METHODOLOGICAL NOTES

PACS numbers: 05.30.-d, 05.70.Ln, 34.10.+x, 42.50.Ct

Quantum models of relaxation

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<u>Abstract.</u> Exactly solvable quantum models for an atom or a molecule interacting with a physical environment are considered. The kinetic description of the random walk of an open system is shown to be generally consistent with the phenomenological relaxation equations found use in spectroscopy. Some aspects of the problem of entropy increase are also revised.

1. Introduction

There is a rich historical background behind the application of kinetic relaxation equations to atomic and molecular spectroscopy. Starting from the semiclassical Bloch [1] and energy spin component [2, 3] equations, the theory of quantum microwave processes have incorporated effective group-theoretical methods [4, 5] and adopted some lines of attack of the fundamental quantum field theory. Given that at the same time much of the current work continues to rely on semiclassical or phenomenological methods of approach, it is sometimes difficult to obtain a unifying picture of the processes which, while seemingly different, are in fact of a similar nature both structurally and physically.

One of the basic achievements of the last two decades is the recognition that physical kinetics makes a part of the general theory of open systems [6-9]. In this context, the irreversible character of relaxation processes results from the interaction of the object with external subsystems (not subject to further observation) and is fundamentally quantummechanical. It seems very instructive — and this is the aim of the present paper — to illustrate this approach step-by-step by using models for which an exact quantum solution is possible.

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Received 22 February 1996 Uspekhi Fizicheskikh Nauk **166** (7) 795–800 (1996) Translated by E G Strel'chenko, edited by A Radzig There is another major reason for addressing this subject. In parallel with the development of quantum kinetics, a principally different approach to the justification of statistical physics has been persistently advocated over the years: the dynamic chaos concept that came to existence and has been developed on the basis of classical nonlinear mechanics [10-12]. Although corresponding quantum models have also been recently proposed [13], still at the heart of the chaos idea is the notion of infinitely tangled phase trajectories of a closed system as the basic cause of randomness. While advancing no constructive ideas for the theory of relaxation, this approach puts in a claim on revealing the profound causes of irreversibility in nature: the claim which we believe is unfounded.

Now this does not mean at all to deny the theoretical and practical significance of the work on discontinuous transformations in nonlinear dynamics. These transformations may be undoubtedly adequate to reflect reality in many problems allowing the macroscopic continuum description of an object (such as nonlinear waves or turbulence theory). But it would of course be naive to take it as the basis for constructing, say, a theory of radioactive decay.

Notice that the history of science abounds with such examples of extending a method beyond the application area where it was first found practically effective. Suffice it to mention the attempts to interpret the Maxwell's equations in terms of elasticity theory; or to describe the hydrogen atom spectrum with the nonlinear theory of vibrations.

To date, the quantum theory of open systems has proved helpful in resolving many of the earlier debated questions on the nature of irreversibility. The clue was provided by the famous Einstein–Podolsky–Rosen [14] paradox, a subject of repeated examination for nearly 60 years now. Unfortunately, the proper realization of what has been achieved on this way is lagging because of the physicists' minds being all too slow to recognize the principle of superposition of quantum states as one of the fundamental principles of reality. It indeed looks like [15] intuition needs some kind of breakthrough which not only would affect the mathematical formulations of the physical laws, but also the logical and philosophical foundations of epistemology (Wissenshaftslehre) itself [16].

2. Quantum kinetic equation

Let us use the language of the theory of open systems to derive the fundamental kinetic equation [9]. Suppose an external medium acts on a quantum system (object) described by the Hamiltonian H_s . We consider the medium to be a collection (uniform flow) of identical physical subsystems with which the object under study successively interacts, and we wish to know how the state of the object evolves in response to this sequence of interactions (Fig. 1). The dashed line in the figure indicates that the states of the corresponding subsystems are no longer statistically independent.



Figure 1.

We denote by H_t the Hamiltonian of a single external subsystem (flow representative) and by H_{int} its interaction with the object at collision, and we require $[H_{int}, H_s + H_t] = 0$ (energy conservation condition). In the special case of an object interacting with thermal radiation, a suitably introduced field oscillator plays the role of a flow representative. A flow of an arbitrary physical nature but with equilibrium statistical properties is also referred to as a thermostat.

Let $\hat{\rho}_0$ and $\hat{\sigma}$ be the density operators for the initial states of the object and a flow representative, respectively, it being assumed that these state are statistically independent. In the usual way [9], the density matrix of the object after the interaction is written as

$$\left\langle n|\rho_1|m\right\rangle = \sum_{\nu} \left\langle n,\nu|U\rho_0\sigma U^+|m,\nu\right\rangle. \tag{1}$$

Here and in what follows we employ Latin and Greek indices for the basis states of the object and the flow representative, respectively. The unitary evolution operator U acts in the Hilbert space extended to include the states of both interacting subsystems (direct product space [17]). Using the energy representation for subsystem operators and the interaction representation for the time evolution of states at collision we should write

$$U = \exp\{iH_{\rm int}t_{\rm c}\},\tag{2}$$

where t_c is the interaction time (we adopt the system of units in which $\hbar = 1$). When analyzing particular models, we shall sometimes specify the evolution operator U directly without explicitly writing out the corresponding H_{int} .

We shall number by *j* the successive interactions of the object with flow representatives, and denote by $\hat{\rho}_j$ its density-of-states operator after the *j*-th interaction. Assuming that the initial mixed states of all the representatives are identical and mutually independent, from (1) it follows

$$\langle n|\rho_{j+1}|m\rangle = \sum_{p,q} (n,m|p,q) \langle p|\rho_j|q\rangle,$$
(3)

where we have defined

$$(n,m|p,q) = \sum_{\mu,\nu,\lambda} \langle n,\mu|U|p,\nu\rangle \langle \nu|\sigma|\lambda\rangle \langle q,\lambda|U^{+}|m,\mu\rangle.$$
(4)

Eqn (3) is the quantum kinetic equation (generalized Markovian chain) with the transition matrix (n, m| p, q) not necessarily interpretable in terms of probability [18].

When applied to the spectroscopy of collisional relaxation, this equation will be used in the following way. We take as our object a single molecule whose excitation by a resonant electromagnetic field we wish to describe. Other molecules in the gas volume will be treated as an external flow which secures the relaxation of the object excitations. We shall assume that the molecules in the flow are in equilibrium, thus neglecting the small perturbing effect the field exerts on the molecular states. As readily verified, only this approximation allows one to employ a linear theory to treat the spectral excitations of the molecule.

To pass to the kinetic equation in differential form, the 'coarse-grain' averaging idea [7] will be used. Assuming that the molecule collides many times during the time dt, and denoting by τ its characteristic time between the collisions, Eqn (3) is rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle n|\rho|m\rangle = \tau^{-1} \left(\sum_{p,q} (n,m|p,q) \langle p|\rho|q\rangle - \langle n|\rho|m\rangle \right).$$
(5)

To make the equation complete we must also add a perturbation due to the exciting field, which we will represent as a time-dependent operator V(t) acting on the states of the object only. In principle, this last requirement is not obligatory because in the framework of the theory of open systems any nonequilibrium influence may be described in exactly the same way as are relaxational thermal collisions (see, e.g., [8]). For most problems in atomic and molecular spectroscopy, however, the classical field approximation proves to be quite sufficient. With the above in mind we finally write

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle n|\rho|m\rangle = \mathrm{i} \langle n|[\rho, V(t)]|m\rangle + \tau^{-1} \left[\sum_{p,q} (n,m|p,q) \langle p|\rho|q\rangle - \langle n|\rho|m\rangle \right].$$
(6)

The discussion which follows will be concerned with the meaning of this equation as applied to physical models amenable to an exact quantum solution.

3. Collisions of two-level atoms

1

A system with two energy levels is perhaps the most traditional quantum transition model in theoretical spectroscopy. Most commonly the model involves equations with phenomenologically introduced longitudinal T_1 and transverse T_2 relaxation times [1], and we shall show below that using Eqn (5) to simulate open system relaxation proves to be fully consistent with this semiempirical approach.

Consider a collision of two identical atoms, each having only two nondegenerate energy levels E_1 and E_2 . The extended system containing two above atoms has three levels: $2E_1$, $E_1 + E_2$ and $2E_2$, one of these being doubly degenerate (Fig. 2). Clearly, owing to the conservation of energy at collision, the quantum amplitudes are zero for all the

$$2E_2 |2,2\rangle$$

$$E_1 + E_2$$

 $|1,1\rangle$

|1, 2|

Figure 2.

transitions except $|1,2\rangle \Leftrightarrow |2,1\rangle$. The nonzero amplitudes can, with no loss of generality, be expressed with the help of a unitary matrix

 $2E_1$

$$\begin{pmatrix} \langle 1, 2|U|1, 2 \rangle & \langle 1, 2|U|2, 1 \rangle \\ \langle 2, 1|U|1, 2 \rangle & \langle 2, 1|U|2, 1 \rangle \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$
(7)

and the identity (to within an unimportant phase) transformation for the nondegenerate singlet states

$$\langle 1, 1|U|1, 1 \rangle = \langle 2, 2|U|2, 2 \rangle = 1.$$
 (8)

To be fully rigorous and general, we should, at this stage, take a unitary matrix of the basis representation of the complex-element group SU(2) to represent the right-hand side of (7). A more elaborate analysis shows, however, that again due to the irrelevance of quantum state phases, a and bcan be chosen to be purely real as well as obeying the condition $a^2 + b^2 = 1$. Thus, in the present model a single parameter b is sufficient to describe the full extent of the transformation the wave functions undergo in a collision act.

We next calculate the relaxation rate of the off-diagonal density matrix element $\langle 1|\rho|2\rangle$. According to (5) and (6), this requires that we find the kinetic coefficient (1, 2|1, 2):

$$(1,2|1,2) = \langle 1,1|U|1,1 \rangle \sigma_1 \langle 2,1|U^+|2,1 \rangle + \langle 1,2|U|1,2 \rangle \sigma_2 \langle 2,2|U^+|2,2 \rangle = a(\sigma_1 + \sigma_2) = (1-b^2)^{1/2}.$$
(9)

In accord with the above discussion, the state $\langle \mu | \sigma | v \rangle$ of the external atom (a flow representative) is taken to be diagonal in the energy representation with the Gibbs eigenvalues

$$\sigma_{1,2} = \frac{1}{Z} \exp\left(-\frac{E_{1,2}}{\Theta}\right), \quad \sigma_1 + \sigma_2 = 1, \quad (10)$$

where Θ is the gas temperature.

Substituting (9) into (5) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle 1|\rho|2\rangle = -(1-a)\tau^{-1}\langle 1|\rho|2\rangle.$$
(11)

It is readily verified that all the other elements (1, 2| p, q) in the relaxation equation for $\langle 1|\rho|2\rangle$ are zero.

Let us find the relaxation coefficients for the diagonal elements $\langle 1|\rho|1\rangle$ and $\langle 2|\rho|2\rangle$. We write

$$(1,1|1,1) = \langle 1,1|U|1,1\rangle\sigma_1\langle 1,1|U^+|1,1\rangle + \langle 1,2|U|1,2\rangle\sigma_2\langle 1,2|U^+|1,2\rangle = \sigma_1 + a^2\sigma_2.$$
(12)

Calculations go through exactly as before to give

$$(1,1|2,2) = b^2 \sigma_1 \,, \tag{13}$$

$$(2,2|1,1) = b^2 \sigma_2 \,, \tag{14}$$

$$(2,2|2,2) = a^2 \sigma_1 + \sigma_2 \,. \tag{15}$$

As customary, it is convenient to introduce the difference between the relative populations, $r = \langle 1|\rho|1 \rangle - \langle 2|\rho|2 \rangle$. Then, substituting the model results (12)–(15) into (5) we arrive at

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -b^2 \tau^{-1} (r - r_0) \,, \tag{16}$$

where $r_0 = \sigma_1 - \sigma_2$ is the equilibrium difference.

Comparing (11) and (16) with the conventional form of the relaxation equations [1] makes it possible to write down the longitudinal and transverse relaxation times for our model:

$$T_1 = \frac{\tau}{b^2}, \qquad T_2 = \frac{\tau}{1 - (1 - b^2)^{1/2}}.$$
 (17)

Of particular interest is the limiting case $|b| \ll 1$. Setting $(1-b^2)^{1/2} \approx 1-b^2/2$ (which corresponds to the first-order perturbation theory) it is found that for 'weak' collisions our model yields $T_2 = 2T_1$. The physical meaning of this result is quite transparent: the 'longitudinal' coefficient characterizes the energy relaxation of the system, whereas the attenuation of the off-diagonal elements describes relaxation of the polarization amplitude (see, e.g., [19], p. 111). Notice that this is exactly the energy-to-polarization relaxation rate ratio typical of a classical system of weakly viscous charged particles.

Despite its extreme simplicity, the model under discussion enables some practically-interesting conclusions to be made. According to familiar gas-kinetic rules, as the temperature Θ increases, the free-running time τ decreases as $\Theta^{-1/2}$. Therefore if, for a fixed gas density (a hermetic vessel containing an atomic vapor), a temperature increase is found to cause a faster decrease in the characteristic relaxation time (as very accurately measured from spectral line broadening [20]), then the only possible explanation is a change in the parameter *b*, which is evidence of an enhanced wave function transformation occurring at collisions. This, in turn, yields useful information on subtleties of the collisional processes involved.

Finally, if the ratio T_2/T_1 is found to differ from 2, this means that either we have entered the 'strong' collision regime or, alternatively, the two-level approximation is inapplicable to the particular spectroscopic object at hand.

4. Relaxation in a thermal field

In a second model we wish to consider, an elementary relaxation act of a (two-level) system presents its interaction with a thermal oscillator. This example corresponds to atomic collisions with thermal collective excitations (phonon field) in a solid and also to relaxation via interaction with equilibrium radiation [21].

Thus, we again consider a two-level object with energies E_1 and E_2 , and analyze, this time, its interaction with a quantum oscillator of the frequency $\omega = E_2 - E_1$ (in our system of units $\hbar = 1$ and frequency has the dimension of

$$H_{t}|\nu\rangle = \omega\left(a^{+}a + \frac{1}{2}\right)|\nu\rangle = \omega\left(\nu + \frac{1}{2}\right)|\nu\rangle, \qquad (18)$$

where a is the photon annihilation operator. Let us next write down the matrix of the dipole interaction between the object and the oscillator. In the energy representation only the elements

$$\langle 1, \nu | H_{\text{int}} | 2, \nu - 1 \rangle = \gamma \sqrt{\nu}$$
 (19)

and their complex conjugates are nonzero. Here the values n = 1, 2 correspond to energies E_1, E_2 of the object, and the constant γ is determined by the off-diagonal element of the dipole moment.

The dipole interaction between a two-level system and a quantum oscillator lends itself to an exact solution [22]. The nonzero elements of the unitary evolution matrix are

$$\begin{pmatrix} \langle 1, \nu | U | 1, \nu \rangle & \langle 1, \nu | U | 2, \nu - 1 \rangle \\ \langle 2, \nu | U | 1, \nu + 1 \rangle & \langle 2, \nu | U | 2, \nu \rangle \end{pmatrix}$$
$$= \begin{pmatrix} \cos(\alpha \sqrt{\nu}) & i \sin(\alpha \sqrt{\nu}) \\ i \sin(\alpha \sqrt{\nu + 1}) & \cos(\alpha \sqrt{\nu + 1}) \end{pmatrix},$$
(20)

where $\alpha = \gamma t_c$. As before, it is assumed that a flow representative (in this case, an oscillator) is in a Gibbs state

$$\langle \mu | \sigma | \nu \rangle = \sigma_{\nu} \delta_{\mu\nu} = [1 - \exp(-\beta)] \exp(-\beta\nu) \delta_{\mu\nu},$$
 (21)

where $\beta = \omega / \Theta$.

Evaluating the 'transverse' relaxation coefficient we find

$$(1,2|1,2) = \sum_{\mu,\nu} \langle 1,\mu|U|1,\nu \rangle \sigma_{\nu} \langle 2,\nu|U^{+}|2,\mu \rangle$$
$$= \sum_{\nu=0}^{\infty} \sigma_{\nu} \cos(\alpha \sqrt{\nu}) \cos(\alpha \sqrt{\nu+1}), \qquad (22)$$

which in conventional small-perturbation approximation $\alpha\sqrt{\nu} \ll 1$ becomes

$$(1,2|1,2) = 1 - \alpha^2 \left(\bar{\nu} + \frac{1}{2} \right),$$
 (23)

where

$$\bar{\nu} = \sum_{\nu} \nu \sigma_{\nu} = \left[\exp(\beta) - 1 \right]^{-1}.$$
(24)

The small perturbation condition can be fulfilled for all *v* values of interest if the temperature Θ is not too high, namely $\alpha^2/\beta \ll 1$.

Substitution of (23) into (5) now gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle 1|\rho|2\rangle = -\alpha^2 \tau^{-1} \left(\bar{\nu} + \frac{1}{2}\right) \langle 1|\rho|2\rangle.$$
(25)

Thus we find that the relaxation rate of an off-diagonal element for the object density matrix is proportional to the mean energy of the thermal oscillator (as defined including the vacuum fluctuations). In this approximation the 'longitudinal' coefficients can be expressed as

$$(1,1|1,1) = 1 - \alpha^2 \bar{\nu}, \tag{26}$$

$$1, 1|2, 2) = \alpha^2(\bar{\nu} + 1), \qquad (27)$$

$$(2,2|1,1) = \alpha^2 \bar{\nu}, \tag{28}$$

$$(2,2|2,2) = 1 - \alpha^2(\bar{\nu}+1).$$
⁽²⁹⁾

We again introduce the difference $r = \langle 1|\rho|1 \rangle - \langle 2|\rho|2 \rangle$, for which, using the basis equation (5), we obtain

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\alpha^2 \tau^{-1} (2\bar{\nu} + 1)(r - r_0) \,, \tag{30}$$

where

$$r_0 = \frac{1}{2\bar{\nu} + 1} = \tanh\frac{\beta}{2} . \tag{31}$$

Comparing (30) and (25) we see that this model also yields $T_2 = 2T_1$ for the extreme case of weak relaxational perturbations.

The numerical values of T_2 and T_1 for atomic relaxation in an equilibrium radiation field are obtained by setting $\alpha^2 \tau^{-1} = 2\omega^3 |\langle 1|d|2 \rangle|^2/3c^3$, where $\langle 1|d|2 \rangle$ is the dipole-moment matrix element. The justification of this statement can be obtained in a variety of ways (see, for example, [23]) and is not considered here.

5. Entropy increase

Study of simple models helps to shed a new light on the ageold problem of statistical physics, heat flow from a hot to a cold body. For this purpose, let us consider once again a single act of interaction between a two-level atom and a thermal oscillator. Although the essence of the matter can, needless to say, be understood with other models — by considering interatomic collisions, for example — it is preferable to model the interaction of subsystems of different physical nature when a question of such generality is discussed.

Thus, let the statistically independent, initial mixed states of the atom and the oscillator are described by Gibbs density operators with different temperatures Θ_s and Θ_t :

$$\langle n|\rho|m\rangle = \left[1 + \exp(-\beta_{\rm s})\right]^{-1} \exp\left(-\frac{E_n}{\Theta_{\rm s}}\right)\delta_{nm},$$
 (32)

$$\langle v|\sigma|\mu\rangle = [1 - \exp(-\beta_t)]\exp(-\beta_t v)\delta_{\mu\nu},$$
 (33)

where again n = 1, 2; $E_2 - E_1 = \omega$; $\beta_{s,t} = \omega/\Theta_{s,t}$. Without any loss of generality, here and in what follows we set $E_1 = 0$.

We next apply the transformation (20) to these states and evaluate the energy transfer in the interaction act within the approximation $\alpha^2 \ll 1$, to obtain

$$\Delta \langle H_{\rm s} \rangle = -\Delta \langle H_{\rm t} \rangle = \sum_{n=1,2} E_n \Delta \rho_n$$
$$= \frac{1}{2} \alpha^2 \omega \left(\tanh \frac{\beta_{\rm s}}{2} \coth \frac{\beta_{\rm t}}{2} - 1 \right). \tag{34}$$

Likewise, the entropy changes are found to be

$$\Delta S_{s} = \Delta \left(-\sum_{n} \rho_{n} \ln \rho_{n} \right) = S_{s}' - S_{s}$$
$$= \alpha^{2} \beta_{s} \left\{ \left[\exp(\beta_{t}) - 1 \right]^{-1} \tanh \frac{\beta_{s}}{2} - \left[\exp(\beta_{s}) + 1 \right]^{-1} \right\},$$
(35)

$$\Delta S_{t} = \Delta \left(-\sum_{\nu} \sigma_{\nu} \ln \sigma_{\nu} \right) = S_{t}' - S_{t}$$
$$= \alpha^{2} \beta_{t} \left\{ \left[\exp(\beta_{s}) + 1 \right]^{-1} - \left[\exp(\beta_{t}) - 1 \right]^{-1} \tanh \frac{\beta_{s}}{2} \right\}$$
(36)

with $\Delta S = \Delta S_s + \Delta S_t$. Notice that the sum of the resulting subsystem entropies, $S'_s + S'_t$, is no longer equal to the quantum entropy of the complete system, which entropy does not change due to the system being closed. The subsystems simply lose their statistical independence when interacting.

Comparing (35) and (36) with (34) we write

$$\Delta S = \Delta \langle H_{\rm s} \rangle \left(\frac{1}{\Theta_{\rm s}} - \frac{1}{\Theta_{\rm t}} \right). \tag{37}$$

It is of fundamental importance that ΔS is always negative. This is precisely the reason why a spontaneous energy transfer is invariably directed toward the colder object, no matter what the nature of the interacting bodies.

The inequality $\Delta S \ge 0$ is readily verified in our model also, but it has a much more general importance. It is proved perfectly rigorously for subsystems of any physical nature and in any initial states, provided only that they are statistically independent prior to the interaction. A similar fundamental inequality for entropies of classical distribution functions has been proved by Shannon in his pioneering work on the theory of information [24]. A proper extension to quantum entropies is obtained by application of recently famous O Klein's lemma (see [9, 25]). It is now unambigously clear that the inequality $\Delta S \ge 0$ as obtained above is just the logical equivalent of the second law of thermodynamics in rigorous quantum theory.

A fairly high level of universality also turns out to be exhibited by expression (37). Consider a physical object which possesses an arbitrary energy spectrum E_n (it does not matter if some of these energies are identical) and interacts with its environment. It is readily shown that if $[H_{int}, H_s + H_t] = 0$ then the interaction does not violate the initial stationarity property $[\rho, H_s] = [\sigma, H_t] = 0$ of the subsystem states. If the initial mixed state of the object is a Gibbsian one at a temperature of Θ and if the density-matrix eigenvalue changes $\Delta \rho_n$ due to interaction with the environment are small, we may write

$$\Delta S_{\rm s} = -\sum_{n} (\Delta \rho_n) \ln \rho_n = \Theta^{-1} \sum_{n} E_n \Delta \rho_n \,. \tag{38}$$

It is perfectly obvious that, up to notation, this is equivalent to the differential definition of entropy as used in thermodynamics [26]. From this, making use of the inequality $\Delta S \ge 0$ and the energy conservation law, the whole of the theory of quasiequilibrium processes can be constructed in a purely axiomatic way. Thus, we cannot perceive seriously publications, appearing sometimes even in our days, which question the statistical justifications of thermodynamics.

6. Conclusions

Hopefully we have demonstrated convincingly enough the efficiency of the quantum theory of open systems as a tool for describing irreversible relaxation processes. Further work along these lines can only be concerned with improving physical models and overcoming mathematical difficulties which will inevitably arise. There exists, however, a great weakness in the above logical scheme which certainly is worth discussing, if only in brief.

The weakness in question is the 'coarse-grain' averaging we used in modelling relaxation process. Let alone the gross simplification caused by the neglect of the natural dissimilarity of individual interaction acts, a consistent quantum theory should hardly at all involve collisions lasting a definite period of time. In fact, if H_{int} is introduced more properly into the Hamiltonian then, with the energy representation so improved, it will no longer be possible to treat the moleculeflow interaction as a sequence of any kind of individual acts.

It appears that the reason runs deep and the problem cannot be resolved by any formal treatment. The point is that in developing a consistent quantum-mechanical approach we are still unable to get rid of the classical time t, along whose metaphysical axis we locate the quantum processes being considered. A more reasonable approach would be to treat time as a certain internal parameter for a set of specifically quantum events, but at present this is beyond our reach. We will restrict ourselves to the following preliminary remarks in this connection.

It is conceivable that rather than the variable t labelling unitary rotations, in a more mature theory the transformations themselves, or more precisely the hierarchy of the extensions of the groups for which these transformations are representations, will play a primary part. The parameterization of time in this hierarchy on the basis of the unitary space metric is a cardinal problem, and some approaches can already be suggested.

It is not within the scope, and inconsistent with the aim, of the present paper to discuss the above ideas in any detail. Besides, being as they are too exotic for the modern scientific mind, there is a danger that discussing them will prevent the reader from grasping the bulk material of the paper. It is our firm believe, however, that sooner or later such a discussion will be not undesirable and indeed necessary.

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