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Cyclotron autoresonance and its applications

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Contents

Abstract. The physical mechanism and general properties of the motion of relativistic charged particles in the cyclotron autoresonance regime are discussed. The motion of particles out of the autoresonance is analyzed. Various methods of maintaining the regime are discussed. Applications of the autoresonance are considered.

1. Introduction

The term 'autoresonance' was first introduced by A A Kolomenskiĭ and A N Lebedev in 1962 [1]. They and V Ya Davydovsky [2] have independently shown that there exists such a regime of motion of a relativistic charged particle in a transverse electromagnetic wave, propagating along a static uniform magnetic field, in which the initial condition of cyclotron resonance of the particle with the wave

$$\omega - kv_z = \frac{\omega_{c0}}{\gamma} \equiv \omega_c \qquad (1)$$

is preserved 'by itself' despite the variation of both relativistic mass and the longitudinal component of the velocity v_z, as the particle moves along the magnetic field. Here $\omega_{c0} = eB_0/(m_0 c)$ is the classical gyrofrequency of the particle with charge e and rest mass m_0, where B_0 is the guide magnetic field directed along the z-axis; γ is the relativistic factor; ω, k are the frequency and wave vector of the wave, respectively.

Among different versions of exact solutions of the problem of motion of a relativistic charged particle in a plane electromagnetic wave, propagating along a uniform magnetic field [1 – 5], the regime of cyclotron autoresonance is a special one, because it is a purely relativistic effect, strictly existing only in the case of wave, whose phase velocity $v_{ph} = \omega/k$ is equal to the speed of light c [1 – 3].

This special feature of cyclotron autoresonance also manifests itself in the fact that in the regime of autoresonance, stochastic instability does not arise [6], and only in this regime can particles be monotonically accelerated or decelerated depending on the initial conditions. At violation of the conditions of autoresonance, the particle energy is found to be periodic function of time [1, 3].

The term 'autoresonance' is also used in other physical situations. Thus, Sloan and Drummond [7] in 1973 proposed a method for ion acceleration, which they named the autoresonant method. According to this method one should use traveling charge density waves, which are excited in a relativistic electron beam, propagating along a strong longitudinal magnetic field in a cylindrically symmetric conducting waveguide. More specifically one uses a cyclotron mode, whose phase velocity depends on the magnitude of the magnetic field. By adiabatically varying the magnetic field in magnitude one can increase the wave phase velocity from zero to the relativistic velocity of the electron beam. As this takes place, by analogy with the traditional linear accelerators, one needs to sustain the synchronism between the velocity of ions and the phase velocity of the accelerating cyclotron wave. In these conditions the energy of the electron beam is automatically extracted both for the acceleration of ions and for sustaining the accelerating wave. A variety of modifications to this autoresonant method of acceleration of ions were considered in a number of works (see, e.g. Refs [8, 9]).

Notice that there is also the term 'autoacceleration' which however has nothing to do with the phenomenon of cyclotron autoresonance [10].

The present review is devoted to the problem of cyclotron autoresonance defined in accordance with the original works [1 – 3]. A brief overview of this problem in connection with the possibility of acceleration of charged particles can be found in

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the Ref. [11]. Problems of generation and amplification of electromagnetic radiation, connected with the phenomenon of cyclotron autoresonance, are fully reviewed in Refs [12 – 15].

2. Motion of a charged particle in the regime of autoresonance

It is known that in the field of a plane electromagnetic wave the energy of a particle does not vary on average [16] (if one neglects weak relativistic effects). However, in the presence of an external uniform magnetic field in the regime of cyclotron autoresonance the mechanism of energy exchange between the wave and the particle essentially varies, and the particle can gain considerable energy [1 – 3].

2.1 General properties of the autoresonant motion of a particle

Neglecting the force of radiation friction, let us consider motion of a relativistic charged particle in the field of a transverse electromagnetic wave $\mathbf{E}^{\sim}, \mathbf{B}^{\sim}$ with an arbitrary elliptic polarization, propagating along a steady magnetic field $\mathbf{B}_0 = (0, 0, B_0)$:

$$\mathbf{E}^{\sim} = (E_1 \cos\theta, E_2 \sin\theta, 0)\,,$$

$$\mathbf{B}^{\sim} = (-NE_2 \sin\theta, NE_1 \cos\theta, 0)\,. \qquad (2)$$

Here $N = kc/\omega \equiv \beta_{\text{ph}}^{-1}$ is the refractive index; $\beta_{\text{ph}} = v_{\text{ph}}/c$. The phase of the wave θ is described, in general, by the equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\omega + kv_z\,. \qquad (3)$$

We can conveniently introduce the ratio of the classical gyrofrequency to the wave frequency $\Omega = eB_0/(m_0 c\omega)$ and the reduced momentum of the particle $\mathbf{P} = \mathbf{p}/(m_0 c)$, time $\tau = \omega t$, and wave amplitudes $\varepsilon_i = eE_i/(m_0 c\omega)$.

There are various methods for solving the equations of motion of a particle in the given fields [1 – 5, 17 – 20]. It is appropriate to extract from the outset the cyclotron rotation of the particle using the formulae

$$P_x = P_\perp \cos\theta_{\text{c}}\,, \qquad P_y = P_\perp \sin\theta_{\text{c}}\,, \qquad (4)$$

where θ_{c} is the phase of cyclotron rotation.

Then the motion equations reduce to

$$\frac{\mathrm{d}P_z}{\mathrm{d}\tau} = \frac{NP_\perp}{2\gamma}\left[(\varepsilon_1 + \varepsilon_2)\cos\theta_- + (\varepsilon_1 - \varepsilon_2)\cos\theta_+\right] = N\,\frac{\mathrm{d}\gamma}{\mathrm{d}\tau}\,, \qquad (5)$$

$$\frac{\mathrm{d}P_\perp}{\mathrm{d}\tau} = \frac{1}{2}\left(1 - \frac{NP_z}{\gamma}\right)\left[(\varepsilon_1 + \varepsilon_2)\cos\theta_- + (\varepsilon_1 - \varepsilon_2)\cos\theta_+\right], \qquad (6)$$

$$\frac{\mathrm{d}\theta_{\text{c}}}{\mathrm{d}\tau} = -\frac{\Omega}{\gamma} + \frac{1}{2P_\perp}\left(1 - \frac{NP_z}{\gamma}\right) \times \left[(\varepsilon_1 + \varepsilon_2)\sin\theta_- - (\varepsilon_1 - \varepsilon_2)\sin\theta_+\right], \qquad (7)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = -1 + \frac{NP_z}{\gamma}\,. \qquad (8)$$

Here $\theta_\pm \equiv \theta \pm \theta_{\text{c}}$.

It follows from (5) that in the case of constant refractive index for arbitrary amplitude and polarization of a transverse wave there exists an exact constant of motion

$$N\gamma - P_z = Y = \text{const}\,, \qquad (9)$$

found first in Refs [1, 2, 21].

This integral is a consequence of the laws of conservation of energy and momentum in the particle – photon system. Really, variations of energy ΔE and momentum Δp_z of a particle are connected with the energy and momentum of an absorbed (or emitted) photon by the relationships $\Delta E = \hbar\omega$, $\Delta p_z = \hbar k$. Therefore, $\Delta E/\Delta p_z = \omega/k$. This gives the integral (9).

For a positively charged particle, exact cyclotron resonance (1) in a steady magnetic field takes place, according to (7) and (8), if the following relation between the wave frequency (with the Doppler shift) and the gyrofrequency is fulfilled

$$\gamma - NP_z = \Omega\,. \qquad (10)$$

The condition (10) for cyclotron resonance is preserved for all the time of particle motion, that is to say, (10) coincides with the motion integral (9) only in the case of the 'vacuum' (luminous) wave, when

$$N = 1\,, \qquad Y = \Omega\,. \qquad (11)$$

This regime has been labelled as autoresonance [1]. Notice that the authors of Ref. [3] labelled such motion of a particle as synchronous.

The physical mechanism of autoresonance is the following: if the exact resonance condition (10) is initially fulfilled, then it may be destroyed for two reasons: due to the Doppler shift of wave frequency and due to the relativistic variation of cyclotron frequency. At $N > 1$ the Doppler shift prevails, at $N < 1$ the effect of relativistic change of particle mass becomes dominant, and only at $N = 1$ do these two competing effects mutually compensate each other. Indeed, with the use of equation (5) and integral (9) for the resonance mismatch

$$\delta = 1 - N\,\frac{P_z}{\gamma} - \frac{\Omega}{\gamma} \equiv \delta_0 + \delta_{\text{D}}$$

we obtain the equation

$$\frac{\mathrm{d}\delta}{\mathrm{d}\tau} = -\frac{1}{\gamma^2}(NY - \Omega)\,\frac{\mathrm{d}\gamma}{\mathrm{d}\tau}\,.$$

Here

$$\delta_0 \equiv 1 - N\,\frac{P_{z0}}{\gamma_0} - \frac{\Omega}{\gamma_0}$$

is the initial resonance mismatch, which leads to the kinematic phase shift, and $\delta_{\text{D}} \equiv -N\Delta V_z - \Delta\Omega/\gamma$ is the resonance mismatch caused by relativistic change in the cyclotron frequency and by the change in the axial velocity of the particle due to the action of wave. It leads to the dynamic phase shift [13]. Hence it follows that at resonance $(Y = \Omega)$ the dynamic shift remains constant only with $N = 1$, and if the initial resonance mismatch is absent $(\delta_0 = 0)$, then $\delta_{\text{D}} = 0$ during all the time of particle's motion.

A more fundamental reason for the existence of autoresonance, only in the case $N = 1$, is the following [22]: at fixed wave frequency the absorption (or emission) of photon by a particle is evidently possible, if the spectrum of absorption (or emission) of a charge in a magnetic field is equidistant. By using laws of conservation of energy and the longitudinal component of momentum during absorption (or emission) of photon and the transition of the particle without change in spin into a new quantum state, one can show [22] that the equidistant spectrum exists only at $N = 1$.

The resonance relation between frequencies (10) corresponds to the phase combination θ_- (for electrons the phase combination θ_+ is resonant). In the case of a circularly polarized wave ($\varepsilon_1 = \varepsilon_2 = \varepsilon$) Eqns (5)–(8) contain the resonance phase as a slow variable and appear to be exact and valid for arbitrary wave amplitude. In the case of a wave with an elliptic polarization, in particular, a linearly polarized wave, the motion equations (5)–(8) contain not only the resonant phase θ_-, but also the nonresonant phase combination θ_+. If the wave amplitude is small enough, then the variables P_z, γ, $P_\perp$, θ_- are slowly varying whereas the nonresonant phase θ_+ should be considered as the fast variable, so one can perform averaging over this phase [23, 24]. Therefore, the energy gained by a particle and the character of its motion depend on the wave polarization: in the autoresonance regime (10) at $N = 1$ in the case of a circularly polarized wave the energy of the particle increases steadily (in accelerating phase), while in the case of a linearly polarized wave one can see fast oscillations about an increasing average value of energy. As this takes place, according to (5) the energy growth rate in a circularly polarized wave is twice as large as that in the case of a linearly polarized wave. This has been noticed in Refs [1, 25].

Therefore the case of the wave with circular polarization seems to be the most optimal one. It allows us to find an exact analytical solution [3]. Let us consider it in detail. In the autoresonant (synchronous) regime, from the motion equations one can obtain the following relationships:

$$P_z = \gamma - \Omega\,,$$
$$P_\perp^2 = 2\Omega\gamma - \Omega^2 - 1\,, \qquad (12)$$
$$P_\perp \sin\theta_- = P_{\perp 0}\sin\theta_{-0} \equiv \Phi\,.$$

Parameter Ω is subject to wide variations: in the case of the light wave it is of the order of 10^{-5} for electrons, and over the range of centimeter waves it may be more than unity.

This parameter according to (12) determines the minimal value of energy $\gamma_{\min} = (1 + \Omega^2)/(2\Omega)$. In this case $P_\perp = 0$. The longitudinal momentum also attains the minimum $P_{z\min} = (1 - \Omega^2)/(2\Omega)$. If $\Omega = 1$ then the minimal energy $\gamma_{\min} = 1$, and $P_{z\min} = 0$. If $\Omega^2 > 1$, then $P_{z\min} < 0$. This means that the particle moves towards the wave, and is then reflected and accelerated in the direction of the wave's propagation [1].

Using the last relationship in (12) one can easily verify that the energy γ obeys the equation

$$\left(\frac{d\gamma}{d\tau}\right)^2 + V(\gamma) = 0\,. \qquad (13)$$

Precisely the same equation was first obtained in Ref. [3], but by a more complicated method.

The 'potential' $V(\gamma)$ is defined by the formula

$$V(\gamma) = -\frac{2\Omega\varepsilon^2}{\gamma^2}(\gamma - \Gamma)\,, \qquad (14)$$

where

$$\Gamma = \gamma_0 - \frac{P_{\perp 0}^2}{2\Omega}\cos^2\theta_{-0} \equiv \frac{1 + \Omega^2 + \Phi^2}{2\Omega}\,. \qquad (15)$$

The label '0' denotes the initial values of the corresponding quantities.

Equation (13) has the exact solution

$$(\gamma - \Gamma)^{3/2} + 3\Gamma(\gamma - \Gamma)^{1/2} = 3\varepsilon\sqrt{\frac{\Omega}{2}}\,\tau + \text{const}\,. \qquad (16)$$

From this solution it follows that the energy gained by the particle grows asymptotically as

$$\gamma \approx \left(3\varepsilon\tau\sqrt{\frac{\Omega}{2}}\right)^{2/3}\,. \qquad (17)$$

In parallel with the energy increase the acceleration rate decreases [11]

$$\frac{d\gamma}{dZ} = \frac{\varepsilon\sqrt{2\Omega(\gamma - \Gamma)}}{\gamma - \Omega} \approx \varepsilon\sqrt{\frac{2\Omega}{\gamma}}\,. \qquad (18)$$

Here $Z = z\omega/c$ is the dimensionless acceleration length. It follows from (18) that the energy gained by the particle over the length Z is determined by a formula similar to (17).

When this result is compared with that for a linear accelerator $\gamma_L \approx \varepsilon Z$, it is apparent that at uniform wave amplitude the linear accelerator is more effective. However at $\varepsilon Z < 4.5$ the energy gained by the particle at autoresonance can exceed γ_L [25].

By introducing the dimensionless radius-vector of the particle $\mathbf{R} = k\mathbf{r}$, one can see that in the autoresonant regime at large values of energy the acceleration length varies in accordance with the law $Z \approx \tau$ while the law for variation of the radius of trajectory $R = (X^2 + Y^2)^{1/2}$ is determined by the asymptotic formula $R \approx (3/2)\sqrt{2\Omega}\left(3\varepsilon\sqrt{\Omega/2}\right)^{-1/3}\tau^{2/3}$. Therefore, the trajectory of the accelerated particle is an unwinding helix with growing step.

In the paper [1] the question of admissible initial conditions at autoresonance has been discussed. It was found that at $|\Omega| > 1$ the injection of particles in the field of the wave may be carried out at any angle, including $\pi/2$, and in doing so each angle of injection corresponds to a definite energy. If $\Omega \ll 1$, then the injection angle α can vary only in the range $-\Omega < \alpha < \Omega$.

It is easy to obtain the equation for the phase $\theta_- \equiv \psi$

$$\frac{d\psi}{d\tau} = -\frac{\varepsilon\Omega}{\Phi}\,\frac{\sin^4\psi}{q + g\sin^2\psi}\,, \qquad (19)$$

where

$$q \equiv \frac{\Phi^2}{2\Omega}\,, \qquad g \equiv \frac{1 + \Omega^2}{2\Omega}\,.$$

The exact solution of (19) is

$$\cot\psi\left(g + q + \frac{q}{3}\cot^2\psi\right)\Bigg|_{\psi_0}^{\psi} = \frac{\varepsilon\Omega}{\Phi}(\tau - \tau_0)\,. \qquad (20)$$

From this solution it follows that the initial accelerating phase $\psi_0 = 0$ does not vary with time. But if $\psi_0 \neq 0$, then the phase ψ tends asymptotically to zero at $\tau \to \infty$. So, over a large enough time interval all the particles are trapped into the regime of autoresonant acceleration without regard to the initial phase.

Autoresonance occurs in both traveling and standing waves. A standing wave can be considered as a sum of two plane circularly polarized waves, propagating in opposite direction. Under some conditions a particle can enter the autoresonant regime with one of the propagating waves as if the other wave were completely absent. At the same time the particle energy grows monotonically [25].

In the regime of autoresonance a particle can not only be accelerated, but also decelerated, depending on its initial phase. If the parameter $\Omega = 1$, then according to (12) we have

$$\gamma = 1 + \frac{P_\perp^2}{2} \,. \tag{21}$$

In this case total deceleration of the particle can occur (the 'arttron' effect [1, 26]): $\gamma \to 1$, $P_\perp, P_z \to 0$. For total deceleration of a particle not only the parameter Ω should be equal 1, but also the initial phase ψ_0 should be equal to π. Then it follows from the exact solution, that the particle comes to rest at the length

$$L_s = \frac{\sqrt{2}}{3\varepsilon}(\gamma_0 - 1)^{3/2} \equiv \frac{P_{\perp 0}^3}{6\varepsilon} \,. \tag{22}$$

In this case the rate of change of energy becomes equal to zero, and if the particle continues to interact with wave, then it appears to be in the accelerating phase and, consequently, it begins to be accelerated.

If the initial decelerating phase $\psi_0 \neq \pi$, then according to the relationship (12), the particle cannot completely lose its energy even at $\Omega = 1$.

In the case of accelerating phase $\psi_0 = 0$ the particle gains energy $\gamma_L = 1 + 2^{2/3}(\gamma_0 - 1)$ over the length L_s at $\Omega = 1$. The energy gain of the accelerated particle $\Delta\gamma_{\rm acc} = \gamma_L - \gamma_0$ appears to be less than the energy $\Delta\gamma_{\rm dec} = \gamma_0 - 1$, which the decelerated particle loses. So, over the deceleration length particles of the beam on average lose more energy than they gain from the wave. Amplification of the wave thereby takes place [26].

2.2 Experimental evidence of the phenomenon of cyclotron autoresonance

The first experiments confirming the existence of the autoresonance mechanism of acceleration of charged particles were reported in Refs [25, 27, 28].

The main difficulties were in the fact that the required high frequency (HF) fields could be realized only in waveguides where the waves are not transverse. Moreover, it was difficult to excite waves traveling along the waveguide axis with circular polarization. So the authors of Ref. [27] tried to use more simple electromagnetic waves, in particular, the H_{11} wave in a waveguide of circular cross section and the H_{01} wave, similar to it in structure, in an waveguide of rectangular cross section.

It was shown that the integral (9) which guarantees the possibility of autoresonance, takes place for an H-wave traveling along the z-axis and does not exist for standing H-waves. Since the wave phase velocity in a waveguide exceeds the speed of light, the continuous acceleration of particles is impossible. Because of this, the effective length of acceleration was determined

$$z_{\rm acc} = \frac{\lambda\beta_{\rm ph}}{2(\beta_{\rm ph} - 1)} \,. \tag{23}$$

During the passage of HF power through the accelerating waveguide, X-ray radiation was observed. For the H_{11}-wave the quantity $\beta_{\rm ph} = 2$. Therefore the acceleration length was $z = 3.6$ cm. If the length of the uniform part of the solenoid > 3.6 cm then the particle energy should periodically change over the waveguide length. To observe this effect, the internal side of the waveguide was covered with a fine layer of luminescent solid. At optimal magnetic field luminous rings were really observed on the walls of the waveguide. The distance between them was $\sim 6-7$ cm, that is $\sim 2z_{\rm acc}$.

The kinetic energy of accelerated electrons was determined from their passing through aluminum foils. It was ~ 700 keV at an electric field strength of $3-5$ kV cm^{-1}. This energy is far in excess of the energy which the particle could gain under these conditions in the case of traditional cyclotron acceleration. A related experiment was realized in Ref. [28], where a wave with $\beta_{\rm ph} = 1.14$ was used.

Therefore the performed experiments proved the possibility of autoresonant acceleration of particles by fast waves in the case when the integral (9) exists. Note, that the integral (9) also takes place in real HF fields of TE-mode. But in the case of TM-mode such an integral only exists when the phase velocity of a wave is equal to the speed of light [15].

In contrast to the waveguide structures of Refs [27, 28] the cavity resonators of circular cross section were used in experiments of Ref. [25]. A low-current, low-energy electron beam was injected into a cavity resonator oscillating in the $TE111$ mode. On the basis of experimental data the parameter $\varepsilon = eE\lambda/(2\pi m_0 c^2)$ was calculated. This provided a way of estimating the value of the gained energy [see (17)]. The estimation appears to agree satisfactorily with experimental value.

The basic conclusion drawn by the authors of Ref. [25] from the obtained results is that significant acceleration of particles can be achieved in conditions different from the ideal situation of autoresonant regime. It is connected with the fact that real HF field in a waveguide or a cavity can be approximated by the field of a plane wave with reasonable accuracy.

Later on the possibility of autoresonant acceleration of particles was experimentally investigated in Refs [29–31]. These studies showed that the character of particle acceleration in a traveling wave (a waveguide) and in a standing wave (a cavity resonator) is considerably different.

Under autoresonant acceleration in a cavity the energy of an accelerated beam is mainly concentrated in the transverse component of velocity, whereas in the traveling wave scheme most of the energy is gained in the axial component of velocity. It follows from Ref. [29] that an autoresonance microwave accelerator is very effective for the production of relativistic rotating electron beams, which can be used, in particular, as a source of coherent radiation [32, 33]. In studies of the interaction between an electron beam and a traveling wave [30, 31] some contradiction inherent in the autoresonance mechanism of acceleration was noticed: on the one hand an increase in the wave electric field strength tends to increase the rate of acceleration,and on the other it leads to the violation of the regime of stable acceleration. The adjustment of optimal parameters is required.

There are also proposals for laser acceleration of electrons and producing an autoresonance laser accelerator — ALA — with a high rate of acceleration [34–38]. The physical mechanism of acceleration in the ALA scheme, in principle, is similar to that of the scheme of the microwave accelerator. However, there are some distinctions between them:

The intensity of the laser radiation is far beyond that of microwaves. This affects the rate of acceleration;

The source of laser radiation is external relative to the region of acceleration. This has an influence on the efficiency of energy transfer from a wave to a particle and removes the difficulties connected with the breakdown and so on;

Laser radiation propagates in the form of beam. This results in the specific character of wave–particle interaction.

The phenomenon of cyclotron autoresonance can also be used for producing sources of high-power millimeter and submillimeter wavelength electromagnetic radiation [12–15]. The first results concerned with the performance of the cyclotron autoresonance maser — CARM — have been reported in Refs [39]. Later on the different schemes of CARM oscillator and CARM amplifier have been subjected to extensive studies and numerical simulation [40].

2.3 Autoresonance in an obliquely propagating wave

It follows from relativistic equations of motion of a particle in the general case of a plane wave, propagating at an angle with respect to the steady magnetic field, that there exists the exact integral [5, 41]

$$\mathbf{N}\gamma - \mathbf{P} + \mathrm{i}\left[\boldsymbol{\varepsilon}\exp(\mathrm{i}\theta) - \boldsymbol{\varepsilon}^{*}\exp(-\mathrm{i}\theta)\right] + \left[\boldsymbol{\rho}\,\boldsymbol{\Omega}\right] \equiv \mathbf{Y} = \mathrm{const}\,. \tag{24}$$

From here on we shall use the reduced parameters: $\mathbf{N} = \mathbf{k}c/\omega$, $\boldsymbol{\rho} = \mathbf{r}\omega/c$, $\boldsymbol{\Omega} = e\mathbf{B}_0/(m_0c\omega)$, $\boldsymbol{\varepsilon} = e\mathbf{E}/(m_0c\omega)$. In the case under consideration one usually introduces the drift variables for the particle [41]. However it seems to be more efficient to separate the cyclotron rotation of the particle with the help of (4) and to use the transformation of the phase θ into a new phase ψ by the formula [42]

$$\psi = \theta + \mu\sin(\psi_{\mathrm{c}} - \phi)\,, \tag{25}$$

where $\mu = N_\perp P_\perp/\Omega$, $N_\perp = (N_x^2 + N_y^2)^{1/2}$, $\tan\phi = N_y/N_x$, $\psi_{\mathrm{c}} \equiv \theta_{\mathrm{c}}$. In this case the equations of particle motion have a rather complicated form. In different conditions they were considered in many works [41–45]. We will take interest in a special case of the particle motion, connected with the existence of integral (24).

In the region of resonance at s-th harmonics of gyrofrequency the difference between the frequencies

$$\Delta_s \equiv \frac{1}{\gamma}(-\gamma + N_zP_z + s\Omega) \tag{26}$$

appears to be small. So, the corresponding combination of phases $\psi_{\mathrm{r}s} \equiv \psi - s\psi_{\mathrm{c}}$ becomes a slow, or semifast variable. That means, that at small enough wave amplitudes the equation of motion may be averaged over fast phases except the resonance phase. In this case one should suppose that separate resonances do not overlap.

It follows from the averaged system that in the range of an s-th resonance there exists the approximate integral [44, 45]

$$N_z\gamma - P_z + \mathrm{i}\left[\varepsilon_z\exp(\mathrm{i}\psi_{\mathrm{r}s}) - \varepsilon_z^{*}\exp(-\mathrm{i}\psi_{\mathrm{r}s})\right]J_s(\mu) = Y = \mathrm{const} + O(\varepsilon^2)\,. \tag{27}$$

Here $J_s(\mu)$ is the Bessel function. The resonance condition $\Delta_s \approx 0$ may be compatible with the integral (27), if

$$N_z = 1\,, \qquad Y = s\Omega\,, \tag{28}$$

while $|\varepsilon_zJ_s| \ll 1$. So, the integral (27) at the conditions (28) guarantees a peculiar autoresonance, or more exactly, synchronous regime of motion of a particle. As large values of energy ($\mu \gg 1$) are approached, the difference $\gamma - P_z$, according to (27), tends to the constant value $\gamma - P_z \to s\Omega$.

Consequently, as the particle energy increases, the condition of cyclotron resonance is fulfilled with increasing accuracy, so the synchronous regime passes to the autoresonance one. Note, that one of the conditions of autoresonance $N_z = 1$ for oblique propagation of an electromagnetic wave was pointed out in Ref. [22].

In the synchronous regime not only acceleration but also deceleration of a particle with full loss of energy is possible. This is evident from the approximate formula (at $s \neq 0$)

$$\gamma = \frac{1}{2s\Omega}\left\{P_\perp^2 + 1 + (s\Omega)^2 + \frac{\mathrm{i}J_s}{s\Omega}\left[\varepsilon_z\exp(\mathrm{i}\psi_{\mathrm{r}s}) - \mathrm{c.c.}\right] \times (P_\perp^2 + 1 - s^2\Omega^2)\right\} + O(\varepsilon^2)\,. \tag{29}$$

Here c.c. stands for the complex conjugation. From this formula it follows that at $s\Omega = 1$ the particle can completely pass its energy to the wave.

In the particular case $N_y = 0$ ($\Phi = 0$), $\varepsilon_x = \varepsilon_1/2$, $\varepsilon_y = -\mathrm{i}\varepsilon_2/2$, $\varepsilon_z = \varepsilon_3/2$ we can obtain the integral

$$G_s\sin\psi_{\mathrm{r}s} = Q_s \equiv \mathrm{const}\,, \tag{30}$$

which is the generalization of the integral (12). Here

$$G_s \equiv \frac{\varepsilon_3P_zN_\perp}{\Omega}J_s(\mu) + \varepsilon_1 sJ_s(\mu) + \varepsilon_2\mu J_s'(\mu)\,, \qquad J_s' \equiv \frac{\mathrm{d}J_s}{\mathrm{d}\mu}\,. \tag{31}$$

As in the case of longitudinal propagation of a transverse wave, there is an equation for the energy of the form (13), where

$$V(\gamma) = -\left(\frac{\Omega}{\gamma N_\perp}\right)^2\left[G_s^2(\gamma) - Q_s^2\right]. \tag{32}$$

From that equation it follows that at large values of energy the rate of acceleration falls as $\gamma^{-1/4}$ without regard to the resonance order. For comparison we can point out that in the case of autoresonance in a transverse vacuum wave propagating along the magnetic field, the rate of acceleration falls as $\gamma^{-1/2}$.

We have considered the possibility of synchronous motion of a particle at resonances of gyrofrequency and its harmonics. Meanwhile, the equations of motion admit a synchronous regime of motion of a particle in a transverse wave at the Cherenkov resonance ($s = 0$). In this case ultrarelativistic particles moving at small pitch-angles may be accelerated without limit. This is evident from the formula

$$\sqrt{\gamma^2 - Q^2} - \sqrt{\gamma_0^2 - Q^2} \approx \varepsilon_3\tau\,, \tag{33}$$

where

$$\gamma_0 = \gamma\Big|_{\tau=0}\,, \qquad \gamma\sin\psi \approx \frac{1 + P_\perp^2}{2\varepsilon_3} \equiv Q = \mathrm{const}\,. \tag{34}$$

The viewed synchronous regime of particle motion exists not only in a transverse wave, but also in a longitudinal wave propagating at an angle to the magnetic field [45]. Unlimited acceleration of a particle is also possible in a purely longitudinal wave, propagating with the speed of light and with no external magnetic field [46].

2.4 Motion of a charged particle at the violation of the autoresonance conditions

Autoresonance does not occur, if one of the conditions (11) ($N = 1$, but $Y \neq \Omega$, or $N \neq 1$) is violated. In any case a frequency mismatch arises, so beats take place, leading in particular to energy oscillations [1, 3].

A qualitative explanation of the energy oscillations is as follows: let us assume that at the initial moment the condition of exact resonance (10) is satisfied, and the velocity of a particle (with positive charge) is parallel to the wave's electric field. In this case the particle energy increases. Due to detuning of the resonance, the angle between the particle velocity and the field strength varies and can become obtuse. Then the particle loses energy, until the angle reverts back to an acute one, and the particle energy again increases. Thus the energy varies in periodic manner.

One can evaluate the value of maximal energy which the particle gains before the resonance is destroyed, when the angle between the transverse velocity of the particle and the electric field strength vector becomes obtuse.

It is convenient to introduce the dimensionless interval s, given by the formula $\mathrm{d}s = \mathrm{d}\tau/\gamma$, instead of the reduced time τ. If at the initial moment the resonance condition $\gamma_0 - NP_{z0} = \Omega$ is fulfilled, then the constant Y in (9) is determined as

$$Y = \Omega + \gamma_0(N^2 - 1)\,. \qquad (35)$$

Detuning of resonance arises because the difference $\gamma - NP_z$ is not a constant of motion in this situation. For estimation we may put $\theta_- \equiv \psi \approx 0$. Then the deviation of the difference $\gamma - NP_z$ from the initial value Ω is described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}s}(\gamma - NP_z) \approx \varepsilon P_\perp (1 - N^2)\,, \qquad (36)$$

where $P_\perp \approx \varepsilon \Omega s$. Hence, the phase variation is

$$\Delta\psi = \frac{\varepsilon^2 \Omega s^3 |1 - N^2|}{6}\,. \qquad (37)$$

Detuning of resonance occurs at $\Delta\psi \approx 1$. It allows us to find $s_{\max}$. Then, taking into account (9), we obtain the desired estimation

$$\gamma_{\max} \approx \left(\frac{3\varepsilon}{|1 - N^2|}\right)^{2/3} \left(\frac{\Omega}{2}\right)^{1/3}. \qquad (38)$$

Let us consider next the relation

$$P_\perp^2 = \gamma^2(1 - N^2) + 2NY\gamma - 1 - Y^2\,. \qquad (39)$$

Quantity $P_\perp^2$ as a function of γ represents a parabola with the coordinates of its vertex

$$\gamma_v = \frac{NY}{N^2 - 1}\,, \qquad P_{\perp v}^2 = \frac{1 - N^2 + Y^2}{N^2 - 1}\,. \qquad (40)$$

Therefore, the character of dependence $P_\perp^2(\gamma)$ is essentially different in the case of $N < 1$ (superluminous waves) and $N > 1$ (subluminous waves) (Fig. 1). At $N < 1$

$$\gamma_{\min} = -\frac{NY + \sqrt{1 - N^2 + Y^2}}{1 - N^2}\,.$$

At $N > 1$

$$\gamma_{\min}^{\max} = \frac{NY \pm \sqrt{1 - N^2 + Y^2}}{N^2 - 1}\,.$$

From the plots given in Fig. 1 it is obvious that in the case of a superluminous wave the particle energy may have values as large as one likes, whereas in the case of subluminous wave the values of the particle energy range between $\gamma_{\min}$ and $\gamma_{\max}$ and with N increasing the region of allowed values of energy becomes narrower.

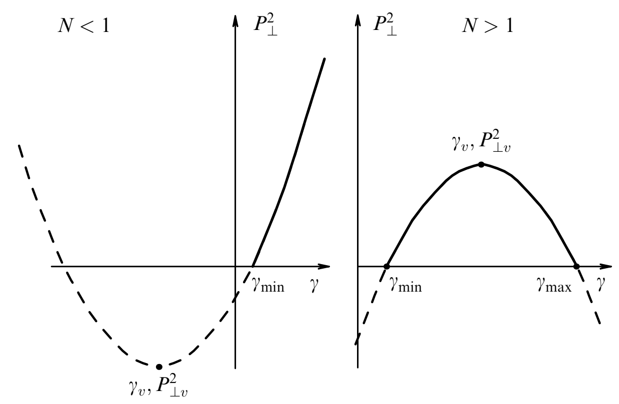

Figure 1. The γ-dependence of $P_\perp^2(\gamma)$. (The dashed lines are nonphysical regions of parabolas).

If the constant $Y = N$, then the minimal value of energy in both cases corresponds to the particle at rest: $\gamma_{\min}^* = 1$ and the maximal value of the particle energy in the case of subluminous wave is equal to $\gamma_{\max}^* = (N^2 + 1)/(N^2 - 1)$.

One can easily obtain from the system (5)–(8)

$$\mathrm{d}(P_\perp \sin\theta) - \frac{\Omega - \gamma + NP_z}{2\varepsilon Y}\,\mathrm{d}P_\perp^2 = 0\,. \qquad (41)$$

In the autoresonance regime ($N = 1$, $Y = \Omega$) this equation leads to the integral (12). At $N = 1$, but at the initial detuning of resonance ($Y \neq \Omega$), we have an integral

$$P_\perp \sin\theta - \frac{\Omega - Y}{2\varepsilon Y}\,P_\perp^2 = \mathrm{const}\,. \qquad (42)$$

Due to the frequency mismatch beats arise. The maximal value of the transverse momentum is $P_{\perp\max} = 2\varepsilon Y/|\Omega - Y|$. It tends to infinity as detuning decreases.

If $N \neq 1$, then (41) gives the integral

$$\varepsilon P_\perp \sin\theta + \frac{\gamma}{2}\left[(1 - N^2)\gamma + 2(NY - \Omega)\right] = W = \mathrm{const}\,. \qquad (43)$$

In combination with (39) this integral determines the family of phase trajectories in the plane (γ, θ).

Using (43), the equation for the energy can be obtained

$$\gamma^2\dot{\gamma}^2 = \varepsilon^2\left[\gamma^2(1-N^2) + 2NY\gamma - 1 - Y^2\right] - \left\{W - \frac{\gamma}{2}\left[(1-N^2)\gamma + 2(NY-\Omega)\right]\right\}^2, \qquad (44)$$

where $\dot{\gamma} \equiv (\mathrm{d}\gamma/\mathrm{d}\tau)$. An identical equation was first obtained by the authors of Ref. [3].

This equation admits a general solution in terms of elliptic integrals [3]. However, examination of this solution is rather complicated, so numerical methods are most often used. A detailed examination of the motion of a particle in a slow or fast electromagnetic wave is contained in Refs [1, 3, 11, 18].

When a particle with zero initial transverse velocity interacts with a slow wave, the sign of the energy change depends on the ratio of the initial longitudinal velocity of the particle v_{z0} to the wave phase velocity v_{ph} according to the Cherenkov mechanism [18]. If $v_{z0} > v_{\mathrm{ph}}$ the particle will lose its energy. At $v_{z0} < v_{\mathrm{ph}}$ the particle, driven by the wave, gains energy. If $v_{z0} = v_{\mathrm{ph}}$ then the energy of the particle does not vary, because in this case the force of the wave electric field is exactly compensated by the magnetic Lorentz force.

Notice that the system (38)–(41) admits a steady solution

$$\theta_s = \pm\frac{\pi}{2}, \qquad \Omega - \gamma_s + NP_{zs} = \pm\frac{\varepsilon}{P_{\perp s}}(\gamma_s - NP_{zs}). \qquad (45)$$

In the paper [47] the stability of the steady state with respect to small deviations from it was discussed. It was shown that the energy of stable steady states increases without limit as $N \to 1$.

2.5 Effect of radiation on the motion of a particle in the autoresonance regime

Resonant motion of a particle with allowance made for the force of radiative friction was discussed in Refs [11, 48, 49]. It was shown that radiation imposes a fundamental limit on the mechanism of autoresonant acceleration although this limit could be sufficiently high.

Using the well known expression for the radiative friction force [16], the equations of motion of a particle in the field of a circularly polarized electromagnetic wave (at $N = 1$) propagating along the magnetic field can be written in the form

$$\frac{\mathrm{d}P_z}{\mathrm{d}s} = \varepsilon P_\perp \cos\psi - \mu\Omega^2 P_z P_\perp^2, \qquad (46)$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}s} = \varepsilon P_\perp \cos\psi - \mu\Omega^2 \gamma P_\perp^2, \qquad (47)$$

$$\frac{\mathrm{d}P_\perp}{\mathrm{d}s} = \varepsilon(\gamma - P_z)\cos\psi - \mu P_\perp \Omega^2(\gamma^2 - P_z^2), \qquad (48)$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}s} = \Omega - \gamma + P_z - \frac{\varepsilon}{P_\perp}(\gamma - P_z)\sin\psi. \qquad (49)$$

Here the previous notations for the reduced variables and parameters are used. Parameter $\mu = 2e^2\omega/(3m_0c^3) \equiv 4\pi r_0/(3\lambda) \ll 1$, where $r_0 = e^2/(m_0c^2)$ is the classical electron radius.

In these equations, small terms of the order of $\mu\varepsilon$ and higher are omitted. One can see that the radiative force has little effect on the evolution of the variables P_z, γ, $P_\perp$ but causes resonance detuning and variation of the resonance phase ψ. This occurs due to a deviation of the difference $\gamma - P_z$ from the constant value Ω.

From (46) and (47) we obtain

$$\frac{\mathrm{d}}{\mathrm{d}s}(\gamma - P_z) \approx -\mu\Omega^2 P_\perp^2(\gamma - P_z). \qquad (50)$$

When radiation is fully excluded the quantity $\gamma - P_z$ remains constant, that is, autoresonance occurs. At $\mu \neq 0$ the resonance of the particle with a wave can be destroyed. If the phase variation $\Delta\psi \ll 1$ the resonance is still preserved. But if $\Delta\psi \approx 1$ then the synchronism between the particle and the wave is violated. It follows from the equations above that the phase variation due to radiation is

$$\Delta\psi \approx \frac{\mu\varepsilon^2\Omega^5 s^4}{12}. \qquad (51)$$

The maximum allowable value of energy which the particle may gain before the resonance is destroyed is determined by the relation

$$\gamma_{\max} \approx \frac{P_{\perp\max}^2}{2\Omega} = \frac{\varepsilon}{\Omega}\sqrt{\frac{3}{\mu\Omega}}. \qquad (52)$$

Such an estimate was obtained in the above quoted papers. It is seen that the maximum allowable energy at autoresonance does not depend on the initial energy of the accelerated particle. As an example, let us consider the quantities $E/B_0 = 10^{-2}$, $B_0 = 300$ kG. In this case, $\gamma_{\max} \approx 3 \times 10^3$.

Thus, consideration of the radiative friction leads to the principle limit on the maximal energy gained by the particle during autoresonance acceleration. However, this limit can be completely removed or strongly suppressed by a longitudinal electrostatic field $E_z(Z)$ [50].

In this case, the additional terms $\varepsilon_0 f\gamma$ and $\varepsilon_0 f P_z$ appear in Eqns (46) and (47), respectively, where

$$\varepsilon_0 = \frac{eE_0}{m_0c\omega}, \qquad f(Z) = \frac{E_z(Z)}{E_0}. \qquad (53)$$

Then, in place of (50) we obtain the equation

$$\frac{\mathrm{d}}{\mathrm{d}s}(\gamma - P_z) = -(\gamma - P_z)(\mu\Omega^2 P_\perp^2 - \varepsilon_0 f). \qquad (54)$$

We can see that the radiative losses at autoresonance are completely suppressed if

$$\varepsilon_0 f(Z) = \mu\Omega^2 P_\perp^2 = \mu\varepsilon^2\Omega^4 s^2 \approx \mu(6\varepsilon)^{2/3}\Omega^{7/3}Z^{2/3}. \qquad (55)$$

Notice that we have met such a law for the variation of particle energy in the autoresonance regime over the acceleration length Z.

If the electrostatic field is constant ($f = 1$), then we can easily obtain an estimate for the phase variation

$$\Delta\psi \approx \Omega\left(\frac{\mu\Omega^4\varepsilon^2 s^4}{12} - \frac{\varepsilon_0 s^2}{2}\right), \qquad (56)$$

as well as for the maximal energy gained

$$\gamma_{\max} \approx \frac{P_{\perp\max}^2}{2\Omega} \approx \frac{3\varepsilon_0}{2\mu\Omega^3}\left(1 + \sqrt{1 + \frac{4\mu\varepsilon^2\Omega^3}{3\varepsilon_0^2}}\right). \qquad (57)$$

This value is considerably above the limit (52), established from the radiative losses. For instance, at $E_0/B_0 = 10^{-4}$, $B_0 = 300$ kG, $\lambda = 10$ cm we have $\gamma_{\max} \approx 3 \times 10^4$.

Notice that the electrostatic field can increase resonance detuning if it is directed the opposite way. In this case efficient particle acceleration becomes impossible.

In conclusion of this section it should be pointed out that the autoresonant motion of a charged particle in its pure form is realized only in the ideal case of a plane electromagnetic wave, traveling along a steady magnetic field with the speed of light in a vacuum. Any deviations from the required conditions violate the synchronism between the particle and the wave and distort the autoresonant regime. So it is natural that searches for electromagnetic structures are carried out where waves travel at the speed of light. Different types of such structures, for example, multipole fields [11], *TEM*-waves in a coaxial waveguide [54], and others have been considered. In such structures the autoresonant motion of a particle is possible both in the acceleration and in the deceleration regimes.

3. Maintenance of synchronism between a charged particle and a slow or fast electromagnetic wave

In the case of a slow ($N > 1$) or fast ($N < 1$) electromagnetic wave, propagating along a steady magnetic field, the initial condition of cyclotron resonance can not be self-maintained throughout the time of the particle's motion. Thus the question arises: is it feasible to force the maintenance of synchronism in such a way that the initial resonance condition will be retained as long as possible? This can be achieved by changing the phase velocity or other parameters of the wave, by profiling of the guide magnetic field or by some kind of external action (for example, the application of an electrostatic field). Motion of a particle under varying conditions is also occasionally known as autoresonant, although it is more suitable to name it synchronous, because in this case synchronism between the particle and wave is maintained not automatically but in a forced manner.

3.1 Maintenance of synchronism in a wave with varying phase velocity

If the wave phase velocity (the refractive index) varies arbitrarily, the integral of the type (9) does not exist, and synchronism between the particle and wave is, in general, impossible. However, if the refractive index varies in a definite way along the direction of wave propagation, then the initial resonance condition would be expected to be retained in the course of the particle motion

$$\gamma_0 - N_0 P_{z0} = \gamma - N(Z) P_z = \Omega \,. \tag{58}$$

Thus, the main problem lies in finding the dependence $N(Z)$ under the condition (58). Different variants of this problem have been considered in Refs [47, 52–54].

If relation (58) is fulfilled, in the case of a circularly polarized wave we have

$$P_z \frac{\mathrm{d}N}{\mathrm{d}Z} = \frac{1}{N}(1 - N^2)\frac{\mathrm{d}P_z}{\mathrm{d}Z} \,. \tag{59}$$

It follows that the index of refraction remains constant only at $N = 1$. If $N \neq 1$, then to maintain synchronism (58) in accordance with (59) one needs to have the relationship between N and P_z

$$P_z\sqrt{|1 - N^2|} = P_{z0}\sqrt{|1 - N_0^2|} \equiv G = \text{const} \,. \tag{60}$$

If $N \to 1$, then $P_z, \gamma \to \infty$. This special case corresponds to the autoresonance regime of particle motion.

Note that the quantity $|1 - N^2|^{-1/2} \equiv \beta_{\text{ph}}|1 - \beta_{\text{ph}}^2|^{-1/2}$ can be viewed as a sort of wave 'momentum'. So, relation (60) shows that in the regime under consideration the longitudinal momentum of a particle should be proportional to the wave 'momentum'. In particular, at $G = 1$ they are equal.

An alternative approach was discussed in Ref. [52], where instead of condition (58), it was supposed that the longitudinal velocity of a particle should be equal to the wave phase velocity.

As in the $N = \text{const}$ case, one can obtain an equation for energy. To do this, one needs to use the integrals (cf.with Ref. [47])

$$P_\perp \sin\psi = \Phi\sqrt{2\Omega} = \text{const} \,, \tag{61}$$

$$\gamma - \frac{P_\perp^2}{2\Omega} = I = \text{const} \,, \tag{62}$$

and the relation

$$N = \frac{\gamma - \Omega}{\sqrt{(\gamma - \Omega)^2 \pm G^2}} \,. \tag{63}$$

It is necessary to note that the constants G, I and the parameter Ω are related through

$$I \to I_\pm = \frac{1 + \Omega^2 \pm G^2}{2\Omega} \,. \tag{64}$$

From here on the upper sign refers to the fast wave, and the lower one to the slow wave.

As a result, one obtains the desired equation of the form of (13), (14), where an explicit dependence on N is absent, and the constant Γ should be changed for $a_\pm = I_\pm + \Phi^2$. From the exact solution it follows that as $N \to 1$ the particle energy increases and scales as in the autoresonance regime, all the particles with the initial energy $\gamma_0 \geqslant a_\pm$ being trapped into the acceleration regime.

To find the explicit dependence $N(Z)$ one has an equation (for the particles with the accelerating phase)

$$\frac{\mathrm{d}N}{\mathrm{d}Z} = \frac{\varepsilon(1 - N^2)}{P_z^2}\sqrt{2\Omega(\gamma - a_\pm)} \,. \tag{65}$$

It follows that in the synchronous acceleration regime in the fast wave, the refractive index has to increase, and in the slow wave it has to decrease in the direction of the wave propagation. However, the explicit expression of $N(Z)$ seems to be rather complicated even in particular cases [53, 54]. Thus, it has little use for applications. In practice, the linear approximation can be useful

$$N(Z) = N_0 + \alpha(Z - Z_0) \,. \tag{66}$$

Assuming that approximation (66) is valid over the acceleration length L, one obtains an estimation for the dephasing

$$\Delta\psi \approx \varepsilon^2\Omega\,\frac{s^3}{6}(1 - N_0^2 - \alpha L N_0) \,. \tag{67}$$

At the same time the inequality $\alpha L/N_0 \ll 1$ should be satisfied. Here s is the dimensionless interval. If

$$|\alpha|LN_0 \approx |1 - N_0^2|\,, \tag{68}$$

then the synchronous regime is preserved over the all acceleration length determined by this relation.

If condition (68) is not fulfilled, then the phase variation may be large. In this case the synchronous regime is destroyed, and the maximal energy gained by the particle over the length L can be estimated by the formula

$$\gamma_{\max} \approx \left(\frac{3\varepsilon}{1 - N_0^2 - \alpha L N_0} \sqrt{\frac{\Omega}{2}} \right)^{2/3}. \tag{69}$$

This value can be larger than that in the case of particle acceleration in a nonluminous wave with constant phase velocity (38).

Note that the regime of synchronous particle deceleration is also possible, in addition to the acceleration regime. However in this case the refractive index must increase in the direction of wave propagation for a slow wave and decrease for a fast wave.

3.2 Maintenance of synchronism by changing the profile of the guide magnetic field

Early ideas on the possibility of sustaining the synchronism between a particle and a nonluminous wave were presented in Ref. [55]. Then this problem became the subject of investigation in a number of Refs [25, 56–60]. In particular, the profile of a nonuniform magnetic field providing autoresonance acceleration of a particle by a circularly polarized TE-wave was determined in Ref. [57] under the additional conditions: (a) the gyroradius of the particle should be constant, i.e. the particle should move over a cylindrical surface, (b) at each instant of time the particle remains in phase with the wave and derives its energy from the wave. Under these conditions two regimes of acceleration were found: (1) where the increase in the particle energy is related, mainly, to the increase of transverse momentum, the pitch of the helix thereby decreasing (in this regime strong magnetic fields are required); (2) where the energy increases, mainly through an increase in longitudinal momentum. Then, the pitch of the helical trajectory grows, and one needs to have a longer acceleration length than in the first regime.

However one can find the profile of the nonuniform magnetic field without additional assumptions about the particle trajectory [58].

If the guide magnetic field is strong enough, so that the drift theory is valid [23, 24], then in the zeroth-order approximation one can neglect the gradient and centrifugal drifts. In this case the averaged equations of motion for the electron in the field of an either fast or slow transverse wave take the form [58]

$$\begin{aligned}
\frac{\mathrm{d}P_\parallel}{\mathrm{d}s} &= \frac{P_\perp^2}{2}\,\nabla\cdot\mathbf{e}_1 - \varepsilon N P_\perp \cos\psi = \frac{P_\perp^2}{2}\,\nabla\cdot\mathbf{e}_1 + N\,\frac{\mathrm{d}\gamma}{\mathrm{d}s}\,,\\
\frac{\mathrm{d}P_\perp}{\mathrm{d}s} &= -\frac{P_\parallel P_\perp}{2}\,\nabla\cdot\mathbf{e}_1 - \varepsilon(\gamma - NP_\parallel)\cos\psi\,,\\
\frac{\mathrm{d}\psi}{\mathrm{d}s} &= \Omega - \gamma + NP_\parallel + \frac{\varepsilon}{P_\perp}(\gamma - NP_\parallel)\sin\psi\,,\\
\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s} &= \mathbf{e}_1 P_\parallel\,.
\end{aligned} \tag{70}$$

Here, the normalized momentum of the particle is $\mathbf{P} = P_\parallel \mathbf{e}_1 + P_\perp(\mathbf{e}_2\cos\theta_c + +\mathbf{e}_3\sin\theta_c)$; $\mathbf{e}_1 = \mathbf{\Omega}/\Omega$, $\mathbf{e}_2$, $\mathbf{e}_3$ is the basic set of the unit vectors, connected with the force line of the guide magnetic field, and

$$P_\parallel \nabla\cdot\mathbf{e}_1 = -\frac{P_\parallel}{\Omega}\,\mathbf{e}_1\cdot\nabla\Omega = -\frac{1}{\Omega}\,\frac{\mathrm{d}\Omega}{\mathrm{d}s}\,. \tag{71}$$

Under the above assumptions one can take

$$\Omega = \Omega(R_\parallel)\,, \tag{72}$$

where $R_\parallel \equiv \mathbf{R}\cdot\mathbf{e}_1 \approx Z$. In what follows we require that the initial condition of cyclotron resonance (at $N = \mathrm{const}$) should be conserved over the course of the particle motion

$$\gamma - NP_\parallel = \Omega(Z)\,. \tag{73}$$

In this case the desired profile $\Omega = \Omega(Z)$ will be determined by the equations of motion (70). So, with constraint (73) one can obtain integrals similar to (61), (62). From these integrals we derive the formula

$$(N^2 - 1)P_\parallel^2 = 1 - 2I\Omega(Z) + \Omega^2(Z)\,. \tag{74}$$

The dependence $P_\parallel^2(\Omega)$ at $N > 1$ is presented in Fig. 2. The vertex of the parabola has coordinates $\Omega_s = I$, $P_{\parallel s}^2 = (1 - I^2)/\,(N^2 - 1)$. In the case of $I \leqslant 1$ the acceleration of the particle takes place in the growing magnetic field at $\Omega \geqslant I$. In the case of $I > 1$ acceleration is possible only at $\Omega \geqslant \Omega_2 \equiv I + \sqrt{I^2 - 1}$.

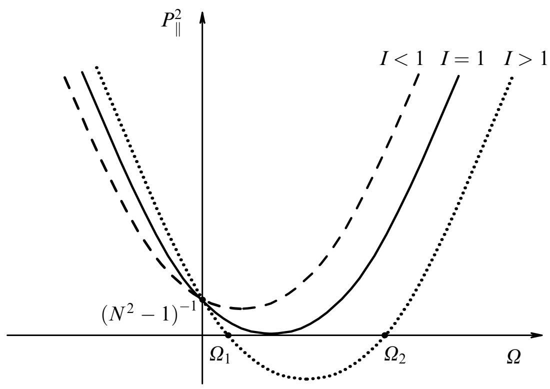

Figure 2. The Ω-dependence of $P_\parallel^2$ at $N > 1$.

In the case of a fast wave ($N < 1$) the dependence $P_\parallel^2(\Omega)$ is shown in Fig. 3. In this case there are no physical solutions at $I \leqslant 1$. If $I > 1$, then a restricted increase in $P_\parallel^2$ from 0 to $(I^2 - 1)/(1 - N^2)$ seems to be possible, when the guide magnetic field increases from $\Omega_1 = I - \sqrt{I^2 - 1}$ to $\Omega_s = I$. On further increasing the magnetic field from $\Omega_s = I$ to Ω_2 the particle reverts to the initial state.

Thus, in the case of a fast wave, the particle's longitudinal momentum varies periodically as the magnetic field grows over a highly restricted range, and as the original value of $P_\parallel^2$ increases, the possible variation of magnetic field narrows. At $P_{\parallel 0}^2 = I$ this range collapses, and the longitudinal momentum does not vary at all.

From the system (70), and considering the above-mentioned integrals one can obtain a formal equation for the

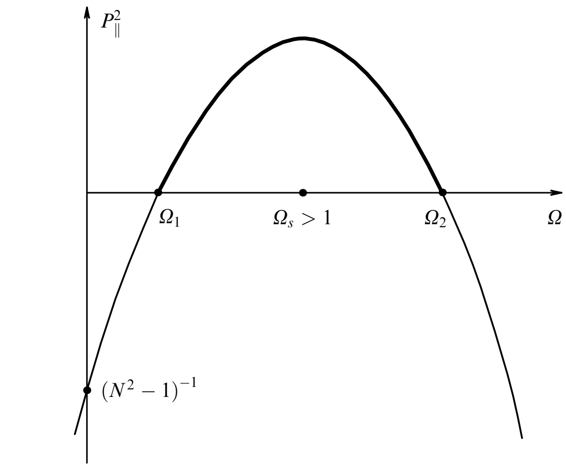

Figure 3. The Ω-dependence of $P_\parallel^2$ at $N<1$.

energy

$$\left(\frac{\mathrm{d}\gamma}{\mathrm{d}Z}\right)^2 = \frac{2\varepsilon^2\Omega(\gamma - I - \Phi^2)N^2}{(\gamma-\Omega)^2}\,. \tag{75}$$

In doing so one needs to use the relation $\Omega = \Omega(\gamma)$ which follows from the equation

$$\Omega^2 - 2\Omega\left[\gamma(1-N^2) + IN^2\right] + \gamma^2(1-N^2) + N^2 = 0\,.$$

However, it is very difficult to obtain an analytical solution of Eqn (75), as well as of the system (70). Therefore numerical methods are used [58]. The results of a numerical solution are presented in Fig. 4. One can see that the regime of the electron motion we are interested in really does have a resonant character. Beginning from some small distance, the dynamical variables of the particle as well as the guide magnetic field grow approximately linearly. The initial dephasing $\psi_0 = \pi$ seems to be stable: at the initial dephasing in the accelerating range from $\pi/2$ to $3\pi/2$ the resonant dephasing tends to π [58].

In other words, phase bunching takes place, and all the particles with initially accelerating phases are trapped into the regime of synchronous acceleration. During the numerical solution the magnetic field was excluded using the resonant condition (73). In this case the dependence $\Omega(Z)$ may be determined from graphic results. However, the profile of the magnetic field may be found analytically using the equation

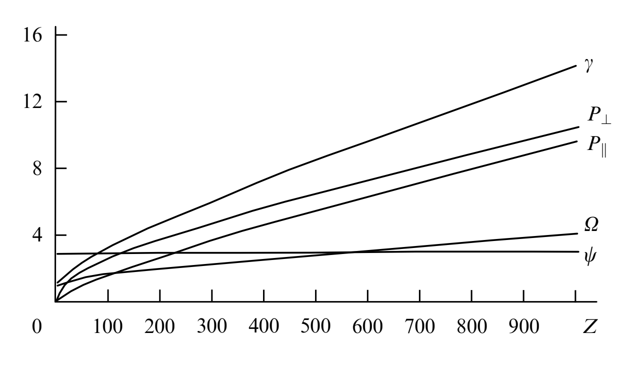

Figure 4. Autoresonance in a synchronizing magnetic field ($\Omega(0)=1$, $P_\parallel = 0.02$, $P_\perp = 0.205$, $\gamma(0) = 1.02$, $\psi(0)=\pi$, $\beta_{\mathrm{ph}} = 0.95$, $\varepsilon = 0.01$).

(for the particles in the accelerating phase)

$$\left[\gamma(1-N^2) - \Omega + IN^2\right]\frac{\mathrm{d}\Omega}{\mathrm{d}Z} = \varepsilon N(N^2-1)\sqrt{2\Omega(\gamma - I - \Phi^2)}\,. \tag{76}$$

In doing so one needs to use the dependence $\gamma = \gamma\{\Omega\}$. Solution of Eqn (76) is possible, but it is too complicated to be realized in practice. Thus, it is reasonable to consider simple profiles of the magnetic field, e.g. in the form of a linear dependence:

$$\Omega(Z) = \Omega_0\left[1 + \alpha(Z - Z_0)\right]\,. \tag{77}$$

It is evident that synchronism can not be preserved in this case as the particle moves along the guide field. At the same time, there is some distance, where motion can be considered as synchronous so the particle gains considerable energy (Fig. 5.)

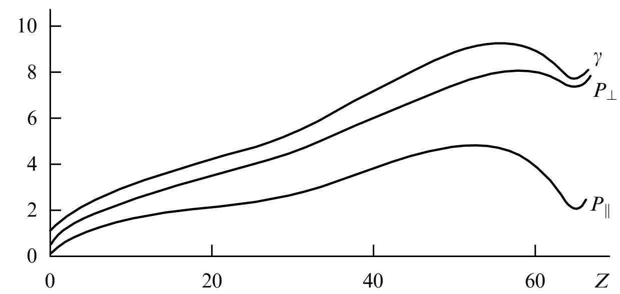

Figure 5. Synchronous regime in the case of a linearly increasing magnetic field ($\Omega(0)=1$, $P_\parallel(0) = 0.02$, $P_\perp(0) = 0.205$, $\gamma(0) = 1.02$, $\psi(0) = \pi$, $\beta_{\mathrm{ph}} = 0.95$, $\varepsilon = 0.1$, $\alpha = 0.05$).

By estimating the resonance mismatch and the phase shift one may find the value of the magnetic field gradient which provides a maximal gain of energy

$$\alpha_{\mathrm{opt}} = \frac{A\varepsilon^2(1-N^2)^2}{N\Omega_0}\,. \tag{78}$$

The parameter A is numerically found to be $A \approx 5\times 10^2$.

Note that to find the magnetic field profile consistently one needs to take into account both the transverse components of the guide field and the particle drift motion. A reduced version of this problem was considered in the paper [59]. On the basis of a numerical solution, the scaling laws were determined for the laser acceleration of electrons: the maximal acceleration length kz_m, depending on the accelerating field, varies according to the law ε^{-1} and depending on the phase velocity it varies as $(\beta_{\mathrm{ph}} - 1)^{-\nu}$, where $\nu \approx 0.6$. The energy of the accelerated particle varies in accordance with the relation $(\gamma_f - \gamma_i)/(\gamma_i - 1) \approx (\beta_{\mathrm{ph}} - 1)^{-\mu}$ where $\mu \approx 0{,}5$, γ_i, γ_f are the initial and final energies respectively.

A somewhat different arrangement of the problem of finding a synchronizing magnetic field was considered in Ref. [61]. The authors proposed a simplified model system, where the particle moves in a plane perpendicular to the magnetic field. The required field was also assumed to vary only in this plane. The case of plane-polarized wave traveling along the magnetic field was considered. It was assumed that along the given trajectory the wave's electric field on average did positive work on the particle. Then it was found that the desired magnetic field was periodic in the direction of wave propagation.

More recently, the idea of using a periodic magnetic field for the purposes of synchronization of a particle with a wave (undulator, wiggler) was applied to a variety of problems connected with the acceleration of charged particles and the generation of electromagnetic radiation [62–64].

3.3 Sustaining synchronism with a longitudinal electrostatic field

The effect of an electrostatic field parallel to a static magnetic field on the resonant interaction between a particle and a transverse electromagnetic wave at $N \neq 1$ was discussed in Refs [65–67]. In particular, the authors of Ref. [67] have considered the effect of an electrostatic field on the induced radiation of nonisochronous oscillators. An adequate profile of an electrostatic field for sustaining the resonance has been found in implicit form in Ref. [66]. It was numerically shown in Ref. [65] that the action of a uniform electrostatic field led to two types of energy gain in a wave with $N \neq 1$: either a steadily increasing oscillatory energy variation, or a step in energy followed by decaying energy oscillations.

Particle motion in a circularly polarized wave is described by Eqns (5)–(8) with additional terms from the electrostatic field (see Section 2.4). We assume that the initial resonance condition is sustained as the particle moves along the guide magnetic field. To do this one needs to have an electrostatic field which will compensate the phase shift arising due to $N \neq 1$. In this case the equations of motion give the relations

$$\varepsilon_0 f(Z) = -\frac{1}{\Omega}\frac{\mathrm{d}}{\mathrm{d}s}(N\gamma - P_z) = \frac{\varepsilon(N^2-1)P_\perp \cos\psi}{P_z - N\gamma} . \quad (79)$$

The desired electrostatic field is thus determined in implicit form [66].

From (79) it follows that there is a singular case $P_z - N\gamma = 0$ in which the electrostatic field is not defined. It happens only at $N < 1$ with the threshold value of energy $\gamma_{\lim} = \Omega/(1-N^2)$ [66]. Excluding this case we can introduce an electrostatic potential $U(Z)$ so that $f(Z) = -\mathrm{d}U/\mathrm{d}Z$, and $f(Z)P_z = -\mathrm{d}U/\mathrm{d}s$.

In the synchronous regime we can obtain separate equations for the determination of both potential and energy. To find these equations let us write integrals of the equations of motion of the particle

$$P_\perp \sin\psi = \Phi = \text{const} , \quad (80)$$

$$\gamma - \frac{P_\perp^2}{2\Omega} + \varepsilon_0 U(Z) = \Gamma = \text{const} , \quad (81)$$

$$(1-N^2)P_z^2 + 2\varepsilon_0 \Omega U(Z) = I = \text{const} . \quad (82)$$

The relativistic relation also takes place

$$(N^2-1)\gamma^2 + 2\Omega\gamma = NP_\perp^2 + N^2 + \Omega^2 . \quad (83)$$

From (82) it follows that particles behave variously in fast and slow waves: with an increase in potential at $\varepsilon_0 > 0$ a particle in a slow wave is accelerated, but in a fast wave it is decelerated. The converse occurs for a decrease in potential.

With the obtained relations we can find the equation for the determination of particle energy (in the accelerating phase)

$$\frac{\gamma(\gamma+q)\,\mathrm{d}\gamma}{\sqrt{(N^2-1)(\gamma+q)^2 - a}} = \varepsilon q N \,\mathrm{d}\tau , \quad (84)$$

where

$$a = \frac{N^2}{N^2-1}\left[N^2 + \Omega^2 - 1 + (N^2-1)\Phi^2\right] , \qquad q \equiv \frac{\Omega}{N^2-1} . \quad (85)$$

From these expressions it follows that the acceleration rate in a slow wave falls with increasing energy faster than in an autoresonant regime in a vacuum wave: $\mathrm{d}\gamma/\mathrm{d}\tau \approx 1/\gamma$.

There is an exact solution of Eqn (84) (at $N > 1$):

$$\left[\xi\sqrt{\xi^2 - a} + \left(a - \frac{\Omega}{\sqrt{N^2-1}}\right)\operatorname{arcosh}\frac{\xi}{\sqrt{a}}\right]\Bigg|_0^\tau = 2\varepsilon\tau\Omega N\sqrt{N^2-1} , \quad (86)$$

where $\xi \equiv (\gamma+q)\sqrt{N^2-1}$. Equation (79) also yields the equation for the determination of the electrostatic field potential with which the synchronous regime of particle motion in the accelerating phase is sustained

$$\varepsilon_0\frac{\mathrm{d}U}{\mathrm{d}Z} = \frac{\varepsilon\sqrt{P_z^2(N^2-1) + 2N\Omega P_z + \Omega^2 - \Phi^2}}{P_z + Nq} . \quad (87)$$

Here P_z must be expressed via U with integral (82). It gives, in principle, the possibility for the determination of $U(Z)$. However the resulting dependence is rather complicated, so it is unlikely that it may be realized experimentally. Therefore it is reasonable to consider a simple, e.g. a linear, dependence of the form $U(Z) = 1 + \alpha Z$. This corresponds to a uniform electrostatic field. This case was studied in detail in Ref. [65].

It is apparent that a uniform electrostatic field cannot maintain synchronism for an extended time interval. As in earlier sections we can estimate a maximal energy gain for the particle before the initial gyro-resonance condition is mismatched

$$\gamma_{\max} \approx \varepsilon_0\alpha N|q|s_{\max} + |q|\left[\varepsilon^2(1-N^2) + (\varepsilon_0\alpha)^2\right]\frac{s_{\max}^2}{2} , \quad (88)$$

where

$$s_{\max} \approx \left\{\frac{6}{\Omega\left[\varepsilon^2(1-N^2) + (\varepsilon_0\alpha)^2\right]}\right\}^{1/3} . \quad (89)$$

In the absence of an electrostatic field this estimate agrees with (37), so the energy of the particle is a periodic function of time. As long as the electrostatic field is small enough, its effect on the character of the energy variation is weak. In this case the energy oscillates and on average increases. There is a value of the field $\varepsilon_0\alpha_{\text{opt}}$, which provides a significant growth of energy. If the field exceeds this value, then the resonance condition is violated sooner, so the particle has no time to gain sufficient energy. One can evaluate the optimal gradient of the electrostatic field

$$\varepsilon_0\alpha_{\text{opt}} = \frac{A}{N}\left(\frac{\varepsilon^2|1-N^2|}{\sqrt{N}}\right)^{2/3} , \quad (90)$$

where the constant A is numerically found to be $A \approx 1,2$ [65].

3.4 Sustaining synchronism by an electrostatic field crossed with the guide magnetic field

Under the action of an electrostatic field, crossed with the guide magnetic field, the character of resonant interaction between the wave and particle changes considerably, so some physical effects appear. In particular, the mechanism of the Cherenkov absorption of a transverse electromagnetic wave, propagating along a steady magnetic field seems to be possible, and this mechanism compares well with that of cyclotron absorption [68]. Peculiar autoresonance in the case of a nonrelativistic charged particle in the field of a plasma wave, traveling at an angle to the external magnetic field has also been revealed [69]. This effect is wholly caused by the slightly inhomogeneous electric field, crossed with the magnetic field.

Consider next the possibility of sustaining the synchronous regime of a charged particle in a nonluminous wave taking into account the electric drift. We take the direction of the steady electric field E_0 to be the y-direction, and the direction of B_0 to be the z-direction.

To extract the cyclotron rotation of a particle we need to make a transformation from the laboratory frame of reference to the frame of reference Σ', moving with the electric drift velocity $V_E = cE_0/B_0$ along the x-axis. Then for the components of the 4-vector velocity in this system $w^i = (w^0, w^1, w^2, w^3)$ we have covariant equations [70], which after averaging over the fast phases yield a simplified system of equations of motion of a particle in the range of cyclotron resonance

$$\frac{dw^0}{ds} = \frac{w_\perp}{2}(\varepsilon_1 + \varepsilon_2\Gamma)\cos\psi\,, \tag{91}$$

$$\frac{dw_\perp}{ds} = \frac{1}{2}\left[\varepsilon_1(w^0 - N\Gamma w^3) + \varepsilon_2\Gamma(w^0 - Nw^3)\right]\cos\psi\,, \tag{92}$$

$$\frac{dw^3}{ds} = \frac{Nw_\perp}{2}(\varepsilon_1\Gamma + \varepsilon_2)\cos\psi\,, \tag{93}$$

$$\frac{d\psi}{ds} = \frac{1}{\Gamma\Omega}\left[\Omega - \Gamma^2\left(w^0 - \frac{N}{\Gamma}w^3\right)\right] - \frac{\sin\psi}{2w_\perp}\left[\varepsilon_1(w^0 - N\Gamma w^3) + \varepsilon_2(\Gamma w^0 - Nw^3)\right]. \tag{94}$$

Here $\Gamma = (1 - V^2)^{-1/2}$; $V = V_E/c$; $\varepsilon_i = E_i/B_0$, $ds = d\tau/\gamma$ is the dimensionless interval. The above equations have the integral

$$w^0 - \frac{\varepsilon_1 + \varepsilon_2\Gamma}{N(\varepsilon_1\Gamma + \varepsilon_2)}\,w^3 = I = \text{const}\,. \tag{95}$$

The exact cyclotron resonance is determined by the relationship

$$\Gamma^2\left(w^0 - \frac{N}{\Gamma}w^3\right) = \Omega\,. \tag{96}$$

It follows that the condition of cyclotron resonance will be preserved as the particle moves, if the quantity $w^0 - (N/\Gamma)w^3 \equiv Y$ is a constant of motion. Thus the condition of cyclotron resonance (96) is preserved if the following relationship is fulfilled

$$\frac{N}{\Gamma} = \frac{\varepsilon_1 + \varepsilon_2\Gamma}{N(\varepsilon_1\Gamma + \varepsilon_2)}\,, \tag{97}$$

with $I = Y = \Omega/\Gamma^2$.

The given synchronous regime of motion of the particle is thus seen to be dependent on the wave polarization. In the general case of a wave with arbitrary elliptic polarization the synchronism between a particle and the wave will be preserved if the crossed electrostatic field is determined by the equation

$$\Gamma = \frac{\varepsilon_1(N^2 - 1) + \sqrt{\varepsilon_1^2(N^2 - 1)^2 + 4N^2\varepsilon_2^2}}{2\varepsilon_2}\,. \tag{98}$$

In the case of a circularly polarized wave ($\varepsilon_1 = \varepsilon_2$) the synchronous regime occurs under the condition

$$\Gamma = N^2\,. \tag{99}$$

In the case of a transverse linearly polarized wave we have two possibilities. The first possibility is

$$(1)\ \ \varepsilon_1 \equiv \varepsilon \neq 0\,, \qquad \varepsilon_2 = 0\,. \tag{100}$$

Then in accordance with (97) the refractive index must be equal to unity. Thus, in the case of a vacuum wave, the autoresonant regime exists in the moving reference system as well, if the wave is polarized in the direction of electric drift. The second possibility is

$$(2)\ \ \varepsilon_1 = 0\,, \qquad \varepsilon_2 \equiv \varepsilon \neq 0\,. \tag{101}$$

It follows from (97) that if a wave is polarized in the direction of the electrostatic field, the synchronous regime is possible only for a slow wave and under the condition

$$\Gamma = N > 1\,. \tag{102}$$

Our calculations show that the case (101) is the optimal one. In this case one can find the equation for energy

$$w^0\,\frac{dw^0}{d\tau'} = \frac{\varepsilon\Omega}{2N}\sqrt{2Yw^0 - a^2}\,, \tag{103}$$

where τ' is the normalized time in the moving frame of reference, $a^2 = 1 + Y^2 + \Phi^2$. It follows that in this case the acceleration rate falls in accordance with a law, similar to that in the case of the autoresonant regime in a vacuum wave.

The exact solution of Eqn (103) is similar to (16).

The energy of reasonably accelerated particles in the moving frame of reference scales as

$$w^0 \approx \left(\frac{3\varepsilon Y\Omega\tau'}{N\sqrt{2}}\right)^{2/3}. \tag{104}$$

The energy gained on average by the particle, is $\Gamma = N$ times as large in the laboratory frame of reference as in the frame of reference moving with the electric drift velocity. The time dependence of the particle energy in the laboratory frame of reference is shown in Fig. 6. From this figure we

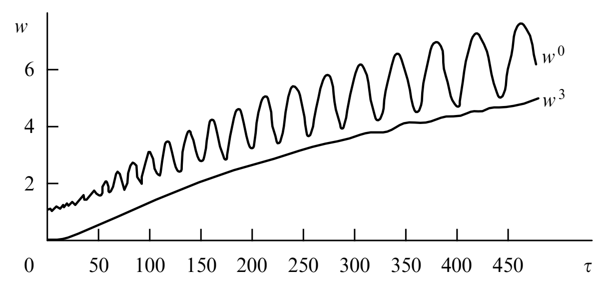

Figure 6. The time dependence of the particle energy in the laboratory frame ($N = 1.1$, $\varepsilon = 0.05$, $V = 0.42$, $\psi_0 = 0$).

notice that although the energy oscillates, it increases on average, as in the autoresonant regime.

Notice that in addition to the acceleration regime of the particles in a slow wave, the regime of their deceleration and stimulated emission is also possible [71]. This is evident from Fig. 7.

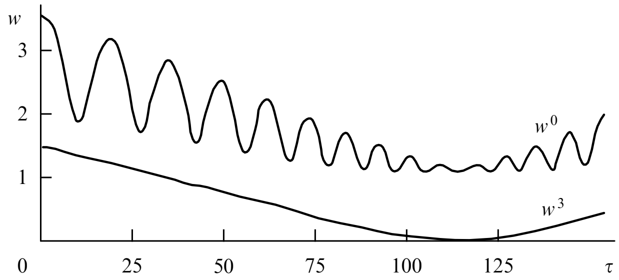

Figure 7. The regime of deceleration of the particle in a slow wave ($N = 1.1$, $\varepsilon = 0.05$, $V = 0.42$, $\psi_0 = \pi$).

4. Conclusions

Our analysis of cyclotron autoresonance is based on the single particle approximation which is valid if the energy of the accelerating field is far in excess of the energy gained by the particle. In this case we may ignore the reaction of an accelerated current on the field. Otherwise it is necessary to use a self-consistent description of the interaction between the particles and the wave. In Ref. [72] the autoresonant acceleration of the particle beam was analyzed on the basis of the self-consistent cold beam model under the assumption that the beam density was constant and the longitudinal velocity depended only on time. It has been shown that a rapid increase in energy occurred initially and then the acceleration rate decreased (as well as in the case of the single-particle approximation). The maximal energy of the particles was determined using laws of conservation of energy and momentum in the wave – beam system. A self-consistent theory of interaction between the relativistic electron beam and the electromagnetic wave was developed in Ref. [73] on the basis of kinetic approach.

If the charged particle beam propagates in a dense plasma, then the problem is rendered rather complicated due to a variety of possible effects. However, as is shown in Ref. [74] there are conditions when the effect of the directional motion of the particles prevails and the beam does not manage to be heated during the time of acceleration. In such a situation, Coulomb's collisions between the beam particles and the plasma particles destroy the phase synchronism between the beam and the wave, so the growth of the beam energy is limited. It is quite similar to the action of radiative friction on the autoresonant motion of a particle.

Acceleration of particles in the regime of autoresonance is also possible in the medium with an inverse population of energetic states in which a transverse wave is generated [75].

In the problem of generation of electromagnetic radiation the autoresonance mechanism is useful for the increase in efficiency of the interaction between the relativistic electron beam and not only single waves but also combination of waves. It is realized in the devices which are known as ubitron and scattron [76, 77].

We will point out some features of the autoresonant mechanism under astrophysical conditions. Calculations on the basis of Eqn (13) show [78] that under the conditions of a high-latitude (65°) magnetospheric plasma at an altitude of 1000 – 2000 km electrons may acquire a considerable energy during a short time and may then be precipitated in the auroral zone. This may be the reason for occasional auroral and geomagnetic phenomena. When strong coherent radiation and strong magnetic fields arise, e.g. in a pulsar or quasar, such a mechanism may lead to the acceleration of cosmic rays. The authors of Ref. [79] considered the simultaneous autoresonant acceleration of guided electrons and positrons by an intensive linearly polarized electromagnetic wave, propagating along an axial magnetic field. It was shown that high-current electron – positron beams can be accelerated to high energies with low radiation losses. This mechanism may act, for example in a pulsar magnetosphere, which consists of a magnetized electron – positron plasma [80].

In parallel with the regular autoresonant mechanism the mechanism of stochastic autoresonant acceleration of charged particles was also discussed [81, 82]. In this case a marked increase in the average kinetic energy of particles may occur.

Due to active experiments in Space new problems on the propagation of the strong electromagnetic pulses in plasmas along the magnetic field are arising [83, 84]. It was shown in [84] that in the autoresonant regime such pulses are longitudinally transverse and have the form of solitons with 'built in' Langmuir oscillation.

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