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METHODOLOGICAL NOTES

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A special case of Brownian motion and Einstein's law

T S Velichkina, I A Yakovlev

Observation of Brownian movement and verification of Einstein's quantitative theory of this phenomenon are indispensable elements in a course of molecular physics. However, a large amount of preliminary work as described by J Perrin [1] is needed to study properly the matter. A major inconvenience of the traditional approach is the necessity to have freshly prepared emulsion in each new observation session for adhesion and precipitation of suspended particles to be avoided.

Another difficulty encountered in a quantitative survey of Brownian movement is related to the three-dimensional motion of these particles. Their vertical displacements cannot be taken properly into account and they disrupt the sequential measurements of horizontal displacements. Moreover, the particles tend to withdraw from the operation area of the microscope lens.

No wonder, reports on the observation of Brownian movement in a way other than that suggested by Perrin are lacking in the literature. It is even more so as regards verification of the Einstein theory.

We have been advised by professor G G Lemmlain that Brownian movement can be observed in a natural object showing two-dimensional Brownian motion and *always ready for the purpose*.

Some natural crystals of beryl ($Be_3Al_2Mg_6O_{18}$) formed in a past geological era contain specific liquid inclusions of salt solutions that gave rise to the mineral. Drops of solution were enclosed during crystallization of beryl and conserved inside resulting crystals. The solution was later separated into the aqueous salt fraction and carbon dioxide released from the solution. At a temperature below 31°C, CO_2 can undergo further partitioning between two phases, liquid and gaseous. The latter phase is a small bubble floating atop the liquid surface.

Such an inclusion in a beryl crystal is shown in Fig. 1 which displays all the above constituent components, viz. aqueous solution of double Al-Mg salt of beryllium, liquid carbon dioxide (*1*), and gaseous CO_2 bubble (*2*).

What is necessary to observe the Brownian movement is a beryl crystal cut into thin slices with polished surfaces. The light CO_2 bubble is always pressed to the top surface by

T S Velichkina, I A Yakovlev Department of Physics,
M V Lomonosov Moscow State University
Vorob'evy Gory, 119899 Moscow, Russia
Tel. (095) 939-14-30
E-mail: iakovlev@crus272phys.msu.su

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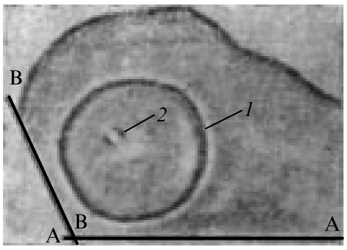

Figure 1.

Archimedes's force, where it remains in *perpetual two-dimensional motion* unaffected by convection and easy to observe with a microscope, the bubble being ca. 1 μm in diameter. One cut section may exhibit more than one inclusion, each containing a bubble of different size. Moving the section over the specimen slide of a microscope, it is easily seen that small bubbles move faster than large ones. Also, it is possible to demonstrate these movements in projection, taking care that the preparation being examined is not overheated above 31 °C by more intensive luminous flux.

A quantitative study of the Brownian movement [2] may be conducted on the same preparation by recording and precisely measuring consecutive displacements of the gas bubble. This is achieved by filming the bubble through a microscope. It appears from the experience that 10 shots are usually enough for successful experimentation.

The developed film section clamped between two glass plates is placed on the object table of a projector. Images of inclusions on a sheet of white paper are of the same size as in Fig 1. Positions of the gas bubble projected from each consecutive shot by manipulating micrometer control knobs of the projector, are marked off with a sharpened pencil and numbered. Each new bubble image is normally displaced relative to the previous one by a few millimetres.

However, it is important to ensure location of true bubble displacements due to Brownian movement and distinguish them from random shifts of images caused by film displacement. There is a special procedure for this purpose. Specifically, the most prominent features of the inclusion are outlined on the paper and each next image is matched to the initial contour by properly fitting the table position with control knobs. This allows the true displacement of the bubble between two film expositions to be recorded.

The entire film section having been looked through distances between the numbered bubble positions, are measured in different combinations using a pair of compasses whose needles are stuck into the pencil marks on the paper sheet showing positions of the bubble. Straight lines between the needles are the distances of interest measured to millimetre-scale.

Conspicuous movements of the gas bubble in our experiment are due to unusually high diffusion coefficient $D \sim 1/r\eta$, where r is the bubble radius and η is the viscosity of CO_2. Very low η near the critical point for CO_2 is as important as r of micron dimensions. Evidently, neither water nor alcohol at room temperature are suitable for the purpose of the experiment.

In order to avoid inaccuracy in data processing, here is a two-column table showing the difference between gas bubble positions recorded in one or two time intervals between fixation of results. The time interval τ between expositions of two adjacent shots is arbitrarily taken equal to unity.

$\tau = 1$			$\tau = 2$		
Point combination	Δs	$(\Delta s)^2$	Point combination	Δs	$(\Delta s)^2$
$1-2$	Δs_{12}	$(\Delta s_{12})^2$	$1-3$	Δs_{13}	$(\Delta s_{13})^2$
$2-3$	Δs_{23}	$(\Delta s_{23})^2$	$2-4$	Δs_{24}	$(\Delta s_{24})^2$
$3-4$	Δs_{34}	$(\Delta s_{34})^2$	$3-5$	Δs_{35}	$(\Delta s_{35})^2$
$4-5$	Δs_{45}	$(\Delta s_{45})^2$	$4-6$	Δs_{46}	$(\Delta s_{46})^2$
...	...	...	...	...	...
$(n-1)-n$	$\Delta s_{n-1,n}$	$(\Delta s_{n-1,n})^2$	$(n-2)-n$	...	...
		$\sum(\Delta s)^2 =$			$\sum(\Delta s)^2 =$
$\overline{(\Delta s)^2_{\tau=1}} = \frac{\sum(\Delta s)^2}{n-1} =$			$\overline{(\Delta s)^2_{\tau=2}} = \frac{\sum(\Delta s)^2}{n-2} =$		

The last stage of analysis is visualized in verification of the Einstein formula for two-dimensional motion $\overline{(\Delta s)^2} = 4Dt$, where t is the time. To this effect, a rectangular coordinate system is plotted at a proper scale on a sheet of paper, with the axes of abscissa and ordinate labelling time intervals $n\tau$ and $\overline{(\Delta s_{ik})^2}$ values, respectively. Einstein's law is obeyed if all the points in the plane of the plot fall on the straight line starting from the origin of the coordinates. This high-quality experiment on filming bubble motion and its conformity with basic principles of the theory ensure that the Einstein's law is obeyed with an accuracy of about 10%.

It should be emphasized that the film once taken can be used infinitely long in further quantitative studies. In the absence of film equipment, they may be confined to the visual observation of Brownian movement of a gas bubble in a natural object followed by manipulations with the previously obtained film.

Also, it is worth noting that the above results may be used in calculating the diffusion coefficient for a bubble in liquid CO_2. Such a possibility was first suggested by S I Vavilov [3] who believed that coefficients of diffusion and viscosity can be derived from the observation of two-dimensional Brownian motion. Following his proposal, A N Kolmogorov and M A Leontovich [4] calculated the so-called *average Brownian area*. However, the results of experimental solution to this problem cannot be discussed here at greater length because of intrinsic mathematical difficulty of these calculations notwithstanding their agreement with experimental data.

To conclude, the observation of Brownian movement and quantitative validation of the related theory under conditions described in this paper were first realized by T S Velichkina and have been used for practical teaching purposes at the Physical Faculty, Moscow State University, for a few years now [5]. The possibility of observing the Brownian movement in a natural mineral will be a good incentive to the development of an artificial glass preparation containing a bubble of CO_2 even if the thrilling opportunity to see the Brownian movement initiated millions of years ago is lost.

References

1. Perrin J *Les Atomes* (Paris: Paris Univ, 1948)
2. Einstein A *Sobranie nauchnykh trudov* (Collected Works) Vol. 3 (Moscow: Nauka, 1965) pp. 118, 149, 155
3. Vavilov S I *Sobranie sochineniĭ* (Collected Works) Vol. 1 (Moscow: AN SSSR Publ., 1956) pp. 320, 330
4. Leontovich M A, Kolmogorov A N *Phys. Zs. Sow.* **4** 1 (1933)
5. Iveronova V I (Ed.) *Fizicheskiĭ Praktikum. Mekhanika i Molekulyarnaya Fizika* (Physical Practicum: Mechanics and Molecular Physics) (Moscow: Nauka, 1967)