkappa^2\left(\ln\varkappa^{-1} + C + \dfrac{1}{2}\ln 3 - \dfrac{37}{12}\right) + \ldots\right], & \varkappa \ll 1, \\ \dfrac{\alpha\Gamma(1/3)}{9\sqrt{3}}(3\varkappa)^{-2/3}\left[1 + \dfrac{6\Gamma(2/3)}{\Gamma(1/3)}(3\varkappa)^{-2/3} + \ldots\right], & \varkappa \gg 1, \end{cases} \tag{4.27}$$

where $C = 0.577\ldots$ is the Euler constant.

Thus, the ratio $\Delta\mu/\mu_{\mathrm{B}}$ is positive in the quasi-classical approximation and with growing $\varkappa$ it decreases in a monotonous way from Schwinger's value down to zero, while in the ultraquantum case ($\varkappa \gg 1$) the magnitude of the electron AMM decreases with growing energy and the field strength by the power-like law ($\propto \varkappa^{-2/3}$).

Upon averaging over the electron spin states, the asymptotics for the radiative electron mass shift subject to the condition (4.23) have the form [64] (see also Refs [9, 10, 75]):

$$\begin{pmatrix} \mathrm{Re}\,\Delta m \\ \mathrm{Im}\,\Delta m \end{pmatrix} = \begin{cases} \dfrac{4\alpha m}{3\pi}\varkappa^2\left[\ln\varkappa^{-1} + C + \dfrac{1}{2}\ln 3 - \dfrac{33}{16}\right], \\ -\dfrac{5\alpha m}{4\sqrt{3}}\varkappa\left[1 - \dfrac{8\sqrt{3}}{15}\varkappa + \dfrac{7}{2}\varkappa^2\right], \end{cases} \quad \varkappa \ll 1\,, \tag{4.28}$$

$$\Delta m = \frac{7\Gamma(2/3)(1 - \mathrm{i}\sqrt{3})\alpha m}{27\sqrt{3}}(3\varkappa)^{2/3}\,, \qquad \varkappa \gg 1\,. \tag{4.29}$$

We emphasize that $\mathrm{Re}\,\Delta m > 0$ and it steadily increases with growing $\varkappa$. For $\alpha\varkappa^{2/3} \sim 1$, when $\mathrm{Re}\,\Delta m \sim m$, the comparison of (4.29) with the two-loop correction to the electron mass shift shows [59, 9] that in the ultraquantum limit $\varkappa \gg 1$ the quantity $\alpha\varkappa^{1/3}\ln\varkappa$ plays the role of the expansion parameter for the perturbation theory in terms of the radiative field intensity, while result (4.29) is valid for $\alpha\varkappa^{1/3}\ln\varkappa \ll 1$.

In superstrong magnetic fields $H \gtrsim H_0$, the above consideration based upon the quasi-classical approximation (4.23) turns out to be invalid and a qualitatively different dependence of the mass shift and the electron AMM was observed [87, 131, 10]: the electron AMM decreases with growing field intensity H, goes through the zero value and changes its sign as compared to Schwinger's value, then in the region $H \gtrsim H_0$ it becomes negative.

In the region $H \gg H_0$, the mass shift of the ground state electron ($n = 0$, $\zeta = -1$) and the AMM in the weakly excited states ($n \sim 1$) are described by the asymptotic formulas [87, 131]

$$\Delta m(n = 0) = \frac{\alpha}{4\pi}\, m \ln^2\left(\frac{2H}{H_0}\right),$$

$$\Delta\mu(n = 1) = -\frac{\alpha e}{4\pi m}\frac{H_0}{H}\ln\left(\frac{2H}{H_0}\right). \tag{4.30}$$

Following [44], we emphasize that the asymptotics (4.30) coincide with that of the vertex operator in QED [79]

$$\Gamma^\mu = \gamma^\mu\left[f(t) - 1\right] - g(t)\,\frac{\sigma^{\mu\nu}k_\nu}{2m} \tag{4.31}$$

with respect to the square of the momentum transferred in the limit $t = -k^2 \gg 4m^2$:

$$\frac{\Delta m(n=0)}{m} = 1 - f(t)\,, \qquad t = eH\,,$$

$$g(t) = -\frac{2m}{e}\,\mu(n \neq 0)\,, \qquad t = 2|e|Hn\,, \qquad n \neq 0\,. \qquad (4.32)$$

Such agreement is not a sheer accident and it reflects the fundamental relation between QED in an intensive field and QED at small distances (with large momenta transferred), which was for the first time found out in Ref. [44], then extended to the scalar QED and deeply analyzed with the help of the renormgroup technique in Ref. [45]. The quantities mentioned in (4.32) describe the common physical phenomena, i.e. the elastic electron scattering by an external field of high intensity or by separate quanta of the external field with large momenta squared of virtual photons. A deep analogy between phenomena under discussion in a strong external field and at large momenta transferred was demonstrated in Refs [44, 45] (see also Ref. [46]) and explained by the identity of the corresponding asymptotics in the main logarithmic approximation to the one-loop $\mathcal{L}^{(1)}$ and two-loop $\mathcal{L}^{(2)}$ corrections (the latter following behind the Heisenberg–Euler correction $\mathcal{L}^{(1)}$) to the Lagrangian $\mathcal{L}^{(0)}$ of a constant electromagnetic field, on the one hand, with the photon polarization functions of the second $\pi^{(2)}$ and the forth $\pi^{(4)}$ orders in e, on the other hand:

$$\begin{bmatrix} \dfrac{\mathcal{L}^{(1)}}{\mathcal{L}^{(0)}} \\[2mm] \dfrac{\mathcal{L}^{(2)}}{\mathcal{L}^{(0)}} \end{bmatrix} \simeq -\begin{bmatrix} \dfrac{1}{3}\dfrac{\alpha}{\pi} \\[2mm] \dfrac{1}{4}\left(\dfrac{\alpha}{\pi}\right)^2 \end{bmatrix} \ln\left(\frac{eH}{m^2}\right) \qquad (4.33)$$

and

$$\begin{bmatrix} \pi^{(2)} \\ \pi^{(4)} \end{bmatrix} \simeq -\begin{bmatrix} \dfrac{1}{3}\dfrac{\alpha}{\pi} \\[2mm] \dfrac{1}{4}\left(\dfrac{\alpha}{\pi}\right)^2 \end{bmatrix} \ln\left(-\frac{k^2}{m^2}\right). \qquad (4.34)$$

Thus, the vacuum polarization by photons with large virtual momenta squared $|k^2| \gg m^2$ behaves in the same way as the vacuum polarization, which is due to the strong external field $H \gg m^2/e$.

The measurement of the anomalous part of the electron magnetic moment by the spin-resonance method of Rabi is known (see, e.g., Ref. [11]) to be based upon using the combination of a constant magnetic field fixing the particle spin magnetic moment projection on its direction and an alternating magnetic field inducing the spin-flip transitions, i.e. transitions with the change of the spin orientation. Therefore we shall dwell on one of the results of Ref. [78] (see also Ref. [10]), where the electron elastic scattering amplitude in the RC field was calculated.

In the limiting case

$$\xi = \frac{eE_\sim}{m\omega} \ll 1\,, \qquad (4.35)$$

when conditions (4.23) are also fulfilled, correction (4.24) to the electron AMM, which is caused by the presence of the plane electromagnetic wave, is determined by the expression [78, 10]

$$\frac{\Delta\mu(\xi)}{\mu_B} = \xi^2\,\frac{\alpha}{2\pi}\left[-1 + \frac{1+3\lambda^2}{(1-\lambda^2)^2} + \frac{2\lambda^4+6\lambda^2}{(1-\lambda^2)^3}\ln\lambda\right], \quad (4.36)$$

where the parameter $\lambda = 2\omega(p^0 - p^3)/m^2 = 2(kp)/m^2$ was introduced. Hence one can see that in the nonstationary electromagnetic field the electron AMM depends nonlinearly on the amplitude and frequency of the field, and radiative correction (4.36) tends to lower the AMM as compared to Schwinger's value.

In the field of a plane electromagnetic wave similar to the case of the photon propagation, the growth of the real part of the electron mass shift is observed in the high-energy region ($\lambda \gg 1$), which is proportional to the logarithm squared of λ [78, 10]:

$$\mathrm{Re}\,\Delta m = \frac{\alpha}{8\pi}\,\frac{e^2E_\sim^2}{m\omega^2}\ln^2\lambda\,, \qquad \ln\lambda \gg 1\,. \qquad (4.37)$$

We also note that in the case of the circular polarization of the wave, the electron AMM in the RC field depends on the direction of the circular polarization of the wave as well.

The ever increasing accuracy of the experimental technique [103] applied to determining the electron AMM demonstrates the urgency about observing its dynamic nature, i.e. the dependence holds:

$$a_e = \frac{\Delta\mu}{\mu_B} = \frac{\alpha}{2\pi}f(E,H)\,. \qquad (4.38)$$

In a constant magnetic field under usual experimental conditions, numerical corrections to unity in the formula $f(E,H) = 1 - \Delta f$ are fairly small. Even when the electron energy is equal to 1 GeV and the magnetic field strength is 4×10^4 G, we have $\Delta f \simeq 5\times10^{-10}$. However, for the electron energy 10 GeV and the field 4×10^4 G we have $\Delta f \simeq 4\times10^{-8}$, which apparently proves the observation to be feasible.

For $\xi = 0.1$ and $\omega \sim 10^{15}$ s^{-1}, which corresponds to modern laser parameters, it follows from Eqn (4.36) that $|a_e^{\mathrm{var}}| \sim 10^{-6} - 10^{-7}$ for the case of relativistic electrons with the energy $p_0 \sim 10^3 m$.

Thus for the above-mentioned values of parameters the pure field contributions to the electron AMM lie in the ranges

$$\left(\frac{\alpha}{\pi}\right)^4 \lesssim |a_e| \lesssim \left(\frac{\alpha}{\pi}\right)^3, \qquad \left(\frac{\alpha}{\pi}\right)^3 \lesssim |a_e^{\mathrm{var}}| \lesssim \left(\frac{\alpha}{\pi}\right)^2. \qquad (4.39)$$

For comparison we note that at present the theoretical computation of the vacuum value of α_e is carried out in the α^4-order of the perturbation theory [104] and the results are in satisfactory agreement with the experimental data [102, 103]:

$$a_e^{\mathrm{theor}} = 1159652133(29)\times10^{-12}\,,$$

$$a_e^{\mathrm{exp}} = 1159652188.4(4.3)\times10^{-12}\,. \qquad (4.40)$$

The feasibility of experimentation on the dynamic nature of AMM was pointed out in Ref. [132], where the electron spin precession was proposed to be measured by using electrons canalized in a bent single crystal. As this takes place, internal fields in the crystal can reach very high values of $F \lesssim 10^{-4}H_0$, and the electron energy can be chosen of the order of several tens of GeV.

The electron mass operator and the amplitude of elastic electron scattering in an arbitrary stationary electromagnetic field were studied in papers [133, 9, 97]. Here we shall confine ourselves to discussion of some most interesting results obtained by Ritus with coworkers in the case of a ground state electron placed in a constant electric field, when $p_\perp^2 = 0$ (the hyperbolic motion case).

In a weak electric field of the strength $E \ll E_0 = m^2/e$, the electron mass shift has the asymptotics [97]:

$$\mathrm{Re}\,\Delta m = m\,\frac{\alpha}{2\pi}\left[-\beta\pi + \beta^2\left(\frac{4}{3}\ln\frac{\gamma}{2\beta} + \frac{4}{9}\right) + \ldots\right],$$

$$\mathrm{Im}\,\Delta m = m\,\frac{\alpha}{2\pi}\left[\beta\left(2\ln\frac{2\beta m}{\gamma\mu} - 1\right) - \beta^2\,\frac{2\pi}{3} + \ldots\right], \quad (4.41)$$

where $\beta = eE/m^2$, μ is the photon mass introduced to avoid infrared divergence, which is related to the infiniteness of motion, $\gamma = \mathrm{e}^C = 1.781\ldots$

The first terms in expansion (4.41) are classical and do not depend on the Planck constant. The term linear in the modulus of the field in $\mathrm{Re}\,\Delta m$ is in this case 2π times greater than that in a magnetic field, where it plays the role of the energy of interaction between the electron AMM and the magnetic field and is an essentially quantum variable. While the classical term of radiation probability entering the imaginary part in (4.41) coincides with the integral of the classical radiation spectral distribution for a charge executing a hyperbolic motion in an electric field [9].

In the ultraquantum limit $\beta \gg 1$, the electron mass shift becomes a nonlinear function of the field [97]:

$$\mathrm{Re}\,\Delta m = m\,\frac{\alpha}{2\pi}\left[-\frac{3}{2}\ln\frac{\beta}{\gamma} + K + \ldots\right],$$

$$\mathrm{Im}\,\Delta m = -m\,\frac{\alpha}{2\pi}\left[\beta\ln\frac{m^2}{\mu^2} - \frac{3\pi}{4}\right], \qquad K = -2.636\,. \quad (4.42)$$

We note that in an electric field $\mathrm{Re}\,\Delta m$ decreases with growing field strength first linearly and then logarithmically for $\beta \gg 1$, while in a magnetic field the mass shift of a ground state electron changes sign with growing field strength and increases as a logarithm of the field strength [see Eqn (4.30)] for $H \gg H_0$. Another qualitative feature of the electron motion in an electric field is that there is no stable state, i.e. the ground state electron in an electric field ($p_\perp^2 = 0$) radiates due to the ability of the electric field to deliver work on the charge, while the electron ground state in a magnetic field ($n = 0$) is stable. One of the classical methods of computing the quantity Δm^{cl} is presented in Ref. [97] and consists in the following.

The correction ΔL to the Lagrangian function of a charged particle in an external field

$$L = -m\sqrt{1 - v^2} + e(\mathbf{v}\mathbf{A}^{\mathrm{ext}}) - eA_0^{\mathrm{ext}}\,, \quad (4.43)$$

allowing for the modification of the charge interaction with the proper field due to an external field (and with the same positions and velocities of the charge at the instant under consideration), is described by the formula

$$\Delta L = \left[\iint \mathrm{d}^3 x\,(\mathbf{j}\mathbf{A}) + \int \mathrm{d}^3 x\,\frac{E^2 - H^2}{2}\right]\Bigg|_0^F, \quad (4.44)$$

where $\mathbf{A}$, $\mathbf{E}$, $\mathbf{H}$ are the potential and strengths of the proper field of the charge, and it determines the classical part of the charge mass shift:

$$\Delta m^{\mathrm{cl}} = -\frac{1}{\sqrt{1 - v^2}}\,\Delta L \equiv -\gamma\Delta L\,. \quad (4.45)$$

Consideration for the explicit form of the retarded field of the uniformly accelerated charge that moves along the z-axis according to the law $z = (t^2 + w_0^{-2})^{1/2}$, where $w_0 = eE/m$ is the charge acceleration, enables the mass shift of the relativistic charge to be represented in the form [97]

$$\Delta m^{\mathrm{cl}} = -\frac{1}{2}\,\alpha w_0 f(v)\,, \qquad f = \frac{\partial}{\partial x}[x\coth x]\,,$$
$$x = 2w_0\tau = 2\,\mathrm{artanh}\,v\,. \quad (4.46)$$

Thus, the classical mass shift of a charge is proportional to the acceleration in the rest frame and is positive while an electron is decelerated by the field, it equals zero in the turning point ($v = 0$), and tends to the value

$$\Delta m^{\mathrm{cl}} = -\frac{1}{2}\,\alpha m\,\frac{eE}{m^2} \quad (4.47)$$

for $v \to 1$, when the electron is moving away from the turning point. The quantum calculation of electron elastic scattering amplitude predicts that the final state mass is shifted, i.e. for $v \to 1$, and it is in this limit that (4.47) coincides with the first term of expansion (4.41). It should be particularly emphasized that the classical part of the mass shift of the uniformly accelerated charge, as it follows from the above calculation scheme, is nonlocal by nature and it shows up only in quantum processes. Modifying the action by the integral of a divergence, the quantity Δm^{cl} does not enter the classical equations of motion.

As an example, we shall consider a two-loop correction to the classical Maxwell Lagrangian in the scalar ($s = 0$) and spinor ($s = 1/2$) electrodynamics [97]. For a weak electric field, when $\eta = eE/m^2 \ll 1$, the radiative correction to the one-loop contribution is completely reduced to the classical mass shift of a charge accelerated by this field:

$$2\,\mathrm{Im}\,(\mathcal{L}^{(1)} + \mathcal{L}^{(2)}) \simeq (2s + 1)\,\frac{(eE)^2}{(2\pi)^3}\exp\left(-\frac{\pi m_*^2}{eE}\right), \quad (4.48)$$

with the expression for $2\,\mathrm{Im}\,\mathcal{L}^{(1)}$ wherein the substitution $m \to m_* = m + \Delta m^{\mathrm{cl}}$ is made, standing in the right-hand side. Since $\Delta m^{\mathrm{cl}} < 0$ and it does not depend on the spin of a charged particle, allowance for the two-loop contribution leads to an increase in the decay rate both in the scalar and in spinor QED.

We shall conclude our consideration of the electron radiative mass shift in a constant electric field by two following important remarks [97].

Firstly, the effective mass of a charged particle (both a fermion and a scalar particle) in a strong electric field decreases similarly to its effective mass $m - \delta m$ in vacuum at short distances. The dependence of the fermion mass shift on the cut-off momentum squared Λ^2 in the main logarithmic approximation is given by the formula [79]

$$\delta m = m\,\frac{\alpha}{2\pi}\,\frac{3}{2}\ln\frac{\Lambda^2}{m^2}\,, \quad (4.49)$$

which coincides up to a sign with dependence (4.42) for $\mathrm{Re}\,\Delta m$.

We have already drawn on a similar analogy in the above discussion of the relation between QED in an intense field and QED behaviour at short distances.

Secondly, the electron AMM behaviour for $p_\perp \ll m$ is also different in a radical way from that in a constant magnetic field or in a crossed field: at first AMM decreases with growing electric field strength, then it reaches minimum at $E \sim m^2/e$ and finally increases, approaching twice the Schwinger value at $E \gg m^2/e$. This qualitative difference in the mass shift behaviour is explained in Ref. [97] by the fact that the effective mass accounts for the dynamic properties of a charged particle interacting with its proper field, hence the dependence of the mass shift on the field strength is quite different for the cases of the electric and magnetic fields.

Electroweak corrections to the electron mass shift and AMM in an external electromagnetic field were studied in Refs [92, 94–96]. Let us consider the case of a constant magnetic field, when conditions (4.23) are fulfilled. In a comparatively weak magnetic field ($\varkappa \ll 1$), one-loop contributions to the real part of the lepton mass shift are quadratic in the parameter $\varkappa$ and strongly suppressed as compared with the electrodynamic contribution, while the vacuum electroweak contribution to the charged lepton AMM is described by the formula [92]

$$\frac{\Delta\mu^i}{\mu_{\rm B}} = \frac{g_i^2}{8\pi^2}\, C_i\,, \qquad i = {\rm H,W,Z}\,, \tag{4.50}$$

where coefficients C_i are equal to

$$C_{\rm H} = \left(\frac{m_{\rm e}}{M_{\rm H}}\right)^2 \left[\ln\left(\frac{M_{\rm H}}{m_{\rm e}}\right)^2 - \frac{7}{12}\right],$$
$$C_{\rm W} = \frac{10}{3}\left(\frac{m_{\rm e}}{M_{\rm W}}\right)^2, \qquad C_{\rm Z} = \frac{2}{3}\left(\frac{m_{\rm e}}{M_{\rm Z}}\right)^2 \tag{4.51}$$

in the leading order in the small parameter $\lambda_i = (m_{\rm e}/M_i)^2$.

In the ultraquantum limit all the contributions are proportional to $\varkappa^{-2/3}$ and the following asymptotics take place [94–96]:

$$\frac{\Delta\mu^{\rm Z}}{\mu_{\rm B}} = -\frac{11}{3}\,\frac{1}{2\pi}\,\frac{G_{\rm F}M_{\rm W}^2}{9\sqrt{6}}\,\frac{\Gamma(1/3)}{(3\varkappa)^{2/3}}\,,$$
$$\frac{\Delta\mu^{\rm W}}{\mu_{\rm B}} = 11\,\frac{1}{2\pi}\,\frac{G_{\rm F}M_{\rm W}^2}{9\sqrt{6}}\,\frac{\Gamma(1/3)}{(3\varkappa)^{2/3}}\,, \qquad \varkappa \gg \lambda_i^{2/3}\,,$$
$$\frac{\Delta\mu^{\rm H}}{\mu_{\rm B}} = \frac{1}{2\pi}\,\frac{7G_{\rm F}m_{\rm e}^2}{18\sqrt{6}}\,\frac{\Gamma(1/3)}{(3\varkappa)^{2/3}}\,, \tag{4.52}$$

where $G_{\rm F} = \sqrt{2}g^2/8M_{\rm W}^2 \simeq 10^{-5}/m_{\rm p}^2$ is the Fermi constant, and the Weinberg angle is approximately taken to be 30°.

Comparison of (4.52) and the asymptotics (4.27) shows that in the ultraquantum case, when $\varkappa \gg (m_{\rm e}/M_i)^3$, the electroweak contribution to the electron AMM exceeds the electrodynamic one:

$$\Delta\mu({\rm QED}) : \Delta\mu({\rm W}) : \Delta\mu({\rm H}^0) = 1 : 12 : 10^{-10}\,. \tag{4.53}$$

In the same ultraquantum limit, the electroweak corrections to the real part of the electron mass shift increase proportionally to $\varkappa^{2/3}$ as for QED and can also exceed the electrodynamic contribution (4.29), with again dominating W-boson contribution.

We note that the one-loop Higgs contribution to the lepton AMM is suppressed as compared to the one descended from the processes $\rm e \to \nu + W \to e$ and $\rm e \to e + Z \to e$ by the factor of order $(m_{\rm e}/M_{\rm H})^2$. This circumstance is inherent in the very structure of the WSG model: the electron and the scalar Higgs H-boson coupling constant is determined by the electron mass and is equal to $m_{\rm e}(\sqrt{2}\,G_{\rm F})^{1/2}$, while the coupling constant for the electron and W- or Z-boson has the order of $g \sim m_{\rm W}\sqrt{G_{\rm F}}$. The greater the mass of other particles, the stronger is the Higgs boson interaction with them. It is due to this fact that among various Higgs boson production mechanisms those with the associative production of Higgs boson with gauge bosons in $\rm e^+e^-$- and hadron collisions are considered to be most promising [134, 135], and moreover the interaction in the sectors of H-, W-, and Z-particles is still described by the perturbation theory under the condition $M_{\rm H} \lesssim M_{\rm W}$ [136].

In Ref. [137], the two-loop contribution ($\rm e \to e + Z + H \to e$) to the electron AMM in a constant magnetic field is calculated and the results obtained are compared with the contribution of the processes $\rm e \to e + Z \to e$ and $\rm e \to e + H \to e$ to the AMM.

For $\varkappa \ll (M_{\rm H} + M_{\rm Z})^2/m^2$, the relation holds [137]:

$$\frac{\Delta\mu({\rm e \to e + Z + H \to e})}{\Delta\mu({\rm e \to e + Z \to e})} \simeq 0.2\alpha Q(M_{\rm Z}, M_{\rm H})\left(\frac{M_{\rm Z}}{M_{\rm Z} + M_{\rm H}}\right)^4, \tag{4.54}$$

where $Q(M_{\rm Z}, M_{\rm H}) \sim 1$ at $M_{\rm H} \sim M_{\rm Z}$, and α is the fine-structure constant. In the other limiting case, when $\varkappa \gg \left[(M_{\rm H} + M_{\rm Z})/m_{\rm e}\right]^3$, the contribution of the process $\rm e \to e + Z + H \to e$ to the lepton AMM decreases proportionally to $\varkappa^{-2/3}\ln\varkappa$ with growing dynamic variable $\varkappa$, while the one-loop contribution of diagrams with the gauge boson exchange falls proportionally to $\varkappa^{-2/3}$.

The one-loop contribution of the diagram with the Higgs boson exchange to the lepton AMM is in this case strongly suppressed as compared to the mechanism discussed.

In the nearest future the accuracy of measuring the muon AMM is planned to be made 20 times higher [92]. Then comparison of theoretical and experimental values of the muon AMM should be drawn with allowance for the weak interaction contribution. This might open up another opportunity to test the Standard WSG model. If measurements of the muon AMM are further conducted with an accuracy increased by 2 or 3 orders, then comparison of theoretical and experimental results has to be made with consideration for the contribution of the process $\rm e \to e + Z + H \to e$ carrying information on the Higgs boson mass. If nothing else, this opens up a new opportunity to obtain some constraints on the Higgs boson mass.

Radiative mass shifts of both the Dirac neutrino (DN) and the Majorana neutrino (MN) in an arbitrary stationary external electromagnetic field were considered in Refs [98, 138–141]. According to general principles of the theory (CP- and CPT-invariance of the WSG model of electroweak interactions), the MN, which is identical to its antiparticle, has neither AMM nor AEM. Therefore we shall discuss in more detail the dynamic nature of the DN mass shift. In the case when the neutrino moves along the direction $\mathbf{E} \uparrow\uparrow \mathbf{H}$ in a special reference frame, the AMM of the massive DN behaves in weak fields ($\varepsilon, \eta \ll 1$) according to the formula [98, 11]

$$\mu_\nu = \mu_\nu^0\left[1 + \frac{4}{9}(\eta^2 - \varepsilon^2)\frac{\ln\lambda}{\lambda^2}\right], \tag{4.55}$$

where $\mu_\nu^0 = 3eG_F/8\pi^2\sqrt{2} \simeq 3\times10^{-19}\, m_\nu\mu_0/1$ eV is the static AMM of the neutrino with the mass m_ν, and the parameter $\lambda = (M_W/m_e)^2$.

In a pure magnetic field ($\varepsilon = 0$), the neutrino AMM increases quadratically with growing magnetic field strength, though with a small numerical factor. With H approaching the 'critical field' $H_0^W = M_W^2/e \simeq 10^{24}$ G, the neutrino AMM diverges logarithmically [98, 11]:

$$\mu_\nu \simeq \mu_\nu^0\,\frac{2}{3}\,\lambda\ln\frac{H_0}{H_0^W - H}\,. \tag{4.56}$$

This divergence is due to instability of the W-boson vacuum in the fields $H \geqslant H_0^W$ within the framework of the perturbation theory, which is developed as a consequence of the tachionic mode appearing in the W-boson energy spectrum [142].

In the range of large transverse neutrino momenta ($p_3 = 0,\ p_\perp \gg M_W$) and comparatively weak fields ($\varepsilon, \eta \ll 1$), the neutrino AMM essentially depends on the dynamic parameter

$$\varkappa = \frac{1}{mH_0}\sqrt{-(F^{\mu\nu}p_\nu)^2}\,,$$

where p_ν is the neutrino 4-momentum, and it has the following asymptotics [98, 11]:

$$\frac{\mu_\nu}{\mu_\nu^0} = \begin{cases} 1 + \dfrac{4}{3}\varkappa^2\,\dfrac{\ln\lambda}{\lambda^3}\,, & \varkappa \ll \lambda\,,\\[2ex] \dfrac{3^{5/6}\Gamma^4(1/3)}{20\pi}\,\dfrac{\lambda}{\varkappa^{2/3}}\,, & \varkappa \gg \lambda^{3/2}\,. \end{cases} \tag{4.57}$$

Unlike the monotone decrease of the electron AMM in QED [see (4.27)], the neutrino AMM first increases according to (4.57), and then falls off in the same fashion ($\sim \varkappa^{-2/3}$) as in the electron case. We note that the electroweak contributions to the electron AMM stemming from virtual W- and Z-bosons, depend on the dynamic parameter $\varkappa$ qualitatively in the same way as $\mu_\nu = \mu_\nu(\varkappa)$ does.

In the same quasi-classical approximation ($p_\perp \gg M_W$, $\varepsilon, \eta \ll 1$) the following asymptotics are valid for the neutrino AEM [11]:

$$\frac{d_\nu}{\mu_\nu^0} = \begin{cases} \dfrac{2}{9}\,\dfrac{\varepsilon\eta}{\lambda}\left[1 + 2\left(\dfrac{\varkappa}{\lambda}\right)^2\right], & \varkappa \ll \lambda\,,\\[2ex] -\dfrac{2}{9}\,\varepsilon\eta\,\dfrac{\lambda}{\varkappa^2}\left(\ln\dfrac{\varkappa}{\sqrt{3}} - C\right), & \varkappa \gg \lambda^{3/2}\,. \end{cases} \tag{4.58}$$

The character of the neutrino AEM variation with growing $\varkappa$ (4.58) and the same of an electron [97] coincide up to a sign for $\varkappa \gg 1$ and differ essentially at $\varkappa \ll 1$, when the electron AEM is proportional to $\ln\varkappa^{-1}$.

In Refs [98, 138, 139], the imaginary part of a neutrino elastic scattering amplitude in a constant external field was also studied in detail. It should be noted that the decay of a free neutrino ($\varepsilon = \eta = 0$) $\nu_e \to e^- + W^+$ is evidently forbidden by the 4-momentum conservation law. In a pure magnetic field ($\varepsilon = 0,\ \eta \neq 0$), the process shows a threshold: the neutrino energy $E_\nu > m_e + M_W$, and moreover, in the particular case when $\mathbf{p} \uparrow\uparrow \mathbf{H}$ the decay rate is equal to zero, $w = 0$. There occurs no threshold in an electric field and $\mathrm{Im}\,\Delta m_\nu \neq 0$ at any $\varepsilon \geqslant m_\nu$. For instance, in the case of the decay of a neutrino with the zero transverse momentum, $p_\perp = 0$, in a weak electric field ($\varepsilon \ll 1,\ \ \eta = 0$) the asymptotics for the decay rate takes the form [98]

$$w = \frac{g^2}{(4\pi)^2}\,\frac{m_\nu^2}{E_\nu}\,\frac{\varepsilon}{\lambda^2}\exp\left(-\frac{\pi}{\varepsilon}\right), \tag{4.59}$$

and in this situation the exponential dependence of the decay rate on the field strength is the same as in the expression for the e^+e^--pair production by a weak electric field in vacuum [see (4.48)].

In conclusion of the present section we shall discuss some physical effects that are due to the AMM of massive DN. With consideration for the radiative effects, the neutrino energy in a constant magnetic field is described in the framework of a linear in $\mu_\nu^0 H$ approximation by the formula [140,11]:

$$E = E_0\left(1 - \zeta\mu_\nu^0 H\,\frac{E_\perp}{E_0^2}\right), \tag{4.60}$$

where

$$E_0^2 = m_\nu^2 + \mathbf{p}^2\,, \qquad E_\perp^2 = m_\nu^2 + p_\perp^2\,, \tag{4.61}$$

and the spin number $\zeta = \pm1$ describes the particle spin orientation along or contrary to the magnetic field $\mathbf{H}$.

First we consider the electromagnetic radiation of a neutrino moving in a magnetic field. According to the energy-momentum conservation laws applied to the emission of a photon with the 4-momentum $k^\mu = (\omega, \mathbf{k})$ and due to (4.60), (4.61), the radiation frequency in the linear in $\mu_\nu^0 H$ approximation is defined by the expression [140]

$$\omega = 2\mu_\nu^0 H\,\frac{(1 - v_z^2)^{1/2}}{1 - v\cos\psi}\,\delta_{\zeta,-1}\delta_{\zeta',1}\,, \tag{4.62}$$

where v_z is the longitudinal component of the neutrino velocity, and $\psi = (\widehat{\mathbf{k},\mathbf{p}})$ stands for the radiation angle.

Hence emission is possible only if the neutrino spin-flip takes place: $\zeta = -1\ (\boldsymbol{\mu}_\nu \uparrow\downarrow \mathbf{H}) \to \zeta' = +1\ (\boldsymbol{\mu}_\nu \uparrow\uparrow \mathbf{H})$. We note that the spin-flip occurs also in the neutron radiation [143], and the formulas for the total probability and the power of electromagnetic radiation by the AMM of a neutrino moving in a magnetic field coincide with the corresponding formulas of Ref. [143] to the lowest order in $\mu_\nu^0 H$, when the replacement $\mu_n \to \mu_\nu^0$ is made in them.

For astrophysical applications the question of the chirality flip in the neutrino motion in a magnetic field is of special interest [144].

The chirality operator

$$h = \frac{(\boldsymbol{\Sigma}\,\mathbf{p})}{|\mathbf{p}|} \tag{4.63}$$

is not an integral of motion, as it does not commute with the Hamiltonian for the stationary neutrino states that follows from Dirac – Schwinger equation (4.13) and in the case of a weak magnetic field takes the form

$$\begin{aligned} &\hat{H} = \hat{H}_D + \hat{V}\,, \qquad \hat{H}_D = \boldsymbol{\alpha}\mathbf{p} + \gamma^0 m_\nu\,,\\ &\hat{V} = -\mu_\nu^0 H\gamma^0\hat{\mu}_3(1 + \gamma^5)\,, \end{aligned} \tag{4.64}$$

where $\hat{\mu}_3 = \gamma^0(E\Sigma_3 + \gamma^5 p_z)/m_\nu$ is the transverse polarization operator [8, 145].

The mean value of operator (4.63) varies with time according to the law [140]:

$$\langle h\rangle = -\left[1 - \frac{\sin^2\theta}{1 - v^2\cos^2\theta}(1 - \cos\omega_{\mathrm{H}}t)\right], \tag{4.65}$$
$$\omega_{\mathrm{H}} = 2\mu_{\nu}^{0}H(1 - v^2\cos^2\theta)^{1/2},$$

where θ is the angle between the neutrino momentum and the magnetic field $\mathbf{H}$, and at $t = 0$ the neutrino is supposed to be the left-chirality one, i.e. $\big\langle h(t=0)\big\rangle = -1$.

It follows from (4.65) that when the left-chirality neutrino moves in the direction perpendicular to the field ($\theta = \pi/2$) it transforms to the right-chirality one in a time $\tau = \pi/2\mu_{\nu}^{0}H$.

This effect of the neutrino chirality flip can be of major importance for the process of the neutron star formation, when the latter exhibits the strong magnetic field $H \sim 10^{13}$ G: one-half the number of left-chirality neutrinos produced in the gravitation collapse process can be transformed into left-chirality ones that are the sterile states practically not interacting with the matter, which in its turn will lead to a decrease in the neutrino momentum observed [144]. This same effect was supposed to underlie the possible explanation for the solar neutrino puzzle, though the magnitude of the neutrino AMM turned out to be too small. When a neutrino moves along the field $\mathbf{H}$ ($\theta = 0$) its chirality is conserved, and it coincides with the spin polarization along the field, given by the operator $\hat{\mu}_3$.

4.3 Anomalous magnetic moments of fermions moving in a medium at finite temperature and nonzero chemical potential in an external magnetic field

A dynamic character of the electron energy shift and AMM in QED at finite temperature in an external magnetic field was studied in Refs [33, 125, 127–129]. In Ref. [33], interaction of fermions with hot vacuum is described in the one-loop approximation basing upon the mass operator constructed from the time Green's functions, and the electron mass operator at finite temperature was considered in Ref. [128] using the temperature Green's functions technique in the framework of the two-dimensional QED approximation. In the real time representation, the electron energy shift in the electron-positron plasma being found in the thermodynamic equilibrium state at temperature T in an external magnetic field $\mathbf{H} \uparrow\uparrow Oz$, is written in the form [33]

$$\Delta E_n(H,T,\mu) = \Delta E_n(H, T=\mu=0) + \Delta E_n^{\mathrm{B}}(H,T) + \Delta E_n^{\mathrm{F}}(H,T,\mu) + \Delta E_n^{\mathrm{F-B}}(H,T,\mu), \tag{4.66}$$

where $\Delta E_n(H, T=\mu=0)$ is the electron radiative energy shift in an external magnetic field at $T=\mu=0$, considered above; $\Delta E_n^{\mathrm{B}}(H,T)$ is the electron energy shift through its interaction with the equilibrium radiation; $\Delta E_n^{\mathrm{F}}(H,T,\mu)$ is the temperature electron energy shift owing to its exchange interaction with electrons and positrons of plasma; the interference term $\Delta E_n^{\mathrm{F-B}}(H,T,\mu)$ is a pure imaginary quantity and thus contributes only to the imaginary part of (4.66).

For the fullness of material presentation it should be mentioned that particular calculations of the finite temperature and density contributions to the electron energy shift are carried out in the rest frame for a medium, as in the case of the photon propagation through an electron-positron plasma [63], while the electron AMM, as in the vacuum case (see, e.g., Refs [9, 11]), is determined by that part of the energy shift $\mathrm{Re}\,\Delta E_n$, which is explicitly dependent on the electron spin orientation, i.e.

$$\mathrm{Re}\,\Delta E_n(\zeta) = -\zeta H \Delta\mu\,. \tag{4.67}$$

Consider first the real part of the temperature electron energy shift in the charge-symmetric case and in the limiting case of comparatively low temperatures ($T \ll m$), when the term $\Delta E_n^{\mathrm{B}}(H,T)$ dominates in (4.66).

The ground state electron mass shift is defined by the formula [33]

$$\mathrm{Re}\big[\Delta E_0^{\mathrm{B}}\big] = \pi\alpha m\left(\frac{T}{m}\right)^2\left\{\frac{1}{3} + \frac{2\pi}{9}\frac{T}{m}\,b + \frac{16}{3}\,b^2F(b) - \frac{16}{15}\,\pi\,\frac{T}{m}\,b^3\big[13F(b) - 48F(2b) + 3bF'(b)\big]\right\}, \tag{4.68}$$

where $F(b)$ is expressed in terms of the Euler Ψ-function:

$$F(b) = \frac{1}{2}\big[\ln b - \mathrm{Re}\,\Psi(\mathrm{i}b)\big], \qquad b = \frac{a}{2\pi}, \tag{4.69}$$

and the parameter $a = eH/mT$ is introduced that determines the relative influence of the magnetic field and temperature on the electron mass shift and which is equal to the ratio of the cyclotron frequency eH/m to the temperature T.

It follows from (4.68) that in a strong magnetic field, when $eH \gg mT$, the temperature correction to the electron mass is three times smaller than the corresponding value in the field-free case [33]:

$$\mathrm{Re}\big[\Delta E_0^{\mathrm{B}}\big] = \begin{cases} \pi\alpha m\left(\dfrac{T}{m}\right)^2\left[\dfrac{1}{3} + \dfrac{2\pi}{9}\dfrac{T}{m}\,b + \dfrac{8}{3}\,b^2(\ln b + C)\right], & b \ll 1\,; \\ \dfrac{1}{9}\,\pi\alpha m\left(\dfrac{T}{m}\right)^2, & b \gg 1\,. \end{cases} \tag{4.70}$$

For the electron excited states in a constant magnetic field, when $nT \ll E_n$ (n is the principle quantum number), the temperature correction to the electron AMM has the asymptotics:

$$\Delta\mu_n(H,T) = \begin{cases} \dfrac{2\pi^2}{15}\,\alpha m\left(\dfrac{T}{E_n}\right)^3\dfrac{1}{H}\left[\dfrac{5}{6}\,\varkappa + \dfrac{\varkappa^3}{2}(35\ln\varkappa + 13\ln 2 + 35C - 2) + 2\pi n\,\dfrac{T}{E_n}\,\varkappa^2\right], & \varkappa \ll 1\,; \\ -\dfrac{4\pi^2}{225}\,\dfrac{\alpha m}{H\varkappa}\left(\dfrac{T}{E_n}\right)^3, & \varkappa \gg 1\,, \end{cases} \tag{4.71}$$

where the parameter $\varkappa = eH/\pi E_n T$ was introduced.

We emphasize that the temperature contribution to the electron AMM, like the case of $T = \mu = 0$, changes sign with growing magnetic field strength, while the field contribution to the electron mass shift at nonvanishing temperature is a sign-variable function of the parameters a and T/m under the condition $eH \ll mT$.

It was shown in Ref. [128] that in a superstrong magnetic field under the conditions

$$eH \gg Tm\,, \qquad T \gg m \tag{4.72}$$

the temperature correction to the electron mass likewise has a negative sign and is determined by the asymptotics

$$\mathrm{Re}\left[\Delta m^{\mathrm{B}}\right] \simeq -\frac{\alpha m}{4\pi} \ln^2\left(\frac{H}{H_0}\right), \tag{4.73}$$

which up to a general sign coincides with asymptotics (4.30).

In the limiting case, when $eH \ll mT$, the above results agree with those of Refs [105–107], and the temperature contribution to the gyromagnetic ratio is described by the formula

$$a_T^{\mathrm{e}} = -\frac{2\pi\alpha}{9}\left(\frac{T}{m}\right)^2. \tag{4.74}$$

Hence at $T = 300$ K there follows $a_T^{\mathrm{e}} \simeq 1.3 \times 10^{-15}$, while at $T = 6000$ K we have $a_T^{\mathrm{e}} \simeq 5 \times 10^{-13}$.

It should be particularly emphasized that the concept of a strong field in studying the physical effects at finite temperature and nonzero chemical potential is essentially different from that in vacuum. In the latter case the role of quantum effects in propagation of, say, an electron or a photon becomes of particular importance when the strength of the external field in the electron rest frame or in the reference system where the photon has the energy of order mc^2 turns out to be comparable to $H_0 = m^2c^3/e\hbar \simeq 4.41 \times 10^{13}$ G. As it yet follows from the above formulas (4.68)–(4.71), even at $H \sim mT/e = H_0 T/m \ll H_0$ all the known free-case [105–111] calculations of the electron mass shift and AMM become invalid and to obtain a correct physical result one has to account exactly for the external field influence. By way of example we mention, for instance, that at room temperature $T = 300$ K the dynamic character of the electron mass shift and AMM becomes apparent in magnetic fields with intensities $H \gtrsim 10^6$ G, which is even lower than the strength of pulsed magnetic fields $H \sim 10^7$ G reached in the laboratory.

This circumstance should be always kept in mind in analyzing the finite temperature and density effects in astrophysics [147, 147], in phenomena that arise in high-energy particle passage through the crystals whose internal electric-field intensities $\lesssim 10^{-4} H_0$ and by far exceed the ordinary laboratory values [22], in processes that accompany the heavy-ion collisions [148, 149], as well as in other branches of physics. For instance, in contemporary models of neutron stars [150] the star crust (of order 0.1 of its radius in thickness) is represented as an ion crystal lattice 'immersed' in a highly degenerate gas of relativistic electrons: the electron number density $n_{\mathrm{e}} \lesssim 10^{38}$ cm^{-3}, the temperature $T \sim 10^6 - 10^9$ K, and the magnetic field intensity $H \sim 10^{12} - 10^{14}$ G.

In the limiting case of a completely degenerate electron gas, the electron mass shift is determined by the second term in (4.66). We shall also assume condition (3.61) to be fulfilled, which is equivalent to the following restriction on the electron concentration:

$$n_{\mathrm{e}} < \frac{\sqrt{2}}{\pi^2}\, m^3 \left(\frac{H}{H_0}\right)^{3/2}. \tag{4.75}$$

Hence for $H \sim (10^{-6} - 10^2) H_0$ there follows that at $n_{\mathrm{e}} < 10^{19} - 10^{31}$ cm^{-3} only the ground level can be occupied with electrons.

Under these conditions the mass shift of the ground state electron ($n = 0$, $p_3 = 0$) is determined by the formula [127]

$$\Delta m = \frac{\alpha}{\pi}\, m \int_0^{\mu/m - 1} \mathrm{d}t\, \frac{(t+1)\exp(bt)}{\sqrt{t(t+1)}}\, \mathrm{Ei}(-bt), \tag{4.76}$$

where the parameter $b = H_0/H$, $\mathrm{Ei}(-\varkappa)$ is the integral exponent function. The similar correction to the electron mass in the field-free case takes the form [110, 127]

$$\Delta m(H=0, T=0, \mu \neq 0) = \frac{\alpha}{2\pi}\, m \left[\sqrt{\left(\frac{\mu}{m}\right)^2 - 1}\left(\frac{\mu}{m} - 2\right) - 3\ln\left(\frac{\mu}{m} + \sqrt{\left(\frac{\mu}{m}\right)^2 - 1}\right)\right], \tag{4.77}$$

where the chemical potential μ is related to the number density of the completely degenerate electron gas by the relation

$$\frac{\mu}{m} = \left[1 + \left(\frac{3\pi^2 n_{\mathrm{e}}}{m^3}\right)^{2/3}\right]^{1/2}. \tag{4.78}$$

Comparing (4.76) and (4.77) we see that in a 'superstrong' magnetic field ($2eH \gg \mu^2 - m^2$) the contribution of finite density effects to the electron mass shift highly exceeds that of (4.77):

$$R = \frac{\Delta m_{(4.76)}}{\Delta m_{(4.77)}} \simeq -\frac{1}{2}\ln\left[\left(\frac{\mu}{m} - 1\right)\frac{H_0}{H}\right]\frac{1}{\sqrt{\mu/m - 1}} \gg 1. \tag{4.79}$$

In the field-free case the one-loop correction to the electron AMM in a completely degenerate electron gas changes sign with the growing chemical potential [108, 111]:

$$\delta a_{\mathrm{e}}^{\mu} = \begin{cases} \dfrac{16\alpha}{3\pi}\,\dfrac{(3\pi^2 n_{\mathrm{e}})^{1/3}}{m_{\mathrm{e}}}, & \dfrac{\mu}{m} - 1 \ll 1; \\ -\dfrac{1}{3}\,\dfrac{\alpha}{\pi}\left(\dfrac{\mu}{m}\right)^2, & \mu \gg m. \end{cases} \tag{4.80}$$

The negative sign of the finite density contribution to the electron AMM takes place in a strong field as well, when condition (4.75) holds [127].

Having in mind that the question of the finite temperature effects influencing the rates of various physical processes [108, 151, 152], as well as the correspondence between imaginary parts (2.23) and (2.24) of the self-energy diagrams [34–36, 153], were recently discussed at length, we shall consider the general structure of the imaginary part of (4.66).

It was shown in Ref. [129] that the quantity $\mathrm{Im}\,\Delta E_n$ [see (4.66)] can be represented as the sum of three terms having the following physical meaning. The first term corresponds to the rate of synchrotron radiation by an electron with the energy E_n ($\mathrm{e}^- \to \mathrm{e}^{-\prime} + \gamma$) in a magnetic field with the statistical weight $(1 + n_{\mathrm{B}})(1 - n_{\mathrm{F}})$ minus the rate of the inverse process $\mathrm{e}^{-\prime} + \gamma \to \mathrm{e}^-$ with the weight $n_{\mathrm{B}} n_{\mathrm{F}}$. The second term corresponds to the difference between the initial electron excitation rate on account of a photon absorption ($\mathrm{e}^- + \gamma \to \mathrm{e}^{-\prime}$) with the statistical weight $n_{\mathrm{B}}(1 - n_{\mathrm{F}})$ and the rate of the inverse process $\mathrm{e}^{-\prime} \to \mathrm{e}^- + \gamma$ with the weight $n_{\mathrm{F}}(1 + n_{\mathrm{B}})$. The third term corresponds to the difference between the rate of the one-photon initial electron annihilation process with the plasma positron ($e^- + e^{+'} \to \gamma$) of the weight $n_F(1+n_B)$ and the inverse process of electron-positron pair production by a photon with the weight $n_B(1-n_F)$.

As a result, the imaginary part of the quantity ΔE_n in the real time representation, or more precisely that of the diagonal matrix element of the mass operator in terms of the time Green's functions, is determined by the formula

$$\operatorname{Im} \Delta E_n = -\frac{1}{2}(\Gamma_d - \Gamma_i), \tag{4.81}$$

where Γ_d is the sum of the rates (together with their statistical weights) characterizing the processes of the electron transition from the state with the energy E_n to other states, and Γ_i is the sum of the rates of all the inverse processes.

Application of the temperature Green's function method with the subsequent analytical continuation to the retarded mass operator results in

$$\operatorname{Im} \Delta E_n^{\mathrm{R}} = -\frac{1}{2}(\Gamma_d + \Gamma_i). \tag{4.82}$$

We emphasize that it is just the imaginary part of the retarded mass operator which has a clear physical meaning: the quantity $(\Gamma_d - \Sigma\Gamma_i)^{-1}$, where $\Sigma = -1$ in the fermionic case, and $\Sigma = +1$ in the bosonic case, determines the relaxation time of the system in the case of small deviations from the equilibrium state [129, 153].

For the electron ground state ($n = 0$, $p_3 = 0$) and low temperatures ($T \ll m$) in the charge-symmetric case, Eqn (4.82) gives rise to the expression correct to the terms exponentially suppressed in the parameter $\exp(-m/T)$ [129]:

$$\operatorname{Im} \Delta E_0 = -\frac{4}{3}\alpha m\left(\frac{T}{m}\right)^2\left(\frac{eH}{mT}\right)^2\left[\exp\left(\frac{eH}{mT}\right) - 1\right]^{-1}. \tag{4.83}$$

This result corresponds to the induced dipole transition $n = 0 \to n' = 1$ in a magnetic field due to absorption of a photon with a frequency equal to the cyclotron frequency $\omega = eH/m$. It is seen from formula (4.83) that the rate of an electron excitation from the ground state through the one-photon absorption at $H = 0$ is, as it should be expected, equal to zero. This contradicts the corresponding result of Ref. [154].

We shall discuss now the dynamic nature of the energy shift and AMM of the massive Dirac neutrino in an electron-positron plasma placed in a constant magnetic field [130].

As it was mentioned above, the vacuum AMM of the Dirac neutrino with the mass m_ν in the Standard Model of electroweak interactions is determined by the expression

$$\mu_\nu^0 = \frac{3eG_F m_\nu}{8\pi^2\sqrt{2}} \simeq 3 \times 10^{-19}\, \frac{m_\nu}{1\,\mathrm{eV}}\, \mu_B, \tag{4.84}$$

and in magnetic fields with the intensity $H \ll M_W^2/e \simeq 10^{24}$ G the field contribution to the neutrino AMM is small as compared with its static value (4.84). In some extended models of electroweak interactions the neutrino magnetic moment is proportional not to neutrino mass but rather to the charged lepton mass and it can reach the values of $\mu_\nu \sim 10^{-13}\mu_B$ [144, 155, 156]. Considerable interest to the 'question of consistency' [155] of a small mass and a large neutrino magnetic moment is related, in particular, to the above-mentioned solar neutrino problem, one of whose possible explanations is accepted by the hypothesis of a resonance enhancement of the neutrino oscillations in the solar matter together with the effect of the neutrino chirality flip. To explain the solar neutrino problem in the framework of the latter hypothesis, the neutrino magnetic moment has to be estimated as $\mu_\nu \gtrsim (10^{-11} - 10^{-10})\mu_B$. Since the Majorana neutrino possesses no intrinsic magnetic moment, then in a situation with the comparatively large magnetic moment of a Dirac neutrino a distinction between DN and MN can be made with the study of their electromagnetic properties.

Though at present the most appropriate explanation for the solar neutrino problem [157] is probably related to the Mikheev–Smirnov–Wolfenstein effect of resonance neutrino oscillations in a nonuniform medium [158–160], investigation of the neutrino electromagnetic interaction, and especially in extremal astrophysical conditions, is of definite interest.

The Dirac neutrino electromagnetic vertex in a collisionless isotropic electron gas (plasma with stationary ions), as well as in an e^-e^+-plasma was considered in a number of papers (see Refs [113–116, 119]). The same problem was considered in Ref. [112] for the case of a classical nonrelativistic magnetoactive plasma.

In Ref. [112], it was mentioned that in contrast to the vacuum situation, the MN moving in a dispersive medium emits electromagnetic waves similar to the DN case, i.e. electromagnetic characteristics of MN and DN in a medium coincide, while an induced magnetic moment does not lead to any alteration of the neutrino chirality.

Description of various effects of neutrino interactions in dispersive media (see, e.g., Ref. [112]) that is based upon the electromagnetic vertex calculated in the above-mentioned papers, becomes invalid in intense external fields, and a dynamic nature of corresponding physical quantities should be taken into account.

The study along this line was initiated in a recent work [130], where a contribution was considered of a weak charged current to the energy shift and the AMM of a DN moving in an electron-positron plasma at finite temperature ($T \ll M_W$) in a constant magnetic field.

We shall discuss here the case when a relativistic neutrino moves in a completely degenerate electron gas in a direction perpendicular to the magnetic field.

With the proviso that

$$H \ll \frac{M_W^2}{e} \simeq 10^{24}\,\mathrm{G}, \qquad M_W \ll E_\nu \ll \frac{M_W^2}{\mu}, \tag{4.85}$$

where μ is the chemical potential of an electron gas, the contribution of the finite density effects to the neutrino AMM has the asymptotics [112, 130]:

$$\Delta\mu_\nu = \begin{cases} -\dfrac{16}{3}\,\dfrac{(3\pi^2 n_e)^{1/3}}{m_\nu}\,\mu_\nu^0, & 2eH \ll \mu^2 - m^2; \\[2ex] \dfrac{32\pi^2}{3}\,\dfrac{n_e}{m_\nu M_W^2}\left(\dfrac{E_\nu}{M_W}\right)^2\mu_\nu^0, & 2eH \gg \mu^2 - m^2. \end{cases} \tag{4.86}$$

Therefore, in a dense electronic medium the induced neutrino magnetic moment can considerably exceed its static value (4.84) both in the cases of relatively strong and weak fields, and being negative in the case of weak fields it increases and becomes positive with growing magnetic field strength.

For instance, in the case of relatively strong fields when

$$n_e \approx \frac{1}{2}\pi^{-2}\left(\frac{H}{H_0}\right)^{3/2} m^3, \qquad H = H_0 \quad \text{and} \quad E_\nu \simeq 10^3 M_W$$

we obtain from (4.86)

$$\Delta\mu_{\mathrm{v}} = 10\mu_{\mathrm{v}}^{0},$$

while in the case of weak fields at $n_{\mathrm{e}} = (10^{24} - 10^{38})$ cm^{-3} it follows from (4.86) that

$$\Delta\mu_{\mathrm{v}} \approx (50 - 5\times 10^{4})\mu_{\mathrm{v}}^{0}.$$

Thus the question about 'compatibility' of a small neutrino mass and its large magnetic moment is answered in the affirmative in this problem and for this purpose one does not need to go beyond the framework of the Standard Model. This is due to the fact that the neutrino AMM is directly proportional to the medium density, and not to the neutrino mass as it holds for the vacuum part of the induced magnetic moment. We note that similar behaviour is also characteristic of the finite density contribution to the electron AMM [see (4.80)]. The generalization of the known Wolfenstein formula [158, 160] to the case of a strong external field, carried out in Ref. [130], may also prove to be important in studying the oscillation characteristics of the neutrino beam in matter.

5. Conclusions

In the current review we considered various physical effects related to the photon and lepton propagation across external electromagnetic fields at finite temperature and nonvanishing density of medium.

In the first place we presented basic methods that are used in the study of the fermion and boson dynamics in the framework of the finite-temperature quantum field theory in the presence of external fields. In particular, general relations between the retarded, advanced and time Green's functions in QFT were determined, which enabled us to put an end to the recent discussion the scientific in literature about the relation between the real and imaginary parts of the self-energy diagrams.

We presented various ways of describing thermodynamic properties of systems in the framework of QFT both by computing Green's functions, and by evaluating the thermodynamic potential with the use of the real and imaginary time methods.

The methods described were used further to describe various radiative effects in an electron-positron plasma. The photon polarization operator and photon dispersion in an external field were studied. The calculations of the radiative shift of the photon mass in a magnetic field, crossed fields and in the field of an electromagnetic wave were also presented. This enabled us to obtain the index of refraction for various modes of electromagnetic waves propagating across an external field, and to describe the polarization vectors of the eigenmodes.

The effect of rotation involving the plane of the wave linear polarization in the field of circularly polarized wave was also analyzed.

Possibility of a photon decay into electron-positron pairs in an external field was demonstrated, and the thresholds pertaining to electron-positron pair production by a photon in a magnetic field were found.

Finally, the finite contribution to the photon polarization operator in a magnetic field was allowed for, and the Debye screening radius as a function of temperature and the magnetic field strength was studied.

In the second part of the review various effects that arise in the fermion propagation across external fields at finite temperature were examined. In particular, contributions of the W- and Z-bosons (the weak interaction mediators) as well as of the Higgs particles together with the photon contribution to the electron mass operator were investigated, and the imaginary part of the mass operator is considered from the point of view of possible electron radiative transitions in an external field.

It was pointed out that an electron in an external field together with the anomalous magnetic moment can also have an anomalous electric moment. Its value is analyzed from the viewpoint of a possible CP-invariance violation and corresponding experimental conditions, as well as of possible models of such violation.

Contributions of finite temperature and density were also considered for.

The radiation mass shift and the anomalous magnetic moment of an electron are investigated as functions of an external magnetic field strength, electromagnetic wave field intensity, and also in the case of their simultaneous application, which is important in connection with the analysis of the results of measuring the electron anomalous magnetic moment by using the spin-resonance Rabi method. The results of calculating the magnetic field and the electromagnetic wave field contributions to the anomalous magnetic moment are compared with the third and forth order radiative corrections in terms of the fine-structure constant, which demonstrates that the modern experimental situation is quite favourable for measurements of the field contributions.

A particular attention was paid to the analysis of the electric field contributions to the electron mass shift both in the quantum and classical regions. A combined action of a strong magnetic field and a hot dense medium on the radiative energy shift and the electron anomalous magnetic moment was investigated.

The fundamental problem of the relation between the real and imaginary parts of the retarded electron mass operator at finite temperature was also considered here, the imaginary part being demonstrated to have a particular physical meaning related to the possible electron transitions.

In a special section we deal with the study of the neutrino electromagnetic properties. In particular, the neutrino anomalous magnetic moment was analyzed as a function of the electric and magnetic fields intensities, as well as of the neutrino energy. The neutrino anomalous electric moment was demonstrated to arise in the combination of electric and magnetic fields. The results of calculating the neutrino decay and the photon emission in electric and magnetic fields were presented. Special attention was paid to the neutrino chirality flip and possible astrophysical applications of this effects in regard both to the solar neutrino, and the supernova neutrino.

Nontrivial results were obtained when the role of the medium in the formation of the neutrino anomalous moment and the radiation shift of its mass was allowed for. Thus, the medium essentially enhances the action of the magnetic field, and in a strong field the neutrino mass radiative correction found in medium substantially exceeds that obtained in the case of a vanishing field.

Furthermore, the very important result is presented that in a dense electronic medium the neutrino anomalous magnetic moment essentially exceeds its static value in a vanishing field and without matter. Thus, it was shown that a small neutrino mass and a comparatively large value of its magnetic moment can be compatible without going beyond the framework of the Standard Model. A generalization of the Wolfenstein formula for the neutrino in medium to the case of a strong magnetic field was also made.

The effects involved were studied with the help of the universal QFT methods in external fields at finite temperature and density of matter. Their further applications can be found in investigating phase transitions in the hadronic medium and in the quark-gluon plasma, as well as the phase transitions and the influence on them of external fields in the unified theory of weak and electromagnetic interactions, and in the study of new particles production in the framework of extended models of elementary particles interaction under extremal conditions of strong fields at finite temperature and in dense medium, etc. A part of this wide range of problems will be considered in our future publications.

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