

FROM THE CURRENT LITERATURE

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Basic principles for determining the components of the Earth's magnetic field on moving ferromagnetic objects (fundamentals of field separation theory)†

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Abstract. A method for determining the Earth's magnetic field components on a moving object having its own magnetic field is presented. A simplified scheme is considered to illustrate the method of problem solution. Equations yielding a correct solution for both the simplified scheme and a real moving ferromagnetic object are derived. Technological problems are listed that can be solved by measuring the Earth's and other magnetic fields onboard a moving object with an arbitrary magnetic field of its own.

Poisson equations are extensively used in studies designed to develop methods for determining the parameters of the Earth's magnetic field (EMF) on moving ferromagnetic objects possessing their own magnetic fields [1]. It is mostly logical to conclude, based on the structure of these equations, that the intrinsic fields of the object ferromagnetic masses (FM) constitute the main obstacle precluding geomagnetic measurements on the object, and that their effect can most rationally be ruled out using the compensation principle. It has long been thought that the problem can be solved provided an effective approach to its practical realization is available. This viewpoint has until recently governed the main line of research in this field. However, all the methods proposed for the purpose have serious drawbacks. The fact is that FM intrinsic fields can only be measured with a certain degree of approximation through unpredicted variations of FM magnetization, which excludes the possibility of constructing a correct solution [2]. Nevertheless, there are currently a large number of original methods which collectively make up a whole theory based on the compensation principle. This theory has to be called the Poisson deviation theory (PDT) since S D Poisson was the first to notice its usefulness for the solution of the problem.

Further studies have led to the development of a method for determining the angular components (MDAC) which is essentially different from those discussed in the previous paragraph [3 – 6]. The method employs the idea of using the information contained in the components of intrinsic fields of magnetically soft FMs magnetized by EMF. The use of this physical concept radically modifies the approach to the solution of the problem. It allows the compensation principle to be abandoned since the FM fields of an object are considered as sources of useful information rather than perturbations. Nor does the presence of FMs near a component magnetometric transducer decreases an error of component EMF measurements. The method of data collection is also different. Measurements are grouped by series, each containing a certain number of simultaneous readings of all measuring systems (a measurement cycle). A moving object is expected to meet the sole condition of reorientation to very small angles in at least any two mutually orthogonal planes. Usually, this condition is satisfied automatically because a moving object spontaneously undergoes such oscillations. Results of each measuring series provide the basis for the system of vector equations whose solution yields parameters of the external (relative to the moving object) magnetic field. Such a field may be the EMF, the interplanetary field, a sum of the EMF and the intrinsic field of an article located at a certain distance from the object (e.g., a pipeline, power cable, various metal items, etc.).

The use of this principle allowed the development of correct methods for solving the problem of the determination of angular EMF components. Major principles of MDAC and results of real studies obtained with the help of this method onboard a ferromagnetic ship were reported in Refs [4 – 6].

However, MDAC is suitable only for measuring angular EMF components, for instance, magnetic inclination and course of the object. This restricts the useful information and allows only the direction of the EMF induction vector to be found in a given system of coordinates. It is argued in Refs [4 – 6] that power components of the field measured in nanoteslas cannot be correctly determined from measurements made only on a ferromagnetic object because the structure of the Poisson vector equation [5] precludes separation of EMF components from similarly oriented components of the object's intrinsic magnetic field (OIMF):

$$\mathbf{T}' = \big(|E| + |P|\big)|S|\mathbf{T} + \mathbf{T}_\mathrm{p}\,, \tag{1}$$

where $\mathbf{T}'$ is the induction vector of the total magnetic field (TMF); $|E|$, $|P|$ are the unit matrix and the Poisson parameter matrix, respectively; $|S| = |S_i||S_j||S_k|$ is the rotation matrix which defines the position of the object after turns to angles i, j, k in the system of coordinates resting in space; $\mathbf{T}$ is the EMF

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induction vector and, finally,

$$\mathbf{T}_{\mathrm{p}} = \begin{vmatrix} X_{\mathrm{p}} \\ Y_{\mathrm{p}} \\ Z_{\mathrm{p}} \end{vmatrix}$$

is the induction vector of the object's constant magnetic field.

It is impossible to differentiate between the components $1+a$, $1+e$, $1+k$ and calculate the power components of the EMF (X, Y, Z) remaining in the framework of the methodology for the collection of information implied by Eqn (1), which may be written in the expanded form as

$$\mathbf{T}' = \begin{vmatrix} 1+a & b & c \\ d & 1+e & f \\ g & h & 1+k \end{vmatrix} |S| \begin{vmatrix} X \\ Y \\ Z \end{vmatrix} + \begin{vmatrix} X_{\mathrm{p}} \\ Y_{\mathrm{p}} \\ Z_{\mathrm{p}} \end{vmatrix}, \qquad (2)$$

where the summarized matrix is

$$|E| + |P| = \begin{vmatrix} 1+a & b & c \\ d & 1+e & f \\ g & h & 1+k \end{vmatrix}.$$

Figure 1 shows three OIMF components oriented similar to the corresponding EMF components. This accounts for the above inference. However, further studies have demonstrated that such a categorical conclusion is erroneous. A more cautious inference appears to be relevant: it is impossible to determine the power components from measurements onboard a moving ferromagnetic object remaining in the framework of MDAC.

The information obtained by MDAC has been used for the development of a more universal technique referred to as the component determination method (CDM), which is based on the possibility of significantly modifying the structure of Poisson equation and allows, in principle, for the differentiation between components of an external magnetic field, in particular, EMF, and the intrinsic magnetic field of a moving object. In fact, MDAC and CDM constitute the basis of a theory which is essentially different from PDT. Suffice it to recall only one important difference mentioned above. In PDT, the intrinsic fields of the object FMs are the main obstacle which interferes with EMF measurements on a moving object, while in the theory being described the same fields are a source of information about the field which magnetize them, EMF in particular. We call it the field separation theory (FST). Further studies have demonstrated that the physical principles of FST are in better agreement with FM magnetization and remagnetization processes as compared to PDT. From this point of view, FSP is superior to PDT. FSP allows both angular and power components of an external magnetic field to be correctly determined from the measurements made onboard a moving object. FST practically rules out the dependence of the accuracy of measuring EMF components on the level and gradient of FM intrinsic fields at the point of magnetic measurements. Also, the method of solution is essentially simplified. There is no limitation on the processes of magnetization and remagnetization of FMs of a moving object, and the problem can be correctly solved in the most general form in compliance with the three criteria introduced by Tikhonov (see, for instance, Ref. [2]): the possibility of a solution, its uniqueness and stability. There is only one requirement that a moving object is expected to meet, namely, it must be subject to reorientation in the field measured in any two mutually orthogonal planes. The reorientation angles are rather small. To meet the current requirement, the angles must be of the order $\pm 0.5°$. Usually, such requirements are satisfied by practically all objects travelling in a 'free' space, namely, under and at the surface of water, in the atmosphere and in space. If necessary, it is possible to narrow the range of required angular changes.

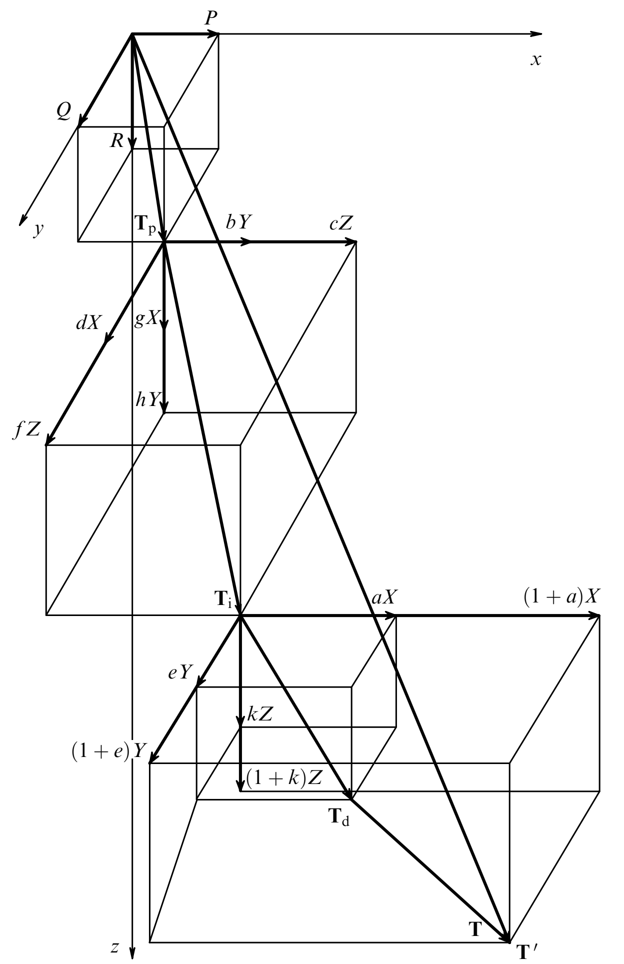

Figure 1. Formation of the total magnetic field on a moving ferromagnetic object (MFO): $\mathbf{T}, \mathbf{T}'$ — EMF and TMF vectors, $\mathbf{T}_{\mathrm{p}}(P, Q, R)$ — vector and components of the MFO constant field, $\mathbf{T}_{\mathrm{i}}(dX, gX, bY, hY, cZ, fZ)$ — vector and components of the induced MFO field, $\mathbf{T}_{\mathrm{d}}\left[(1+a)X, (1+e)Y, (1+k)Z\right]$ — vector and components of the field representing the sum of EMF components and those of induced MFO intrinsic field of the same direction.

FSP implies the solution of the problem formulated in the following way. Let an arbitrarily moving (in space) object experience a certain vector (external) field which may have a nature other than electromagnetic. Furthermore, let the object contain elements which interact with this field and generate the intrinsic field consisting of a number of components (in the general case, nine) whose moduli and directions depend on the modulus and direction of the external field. The transformation law for the external field is given by a matrix operator (usually, a second-rank tensor) with coefficients which, in general, are different and randomly vary in time. The object is able to generate and emit a certain constant intrinsic field of the same nature (residual field). Therefore, at any point of space where the object's field is present, it is added to the external field. This process is responsible for the formation of the total vector field. It is necessary to find all the components of the external field and the object's intrinsic field based on the results of measuring the total vector field. Let us consider an approach to the solution of this problem using, as a case in point, the interaction between EMF and a moving ferromagnetic object.

The physical principles of this approach may be illustrated by considering the formation of the combined magnetic field of an instrument, which consists of a ferromagnetic rod (FR) carrying a rigidly attached transducer magnetically sensitive to the field components (MST) and an additional ferromagnetic mass (AFM) the intrinsic field of which influences the FR. Therefore, the total magnetic field consists of the EMF, the FR's intrinsic field, and the AFM's intrinsic field. Let us assume that measurements are made in the plane of the geomagnetic meridian, with the EMF horizontal component H and vertical component Z. The FR can be reoriented in this plane to an angle j (Fig 2). The AFM moves together with FR and changes its position with respect to the FR. Let us further assume that the FR's intrinsic field at the MST fixation point depends on the sole Poisson parameter a. The AFM's intrinsic field is also determined by one Poisson parameter a_1, which becomes a_2 when AFM changes its position relative to MST. With these assumptions, the Poisson equation for the initial AFM position will take the form

$$T_1' = (1 + a + a_1)H\cos j + (1 + a + a_1)Z\cos j + P + P_1\,, \tag{3}$$

where P, P_1 are constant components of the FR and AFM intrinsic fields, respectively.

If the AFM position relative to MST is modulated much faster than the FR orientation in the EMF (a change of angle j), the similar equation for another AFM position has the form

$$T_2' = (1 + a + a_2)H\cos j + (1 + a + a_2)Z\sin j + P + P_2\,. \tag{4}$$

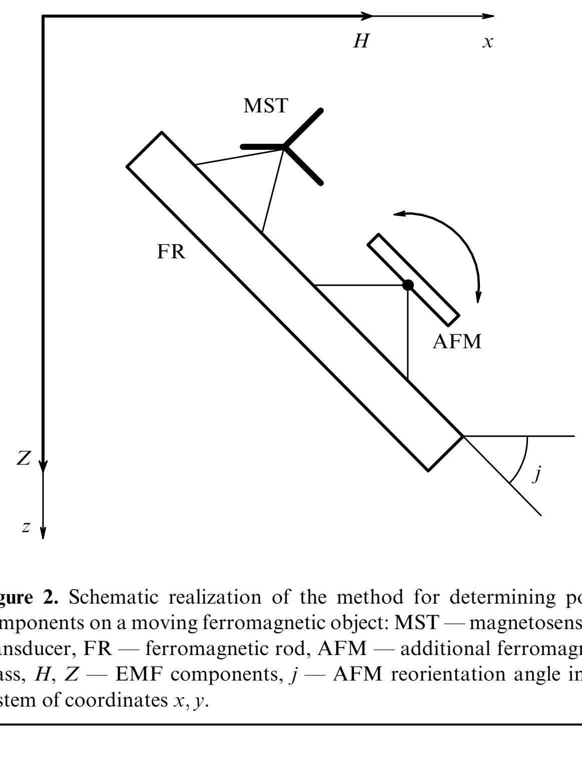

Figure 2. Schematic realization of the method for determining power components on a moving ferromagnetic object: MST — magnetosensitive transducer, FR — ferromagnetic rod, AFM — additional ferromagnetic mass, H, Z — EMF components, j — AFM reorientation angle in the system of coordinates x, y.

The difference between (3) and (4) is

$$T_1' - T_2' = (a_1 - a_2)H\cos j + (a_1 - a_2)Z\sin j + P_1 - P_2\,. \tag{5}$$

It follows from (5) that *the difference between the combined fields does not depend on the FR's intrinsic magnetic field.* Three cycles of such measurements lead to a system of equations whose solution yields the following quantities:

$$H_\mathrm{r} = (a_1 - a_2)H\,,$$
$$Z_\mathrm{r} = (a_1 - a_2)Z\,,$$
$$P_\mathrm{r} = P_1 - P_2\,. \tag{6}$$

These, in turn, are used to obtain the EMF angular components in compliance with the principles of MDAC [3–5].

For the power components H and Z to be determined, one needs a method of separation between the co-factors $a_1 - a_2$ and H, $a_1 - a_2$ and Z. For this, measurements are made when the AMF experiences a known magnetic field X_s. In this case, Eqns (3) and (4) take the form

$$T_{1\mathrm{s}}' = (1 + a)H\cos j + (1 + a)Z\sin j + a_1(H\cos j + Z\cos j + X_\mathrm{s}) + P + P_1\,, \tag{7}$$

$$T_{2\mathrm{s}}' = (1 + a)H\cos j + (1 + a)Z\sin j + a_2(H\cos j + Z\sin j + X_\mathrm{s}) + P + P_2\,. \tag{8}$$

The difference between (7) and (8) appears as

$$T_{1\mathrm{s}}' - T_{2\mathrm{s}}' = (a_1 - a_2)H\cos j + (a_1 - a_2)Z\sin j + (a_1 - a_2)X_\mathrm{s} + P_1 - P_2\,. \tag{9}$$

The difference between (8) and (5) is of the form

$$(T_{1\mathrm{s}}' - T_{2\mathrm{s}}') - (T_1' - T_2') = (a_1 - a_2)X_\mathrm{s}\,. \tag{10}$$

The difference between the Poisson parameters calculated from Eqn (10) will look like

$$a_1 - a_2 = \frac{(T_{1\mathrm{s}}' - T_{2\mathrm{s}}') - (T_1' - T_2')}{X_\mathrm{s}}\,. \tag{11}$$

It follows from (11) that a series of measurements by the above method may be effectively used to find the difference between Poisson parameters entering the system of equations based on Eqns (5) and having the solution (6). The latter serves to calculate *the power components of the EMF or any other magnetic field external in respect to the moving ferromagnetic object*:

$$H = \frac{H_\mathrm{r} X_\mathrm{s}}{\Delta T_\mathrm{s} - \Delta T_1}\,, \tag{12}$$

where $\Delta T_\mathrm{s} = T_{1\mathrm{s}}' - T_{2\mathrm{s}}'$, and $\Delta T_1 = T_1' - T_2'$. The components (aH and aZ) of the induced magnetic intrinsic field of FR and the constant field intensity P are calculated based on the principles of MDAC.

Let us now consider the possibility of solving the same problem for the case of a real moving object without any limitation on the problem's generalization. Suppose that a moving ferromagnetic object (MFO) has an MST fixed on it together with an AFM, whose intrinsic field affects MST. The AFM can be reoriented relative to the MST. In this case, the Poisson vector equation (2) for one of the AFM positions will have the form

$$\mathbf{T}_1' = \begin{vmatrix} 1+a+a_1 & b & c \\ d & 1+e+e_1 & f \\ g & h & 1+k+k_1 \end{vmatrix} |S|\mathbf{T} + (\mathbf{T}_{\mathrm{p}} - \mathbf{T}_{1\mathrm{p}})\,. \tag{13}$$

The same equation (13) for another AFM position takes the form

$$\mathbf{T}_2' = \begin{vmatrix} 1+a+a_2 & b & c \\ d & 1+e+e_2 & f \\ g & h & 1+k+k_2 \end{vmatrix} |S|\mathbf{T} + (\mathbf{T}_{\mathrm{p}} - \mathbf{T}_{2\mathrm{p}})\,. \tag{14}$$

If the measurements are made in such a way that the equality of matrices $|S|$ in Eqns (13) and (14) is conserved, then we arrive at the following relationship

$$\mathbf{T}_1' - \mathbf{T}_2' = \begin{vmatrix} a_1-a_2 & 0 & 0 \\ 0 & e_1-e_2 & 0 \\ 0 & 0 & k_1-k_2 \end{vmatrix} |S|\mathbf{T} + (\mathbf{T}_{1\mathrm{p}} - \mathbf{T}_{2\mathrm{p}})\,. \tag{15}$$

A series of measurements taken by this method yields a system of vector equations whose solution is markedly simplified due to the transformation of the square matrix in Eqn (2) to the diagonal matrix of Eqn (15). Moreover, the matrix of Eqn (15) consists of Poisson parameters formed by magnetization of the AFM rather than the object's FM.

Therefore, the solution of the system based on Eqns (15) leads to the following quantities adequate to the constituents of the AFM intrinsic field:

$$\begin{aligned} X_{\mathrm{r}} &= (a_1-a_2)X\,, \\ Y_{\mathrm{r}} &= (e_1-e_2)Y\,, \\ Z_{\mathrm{r}} &= (k_1-k_2)Z\,, \\ T_{\mathrm{r}} &= T_{1\mathrm{p}} - T_{2\mathrm{p}}\,. \end{aligned} \tag{16}$$

It has been demonstrated that the power components X, Y, Z can be found by separating the co-factors of the products (16). Let us employ the method using the calibrated fields X_{s}, Y_{s}, Z_{s}.

First, let us include the calibration field sources closely associated with the AFM to the measuring system. These fields in conjunction with the corresponding EMF components affect the AFM and induce a magnetic moment in the latter, which is in turn responsible for the magnetic field differing from the EMF by the calibration field component. Measurements of the total magnetic field are feasible when the source of the calibration field is either switched off or switched on. In the latter case, the difference between the Poisson equations has the form

$$\mathbf{T}_{1\mathrm{s}}' - \mathbf{T}_{2\mathrm{s}}' = \begin{vmatrix} a_1-a_2 & 0 & 0 \\ 0 & b_1-b_2 & 0 \\ 0 & 0 & c_1-c_2 \end{vmatrix} |S|(\mathbf{T}+\mathbf{T}_{\mathrm{s}}) + (\mathbf{T}_{1\mathrm{p}} - \mathbf{T}_{2\mathrm{p}})\,, \tag{17}$$

by analogy with the equation difference (15). The solution to the system of (17)-like equations allows the components of the OIMF to be determined as follows:

$$\begin{aligned} X_{\mathrm{s}}' &= (a_1-a_2)(X+X_{\mathrm{s}})\,, \\ Y_{\mathrm{s}}' &= (b_1-b_2)(Y+Y_{\mathrm{s}})\,, \\ Z_{\mathrm{s}}' &= (c_1-c_2)(Z+Z_{\mathrm{s}})\,. \end{aligned} \tag{18}$$

Having found the difference between the corresponding components in expressions (16) and (18), one obtains

$$\begin{aligned} X_{\mathrm{s}}' - X_{\mathrm{r}}' &= (a_1-a_2)X_{\mathrm{s}}\,, \\ Y_{\mathrm{s}}' - Y_{\mathrm{r}}' &= (b_1-b_2)Y_{\mathrm{s}}\,, \\ Z_{\mathrm{s}}' - Z_{\mathrm{r}}' &= (c_1-c_2)Z_{\mathrm{s}}\,. \end{aligned} \tag{19}$$

This expression may be used to calculate the difference between the Poisson parameters for the AFM available at the time of measurement. Their known values serve to determine the EMF power components

$$\begin{aligned} a_1-a_2 &= \frac{X_{\mathrm{s}}' - X_{\mathrm{r}}'}{X_{\mathrm{s}}}\,, \\ b_1-b_2 &= \frac{Y_{\mathrm{s}}' - Y_{\mathrm{r}}'}{Y_{\mathrm{s}}}\,, \\ c_1-c_2 &= \frac{Z_{\mathrm{s}}' - Z_{\mathrm{r}}'}{Z_{\mathrm{s}}}\,. \end{aligned} \tag{20}$$

If the difference between the Poisson parameters for the AFM is found from (20), then it is possible to calculate the MST power components using the corresponding Eqns (16):

$$\begin{aligned} X &= \frac{X_{\mathrm{r}}X_{\mathrm{s}}}{X_{\mathrm{s}}' - X_{\mathrm{r}}'}\,, \\ Y &= \frac{Y_{\mathrm{r}}Y_{\mathrm{s}}}{Y_{\mathrm{s}}' - Y_{\mathrm{r}}'}\,, \\ Z &= \frac{Z_{\mathrm{r}}Z_{\mathrm{s}}}{Z_{\mathrm{s}}' - Z_{\mathrm{r}}'}\,. \end{aligned} \tag{21}$$

A comparison of Eqns (12) and (21) leads to an interesting conclusion that the equations for determining the power components of an FR with the simplest intrinsic magnetic field are similar in terms of complexity to the equations for determining the power components on a ferromagnetic object having an intrinsic magnetic field with absolutely unpredictable characteristics. What is necessary is to follow the technology developed for the realization of MDAC and take into account the processes of FM remagnetization on moving ferromagnetic objects.

The principal advantage of the FST is the possibility of studying geomagnetic and cosmic magnetic fields using practically any type of moving objects. There are no special requirements for the construction of this apparatus in terms of intrinsic field properties. Moreover, with the use of this technique, the effect of the intrinsic fields of moving objects on external magnetic field measurements may be utterly disregarded.

Apart from basic EMF and interplanetary magnetic field studies, there is now ample opportunity to solve a variety of applied problems by measuring the magnetic field *external* with respect to the moving apparatus, which is the sum of the

EMF and the field of the ferromagnetic object with the measuring device mounted on it. These fields are easy to separate based on the difference between their spectral characteristics. Analysis of the constituents of an object field allows for the determination of both the standard field intrinsic in the newly designed construction and deviations from it during exploitation. Examples of applied problems which can be solved with the help of magnetometric techniques are listed below:

(1) geomagnetic studies on MFOs;

(2) determination of the magnetic course of the MFO;

(3) terrestrial and underwater geomagnetic navigation of an MFO;

(4) detection and mapping of main gas and oil pipelines;

(5) technical assessment of the gas and oil pipelines and the discovery of accidents;

(6) detection and mapping of power cable lines;

(7) technical assessment of power cable lines and the discovery of breakage and insulation damage;

(8) demagnetization of various objects (ships);

(9) positioning of semisubmerged boring machines (commissioning, fixation of the critical deviation of the boring rig axis from that of the submarine bore-hole (SBH), the secondary search for a SBH following its scheduled or emergency closure; all power and control systems may be installed on the boring rig);

(10) search for underwater and underground metal objects directly from an MFO;

(11) exploratory drilling for gas and oil.

All these operations may be carried out using practically any type of MFO including motorvessels and submarines, diversified aircraft (even very small ones), satellites and other spacecraft. It has been mentioned above that no special requirements are established with respect to the MFO intrinsic field level and gradient.

At present, a measuring system is available which was previously used to develop and improve techniques for geomagnetic measurements on an MFO (a ferromagnetic ship). The testing program included measurements on a calm sea surface and during a heavy gale (wind force 8). This allowed methods to be devised for the solution of the above problems. The development of a concrete measuring system implies a coordination between the technical characteristics of the instrument and the type of moving object designed to carry it. There are always several solutions which make it crucial to choose the most rational one in terms of precision, reliability, and operative costs. The solution of any of the above problems implies the construction of a cheap working model whose technical parameters meet the customer's requirements (which usually broaden with the project's progress). All the scheduled tests being completed, the model may be installed on the moving object.

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