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Spin wave amplification in magnetically ordered crystals

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Abstract. The review deals with the macroscopic approach to the s – f exchange interaction of electrons in ordered and disordered magnets. Attention is focused on the exchange amplification of spin waves in a crystal with antiferromagnetic order under electron drift. Various accompanying effects as well as the radioelectric effect due to s – f exchange and magnetoplasma phenomena, are treated. A microscopic theory is constructed to explain the amplification of spin waves in a quantizing magnetic field.

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1. Introduction

Many of the properties of condensed systems can be described just in terms of a general phenomenological model of the condensed state. The basic objective of the review is to present in a systematic manner the most important consequences resulting from a synthesis of the two phenomenological models used in solid state physics, viz., the model of mobile electrons as a *solid state* plasma and that of the phenomenological magnetic medium, coupled by strong exchange interaction. The solid state plasma model allows the description of electrodynamic, magnetohydrodynamic and magneto-optical phenomena related to a continuous medium of mobile charge carriers in metals and semiconductors [1 – 3]. In the phenomenological model a wide range of phenomena arising due to the high-frequency and resonance properties of magnetic dielectrics is explained on the basis of the symmetry properties and certain other general concepts concerning the magnetic state [4 – 6].

From the phenomenological viewpoint, the model which describes the properties of a magnet having current carriers (for instance, ferromagnetic and antiferromagnetic metals and semiconductors) is represented by the localised magnetic

moments of small magnets embedded in a medium of mobile current carriers. The fundamental difficulty in this model is the lack of a macroscopic analogue of exchange interaction between current carriers and localised moments in these media. One possible way of describing the physical properties of magnetic semiconductors and metals is the microscopic approach employing the s–f exchange model [7, 8]. For certain basic reasons many important properties are not considered in this model. For instance, in the framework of microscopic approach even such a simple task as the description of the interaction between magnetisation oscillations and the electron conductivity encounters serious obstacles. Indeed, scattering of spin waves by current carriers whose interactions are described by the microscopic s–f exchange Hamiltonian can be evaluated in the microscopic approach only if the electron gas is regarded as a collisionless plasma. However, in a majority of real magnetically ordered crystals, the carrier mobility is as a rule rather low, i.e., the free path length is very small. In such a situation the collisionless limit is inapplicable to an overwhelming number of spin waves and the interaction between spin waves and current carriers is a phenomenon of collective nature. Consequently, interpretation of the interaction of a wave with current carriers as with a continuous medium (i.e., phenomenologically) requires the use of macroscopic description of s–f exchange.

These problems are given a consistent treatment in Section 2. We define the generalised magnetic susceptibility of a conducting isotropic antiferromagnet on the basis of macroscopic hydrodynamic approach, asymptotically exact in the limit $kl \ll 1$ (where k is the wave vector of the spin wave and l is the electron free path). The use of the macroscopic description for the s–f exchange interaction in calculating the magnetic susceptibility is necessitated by the introduction of an effective field due to the s–f exchange, in addition to the effective field inherent in a crystal in the absence of current carriers, into the Landau–Lifshits equation for magnetisation. In such a formulation the approach developed in this section may serve as a basis for explaining the processes in which the classical description of the s–f exchange interaction is unavoidable. The results derived are equally applicable to a wide range of phenomena in antiferromagnetic semimetals, degenerate and nondegenerate magnetic semiconductors whose properties are largely determined by the s–f exchange interaction.

In Section 3 we shall apply our results to one of still unresolved problems — the amplification of spin waves. Although the amplification of spin waves by a fast moving stream of charged particles was predicted about thirty years ago [9], nonetheless the problem is even today defying practical realisation.

The central result of that section, an outcome of the unification of two models, is that in ferromagnetic semiconductors, on which all efforts were concentrated in an attempt to realise spin wave amplification by electron drift, the amplification coefficient due to s–f exchange interaction vanishes identically. Nonzero contribution in this case may come only from the relativistic interaction of an electron beam with magnetisation. Unlike in a ferromagnet, in an antiferromagnet major contribution to spin wave amplification coefficient is made by the s–f exchange interaction. This distinction is evident even from very general considerations pertaining to the law of conservation of total magnetic moment. The s–f exchange interaction, by its nature, is electrostatic; in other words, it is an interaction which preserves the magnetic moment of a crystal. Consequently, Cherenkov's radiation from a fast electron moving in a ferromagnet is forbidden by the law of conservation. But in an antiferromagnet the total number of magnons is not an integral of motion because the magnet has two sublattices. Consequently, if the Cherenkov condition is satisfied, an electron moving in an antiferromagnet becomes a source for magnon emission [10].

Using the general dispersion equation describing the propagation of a spin wave in the presence of electron drift that is derived from the expression for the generalised susceptibility, in Sections 3.2–3.4 we shall evaluate the coefficient of spin wave amplification by electron drift in an isotropic antiferromagnet for various configurations of electric and magnetic fields. When there is no drift, these results determine the contribution of s–f exchange to spin wave attenuation, the width of nonuniform antiferromagnetic resonance, the threshold of parametric excitation of spin waves, specific features of the exchange mechanism for magnon relaxation in a superconducting antiferromagnet and other allied effects stimulated by s–f exchange. In this section we shall also derive a dispersion equation for the joint propagation of sound and spin waves coupled through the conduction electrons in an antiferromagnet. Other effects which owe their origin to such a coupling are also discussed.

In Section 4 we shall study the amplification and attenuation of spin waves in an anisotropic antiferromagnet. Most of the antiferromagnets are, as regards magnetism, strongly anisotropic crystals. Depending on the type of anisotropy and the orientation of the external magnetic fields relative to the anisotropy axis, the magnon frequency and the antiferromagnetic resonance frequency vary. Accordingly, so do the conditions for the relaxation and excitation of spin waves. As an example, an antiferromagnet with easy plane anisotropy is studied. The contribution of s–f exchange to relaxation of magnetic moment in this case is found to be strongly anisotropic with respect to the direction of the external magnetic field. The spin wave amplification coefficient also exhibits a corresponding anisotropy. These results may be used to calculate different effects due to conduction electrons which are regarded as collision plasma in a crystal with an arbitrary anisotropy.

Section 5 deals with magnetoplasma phenomena in antiferromagnetic semiconductors and metals. Several important magnetoplasma phenomena do not yield to interpretation in the framework of hydrodynamic description of electron liquid. For this purpose, in place of the hydrodynamic description used in the previous sections, we apply a more general approach based on kinetic equation. In this approach the plasma subsystem is described fully in terms of conductivity tensor (or dielectric permittivity tensor) formalism; the results thus derived in the hydrodynamic limit $kl \ll 1$ pass into the results obtained in Sections 2–4.

The theory developed in Section 5 gives a regular method for calculating various magnetoplasma effects under s–f exchange interaction between electrons and the magnetic subsystem. Resonance effects in degenerate semiconductors and semimetals are examined as an illustration. General expressions are derived for the attenuation and amplification of spin waves under cyclotron and geometric resonances.

Conditions are found for the attenuation or amplification of spin waves to be oscillating functions of the wave vector and external magnetic field.

Section 6 is concerned with the generalisation of the previous results for several types of current carriers. For a particular case when a crystal has both conduction electrons and holes (doped and proper semiconductors and semimetals), expressions are derived for the amplification and attenuation of spin waves due to s–f exchange. The results are applied to study the case where there is totally no polarisation of electron spin, i.e., the case of weak magnetic fields. In the macroscopic description this corresponds to the presence of two types of current carriers, viz., carriers with spin oriented along and against the magnetic field direction. Under these conditions the contribution of s–f exchange to the effects (discussed in the previous sections) is a linear function of the magnetic field and vanishes when there is no magnetic field.

Section 7 is devoted to s–f exchange mechanism of the amplification of spin waves in disordered magnets. For various reasons this case is of special interest both from theoretical and from practical points of view:

1) The conventional approach based on the solution of the Landau–Lifshits equations for the sublattices of a magnetically ordered crystal is not applicable to this case.

2) Disordered state is typical of a vast majority of conducting magnets having an extremely high carrier mobility.

In this section, amplification of spin waves in spin glasses is examined as an example.

Section 8 studies the effect which is opposite to spin wave amplification by a drifting electron stream, viz., the generation of a potential difference when a spin wave travels through a specimen, entraining thereby the current carriers. An essentially simplified condition for this effect to be observed is that there should not be any need for creating drifting carrier streams by means of an external electric field. Owing to the high radioelectric effect induced by the s–f exchange interaction in antiferromagnets, this effect deserves special study both from the viewpoint of practical use in VHF electronics and as an efficient tool for detection of processes related to electron–magnon interaction in semiconductors and semimetals.

Section 9 — the concluding one — is devoted to quantum mechanical interpretation of the amplification of spin waves by Cherenkov electrons when $kl \gg 1$, a limit opposite to that studied in the previous sections. This limiting case may hold for the shortwave part in the magnon spectrum of crystals having a high carrier mobility. For the case of an antiferromagnet with uniaxial anisotropy, expressions are derived for the coefficient of spin wave amplification both in a quantizing and in a nonquantizing magnetic field.

Figure 1 shows a schematic representation for various types of magnetically ordered structures studied in the review.

2. Magnetic susceptibility

A wide range of phenomena occurring in conducting antiferromagnets can be explained in terms of a simple two-sublattice phenomenological model in which magnetic moments of the sublattices are treated as classical vectors. The general approach used below is based on the calculation of generalised magnetic susceptibility which depends on the wave frequency and wave vector.

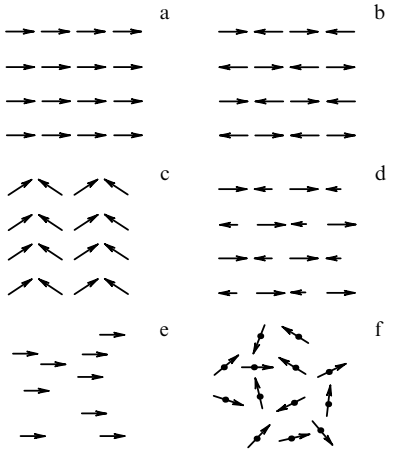

Figure 1. Schematic representation of spin arrangement in certain magnets: (a) ferromagnet; (b) antiferromagnet; (c) antiferromagnet in a magnetic field, weak ferromagnet; (d) ferrimagnet; (e) amorphous ferromagnet; (f) spin glass.

2.1 Magnetic susceptibility of an antiferromagnet in the absence of s–f exchange

In what follows we need certain results concerning magnetic susceptibility and spin wave spectrum in an isotropic antiferromagnet that ensue from the phenomenological consideration. In the macroscopic approach, the motions of sublattice magnetisation $\mathbf{M}_1$ and $\mathbf{M}_2$ of an antiferromagnet are described by the Landau–Lifshits equations:

$$\frac{\partial \mathbf{M}_1}{\partial t} = g(\mathbf{M}_1 \times \mathbf{H}_1)\,, \qquad \frac{\partial \mathbf{M}_2}{\partial t} = g(\mathbf{M}_2 \times \mathbf{H}_2)\,, \tag{1}$$

where $g = 2\mu_0/\hbar$, $\mu_0 = e\hbar/2m_0c$ is the Bohr magneton, $\mathbf{H}_1$ and $\mathbf{H}_2$ are the effective magnetic fields acting on the first and second sublattice, respectively. Fields depend on the variations of the total energy functional Φ of the antiferromagnet with respect to sublattice magnetisations:

$$\mathbf{H}_1 = -\frac{\delta \Phi}{\delta \mathbf{M}_1}\,, \qquad \mathbf{H}_2 = -\frac{\delta \Phi}{\delta \mathbf{M}_2}\,, \tag{2}$$

where

$$\begin{aligned}\Phi = {} & \int \delta_0\, \mathbf{M}_1 \cdot \mathbf{M}_2\, \mathrm{d}^3 r + \int \alpha_{12}\, \frac{\partial \mathbf{M}_1}{\partial x_i} \cdot \frac{\partial \mathbf{M}_2}{\partial x_i}\, \mathrm{d}^3 r \\ & - \frac{1}{2} \int \mathbf{H}^{(m)} \cdot (\mathbf{M}_1 + \mathbf{M}_2)\, \mathrm{d}^3 r - \int \mathbf{H} \cdot (\mathbf{M}_1 + \mathbf{M}_2)\, \mathrm{d}^3 r \\ & + \frac{1}{2}\, \alpha \int \left(\frac{\partial \mathbf{M}_1}{\partial x_i} \cdot \frac{\partial \mathbf{M}_1}{\partial x_i} + \frac{\partial \mathbf{M}_2}{\partial x_i} \cdot \frac{\partial \mathbf{M}_2}{\partial x_i} \right) \mathrm{d}^3 r \end{aligned} \tag{3}$$

is the total energy functional of infinite isotropic antiferromagnet. Here δ_0, α, and α_{12} are phenomenological constants, $\mathbf{H}^{(m)}$ is the magnetic field due to magnetisation of the antiferromagnet and $\mathbf{H}$ is the external magnetic field. By virtue of (2) and (3) the effective fields are

$$\begin{aligned} \mathbf{H}_1 &= -\delta_0(\mathbf{M}_1 + \mathbf{M}_2) + \alpha \nabla^2 \mathbf{M}_1 + \alpha_{12} \nabla^2 \mathbf{M}_2 + \mathbf{H} + \mathbf{H}^{(m)}, \\ \mathbf{H}_2 &= -\delta_0(\mathbf{M}_1 + \mathbf{M}_2) + \alpha \nabla^2 \mathbf{M}_2 + \alpha_{12} \nabla^2 \mathbf{M}_1 + \mathbf{H} + \mathbf{H}^{(m)}. \end{aligned} \tag{4}$$

Expanding $\mathbf{H}_1$ and $\mathbf{H}_2$ into series in small deviations of magnetisations $\mathbf{m}_1$, $\mathbf{m}_2$ and fields $\delta\mathbf{H}_1$, $\delta\mathbf{H}_2$ in the neighbour-

hood of equilibrium positions

$$\mathbf{H}_1^0 = \mathbf{H}_2^0 = -\delta_0(\mathbf{M}_{10} + \mathbf{M}_{20}) + \mathbf{H} \tag{5}$$

and replacing $m(r,t)$ and $H^{(m)}(r,t)$ by their Fourier components, we get

$$\mathbf{m}_1(\mathbf{r},t) = \int \mathbf{m}_1(\mathbf{k},\omega)\exp\bigl[\mathrm{i}(kr-\omega t)\bigr]\,\mathrm{d}^3k\,\mathrm{d}\omega\,,$$

$$\mathbf{H}^{(m)}(\mathbf{r},t) = \int \mathbf{h}(\mathbf{k},\omega)\exp\bigl[\mathrm{i}(kr-\omega t)\bigr]\,\mathrm{d}^3k\,\mathrm{d}\omega\,. \tag{6}$$

Applying (4)–(6), we obtain the Landau–Lifshits equation for the Fourier components of the deviations of the field and magnetisation:

$$\begin{aligned} -\mathrm{i}\omega\mathbf{m}_1 &= g\Bigl(\mathbf{M}_{10}\times\bigl[\mathbf{h}-(\delta_0+\alpha k^2)\mathbf{m}_1-(\delta_0+\alpha_{12}k^2)\mathbf{m}_2\bigr]\Bigr)\,,\\ -\mathrm{i}\omega\mathbf{m}_2 &= g\Bigl(\mathbf{M}_{20}\times\bigl[\mathbf{h}-(\delta_0+\alpha k^2)\mathbf{m}_2-(\delta_0+\alpha_{12}k^2)\mathbf{m}_1\bigr]\Bigr)\,. \end{aligned} \tag{7}$$

Solving the system (7) and introducing the magnetic susceptibility tensor for an antiferromagnet

$$\mathbf{m}(\mathbf{k},\omega) = \mathbf{m}_1(\mathbf{k},\omega) + \mathbf{m}_2(\mathbf{k},\omega) = \hat{\chi}(\mathbf{k},\omega)\mathbf{h}(\mathbf{k},\omega)\,, \tag{8}$$

we obtain:

$$\hat{\chi} = \begin{pmatrix} \chi_{xx} & \chi_{xy} & 0\\ \chi_{yx} & \chi_{yy} & 0\\ 0 & 0 & \chi_{zz}\end{pmatrix}, \tag{9}$$

where

$$\begin{aligned} &\chi_{xx} = \frac{2a(A+B)}{\omega_+^2-\omega^2}\,, \qquad \chi_{yy} = \frac{2a(A+B)+2d(C-D)}{\omega_+^2-\omega^2}\,,\\ &\chi_{zz} = \frac{2d(C-D)}{\omega_-^2-\omega^2}\,, \qquad \chi_{xy} = -\frac{2\mathrm{i}a\omega}{\omega_+^2-\omega^2}\,,\\ &\chi_{yz} = 0\,, \qquad \chi_{zx} = 0\,, \qquad \chi_{yx} = -\chi_{xy}\,,\\ &\omega_\pm^2 = (A\pm B)^2 + C^2 - D^2\,; \end{aligned} \tag{10}$$

$$\begin{aligned} &A = -g(\delta_0+\alpha k^2)M_0\,\frac{H}{H_\mathrm{c}}\,, \qquad B = -g(\delta_0+\alpha_{12}k^2)M_0\,\frac{H}{H_\mathrm{c}}\,,\\ &C = -gM_0(\delta_0+\alpha k^2)\sqrt{1-\frac{H^2}{H_\mathrm{c}^2}}\,,\\ &D = -gM_0(\delta_0+\alpha_{12}k^2)\sqrt{1-\frac{H^2}{H_\mathrm{c}^2}}\,,\\ &\alpha = -gM_0\,\frac{H}{H_\mathrm{c}}\,, \qquad d = -gM_0\sqrt{1-\frac{H^2}{H_\mathrm{c}^2}}\,. \end{aligned} \tag{11}$$

Here $H_\mathrm{c} = 2\delta_0 M_0$ stands for the sublattice collapse field of antiferromagnet.

In the longwave limit ($k \to 0$) expression (10) takes the form:

$$\begin{aligned} &\chi_{xx} = \frac{\omega_+^2}{\omega_+^2-\omega^2}\,\chi_0\,, \qquad \chi_{yy} = \frac{\omega_+^2+\omega_-^2}{\omega_+^2-\omega^2}\,\chi_0\,,\\ &\chi_{zz} = \frac{\omega_-^2}{\omega_-^2-\omega^2}\,\chi_0\,, \qquad \chi_{xy} = -\frac{\mathrm{i}\omega\omega_+}{\omega_+^2-\omega^2}\,\chi_0\,; \end{aligned} \tag{12}$$

$$\begin{aligned} &\omega_+^2 = (gH)^2\,, \qquad \chi_0 = \frac{1}{\delta_0}\,,\\ &\omega_-^2 = 2(gM_0)^2\delta_0(\alpha-\alpha_{12})k^2\left(1-\frac{H^2}{H_\mathrm{c}^2}\right). \end{aligned}$$

The spin wave spectrum is determined from the dispersion equation [4]:

$$k^2 + 4\pi k_i k_j \chi_{ij}(\mathbf{k},\omega) = 0\,, \tag{13}$$

and as χ_0 is small, the spectrum also depends on the poles of the tensor $\hat{\chi}$. Thus, the frequencies ω_+ and ω_- define two branches (ω_+ is the optical branch and ω_- is the acoustic branch) of the excitations of antiferromagnet.

2.2 Landau–Lifshits equation for an antiferromagnet in the presence of s–f exchange

Regarding the spin of a magnetic atom as a classical vector, let us express the Hamiltonian of exchange interaction of conduction electrons with the magnetic subsystem of antiferromagnet as follows:

$$H_\mathrm{int} = -\frac{A}{2}(\mathbf{S}_1+\mathbf{S}_2)\cdot\mathbf{l}_z\,, \tag{14}$$

where $\mathbf{S}_1$ and $\mathbf{S}_2$ are the spin of magnetic atoms in the first and second sublattices, respectively, A is the s–f exchange integral and $\mathbf{l}_z$ is a unit vector along the z-axis. Therefore the total energy functional $\tilde{\Phi}$, in the presence of conduction electrons in an antiferromagnet, assumes the form:

$$\tilde{\Phi} = \Phi - \frac{A}{2}\int(S_1^z+S_2^z)n(r)\,\mathrm{d}^3r\,, \tag{15}$$

where $n(r)$ is the electron concentration in the conduction band. Here it is assumed that all electron spins are polarised. For this it is necessary that

$$\frac{1}{2}\,A(S_1^z+S_2^z) > \varepsilon\,,$$

where $\varepsilon = \varepsilon_\mathrm{F}$ is the Fermi energy of a degenerate electron gas and $\varepsilon = k_\mathrm{B}T$ is that of a nondegenerate electron gas (k_B is the Boltzmann constant). By virtue of the relation between sublattice magnetisation and magnetic atom spin: $\mathbf{M}_1 = (2\mu_0/a^3)\mathbf{S}_1$ (where a is the lattice constant), we obtain the effective magnetic fields:

$$\tilde{\mathbf{H}}_1 = \mathbf{H}_1 + \mathbf{H}_{1,\mathrm{s-f}}\,, \qquad \mathbf{H}_{1,\mathrm{s-f}} = \frac{A}{4\mu_0}\,a^3 n(r)\mathbf{l}_z\,. \tag{16}$$

Hence, in the presence of s–f exchange, the Landau–Lifshits equations take the form:

$$\begin{aligned} &\frac{\partial\mathbf{M}_1}{\partial t} = g\bigl(\mathbf{M}_1\times\mathbf{H}_1\bigr) + g\bigl(\mathbf{M}_1\times A_\mathrm{s}n(r)\mathbf{l}_z\bigr)\,,\\ &\frac{\partial\mathbf{M}_2}{\partial t} = g\bigl(\mathbf{M}_2\times\mathbf{H}_2\bigr) + g\bigl(\mathbf{M}_2\times A_\mathrm{s}n(r)\mathbf{l}_z\bigr)\,,\\ &A_\mathrm{s} = \frac{A}{4\mu_0}\,a^3\,. \end{aligned} \tag{17}$$

Expressions (16) and (17) demonstrate that the s–f exchange contribution can be regarded as an additional magnetic field with $\mathbf{H}$ replaced by $\mathbf{H} + A_\mathrm{s}n(r,t)\mathbf{l}_z$ in Eqns (1). Linearising the additional terms in (17) near the equilibrium values of magnetisation and concentration, we

obtain

$$g\left[\mathbf{M}_1 \times A_s n(r)\mathbf{l}_z\right] = g\left[\mathbf{M}_{10} \times A_s n_0 \mathbf{l}_z\right] + g\left[\mathbf{M}_1^0 \times A_s n_1 \mathbf{l}_z\right] + g\left[\mathbf{m}_1 \times A_s n_0 \mathbf{l}_z\right]. \quad (18)$$

From (17) and (18) it follows that the first term on the right-hand side of (18) contributes into the equilibrium magnetic field:

$$\tilde{\mathbf{H}}_1^0 = -\delta_0(\mathbf{M}_2 + \mathbf{M}_1) + \mathbf{H}_1^0 + gA_s n_0 \mathbf{l}_z . \quad (19)$$

Owing to the last term on the right-hand side of (18), the magnon frequencies $\omega_\pm$ and the magnetic field $\mathbf{H}$ suffer renormalisation and $\mathbf{H}$ is to be substituted for $\mathbf{H} + gA_s n_0 \mathbf{l}_z$ in (10) and (12). Hence, in particular, it follows that if conduction electrons are present in an isotropic antiferromagnet, weak ferromagnetism arises for any arbitrarily low carrier concentration. Another important conclusion follows from the type of the renormalised magnon frequencies $\omega_\pm$ (12) which contain the renormalised field. Expressions (17) and (18) show that $\omega_\pm > 0$ for any current carrier concentration right up to the critical value $n_{cr} = H_c/gA_s$ at which the sublattices of the antiferromagnet collapse if there is no external magnetic field. Thus, the uniform state of magnetisation and carrier concentration proves to be stable to longwave fluctuations. At subcritical concentrations $n_1 < n_{cr}$ a nonuniform equilibrium distribution may in general arise in an antiferromagnet in a sufficiently weak external magnetic field $H \ll H_c$ [8]. Such nonuniform states are always separated, as noticed in Ref. [11], from the uniform states by a potential barrier due to the short-range nature of the s–f exchange interaction. As a result, a uniform state can be unstable only to shortwave fluctuations. But the initial model (15) used in Ref. [8] to analyze this situation is no longer adequate. Hence, in what follows, we only study equilibrium distributions of magnetisations and current carriers over the whole range of variation of the magnetic field.

By virtue of these remarks, it is sufficient to retain only the second term on the right-hand side of the linearised Landau–Lifshits equation (18), after renormalising the frequency and field appropriately. We find that the second term in (18),

$$g\left[\mathbf{M}_{10} \times A_s n_1(r,t)\mathbf{l}_z\right], \quad (20)$$

vanishes identically for a ferromagnet because $\mathbf{M}_1^0 = \mathbf{M}_0 \parallel \mathbf{l}_z$, but for an antiferromagnet it vanishes only if $H \geqslant H_c$. In Section 3 we shall show that at zero lattice temperature this is a consequence of the law of the total magnon number conservation under ferromagnetic ordering.

Expanding the deviation of the carrier concentration $n_1(r,t)$ from its equilibrium value in the Fourier series

$$n_1(\mathbf{r},t) + \sum n'(\mathbf{k},\omega)\exp\left[\mathrm{i}(kr - \omega t)\right], \quad (21)$$

we obtain, instead of (17), a system of equations for the Fourier components of the field and carrier concentration:

$$\begin{aligned} -\mathrm{i}\omega\mathbf{m}_1 &= g\Big(\mathbf{M}_{10} \times \left[\mathbf{h} + A_s n'\mathbf{l}_z - (\delta_0 + \alpha k^2)\mathbf{m}_1 - (\delta_0 + \alpha_{12}k^2)\mathbf{m}_2\right]\Big), \\ -\mathrm{i}\omega\mathbf{m}_2 &= g\Big(\mathbf{M}_{20} \times \left[\mathbf{h} + A_s n'\mathbf{l}_z - (\delta_0 + \alpha k^2)\mathbf{m}_2 - (\delta_0 + \alpha_{12}k^2)\mathbf{m}_1\right]\Big), \end{aligned} \quad (22)$$

with due regard for the remarks made above.

2.3 Hydrodynamic theory of magnetic susceptibility in the presence of s–f exchange

In the macroscopic model of s–f exchange interaction of conduction electrons with spin waves, we assume that a force,

$$-\frac{\partial}{\partial z} H_{int},$$

acts on an electron when a spin wave propagates in the crystal along the z-axis, where H_{int} is the Hamiltonian of s–f exchange interaction. This force can be regarded as an external one (or an ems) and, according to (14), characterised by the electric field strength:

$$E_z = -\frac{A_s}{e}\left(\frac{\partial M_1^z}{\partial z} + \frac{\partial M_2^z}{\partial z}\right), \quad (23)$$

where e is the electron charge. Hence the electrostatic induction $\mathbf{D}$ is

$$D_z = \varepsilon E_z + \frac{A_s \varepsilon}{e}\left(\frac{\partial S_1^z}{\partial z} + \frac{\partial S_2^z}{\partial z}\right), \quad (24)$$

where ε is the dielectric permittivity of the crystal. To obtain a closed system of equations we have to supplement (23) and (24) with Maxwell's equations and equations of hydrodynamics for electron liquid in a medium:

$$\operatorname{rot}\mathbf{H}^{(m)} = 0, \quad (25)$$

$$\operatorname{div}\mathbf{H}^{(m)} = -4\pi \operatorname{div}(\mathbf{M}_1 + \mathbf{M}_2), \quad (26)$$

$$\operatorname{div}\mathbf{D} = en_1, \quad (27)$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{e}{m}\left(\mathbf{E} + \frac{1}{c}\,\mathbf{v} \times \mathbf{B}\right) - \nu\mathbf{v} + \frac{T}{m}\frac{1}{\rho}\nabla_r \rho, \quad (28)$$

$$\operatorname{div}\mathbf{j} + \frac{\partial \rho}{\partial t} = 0, \quad (29)$$

where $\rho = -en$ and $\mathbf{j} = -\rho\mathbf{v}$ are the charge density and current, respectively. Equations (25) and (26) describe the magnetisation oscillations of a crystal in magnetostatic approximation. If there are no conduction electrons, Eqns (25) and (26), together with the Landau–Lifshits equations, yield the dispersion equation (13) for spin waves in magnetostatic approximation. Equation (28) gives the drift velocity of electrons in a magnetic field with due regard for collisions (ν is the electron collision frequency) and for the contribution of the intrinsic pressure tensor (the last term on the right-hand side of (28)). The equation of continuity (29) is the closure of the system consisting of Eqns (17), (24) and (25)–(29).

Substituting the Fourier components (6) and (21) into (25)–(29), and using the expression

$$\mathbf{v}(\mathbf{r},t) = \mathbf{v}_0 + \int \mathbf{v}'(\mathbf{k},\omega)\exp\left[\mathrm{i}(kr - \omega t)\right]\mathrm{d}\omega\,\mathrm{d}^3k \quad (30)$$

for the velocity (v_0 is the carrier drift velocity), we find from (29) the deviation of the equilibrium concentration

$$n'(k,\omega) = \frac{v'(k,\omega)n_0 k}{\omega - kv_0}. \quad (31)$$

Let us consider the motion of electrons in the spin wave propagation direction, paying attention to scattering and diffusion. For a nonzero Fourier component of the field E_z' Eqn (28) gives a relation between the components of E_z' and

$v'(k,\omega)$:

$$v'_z(k,\omega) = \mathrm{i}\,\frac{e}{m}\,\frac{E'_z}{L(\omega,k)}\,,$$

$$L(\omega,k) = kv_0 - \omega - \mathrm{i}\nu + \frac{Dvk^2}{\omega - kv_0} + \frac{\omega_c^2 \sin^2\theta(ku_0 - \omega - \mathrm{i}\nu)}{\omega_c^2\cos^2\theta - (kv_0 - \omega - \mathrm{i}\nu)^2}\,, \qquad (32)$$

where θ is the angle between the directions of the magnetic field and electron drift velocity, $D = k_B T/mv$ is the electron diffusion coefficient and $\omega_c = eH/mc$ is the cyclotron frequency.

From the relation between $D(k,\omega)$ and $n'(k,\omega)$

$$\mathrm{i}kD(k,\omega) = -en'(k,\omega)\,, \qquad (33)$$

which follows from (27), and the relation between D and $\mathbf{m}_1(k,\omega)$ and $\mathbf{m}_2(k,\omega)$ (which follows from (24)), we get

$$D(k,\omega) = \varepsilon E(k,\omega) + \mathrm{i}k\,\frac{A_s}{e}\left[m_1(k,\omega) + m_2(k,\omega)\right]. \qquad (34)$$

From (31)–(34), we find that

$$n'(k,\omega) = \frac{k^2\left[m_1(k,\omega) + m_2(k,\omega)\right]A_s n_0}{\left[v\omega_R + (\omega - kv_0)L(\omega,k)\right]m}\,, \qquad (35)$$

where $\omega_R = e^2 n/\varepsilon m v$ is the dielectric relaxation frequency.

Substituting (35) into (22), we get a system whose solution yields the magnetic susceptibility tensor $\hat{\chi}^F$ which, in structure, coincides with the susceptibility (9) introduced for an antiferromagnet in the absence of conduction electrons with components χ_{xx}, χ_{yy}, χ_{yx} (10) and with χ_{zz} of the form:

$$\chi_{zz} = \frac{2d(C-D)}{\omega_-^2 - \omega^2 + 2F(C-D)}\,, \qquad (36)$$

where

$$F = gM_0\sqrt{1 - \frac{H^2}{H_c^2}}\,\frac{k^2 A_s^2 n_0}{\left[v\omega_R + (\omega - kv_0)L(\omega,k)\right]m}\,. \qquad (37)$$

In the longwave limit $k \to 0$, using the definition (37), we can write the components of the generalised susceptibility tensor as

$$\chi_{zz} = \frac{\omega_-^2}{\omega_-^2 - \omega^2 + 2F(C-D)}\,\chi_0\,,$$

$$2F(C-D) = Fv_s k^2\sqrt{\frac{\alpha - \alpha_{12}}{2\delta_0}}\,, \qquad (38)$$

where v_s is the velocity of spin waves.

Formulas (36) and (38) completely determine the magnetic susceptibility tensor of an antiferromagnet in the presence of electron drift. If $k = 0$, i.e., if there is no spatial distribution, Eqn (38) shows that the magnetic susceptibility is of the same type as in the absence of current carriers, the only difference being that (36) and (38) contain renormalised frequencies and field (Section 2.2). This conclusion is general and does not depend on the model chosen (in our case, an isotropic antiferromagnet).

2.4 Susceptibility tensor $\chi^F(\mathbf{k},\omega)$: its properties and applications

The susceptibility tensor $\hat{\chi}^F(\mathbf{k},\omega)$ is one the fundamental quantities which determine the properties of a medium. Its components $\hat{\chi}^F(\mathbf{k},\omega)$ given by (36) and (38) satisfy the Kramers–Kronig relations and are particularly useful in determining the contribution of the current carriers to attenuation, static susceptibility and certain other quantities important from practical viewpoint. The knowledge of the susceptibility tensor $\hat{\chi}^F(\mathbf{k},\omega)$ gives full information about the spectral distribution of magnetic moment density fluctuations [4] which is useful in finding the contribution of current carriers to the temperature dependence of equilibrium magnetisation. The magnetic moment correlation functions determine the cross-section $\mathrm{d}\sigma$ of neutron scattering. As in the case of neutron scattering, a knowledge of magnetic moment correlators is sufficient for uniquely determining the scattering of light by magnetisation oscillations in an antiferromagnet. In all cases Eqns (36) and (38) give the contribution of current carriers to the effects discussed above.

The expressions of generalised susceptibility (36) and (38) can be used to describe a variety of media, e.g., antiferromagnetic metals, degenerate and nondegenerate magnetic semiconductors in which s–f exchange interaction cannot be disregarded. Indeed, Eqns (36) and (38) govern the electrodynamics of such media when dispersion plays a decisive part. The dispersion equation

$$\det\left|\frac{\omega^2}{c^2}\,\varepsilon_{il}\,\mu_{lj}^F - k^2\delta_{ij} + k_i k_j\right| = 0 \qquad (39)$$

gives all possible branches of electromagnetic waves in these media: dielectric permittivity $\hat{\varepsilon}$ and magnetic permeability $\hat{\mu}$ are the only parameters that are contained in (39). We shall apply these results in the next section to study the influence of electron drift on the spin wave spectrum.

3. s–f exchange amplification of spin waves in magnetically ordered crystals

3.1 Amplification of spin waves in antiferromagnets and ferromagnets: qualitative differences

Amplification of spin waves in magnetically ordered crystals by a stream of drifting electrons has since long been attracting the attention of researchers. The ability of electrons moving at a velocity greater than the phase velocity of spin waves to amplify spin waves was first noticed in Ref. [9]. That in a ferromagnetic semiconductor spin waves could be amplified by conduction electrons was theoretically studied in Refs [12–18]. Though attempts made to detect amplification in laboratory do not unambiguously demonstrate the existence of amplification, they give enough grounds for believing that it could be implemented in practice. Thus, attenuation of VHF signals in the ferromagnet $CdCr_2Se_4$ was found to be moderated under the action of current carrier drift in a strong electric field [19]. Subsequently, efforts were made to amplify magnetostatic waves in ferrite-semiconductor sandwich systems: electric field was also found to reduce the attenuation effect [20–22]. However, to date there is no reliable experimental evidence to confirm the existence of spin wave amplification by current carrier.

The basic difficulty in creating conditions favourable for the observation of amplification in ferromagnets is that in

these crystals magnons are mainly generated through relativistic interaction of current carriers with the magnetic moment of the crystal:

$$H = -\frac{e}{mc}\,\mathbf{A}(r)\cdot\hat{\mathbf{P}}\,, \tag{40}$$

where $\mathbf{A}(r)$ is the vector potential due to magnetisation oscillators, $\hat{\mathbf{P}}$ and m are the momentum and effective mass of the carrier, respectively. As a result, in order to observe perceptible amplification we require a beam of electrons moving with velocities comparable to the velocity of light [4] (as a rule, only the electrons moving in vacuum over the surface of a ferromagnetic metal are considered). The essential limitation on the magnitude of spin wave amplification coefficient (disregarding for a moment the technical difficulties) is the constraint on the density of electrons in the beam. Even a concentration $n \sim 10^{10}$ cm^{-3} is rather difficult to attain in laboratory. So, an idea was advanced in Refs [12–14] to amplify spin waves in a ferromagnetic semiconductor with the help of conduction electrons. It was believed that it would be possible to enhance greatly the concentration of current carriers responsible for amplification. Unfortunately, the trouble is that it is impossible to create an electron stream drifting with a sufficiently high velocity (even a velocity three to four orders less than the velocity of light is rather difficult to attain in laboratory). Consequently, in this case, too, the coefficient of amplification of spin waves in a ferromagnetic semiconductor is small (practical attempt at amplification was made in Ref. [19]).

Microscopic approach to the problem has revealed, as demonstrated first in Ref. [10], that spin waves are generated by conduction electrons rather differently in antiferromagnets than in ferromagnets. According to Ref. [10], in antiferromagnets amplification is effected not through relativistic interaction (40), but essentially via strong exchange interaction of conduction electrons with the spins of magnetic sublattices. This gives an amplification coefficient larger by several orders of magnitude than in the case of relativistic mechanism and, consequently, provides a way for the practical realisation of amplification in antiferromagnetic crystals. This qualitative distinction of antiferromagnets from ferromagnets can be interpreted on the following considerations. Both in antiferromagnets and in ferromagnets the interaction of current carriers with the spin of magnetic atoms is described by the Hamiltonian of the s–f exchange:

$$H = -A_{\mathrm{s}} M_r^z \sigma_r^z\,, \tag{41}$$

where A_{s} is the s–f exchange constant (17), M_r^z is the magnetisation of the crystal and σ_r^z is the electron spin. While in an antiferromagnet a spin density wave may be generated without any change in the total momentum of the system (Fig. 2), in a ferromagnet the presence of this wave is necessarily related to the violation of the total momentum of the system. Since the interaction (41) conserves the total momentum, such a process is forbidden in a ferromagnet.†

† In Refs [23, 24] the s–f exchange has been found to make nonzero contribution to spin wave amplification coefficient in ferromagnets. The reason is in discarding the terms of second order in magnon operators of the s–f exchange Hamiltonian that totally cancel the contribution to amplification from the terms linear in magnon operators (at zero lattice temperature). A consistent microscopic approach for antiferromagnets is given in Section 9 of Ref. [25].

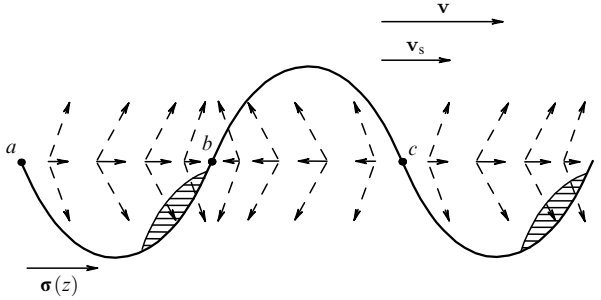

Figure 2. Broken arrows show the deviations of spins of magnetic atoms in an antiferromagnet such that the total variation of magnetic moment $\Delta M_r = 0$. Shaded areas represent the electron clusters formed as a result of grouping of current carriers.

In antiferromagnet, for the electron with spin σ^z directed along the axis the part of wave (a, b) in Fig. 2 is attractive because it corresponds to the electron energy gain (41). At the same time the part (b, c) is repulsive. In essence, the considered situation in antiferromagnet is analogous to the case of interaction between conductivity electrons and elastic waves: the role of deformation potential in antiferromagnet plays the s–f exchange constant A and the role of elastic deformation plays the deviation of magnetic moment M_r^z [26]. Spin wave amplification in this case occurs if condition $v > v_{\mathrm{s}}$ is fulfilled.

3.2 Amplification of spin waves in a longitudinal field

We shall apply the results of Section 2 to calculate the coefficient of spin wave amplification by electron drift. Substituting (36) into (13), we obtain a dispersion equation for the oscillations of magnetisation of an antiferromagnet in the presence of s–f exchange:

$$(\omega_+^2 - \omega^2)\left[(\omega_-^2 - \omega^2) + 2F(C - D)\right] = 0\,. \tag{42}$$

This expression shows that the influence of s–f exchange on the optical branch consists only in frequency renormalisation (Section 2.2) and that s–f exchange does not contribute to attenuation. It is easy to understand the physical reason for this conclusion. In an isotropic antiferromagnet the ω_+ branch corresponds to coherent rotation of sublattice magnetisation vectors (without any change in the angle between sublattice magnetisation). The total magnetisation in such a case exhibits circular oscillations in a plane perpendicular to the z-axis (see Fig. 2), i.e., the projection M_z remains unchanged in magnitude and the electric field E_z, according to (23), vanishes. Therefore, oscillations of this type do not affect the motion of current carriers. On the contrary, sublattice magnetisation oscillations for the ω_- branch are in antiphase (with a change in the angle between magnetisation directions) so that the total magnetisation periodically varies along the z-axis, exerting thereby influence on the motion of current carriers (see Fig. 2).

Now we consider a case when the magnetic field is directed along the current carrier drift direction. Using the expressions for C and D in (11) and for F in (37), we obtain from (42) in longwave approximation a dispersion equation for an antiferromagnet containing drifting electrons:

$$(\omega^2 - v_{\mathrm{s}}^2 k^2)\left[v\omega_{\mathrm{R}} + (\omega - kv_0)(kv_0 - \omega - \mathrm{i}\nu) + Dvk^2\right] = -\tilde{A}k^4 v_{\mathrm{s}} v\,, \tag{43}$$

where

$$\tilde{A} = gM_0\sqrt{1-\frac{H^2}{H_c^2}}\,\omega_R\,\frac{\varepsilon}{4e^2}\left(\frac{a^3}{2\mu_0}\right)^2 A^2\sqrt{\frac{\alpha-\alpha_{12}}{2\delta_0}}\,, \qquad (44)$$

$$v_s = gM_0\sqrt{2\delta_0(\alpha-\alpha_{12})\left(1-\frac{H^2}{H_c^2}\right)}\,, \qquad (45)$$

v_s is the phase velocity of spin waves for the ω_- branch introduced in correspondence with the expression for ω_- in the longwave approximation (12).

Equation (43) holds asymptotically exact only in the limit of frequent collisions $\nu \gg \omega$, $\omega - kv_0$, i.e., only if the hydrodynamic description is valid in our approach. In this limit Eqn (43) reduces to

$$(\omega^2 - k^2v_s^2)\left[\omega_R - i(\omega - kv_0 + iDk^2)\right] = -\tilde{A}k^4v_s\,. \qquad (46)$$

Putting

$$\frac{kv_s}{\omega} = 1 + i\alpha\,, \qquad (47)$$

from (46) and (47) we obtain the attenuation of spin waves:

$$\mathrm{Re}\,\alpha = -\frac{1}{2}\,\frac{\tilde{A}\gamma\omega/v_s^3}{(\omega_R/\omega)^2(1+\omega^2/\omega_R\omega_D)^2+\gamma^2}\,, \quad \omega_D = \frac{v_s^2}{D}\,, \qquad (48)$$

where $\gamma = v_0/v_s - 1$ and ω_D is the diffusion frequency. According to [4], we have

$$\sqrt{\frac{\alpha-\alpha_{12}}{2\delta_0}} \sim a\,. \qquad (49)$$

Using (44), (49) and the relations $g = 2\mu_0/\hbar$, $M_0 = 2\mu_0 S/a^3$, let us represent (48) as follows

$$\mathrm{Re}\,\alpha(\omega) \sim -\frac{4\omega^3a^3/v_s^3(v_0/v_s-1)Q\omega_R/\omega}{(\omega_R/\omega)^2(1+\omega^2/\omega_R\omega_D)^2+(v_0/v_s-1)^2}\,, \qquad (50)$$

where

$$Q = \frac{1}{32}\,\frac{A^2S}{\hbar\omega}\,\frac{\varepsilon a}{e^2}\sqrt{1-\frac{H^2}{H_c^2}}\,.$$

Expressions (48) and (50) resemble the formulas for sound amplification [2] and were derived for spin waves first in Refs [26–28].

If $v_0 < v_s$, Eqns (48) and (50) give the attenuation of spin waves due to the interaction with conduction electrons via s–f exchange mechanism. In those cases where the s–f mechanism prevails over relaxation mechanisms (say, magnon–magnon or magnon–phonon interaction) it determines such parameters like antiferromagnetic resonance line width, parametric spin wave excitation threshold, full width of scattered neutron intensity and many others.

Formulas (48) and (50) show that for $v_0 \geqslant v_s$ spin waves experience amplification instead of attenuation. For a fixed drift velocity v_0, amplification per unit length $\alpha\omega/v_s$ steadily increases with the wave frequency. For a fixed frequency, the amplification coefficient is maximum when the drift velocity is

$$v_{max} = v_s\left(1+\frac{\omega_R}{\omega}+\frac{\omega}{\omega_D}\right). \qquad (51)$$

Figure 3 shows spin wave amplification as a function of the drift velocity. From (50) and (51), we obtain the maximum of the amplification coefficient:

$$\alpha_{max}(\omega) \sim Q\,\frac{\omega^3a^3}{v_s^3}\left(1+\frac{\omega^2}{\omega_R\omega_D}\right)^{-1}. \qquad (52)$$

The values of the frequencies ω_R and ω_D characteristic of solids to be substituted in (52) are equal to 10^8-10^{11} s^{-1} and 10^7-10^{10} s^{-1}, respectively. Substituting these values into (51), we find that in real conditions v_{max} as a rule is far greater than the spin wave phase velocity: $v_{max} \gg v_s$. This is the reason why the amplification cannot be easily realised at v_{max} in practice; in most of the antiferromagnets $v_s > 10^6$ cm s^{-1}. This obstacle can however be overcome by applying a transverse magnetic field (Section 3.3) in place of a longitudinal field.

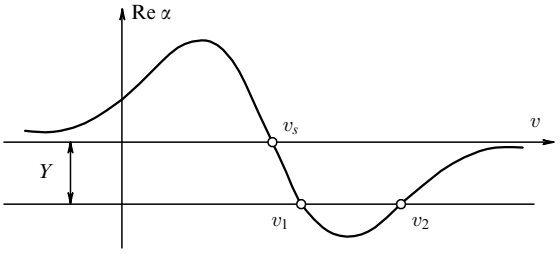

Figure 3. Spin wave amplification coefficient versus drift velocity of conduction electrons. The $\mathrm{Re}\,\alpha(v)$ plot passes through zero for $v_0 = v_s$ ($Y = 0$) and $v_0 = v_1$ ($Y \neq 0$), where Y is the lattice absorption coefficient for spin waves (Section 3.4).

Expressions (48) and (50) show in addition that at low carrier concentrations the amplification coefficient α is proportional to n_0. So, for every frequency ω, there is an intermediate concentration $n(\omega, v_0)$ at which the amplification coefficient α is maximum.

For $A = 0.5$ eV, $S = 2$, $\omega \approx 10^{11}$ s^{-1}, $\varepsilon = 20$, $a = 3\times 10^{-8}$ cm, and $\omega^2 \sim \omega_R\omega_D$, Eqn (52) gives $\alpha \sim 0.1$, in other words, maximum amplification coefficient for an antiferromagnet is far greater than the characteristic magnitude of spin wave attenuation for a magnetically ordered crystal ($Y \sim 10^{-2}-10^{-3}$).

3.3 Amplification of spin waves in crossed fields

If there is a component of the magnetic field **H** perpendicular to the electric field **E**, it is easier to create conditions for the observation of spin wave amplification in antiferromagnets. Under such conditions the magnetic field plays a dual role. If there is no magnetic field or if the magnetic and electric fields are parallel, the drift velocity of current carriers is wholly determined by the carrier mobility and the electric field strength: $v = uE$. As mentioned in the Introduction, in a majority of antiferromagnets the carrier mobility is low ($u < 10^2$ cm^2 V^{-1} s^{-1}), so an electric field of sufficiently high intensity (right up to its breakdown value) is needed to produce the critical velocity ($v > 10^6$ cm s^{-1}) at which amplification is possible. On the other hand, in crossed electric and magnetic fields the drift velocity, if $\omega_c/\nu \gg 1$, does not generally depend on the electron mobility and is equal to

$$v = c\,\frac{\mathbf{E}\times\mathbf{H}}{H^2} \qquad (53)$$

(depending on the effective electron mass, the value of ω_c for $H \sim 10^3$ Oe is from 10^{10} to 10^{12} s^{-1}).

Therefore, in semiconductors with a low mobility, a large drift velocity can be obtained in a perpendicular magnetic field, provided its intensity is high enough.

A nonquantizing field is studied on the same lines as the previous case of longitudinal field. If the magnetic field is not directed along **E**, in longwave approximation the dispersion equation (42) takes the form:

$$(\omega^2 - v_s^2 k^2)\left[\nu\omega_R + (\omega - kv_0)L(\omega, k)\right] = -\tilde{A}k^4 v_s \,. \qquad (54)$$

In the limit of frequent collisions, (32) yields

$$L(\omega, k) = -i\nu + \frac{D\nu k^2}{\omega - kv_0} - i\,\frac{\nu\omega_c^2 \sin^2\theta}{\omega_c^2\cos^2\theta + \nu^2} \,. \qquad (55)$$

Substituting (55) into (54), we obtain the dispersion equation:

$$(\omega^2 - v_s^2 k^2)\left[\nu\left(\frac{\omega_R}{\xi}\right) - i\nu(\omega - kv_0) + \nu k^2\left(\frac{D}{\xi}\right)\right] = -k^4 v_s \frac{\tilde{A}}{\xi} \,, \qquad (56)$$

$$\xi = \left(1 + \frac{\omega_c^2}{\nu^2}\right)\left(1 + \frac{\omega_c^2\cos^2\theta}{\nu^2}\right)^{-1} \,. \qquad (57)$$

In deriving the amplification coefficient with the help of (56), we arrive at (50) containing, in place of ω_R and ω_D, the frequencies $\omega'_R = \omega_R/\xi$ and $\omega'_D = \omega_D\xi$, respectively. For a parallel field, we have $\xi = 1$, i.e., the result obtained earlier in Section 3.2. In a perpendicular field

$$\xi = 1 + \frac{\omega_c^2}{\nu^2} \,. \qquad (58)$$

Thus, the other important part played by a transverse magnetic field lies in reducing the maximum drift velocity v_{max} (51) at which amplification is maximum.

3.4 Attenuation of spin waves: its effects

In the preceding sections we studied the amplification of spin waves, disregarding the relaxation of spin waves. In the macroscopic antiferromagnet model, allowance for relaxation can be made by adding terms describing attenuation to the equation of motion of magnetisations (1):

$$-g\,\frac{\lambda}{M_0}\left(\mathbf{M}_j \times (\mathbf{M}_j \times \mathbf{H})\right) \,, \qquad (59)$$

where $\mathbf{M}_j$ is the magnetisation of the j-th sublattice of the antiferromagnet. Expression (59) corresponds to the Landau–Lifshits absorption which phenomenologically accounts for the scattering of spin waves by lattice vibrations, defects, impurities etc. So, the Landau–Lifshits equations, in the presence of s–f exchange and spin wave attenuation, assume the form:

$$\frac{\partial \mathbf{M}_j}{\partial t} = g(\mathbf{M}_j \times \mathbf{H}_j) + g\left(\mathbf{M}_j \times A_s n(r)\mathbf{l}_z\right) - g\,\frac{\lambda}{M_0}\left(\mathbf{M}_j \times (\mathbf{M}_j \times \mathbf{H}_j)\right) \,. \qquad (60)$$

Linearising (60) and confining ourselves to the case $H/H_c \ll 1$, from (60) we obtain, by virtue of the results found in Section 2.3, the following expressions for the components of the generalised susceptibility tensor in the longwave approximation:

$$\chi_{xx} = \chi_0\,\frac{\omega_+^2 + 2i\omega Y}{\omega_+^2 - \omega^2 - 2i\omega Y} \,,$$

$$\chi_{yy} = \chi_0\,\frac{\omega_+^2 + \omega_-^2}{\omega_+^2 - \omega^2 - 2i\omega Y} \,,$$

$$\chi_{zz} = \chi_0\,\frac{\omega_-^2 + 2i\omega Y}{\omega_-^2 - \omega^2 - 2i\omega Y + 2F(C - D)} \,,$$

$$\chi_{xy} = \frac{-\omega\omega_+}{\omega_+^2 - \omega^2 - 2i\omega Y} \,, \qquad (61)$$

$$\chi_{yx} = -\chi_{xy} \,, \qquad \chi_{xz} = \chi_{zx} = 0 \,, \qquad Y = g\lambda M_0\delta_0 \,. \qquad (62)$$

In the absence of s–f exchange, Eqns (61) give the attenuating branches of spin waves: $\omega_\pm - iY$. In the presence of s–f exchange, the high-frequency branch is always a decaying one. Substituting (61) into (13), we obtain the condition for the amplification of the low-frequency branch:

$$-\mathrm{Re}\,\alpha(\omega) > \frac{Y}{\omega} \,, \qquad (63)$$

where $\alpha(\omega)$ is given by (48). By virtue of the definition (61), inequality (63) takes the form:

$$-\mathrm{Re}\,\alpha(\omega) > \frac{gM_0\lambda\delta_0}{\omega} \,. \qquad (64)$$

The factor λ is related to the width of the antiferromagnetic resonance line as follows [5]:

$$\Delta\omega = \lambda\omega_E \,, \qquad (65)$$

where ω_E is the frequency corresponding to the exchange field. For instance, for an antiferromagnet with $H_E \sim 10^5$ Oe, $\Delta H \sim 10$ Oe, the formula gives $\lambda \sim 10^{-4}$. For $\delta_0 \sim 100$ and a frequency of $gM_0/\omega \sim 0.1$ (i.e., for $\omega \sim 10^{11}$ s^{-1}), Eqn (64) yields an amplification coefficient of $|\alpha| > 10^{-3}$.

3.5 Electron heating: its effect

In the preceding sections we assumed that the electron and lattice temperatures are the same. But in reality, even a relatively low-strength electric field needed to induce the drift would result in a strong heating of the current carriers. Supposing that in an antiferromagnet electrons are scattered basically by magnons, we find that the electron temperature is related to mobility as follows:

$$T_e \propto u^{-2} \,. \qquad (66)$$

This temperature dependence $T_e(u)$ has a similar form for electron scattering on acoustic phonons and magnons. From (66) it follows that in a magnetic semiconductor with a low mobility placed in an electric field of a strength usually used in the laboratory, electron heating may be quite perceptible. The actual electron temperature may exceed by one order or more the lattice temperature.

If electron heating is taken into account, in (48) and (50) we have to substitute $\omega_D(T)$ for $\omega_D(T_e)$:

$$\omega_D(T_e) = \omega_D(T)\,\frac{T}{T_e} \,, \qquad (67)$$

where T_e is the electron temperature. From (48), (50) and (67), we find that electron heating results in a reduced amplification coefficient; in the limit of strong heating we may take

$$\alpha(T_e) \propto T_e^{-2} . \tag{68}$$

Physically, electron heating diminishes the spin wave amplification coefficient exactly in the same way the electron drift suppresses sound amplification. In our case, for electrons, the spin density wave (see Fig. 2) is a periodic sequence of potential wells such that the ratio of the potential well depth to the electron energy decreases rapidly with increasing electron energy due to heating. Consequently, lesser influence is exerted on the motion of electrons. This fact has to be taken into account in experimentally studying amplification, particularly, by using crystals with a sufficiently high electron mobility and quite strong magnetic fields for suppressing the dielectric heating effects.

3.6 Antiferromagnetic resonance in superconducting antiferromagnets: specific features

Antiferromagnetism and superconductivity are typical of many conducting antiferromagnets, say, of triborides $ReRh_4B_4$ (where Re stands for Nd, Tm or Sm), chalcogenides $ReMo_6S_8$ (Tb, Dy, Cd, or Nd,) [29] as well as of high-temperature superconducting antiferromagnets $YBa_2Cu_3O_{7-x}$ and $La_{2-y}Sr_yCuO_4$.

Study of the antiferromagnetic resonance in these crystals is a direct method for investigating the interaction of current carriers with spin waves. Of special interest is the case when conduction electrons make the major contribution to the width of antiferromagnetic resonance line. Here, if $T_N > T_c$ (i.e., transition into antiferromagnetic phase takes place at a far higher temperature than that of transition into the superconducting phase), the line width in the temperature range $(T_c,\ T_N)$ is determined by the contribution of conducting electrons. In the superconducting phase the contribution of s–f exchange to the total line width vanishes in the temperature range $(0,\ T_c)$ if electrons are paired with unlike spins. When electrons make a prevailing contribution to the absorption, the antiferromagnetic resonance line width should suffer a sharp reduction.

If $T_c > T_N$ a similar effect should be observed in a magnetic field in the range $(0,\ T_c)$. Even a relatively weak magnetic field destroys singlet superconductivity (due to the generation of the Zeeman s–f exchange splitting induced in the spin-flop phase by the magnetic field) as a result of steep broadening of the antiferromagnetic resonance line.

This effect may not be present in high-temperature superconducting antiferromagnets if electron pairing is caused by s–f exchange and the s–f exchange integrals for oppositely directed spins of the electrons in a pair have unlike signs.

3.7 Propagation of sound in conducting magnets

The propagation of spin and sound waves which interact with the conduction electrons in an isotropic antiferromagnet via s–f exchange and deformation potential mechanisms is described by the following equations:

$$\frac{\partial \mathbf{M}_1}{\partial t} = g(\mathbf{M}_1 \times \mathbf{H}_1) , \tag{69}$$

$$\frac{\partial \mathbf{M}_2}{\partial t} = g(\mathbf{M}_2 \times \mathbf{H}_2) , \tag{70}$$

$$\operatorname{rot} \mathbf{H}^{(m)} = 0 , \tag{71}$$

$$\operatorname{div} \mathbf{H}^{(m)} = -4\pi \operatorname{div} (\mathbf{M}_1 + \mathbf{M}_2) , \tag{72}$$

$$\operatorname{div} \mathbf{D} = -en_1 , \tag{73}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{e}{m}\left(\mathbf{E} + \frac{1}{2}\,\mathbf{v} \times \mathbf{B}\right) - \nu\mathbf{v} + \frac{T}{m}\,\frac{1}{n_0}\,\nabla_r n_1 , \tag{74}$$

$$\operatorname{div} \mathbf{j} - e\,\frac{\partial n_1}{\partial t} = 0 , \tag{75}$$

$$\rho\,\frac{\partial^2 u_z}{\partial t^2} = \frac{\partial T_{zz}}{\partial z} , \tag{76}$$

$$T_{zz} = \lambda\,\frac{\partial u}{\partial z} - G\,\frac{\varepsilon}{e}\,\frac{\partial E_z}{\partial z} , \tag{77}$$

$$D_z = \varepsilon E_z - \frac{G\varepsilon}{e}\,\frac{\partial^2 u_z}{\partial z^2} + \frac{A\varepsilon}{2e}\left(\frac{\partial S_1^z}{\partial z} + \frac{\partial S_2^z}{\partial z}\right) , \tag{78}$$

where ρ is the crystal density, λ is the elasticity constant, T_{zz} is the elastic stress tensor and G is the deformation potential constant. The sound and spin waves are assumed to propagate along the z-axis. Equations (69)–(75) and relation (78) (for $G = 0$) which govern the propagation of spin waves in the presence of s–f exchange interaction have already been studied in the preceding sections. Equations (73)–(77) and relation (78) (for $A = 0$) govern the propagation of sound waves interacting with the electron subsystem via deformation potential [2]. For an isotropic antiferromagnet the effective magnetic fields in (69) and (70) are given by expressions (4). Expanding the displacement $\mathbf{u}$ in the Fourier series with the help of the results of Section 2, we obtain from (69)–(78) a dispersion equation for coupled spin and sound waves:

$$\begin{aligned}(\alpha^2k^2 - \omega^2)\Big\{&(\omega^2 - k^2v_s^2)\left[\omega_R - \mathrm{i}(\omega - kv_0) + Dk^2\right] \\ &+ \tilde{A}k^4v_s\Big\} = (\omega^2 - k^2v_s^2)(\omega - kv_0 + \mathrm{i}Dk^2)\,\mathrm{i}\tilde{G}(k) \\ &- \tilde{A}k^4v_s\tilde{G}(k) ,\end{aligned} \tag{79}$$

$$\tilde{G}(k) = \frac{G^2\varepsilon}{\lambda e^2}\,\alpha^2k^4 , \tag{80}$$

where α is the velocity of sound. Without going into a detailed analysis, we only mention that the dispersion equation (79) describes the coupled oscillations of sound and spin waves. In the absence of synchronism between sound and spin waves, this coupling leads to renormalisation of the sound velocity and, in particular to its dependence on the longitudinal magnetic field (in a nonmagnetic semiconductor or a metal the sound velocity is independent of the longitudinal magnetic field [2, 30, 31]). Accordingly, the sound absorption coefficient suffers renormalisation and becomes dependent on the magnetic field. From (79) we also find that the s–f exchange inhibits sound absorption for $\alpha > v_s$ and enhances it for $v_s < \alpha$. On the contrary, for the spin wave branch, when the contribution of sound vibrations is taken into account, absorption coefficient increases for $\alpha > v_s$ and decreases for $v_s > \alpha$. When the Cherenkov condition holds, what has been said above applies to amplification rather than to attenuation of the corresponding branch.

If the synchronism condition, $\alpha = v_s$, holds, the branches of Eqn (79) can no longer be regarded as modified by the interaction of spin and sound waves. In the neighbourhood of

the synchronism point, excitation of a sound wave leads to the generation of an equally intensive spin wave and vice versa. In reality, this effect resembles magneto-acoustic resonance [4] and is induced, not by magnetoelastic interaction, but by the interaction between magnetic and elastic oscillations via conduction electrons.

To make allowance for the magnetoelastic interaction, we have to replace the functional $\tilde{\Phi}$ (15) by $\tilde{\tilde{\Phi}}$ [32]:

$$\tilde{\tilde{\Phi}} = \tilde{\Phi} + \int F_{ik}(\mathbf{M}_1, \mathbf{M}_2) u_{ik}\, \mathrm{d}v\,, \tag{81}$$

$$\begin{aligned} F_{ik}(\mathbf{M}_1, \mathbf{M}_2) = {} & \delta_1(M_{1i}M_{1k} + M_{2i}M_{2k}) \\ & + \delta_2(M_{i1}M_{2k} + M_{1k}M_{2i}) \\ & + \delta_{ik}\left[\delta_3(\mathbf{M}_1 \cdot \mathbf{M}_2) + \delta_4(\mathbf{M}_1^2 + \mathbf{M}_2^2)\right], \end{aligned} \tag{82}$$

where δ_1, δ_2, δ_3, and δ_4 are phenomenological constants of a medium with isotropic magnetostriction properties which are independent of M_1^2, M_2^2, and $\mathbf{M}_1 \cdot \mathbf{M}_2$. Accordingly, Eqns (69)–(78) have to be modified. Magnetoelastic interaction, if taken into account, gives rise to several additional effects, for instance, low-frequency quasiacoustic branches and allied phenomena.

3.8 Nonexchange mechanisms of spin wave amplification

We shall discuss the contribution made by other mechanisms to spin wave attenuation and their interaction with the electron subsystem of a crystal. According to Section 3.1, the relativistic interaction (40) is one of the basic mechanisms for the attenuation of spin waves in a ferromagnet. Its contribution to spin wave attenuation was studied (beginning with Ref. [9]) by several authors [12–17]. The contribution of the relativistic interaction in the case of an antiferromagnet is considered in Ref. [12].

In the macroscopic approach the spin wave attenuation due to (40) is calculated by jointly solving Maxwell's equations, equations of motion for the current carrier and the Landau–Lifshits equations.

The motion of current carriers under the action of spin wave with a vector potential A (40) is described by the equation:

$$m\,\frac{\mathrm{d}\mathbf{v}_1}{\mathrm{d}t} = -e\mathbf{e} + \frac{e}{c}\,\mathbf{v}_0 \times \mathbf{b}\,, \tag{83}$$

$$\mathbf{e} = -\frac{1}{c}\,\frac{\partial \mathbf{A}}{\partial t}\,, \tag{84}$$

$$\mathbf{b} = \mathrm{rot}\,\mathbf{A}\,, \tag{85}$$

where $\mathbf{e}$ and $\mathbf{b}$ given by Eqns (84) and (85), respectively, are the variable electric field and induction due to the spin wave. Equations (84) and (85) can be rewritten as

$$\mathrm{rot}\,\mathbf{e} = -\frac{1}{c}\,\frac{\partial \mathbf{b}}{\partial t}\,, \tag{86}$$

$$\mathrm{rot}\,\mathbf{h} = \frac{1}{c}\,\frac{\partial \mathbf{d}}{\partial t}\,, \tag{87}$$

$$\mathbf{b} = \hat{\mu}\mathbf{h}\,, \qquad \mathbf{d} = \hat{\varepsilon}\,\mathbf{e}\,, \tag{88}$$

where $\hat{\mu}(\omega, k)$ and $\hat{\varepsilon}(\omega, k)$ are the magnetic permeability and dielectric permittivity tensor which are to be computed. The dielectric permittivity tensor is found from a joint solution of the equation of motion (83) [after substituting $\mathbf{b}$ from (86)] and the equation of continuity of current:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,\mathbf{j} = 0\,. \tag{89}$$

The tensor $\hat{\varepsilon}(\omega, k)$ takes the simplest form when the quasi-neutrality condition is satisfied, i.e., when $\mathrm{div}\,\mathbf{j} = 0$. In this case the joint solution of (83), (86) and (89) gives the following expression for ε_{ik}:

$$\varepsilon_{ik} = \varepsilon_0\delta_{ik} + \frac{4\pi}{\omega}\,\mathrm{i}\sigma_{ik}\,, \tag{90}$$

$$\sigma_{ik} = \sigma_0\left(1 - \frac{\mathbf{k}\cdot\mathbf{v}_0}{\omega}\right) S_{ik}\,,$$

$$\hat{S} = \begin{pmatrix} \dfrac{k_z^2}{k^2} & 0 & -\dfrac{k_x k_z}{k^2} \\ 0 & 1 & 0 \\ -\dfrac{k_x k_z}{k^2} & 0 & \dfrac{k_x^2}{k^2} \end{pmatrix},$$

where $\sigma_0 = e^2 n / mv$. For a uniaxial antiferromagnet considered below, the magnetic susceptibility tensor is of the form [12]:

$$\hat{\mu} = \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{pmatrix}, \tag{91}$$

$$\mu_{xx} = \frac{\Omega_2^2 - \omega^2}{\Omega_1^2 - \omega^2}\,, \qquad \mu_{zz} = 1\,, \qquad \mu_{xx} = \mu_{yy}\,,$$

$$\Omega_1^2 = 2(gM_0)^2\delta_0\left[\beta + (\alpha - \alpha_{12})k^2\right],$$

$$\Omega_2^2 = \Omega_1^2 + 8\pi\,(gM_0)^2\left[\beta + (\alpha - \alpha_{12})k^2\right]. \tag{92}$$

Substituting (90), (91) and (92) into (86)–(88), we obtain the dispersion equation:

$$\begin{aligned} \left(\frac{k_z^2}{\mu_{xx}} + \frac{k_x^2}{\mu_{zz}} - \frac{\omega^2}{c^2}\,\varepsilon_{zz}\right)\Bigg[\frac{1}{\mu_{xx}}(\varepsilon_{xx}k_x^2 + \varepsilon_{zz}k_z^2 + 2\varepsilon_{zx}k_xk_z) \\ - \frac{\omega^2}{c^2}(\varepsilon_{zz}\varepsilon_{xx} - \varepsilon_{zx}^2)\Bigg] = 0\,. \end{aligned} \tag{93}$$

In the low-frequency limit we have $4\pi\sigma_0\omega/c^2k^2 \ll 1$ and $4\pi\sigma_0/\omega \gg \varepsilon_0$. Hence from (93) we find the spin wave attenuation coefficient for the first branch of the antiferromagnet

$$\alpha_1(\omega) = -\frac{v_s^2}{c^2}\,\varepsilon_0\,\frac{\omega_M\omega_R}{\omega^2}\left[\beta + (\alpha - \alpha_{12})k^2\right]\left(1 - \frac{kv_0}{\omega_1}\right)\frac{k_z}{k}\,, \tag{94}$$

$$\omega_M = 4\pi g M_0\,,$$

$$\omega_1^2 = \Omega_1^2 + 8\pi(gM_0)^2\left[\beta + (\alpha - \alpha_{12})k^2\right]\sin\theta, \quad \sin\theta = \frac{k_x}{k}\,,$$

and for the second branch of the antiferromagnet

$$\alpha_2(\omega) = -\frac{v_s^2}{c^2}\,\varepsilon_0\,\frac{\omega_R\omega_M}{\omega^2}\left[\beta + (\alpha - \alpha_{12})k^2\right]\left(1 - \frac{\mathbf{k}\cdot\mathbf{v}_0}{\Omega_1}\right). \tag{95}$$

The formulas (94) and (95) were first derived in Ref. [12]. If the Cherenkov condition holds, they describe the spin wave amplification due to the relativistic interaction (40).

Comparing the relativistic mechanism (94) and (95) with the exchange mechanism (48) and (50) in the region where the dispersion of spin wave is important ($\alpha k^2 \gg \beta$), we find $\alpha_{\rm rel}/\alpha_{\rm s-f} \sim 10^{-8}$ for $\delta_0 \sim 100$. In the region of small k ($k < 100\ {\rm cm}^{-1}$), $\alpha_{\rm rel}$ is comparable to, and may even exceed, $\alpha_{\rm s-f}$. In this case (of small k), the obstacle in the way of spin wave amplification is that the necessary drift velocity cannot be created.

Besides the exchange and relativistic mechanisms, in magnetic semiconductors or metals there exists the s – f dipole mechanism of interaction of conduction electrons with the magnetic subsystem:

$$H_{\rm int} = -2\mu_0 \mathbf{S}\cdot\mathbf{H}^{(m)}\,, \tag{96}$$

where $\mathbf{H}^{(m)}$ is the field generated by the spin wave. The contribution of (96) to spin wave amplification was studied in Ref. [33]. As a rule, it is three or four orders of magnitude less than that of the exchange mechanism. A magnetoelastic mechanism is also considered in Ref. [34] for the amplification of spin waves in an easy plane antiferromagnet. The amplification coefficient α in this case is $\sim 10^{-4}$, i.e., several orders of magnitude less than for the exchange amplification.

4. Amplification of spin waves in anisotropic antiferromagnets: a macroscopic theory

A majority of magnetically ordered crystals exhibit magnetic anisotropy which exerts perceptible influence on their magnetic properties. The anisotropy– related properties may play a key role in studying the influence of current carriers on the propagation of spin waves in these crystals. In this section we shall examine an s – f exchange mechanism for the amplification of spin waves in easy plane antiferromagnets.

4.1 Amplification of spin waves in an easy plane antiferromagnet in a magnetic field parallel to the anisotropy axis

The total energy functional of a uniaxial antiferromagnet, according to Ref. [4], is

$$\tilde{\Phi} = \Phi - \frac{\beta}{2}\int\left[(\mathbf{M}_1\cdot\mathbf{n})^2 + (\mathbf{M}_2\cdot\mathbf{n})^2\right]{\rm d}^3r - \beta'\int(\mathbf{M}_1\cdot\mathbf{n})(\mathbf{M}_2\cdot\mathbf{n})\,{\rm d}^3r\,, \tag{97}$$

where Φ is the isotropic part of the total energy functional (3) and $\mathbf{n}$ is a unit vector along the anisotropy axis. Let us consider a case where the external magnetic field is parallel to the anisotropy axis: $\mathbf{n}\,\|\,\mathbf{H}$. According to (97), the effective field for an easy plane antiferromagnet ($\beta - \beta' < 0$) is

$$\tilde{\mathbf{H}}_1 = \mathbf{H}_1 + \beta(\mathbf{M}_{10}\cdot\mathbf{n})\mathbf{n} + \beta'(\mathbf{M}_{20}\cdot\mathbf{n})\mathbf{n}\,,$$
$$\tilde{\mathbf{H}}_2 = \mathbf{H}_2 + \beta(\mathbf{M}_{20}\cdot\mathbf{n})\mathbf{n} + \beta'(\mathbf{M}_{10}\cdot\mathbf{n})\mathbf{n}\,, \tag{98}$$

where $\mathbf{H}_1$ and $\mathbf{H}_2$ are the effective fields in the absence of anisotropy (4). Applying (98) and the results of Section 2.2, we find that the Landau – Lifshits equations for the Fourier components of magnetisation deviations, when anisotropy and s – f exchange are taken into account, assume the form:

$$-{\rm i}\omega\mathbf{m}_1 = g\Big(\mathbf{M}_{10}\times\big[\mathbf{h} + A_s n'\mathbf{l}_z - (\delta_0+\alpha k^2)\mathbf{m}_1 - (\delta_0+\alpha_{12}k^2)\mathbf{m}_2\big]\Big) + g\Big(\mathbf{M}_{10}\times\big[\beta(\mathbf{m}_1\cdot\mathbf{n})\mathbf{n} + \beta'(\mathbf{m}_2\cdot\mathbf{n})\mathbf{n}\big]\Big)\,,$$
$$-{\rm i}\omega\mathbf{m}_2 = g\Big(\mathbf{M}_{20}\times\big[\mathbf{h} + A_s n'\mathbf{l}_z - (\delta_0+\alpha k^2)\mathbf{m}_2 - (\delta_0+\alpha_{12}k^2)\mathbf{m}_1\big]\Big) + g\Big(\mathbf{M}_{20}\times\big[\beta(\mathbf{m}_2\cdot\mathbf{n})\mathbf{n} + \beta'(\mathbf{m}_1\cdot\mathbf{n})\mathbf{n}\big]\Big) \tag{99}$$

Solving the system (99) under the assumption that there is no s – f exchange, we obtain the susceptibility tensor: its structure and form are given in Ref. [4]. For $A_s = 0$ we obtain the frequencies of spin wave branches for an antiferromagnet:

$$\omega_+^2 = (A+B)^2 + C^2 - D^2 + (D-C)(E'-E)\,,$$
$$\omega_-^2 = (A-B)^2 + C^2 - D^2 + (C-D)(E'+E)\,, \tag{100}$$

where A, B, C, and D are given by (11);

$$E = gM_0\sqrt{1-\frac{H^2}{H_c^2}}\,\beta\,, \qquad E' = gM_0\sqrt{1-\frac{H^2}{H_c^2}}\,\beta'\,.$$

In the presence of $s\,f$ exchange between conduction electrons and magnetisations, expression (99) yields the dispersion equation:

$$(\omega^2-\omega_+^2)\left[\omega^2-\omega_-^2-2F(C-D)\right] = 0\,, \tag{101}$$

where F is given by (37). Equation (101) shows that for the case under study electrons interact only with the low frequency branch ω_-. Equation (101) coincides, in structure, with (43). In the long-wave approximation, from (100) we find that

$$\omega_- = gM_0\sqrt{(2\delta_0-\beta-\beta')(\alpha-\alpha_{12})k^2\left(1-\frac{H^2}{H_c^2}\right)}\,. \tag{102}$$

Expression (102) differs from Eqn (12) for the isotropic case in that it contains $2\delta_0-\beta-\beta'$ in place of $2\delta_0$. Thus, except for this unimportant substitution of $2\delta_0$ for $2\delta_0-\beta-\beta'$, all the conclusions derived for the amplification coefficient for an isotropic antiferromagnet (Section 3) also hold for an easy plane antiferromagnet placed in a magnetic field parallel to the anisotropy axis.

If the anisotropy in the basal plane of an easy plane antiferromagnet is taken into account, the situation is quite different. In particular, this gives rise to a finite energy gap in the spectrum (102) for $k = 0$.

For an arbitrary direction of the magnetic field the spin wave spectrum was calculated in Ref. [34] under the assumption that there is anisotropy in the basal plane. This was done with the help of the following functional:

$$\tilde{\Phi} = \Phi - \frac{\beta}{2}\int\left[(M_1^z)^2 + (M_2^z)^2\right]{\rm d}^3r + \beta\int M_1^z M_2^z\,{\rm d}^3r - \frac{K_1}{2}\int\left[M_1^y - M_2^y\right]^2{\rm d}^3r\,, \tag{103}$$

where K_1 is the intraplane anisotropy constant. Using (103), the expression for the generalised susceptibility tensor χ_{ij} was derived:

$$\hat{\chi}(\omega,0)=\frac{M_0}{\Delta}\times\begin{pmatrix} B_2\Delta_2-D^2C_2 & \mathrm{i}\,\dfrac{\omega}{g}\,\Delta_2\cos\theta & -\left(\dfrac{\omega}{g}\right)^2 D\sin\theta \\ -\mathrm{i}\,\dfrac{\omega}{g}\,\Delta_2\cos\theta & B_1\Delta_2\cos^2\theta & \dfrac{\mathrm{i}}{2}\,\dfrac{\omega}{g}\,B_1D\sin 2\theta \\ -\left(\dfrac{\omega}{g}\right)^2 D\sin\theta & -\dfrac{\mathrm{i}}{2}\,\dfrac{\omega}{g}\,B_1D\sin^2\theta & (C_1\Delta_1-B_1D^2)\sin^2\theta \end{pmatrix}, \tag{104}$$

where

$$B_1=H_c+H_a\sin^2\theta\,,$$

$$B_2=H_c\cos^2\theta+H_A\sin^2\theta\cos^2\varphi+H_a\sin^2\theta\,,$$

$$C_1=H_A\sin^2\varphi+H_a\sin^2\theta\,,\quad C_2=H_c\sin^2\theta-H_a\cos 2\theta\,,$$

$$D=\frac{1}{2}\,H_A\sin 2\varphi\sin\theta\,,\qquad \Delta_1=B_1B_2-\left(\frac{\omega}{g}\right)^2,$$

$$\Delta_2=C_1C_2-\left(\frac{\omega}{g}\right)^2,\quad \Delta=(\omega_-^2-\omega^2)(\omega_+^2-\omega^2)\,, \tag{105}$$

and the frequencies ω_- and ω_+ in (105) obey the equation:

$$\left(\frac{\omega}{g}\right)^4-\left(\frac{\omega}{g}\right)^2(B_1B_2+C_1C_2)+B_1C_2(B_2C_1-D^2)=0\,, \tag{106}$$

where φ is the angle between the magnetic field and the anisotropy axis, 2θ is the angle between the sublattice magnetic moments $H_A^2=\beta^2M_0^2$ and $H_a=K_1M_0$. Figures 4 and 5 drawn from Ref. [34] give the experimental and calculated spectra of EuTe for certain directions. For $\varphi=0$,

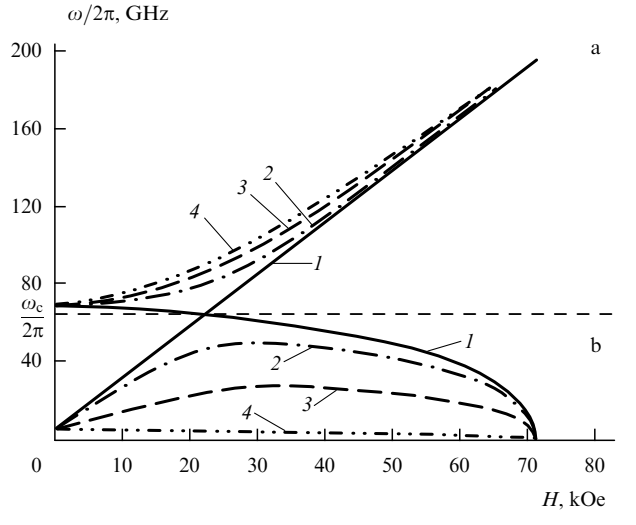

Figure 4. Calculated antiferromagnetic resonance spectrum. (a) hf branch, (b) lf branch in EuTe for four different directions of the magnetic field **H** in the plane (112): $\varphi=90°$ (*1*); 60° (*2*); 30° (*3*); 0 (*4*).

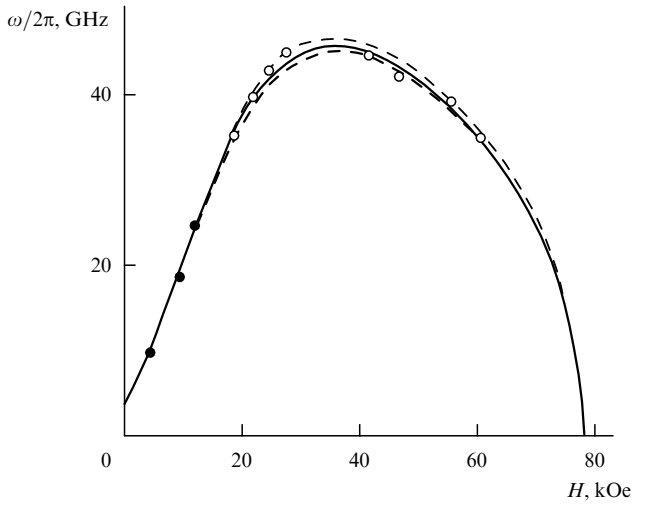

Figure 5. Low-frequency antiferromagnetic resonance branch in EuTe for $\mathbf{H}\parallel[100]$ ($\varphi=54.7°$). Solid line is the antiferromagnetic resonance spectrum calculated with the following demagnetising factors: $N_x=N_y=0.25$ and $N_3=0.5$.

(104) and (105) yield

$$\omega_-=g\sqrt{H_cH_a}\left[1-\left(\frac{H}{H_c}\right)^2\right],$$

$$\omega_+=g\sqrt{H_cH_A+H^2}\,. \tag{107}$$

Thus, in the presence of anisotropy in the basal plane, as already mentioned, an energy gap arises even if the magnetic field is parallel to the anisotropy axis. In EuTe this gap is, according to Ref. [34], very small ($\omega_-\ll\omega_+$) because $H_a\approx 8$ Oe (while $H_A\approx 10$ kOe). Nevertheless, if $k<100$ cm^{-1}, the phase velocity v_s of spin waves along the magnetic field parallel to the anisotropy axis is, because of the gap, greater than 10^7 cm s^{-1}.

According to Section 2, in the presence of conduction electrons, Eqn (107) contains, in place of the magnetic field H, the renormalised field:

$$H\to H+gA_sn_0\,. \tag{108}$$

So, current carrier contribute to the uniform antiferromagnetic resonance of both branches of the antiferromagnet. The measurement of carrier concentration dependence on the resonance frequency gives, on one hand, an independent way for determining the parameters responsible for the magnetic properties (s–f exchange integral) and, on the other hand, a method for determining the semiconductor properties of magnets (concentration).

4.2 Amplification of spin waves in an easy plane antiferromagnet in a magnetic field perpendicular to the anisotropy axis

For an easy plane antiferromagnet in an external magnetic field perpendicular to the anisotropy axis, the equilibrium effective fields are given by the same formulas as for an isotropic antiferromagnet (5). In the presence of s–f exchange, the Landau–Lifshits equations are of the type (99) and for the field geometry under consideration, from Eqn

(99) we get the projections of the components of deviations:

$$(\omega^2-\omega_+^2)\left[\omega_-^2-\omega^2+2F(E'-E)\right]=0\,, \tag{109}$$

where

$$\omega_+=gM_0\sqrt{2\delta_0(\alpha-\alpha_{12})k^2+\left(\frac{H}{M_0}\right)^2}\,,$$

$$\omega_-=gM_0\sqrt{2\delta_0(\alpha-\alpha_{12})k^2+\left(\frac{H_1}{M_0}\right)^2}\,,$$

$$H_1=M_0\sqrt{2\delta_0(\beta-\beta')}\,. \tag{110}$$

Moreover, in deriving (109), only the zero order terms were retained in the expansion in k of the term $\sim F$.

From (110) it is clear that the high-frequency branch is amplified if $H<H_1$, whereas the low-frequency branch is amplified if $H>H_1$. If the drift takes place along the magnetic field, the dispersion equation (110), by virtue of (37) for F, takes the form:

$$(\omega^2-v_s^2k^2)\left[\omega_R-i(\omega-kv_0+iDk^2)\right]=\tilde{\tilde{A}}k^2(E'-E)\,, \tag{111}$$

where

$$\tilde{\tilde{A}}=gM_0\omega_R\,\frac{A_s^2\varepsilon}{e^2}\,,\qquad v_s=\frac{\omega_-}{k}\,. \tag{112}$$

From (111) we obtain the coefficient of spin wave attenuation in an easy plane antiferromagnet

$$\mathrm{Re}\,\alpha(\omega)=-\frac{1}{2}\,\frac{\tilde{\tilde{A}}(E-E')\gamma/v_s^2\omega}{(\omega_R/\omega)^2(1+\omega^2/\omega_R\omega_D)^2+\gamma^2}\,. \tag{113}$$

It follows from (113) that for $v>v_s$ attenuation is superseded by amplification. But, unlike in an isotropic case or in the case of easy plane antiferromagnet placed in a magnetic field parallel to the anisotropy axis (in the absence of anisotropy in the basal plane), the amplification in a perpendicular magnetic field is possible only for finite k ($v_s\to\infty$, $k\to 0$). It is therefore desirable to compare the amplification coefficients for an isotropic antiferromagnet α_{isotr} (48) and for an easy plane antiferromagnet $\alpha_{anisotr}$ (113). For identical drift velocities and wave frequencies, from (113) and (48), we obtain the ratio

$$\frac{\alpha_{anisotr}}{\alpha_{isotr}}\sim\frac{1}{ka}\sqrt{\frac{\beta-\beta'}{\delta_0}}\,.$$

So, depending on the values of the parameters, the ratio of amplification coefficient for an anisotropic antiferromagnet to the amplification coefficient for an isotropic one may take a wide range of values.

From (113) it is clear that for a fixed wave frequency, amplification is maximum at the same drift velocity as for an isotropic antiferromagnet (51). Maximum amplification coefficient is

$$\alpha_{max}(\omega)=\frac{1}{4}\,\frac{\tilde{\tilde{A}}(E-E')/v_s^2\omega}{(\omega_R/\omega)(1+\omega^2/\omega_R\omega_D)}\,. \tag{114}$$

In case drift takes place in a direction other than the field direction, the results obtained for an isotropic antiferromagnet are equally applicable to an easy plane antiferromagnet: in Eqns (113) and (114) we have to substitute, in place of ω_R and ω_D, the expressions ω_R/ξ and $\omega_D\xi$, respectively.

Expressions (113) and (114) are helpful in evaluating amplification in magnetic semiconductors whose electric and magnetic properties have at present been studied quite thoroughly: for example, for EuTe considered in the previous section. Current carrier mobility measured at $T\leqslant T_N$ is $u\sim 100\ \mathrm{cm^2\,V^{-1}\,s^{-1}}$ [35] (i.e., the hydrodynamic approximation is applicable). And the anisotropy field is $H_A\approx 10$ kOe [34]. We shall evaluate the amplification coefficient for a spin wave of wavelength $k\approx 10^5\ \mathrm{cm^{-1}}$ (spin waves of such a wavelength are usually used in experiments on inelastic scattering of light in EuTe [34, 36]). For such a wave, from (109) we obtain a phase velocity $v_s\sim 10^6\ \mathrm{cm\,s^{-1}}$. In order to evaluate the maximum amplification coefficient for a magnetic field perpendicular to the anisotropy axis, let us express (114) as follows:

$$\mathrm{Re}\,\alpha_{max}=\frac{1}{2}\,\frac{\tilde{Q}\gamma}{1+\omega^2/\omega_R\omega_D}\,, \tag{115}$$

$$\tilde{Q}=\frac{1}{8}\,\frac{\varepsilon a}{e^2}\,\frac{A^2S}{\hbar}\,\frac{a^2}{v_s^2}(E-E')\,. \tag{116}$$

For EuTe taking $\varepsilon=20$, $a\approx 3\times 10^{-8}$ cm, $\omega^2\approx\omega_R\omega_D$ (for the low-frequency branch $\omega\sim 10^{11}\ \mathrm{s^{-1}}$), from (115) we find that $\mathrm{Re}\,\alpha_{max}\sim 10^{-2}$. The obtained results where also confirmed in Ref. [37].

5. Magnetoplasma effects in s – f exchange amplification of spin waves

In this section we shall study the dispersion equations describing the propagation of spin waves in conducting crystals in which the interaction of spin waves with the current carriers is wholly determined by the complex conductivity tensor $\sigma_{ij}(\omega,k)$ of plasma carriers. This approach allows us to obtain not only the results that are based on simple hydrodynamic description of a charged liquid [Eqns (28) and (29)] considered above but also those based on the kinetic equation. This makes it possible to study a wider range of magnetoplasma effects accompanying electron amplification (attenuation) of spin waves in the presence of s – f exchange.

5.1 Magnetic susceptibility tensor and conductivity

Since everywhere above we discarded the coupling between spin waves and electromagnetic waves, the knowledge of the longitudinal component alone

$$\sigma_\parallel(\omega,k)=\frac{k_ik_j}{k^2}\,\sigma_{ij}(\omega,k)$$

of the tensor $\sigma_{ij}(\omega,k)$ was adequate for investigating spin wave amplification. Thus, for longitudinal waves, we have

$$j_\sim(\omega,k)=\sigma_\sim(\omega,k)E_\sim\,,$$

$$\mathbf{E}_\sim(z,t)=\mathbf{E}_\sim\exp(-i\omega t+ikz)\,,\qquad \mathbf{E}_\sim\parallel\mathbf{k}\,. \tag{117}$$

This equation together with the equation of continuity (29) yields

$$k\sigma_{\sim}(\omega,k)E_{\sim}=\omega\rho_1(\omega,k)\,, \tag{118}$$

where $\rho_1(\omega,k)$, by virtue of (34), is given by the Poisson equation (27):

$$\rho_1(\omega,k)=\mathrm{i}k\Big\{\varepsilon E_{\sim}+\mathrm{i}k\big[m_1(k,\omega)+m_2(k,\omega)\big]\Big\}\frac{A_s\varepsilon}{e}\,. \tag{119}$$

From (118) and (119) we find the equation for the non-equilibrium correction to the current carrier concentration

$$n_1(\omega,k)=\frac{k^2\sigma_{\sim}\big[m_1(k,\omega)+m_2(k,\omega)\big]A_s/e^2}{-\mathrm{i}\omega+\sigma_{\sim}/\varepsilon}\,. \tag{120}$$

Substituting (120) into the Landau–Lifshits equation (22), we obtain the magnetic susceptibility tensor (36) in which F stands for the expression:

$$F=gM_0\sqrt{1-\frac{H^2}{H_c^2}}\,\frac{k^2\sigma_{\sim}A_s^2/e^2}{-\mathrm{i}\omega+\sigma_{\sim}/\varepsilon}\,. \tag{121}$$

If the type of the longitudinal conductivity $\sigma_{\sim}$ of the medium is known, then (36) and (121) completely determine the magnetic tensor. For instance, in the simple case considered above in which the electron or hole plasma can be described by the hydrodynamic equations (28) and (29) for a charged liquid,

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v}=-\frac{e}{m}\,\mathbf{E}-\nu\mathbf{v}+\frac{T}{m}\,\frac{1}{\rho}\,\nabla_r\rho\,, \tag{122}$$

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\mathbf{j}=0\,, \tag{123}$$

the longitudinal conductivity of the medium is of the form:

$$\sigma_{\sim}(\omega,k)=\sigma_0\left[1-\frac{\mathbf{k}\cdot\mathbf{v}_0}{\omega}+\mathrm{i}\,\frac{k^2v_T^2}{\omega\nu}\right]^{-1}\,, \tag{124}$$

where $v_T=\sqrt{T/m}$ is the thermal velocity. Substituting (124) into (121), we obtain an expression which coincides with (37) in the frequent collision limit (i.e., when the hydrodynamic description holds).

We shall consider some practically important cases whose description is based on kinetic approach for conductivity calculations.

5.2 Results of kinetic approach

To calculate the conductivity of a medium under electron drift, we use the kinetic equation for the electron gas distribution function:

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial\mathbf{r}}+\frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial\mathbf{v}}=I[f]\,, \tag{125}$$

$$\mathbf{F}=-e\left(\mathbf{E}_{\mathrm{d}}+\mathbf{E}_{\sim}+\frac{\mathbf{v}}{c}\times\mathbf{H}\right)\,,$$

where $\mathbf{E}_{\mathrm{d}}$ is the constant field responsible for the electron drift and $\mathbf{E}_{\sim}$ is the wave field. The collision integral is expressed through the relaxation time:

$$I[f]=-\frac{f-f_0}{\tau}\,, \tag{126}$$

where f_0 is the symmetric electron distribution function. Let us consider the case of crossed electric and magnetic fields for the geometry studied in Section 3.3.

The solution to Eqns (125) and (126) in the linear field approximation is presented in Ref. [38]. In particular, if the wavelength of spin waves is of the same order of magnitude as the Larmor radius, the geometric resonance arises. In this case, i.e., if $T\ll\varepsilon_{\mathrm{F}}$ and $kr_{\mathrm{L}}\approx1$, where $r_{\mathrm{L}}=v_{\mathrm{F}}/\omega_{\mathrm{c}}$, from (125), (126) we obtain the longitudinal conductivity:

$$\sigma_{\sim}(\omega,k)=\frac{3\sigma_0}{(kl)^2}\,\frac{\lambda\big[1-g_0(kr_{\mathrm{L}})\big]}{1-kv_{\mathrm{d}}/\omega-(\mathrm{i}/\omega\tau)\big[1-g_0(kr_{\mathrm{L}})\big]}\,,$$

$$\lambda=1+\mathrm{i}\omega\tau\left(1-\frac{kv_{\mathrm{d}}}{\omega}\right)\,,$$

$$g_0(x)=\int_0^{\pi/2}J_0^2(x\sin\theta)\sin\theta\,\mathrm{d}\theta\,. \tag{127}$$

The function $\sigma_{\sim}(\omega,k)$ has resonant peaks due to the oscillations of the function $g_0(x=kr_{\mathrm{L}})$. The conductivity oscillations are responsible for the resonance behaviour of the magnetic susceptibility (121), (36) and, as a consequence, for the resonance variations in the amplification and attenuation of spin waves.

If the electrons in a magnetic field spin with a frequency ω_{c} same as the wave frequency ω, cyclotron resonance arises. Owing to electron drift, the resonance cyclotron attenuation condition is of the type $\omega_{\mathrm{c}}=\omega'$, where $\omega'=\omega(1-v_{\mathrm{d}}/v_{\mathrm{F}})$ is the Doppler shifted wave frequency.

Near the cyclotron resonance $kr_{\mathrm{L}}=v_{\mathrm{F}}\omega/v_s\omega_{\mathrm{c}}$; since v_s is of the order of the spin wave velocity ($v_{\mathrm{F}}\gg v_s$), we find $kr_{\mathrm{L}}\gg1$. Under this condition, from (125) and (126) we obtain the following expression for conductivity:

$$\sigma_{\sim}(\omega,k)=\frac{3\pi}{2}\,\sigma_0\,\frac{\omega\tau}{(kl)^3}\left(1-\frac{kv_{\mathrm{d}}}{\omega}\right)\coth\frac{\pi\lambda}{\omega_{\mathrm{c}}\tau}\,. \tag{128}$$

As distinguished from geometric resonance, cyclotron resonance is possible even in nondegenerate semiconductors because the electron spin frequency in a magnetic field does not depend on the electron velocity. Therefore the expression (128) is of the same form as before, except for the difference that here $3\pi/2$ is to be substituted for $2\sqrt{\pi}$ and the free path length $l=v_{\mathrm{F}}\tau$ is to be interpreted as $l=v_T\tau$, where v_T is the thermal velocity of electrons.

Depending on the magnetic field, the real part of the conductivity (128) suffers a series of resonances (Fig. 6) [38] whose locations are defined by the condition:

$$n\omega'=n\left(1-\frac{v_{\mathrm{d}}}{v_s}\right)\omega=\omega_{\mathrm{c}}\qquad(n=1,2,3,\ldots)\,. \tag{129}$$

This is responsible for the oscillatory character of the magnetic susceptibility and the specific behaviour of the attenuation (amplification) of spin waves (Section 5.3).

In a strong magnetic field where Landau quantization is possible, attenuation (amplification) of spin waves may exhibit oscillatory behaviour, depending on the magnetic field strength. The case of quantizing fields is discussed in Section 9.

5.3 Resonance effects

Substituting (127) into (120) and (121), we find that the spin wave amplification coefficient experiences oscillations under geometric resonance.

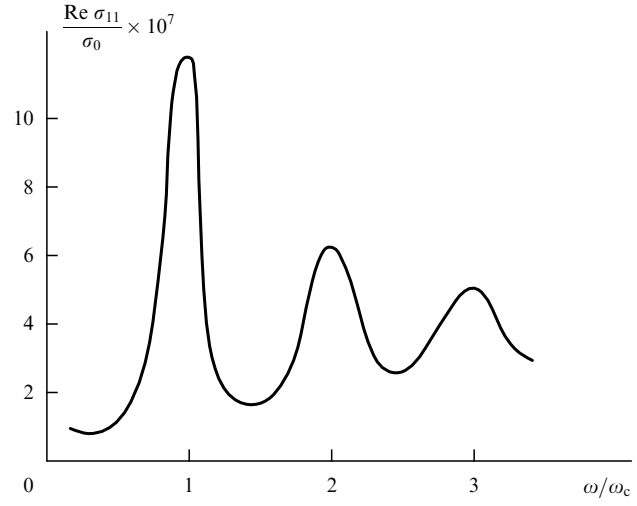

Figure 6. Conductivity versus the ratio ω/ω_c under cyclotron resonance.

As already mentioned, from the practical point of view, it is easier to realise the cyclotron resonance. Substituting (120) into (22), we obtain the amplification coefficient:

$$\alpha = -\frac{\pi}{4e^2}\frac{\omega}{v_s^3\hbar}A^2Sa^4\sqrt{1-\frac{H^2}{H_c^2}}\frac{\varepsilon_0^2}{\left|\varepsilon(\omega,k)\right|^2}\operatorname{Re}\sigma_\sim . \qquad (130)$$

Using (130) and (128) for the amplification coefficient under cyclotron resonance, we obtain

$$\alpha = -\frac{\pi}{96}\left(1-\frac{v_D}{v_s}\right)\frac{\omega_p^2A^2Sa^4}{e^2v_s^2v_F^2\hbar}\left(1-\frac{H^2}{H_c^2}\right) \times \frac{kr_D}{1+(\varepsilon_0/3)k^2r_D^2}\operatorname{Re}\left(\cot\frac{\pi\lambda}{\omega_c\tau}\right), \qquad (131)$$

where r_D is the Debye radius, ω_p is the plasma frequency. For $v_D > v_s$, Eqn (131) shows that the spin waves experience resonance amplification provided $n\omega(1-v_D/v_s) = \omega_c$. Accordingly, for $v_D < v_s$, spin waves suffer resonance attenuation.

In the strong degeneration limit, which holds for magnetically ordered metals, the main obstacle for spin wave amplification is that a sufficiently high drift velocity cannot be easily created because this would require a strong electric field to be applied to the metal. The result obtained here for the spin wave attenuation coefficient may play a key role in experimental investigation of antiferromagnetic resonance in metals and semiconductors. As a result of the skin effect, the electromagnetic field penetrates into the metal to the depth of the skin layer. Consequently, the resonance phenomena related to spin wave attenuation would be manifested in the special behaviour of surface impedance which determines the reflection and attenuation of electromagnetic waves in a metal. Thus, oscillations in attenuation (131) lead to oscillations in the half-width; and the location and the shape of the curve of electromagnetic wave absorption under antiferromagnetic resonance can be detected in experiments. Magnetic susceptibility (38) shows that the s–f exchange contribution to susceptibility is wholly dependent on the dispersion of susceptibility. We know, that under magnetic resonance conditions in a metal, dispersion may play a decisive role in the formation of the shape and position of electromagnetic wave absorption curve [39]. Under these conditions, antiferromagnetic metals may exhibit radioelectric effect (radioelectric current, Section 8), particularly, oscillations of radioelectric current as demonstrated in Section 8.

6. Multiple-carrier theory

In the previous sections we studied magnetically ordered crystals having only one type of carriers. In this section we shall extend the results to a crystal with several types of carriers.

6.1 Two types of carriers

Now we take up the most important case, namely, a crystal containing carriers of unlike sings — electrons and holes — which interact with the crystal magnetisation via s–f exchange mechanism. The total energy functional of an antiferromagnet containing electrons and holes is

$$\tilde{\Phi} = \Phi - \frac{A_e}{2}\int(S_1^z+S_2^z)n_e(r)\,d^3r - \frac{A_h}{2}\int(S_1^z+S_2^z)n_h(r)\,d^3r , \qquad (132)$$

where $n_e(r)$ and $n_h(r)$ are the concentration of electrons and holes, respectively, and A_e and A_h are the corresponding constants.

In order to generalise the expressions for magnetic susceptibility (36) and (38) to the case of two (or several) different types of carriers, we shall regard the force acting on electrons and holes due to spin waves as an additional electric field in the expression for current. By virtue of (132), the complete system of equations describing the interaction of electrons and holes with a spin wave takes the form:

$$-i\omega\mathbf{m}_1 = g\Big(\mathbf{M}_{10}\times\big[\mathbf{h}+A_s^e n_e'\mathbf{l}_z + A_s^h n_h'\mathbf{l}_z - (\delta_0+\alpha k^2)\mathbf{m}_1 - (\delta_0+\alpha_{12}k^2)\mathbf{m}_2\big]\Big),$$

$$-i\omega\mathbf{m}_2 = g\Big(\mathbf{M}_{20}\times\big[\mathbf{h}+A_s^e n_e'\mathbf{l}_z + A_s^h n_h'\mathbf{l}_z - (\delta_0+\alpha k^2)\mathbf{m}_2 - (\delta_0+\alpha_{12}k^2)\mathbf{m}_1\big]\Big); \qquad (133)$$

$$ik\sigma_e\left[E_\sim - \frac{A_s^e}{|e|}(m_1+m_2)\,ik\right] - i\omega\rho_e' = 0 ,$$

$$ik\sigma_h\left[E_\sim + \frac{A_s^h}{|e|}(m_1+m_2)\,ik\right] - i\omega\rho_h' = 0 ; \qquad (134)$$

$$A_s^{e,h} = A^{e,h}\frac{a^3}{4\mu_0}, \qquad ik\varepsilon E_\sim = -|e|n_e' + |e|n_h' . \qquad (135)$$

Equation (133) is a generalisation of (22) to the case of two types of carriers. Equations (134) are the equations of continuity for electron and hole currents, respectively, and in the framework of our approach, by what has been said above, the expressions for currents take the form:

$$j_e = \sigma_e\left[E_\sim - \frac{A_s^e}{|e|}(m_1+m_2)\,ik\right],$$

$$j_h = \sigma_h\left[E_\sim - \frac{A_s^h}{|e|}(m_1+m_2)\,ik\right]. \qquad (136)$$

Finally, Eqn (135) is the Poisson equation for two types of carriers. Eliminating the field $E_\sim$ from (134) and (135) we

obtain the nonequilibrium correction to the concentration of electrons due to their interaction with a spin waves:

$$n'_{\mathrm{e}} = k^2(m_1+m_2)\frac{-\sigma_{\mathrm{e}}\sigma_{\mathrm{h}}/\varepsilon + A_{\mathrm{s}}^{\mathrm{e}} + A_{\mathrm{s}}^{\mathrm{h}} + \mathrm{i}\omega\sigma_{\mathrm{e}}A_{\mathrm{s}}^{\mathrm{e}}}{(\mathrm{i}\omega e^2/\varepsilon)(\sigma_{\mathrm{e}}+\sigma_{\mathrm{h}}) + \omega^2 e^2} \qquad (137)$$

and a similar expression for the correction to hole concentration n'_{h} is obtained on substituting e for h in (137). Substituting (137) into (133), we obtain a dispersion equation resembling (42) with the difference that F is here equal to

$$F = gM_0\sqrt{1-\frac{H^2}{H_{\mathrm{c}}^2}}\,\frac{k^2}{e^2} \times \left[\frac{\sigma_{\mathrm{e}}\sigma_{\mathrm{h}}}{\varepsilon}(A_{\mathrm{s}}^{\mathrm{e}}+A_{\mathrm{s}}^{\mathrm{h}})^2 + \mathrm{i}\omega\sigma_{\mathrm{e}}(A_{\mathrm{s}}^{\mathrm{e}})^2 + \mathrm{i}\omega\sigma_{\mathrm{h}}(A_{\mathrm{s}}^{\mathrm{h}})^2\right] \times \left[\frac{\mathrm{i}\omega(\sigma_{\mathrm{e}}+\sigma_{\mathrm{h}})}{\varepsilon}+\omega^2\right]^{-1}. \qquad (138)$$

The expressions (36), (38) and (138) completely define the magnetic susceptibility tensor of an antiferromagnet containing two types of carriers.

Putting $kv_{\mathrm{s}}/\omega = 1+\mathrm{i}\alpha$, from (42) and (138), we obtain a general expression for the spin wave attenuation coefficient:

$$\mathrm{Re}\,\alpha = -\frac{P_1 k^4}{2\omega^3\left|\varepsilon(\omega,k)\right|^2}\left\{A_{\mathrm{e}}^2\left[\left|\varepsilon-\frac{\sigma_{\mathrm{h}}}{\mathrm{i}\omega}\right|^2\mathrm{Re}\,\sigma_{\mathrm{e}} + |\sigma_{\mathrm{e}}|^2\frac{\mathrm{Re}\,\sigma_{\mathrm{h}}}{\omega^2}\right] + A_{\mathrm{h}}^2\left[\left|\varepsilon-\frac{\sigma_{\mathrm{e}}}{\mathrm{i}\omega}\right|^2\mathrm{Re}\,\sigma_{\mathrm{h}} + |\sigma_{\mathrm{h}}|^2\frac{\mathrm{Re}\,\sigma_{\mathrm{e}}}{\omega^2}\right] + 2A_{\mathrm{h}}A_{\mathrm{e}}\left[\frac{|\sigma_{\mathrm{h}}|^2\,\mathrm{Re}\,\sigma_{\mathrm{h}}}{\omega^2}+\frac{|\sigma_{\mathrm{e}}|^2\,\mathrm{Re}\,\sigma_{\mathrm{e}}}{\omega^2}-2\frac{\varepsilon}{\omega}\mathrm{Im}(\sigma_{\mathrm{e}}\sigma_{\mathrm{h}})\right]\right\}, \qquad (139)$$

where

$$\varepsilon(\omega,k) = \varepsilon - \frac{1}{\mathrm{i}\omega}(\sigma_{\mathrm{e}}+\sigma_{\mathrm{h}}), \qquad (140)$$

$$P_1 = v_{\mathrm{s}}\sqrt{\frac{\alpha-\alpha_{12}}{2\sigma_0}}\,gM_0\sqrt{1-\frac{H^2}{H_{\mathrm{c}}^2}}\,\frac{1}{e^2}\left(\frac{a^3}{4\mu_0}\right)^2. \qquad (141)$$

The expressions (139)–(141) show that, if there are two types of carriers, the expression (139) for spin wave attenuation when $A_{\mathrm{h}} = 0$, i.e., when only electrons interact with the spin wave, differs from the expression derived earlier for one type of carrier. The reason for this is that, when electrons interact with a spin wave, the local electron density is perturbed and, consequently, an electric field is generated with which the holes interact.

If there is no magnetic field, it is much more difficult to create conditions for the amplification of spin waves in a crystal with two types of carriers than in a crystal with only one type of carrier because electrons and holes move in opposite directions under the action of the drifting electric field. Hence, when there are two types of carriers, holes influence substantially the conditions for the amplification of spin waves. However, amplification is possible only if the electron part of the amplification is greater than the attenuation due to holes. Finally, we may note that in the low-frequency limit, i.e., if $|\sigma| \gg \omega$, from (139)–(141) follows the spin wave attenuation coefficient:

$$\mathrm{Re}\,\alpha = -\frac{1}{8}\frac{a^3\omega^3}{v_{\mathrm{s}}^3}\frac{a}{e^2}\frac{1}{\hbar\omega}\sqrt{1-\frac{H^2}{H_{\mathrm{c}}^2}}\,S(A_{\mathrm{e}}+A_{\mathrm{h}})^2\frac{1}{\omega}\,\mathrm{Re}\,\frac{\sigma_{\mathrm{e}}\sigma_{\mathrm{h}}}{\sigma_{\mathrm{e}}+\sigma_{\mathrm{h}}}. \qquad (142)$$

6.2 Amplification of spin waves in crossed fields

As mentioned in the previous section, in the absence of a magnetic field, it is far more difficult to observe amplification in a system with two types of carriers than in a system with only one type of carrier. Here, in full analogy with sound amplification [38], in order to attain amplification in a two-carrier system due both to electrons and holes, crossed electric and magnetic field have to be applied in such a way that electrons and holes drift in the direction of spin wave. The magnetic field should be strong enough both for electrons and for holes:

$$\omega_{\mathrm{c}}^{\mathrm{e,h}}\,\tau^{\mathrm{e,h}} \gg 1. \qquad (143)$$

If this condition is satisfied, the electron and hole conductivities are given by the expression:

$$\sigma_{\mathrm{e,h}} = \sigma_0^{\mathrm{e,h}}\left[1-\frac{kv_{\mathrm{D}}^{\mathrm{e,h}}}{\omega}+\mathrm{i}\,\frac{k^2(v_T^{\mathrm{e,h}})^2}{\omega\nu_{\mathrm{eff}}^{\mathrm{e,h}}}\right]^{-1}, \qquad (144)$$

where

$$v_T^{\mathrm{e,h}} = \sqrt{\frac{T}{m_{\mathrm{e,h}}}}, \qquad v_{\mathrm{D}}^{\mathrm{e,h}} = \frac{cE}{H},$$
$$\nu_{\mathrm{eff}}^{\mathrm{e,h}} = \omega_{\mathrm{c(e,h)}}^2\tau_{\mathrm{e,h}}, \qquad \sigma_0^{\mathrm{e,h}} = \frac{e^2 n_{\mathrm{e,h}}}{m_{\mathrm{e,h}}\nu^{\mathrm{e,h}}}. \qquad (145)$$

In what follows we shall restrict ourselves to proper semiconductors and semimetals in which the current carriers occur in equal concentrations and use the low-frequency limit:

$$\left|\sigma^{\mathrm{e,h}}(\omega,k)\right| \gg \omega\varepsilon. \qquad (146)$$

If the condition (146) is satisfied, then the spin wave amplification coefficient is given by (142). Substituting (145) into (142), we obtain the amplification coefficient for a degenerate case:

$$\mathrm{Re}\,\alpha = -\frac{1}{8}\frac{a^3\omega^3}{v_{\mathrm{s}}^3}\frac{a}{e^2}\frac{1}{\hbar\omega}\sqrt{1-\frac{H^2}{H_{\mathrm{c}}^2}}\,S(A_{\mathrm{e}}+A_{\mathrm{h}})^2\frac{1}{\omega}\frac{\sigma_0^{\mathrm{e}}\sigma_0^{\mathrm{h}}}{\sigma_0^{\mathrm{e}}+\sigma_0^{\mathrm{h}}} \times\left(1-\frac{v_{\mathrm{D}}}{v_{\mathrm{s}}}\right)\left[\left(1-\frac{v_{\mathrm{D}}}{v_{\mathrm{s}}}\right)^2+\left(\frac{k^2p_{\mathrm{F}}^2c^2}{3\omega e^2H^2}\frac{m_{\mathrm{e}}+m_{\mathrm{h}}}{m_{\mathrm{e}}\tau_{\mathrm{e}}+m_{\mathrm{h}}\tau_{\mathrm{h}}}\right)^2\right]^{-1}, \qquad (147)$$

where p_{F} is the Fermi momentum for equal concentrations of electrons and holes. Accordingly, for a nondegenerate case

$$\mathrm{Re}\,\alpha = -\frac{1}{8}\frac{a^3\omega^3}{v_{\mathrm{s}}^3}\frac{a}{e^2}\frac{1}{\hbar\omega}\sqrt{1-\frac{H^2}{H_{\mathrm{c}}^2}}\,S(A_{\mathrm{e}}+A_{\mathrm{h}})^2\frac{1}{\omega}\frac{\sigma_0^{\mathrm{e}}\sigma_0^{\mathrm{h}}}{\sigma_0^{\mathrm{e}}+\sigma_0^{\mathrm{h}}} \times\left(1-\frac{v_{\mathrm{D}}}{v_{\mathrm{s}}}\right)\left[\left(1-\frac{v_{\mathrm{D}}}{v_{\mathrm{s}}}\right)^2+\left(\frac{k^2T}{\omega}\frac{c^2}{e^2H^2}\frac{m_{\mathrm{e}}+m_{\mathrm{h}}}{m_{\mathrm{e}}\tau_{\mathrm{h}}+m_{\mathrm{h}}\tau_{\mathrm{e}}}\right)^2\right]^{-1}. \qquad (148)$$

6.3 Amplification of spin waves in a weak magnetic field

The approach developed in this section can be used to study the case of one carrier in a weak magnetic field, i.e., when

$$\frac{A}{2}(S_1^z + S_2^z) < \varepsilon\,, \tag{149}$$

where $\varepsilon = k_B T$ for a nondegenerate semiconductor and $\varepsilon = \varepsilon_F$ for a degenerate semiconductor. Condition (149) is of special interest because in many cases a strong magnetic field is needed (as a rule, $H \gg 10$ kOe) for the opposite of inequality (149) to hold. This complicates the experimental conditions.

In the macroscopic approach considered here, the results are generalised to the case of a semiconductor containing electrons with opposite spin projections on the magnetic field direction by modifying the expression for the total energy:

$$\tilde{\Phi} = \Phi - \frac{A_\uparrow}{2}\int (S_1^z + S_2^z) n_\uparrow(r)\,\mathrm{d}^3 r - \frac{A_\downarrow}{2}\int (S_1^z + S_2^z) n_\downarrow(r)\,\mathrm{d}^3 r\,, \tag{150}$$

where $n_\uparrow$ and $n_\downarrow$ are the concentrations of electrons with spin projections along and against the magnetic field, respectively.

So, if the inequality (149) is satisfied, a one-carrier system can be regarded as a particular case of two-carrier system with concentrations $n_\uparrow$ and $n_\downarrow$, respectively, and in contrast to electron – hole plasma, with charges of the same sign.

Accordingly, Eqns (133) – (135) assume the form:

$$\begin{aligned}
-\mathrm{i}\omega \mathbf{m}_1 &= g\Big(\mathbf{M}_{10} \times \big[\mathbf{h} + A_\uparrow n_\uparrow' \mathbf{l}_z + A_\downarrow n_\downarrow' \mathbf{l}_z \\
&\quad - (\delta_0 + \alpha k^2)\mathbf{m}_1 - (\delta_0 + \alpha_{12} k^2)\mathbf{m}_2\big]\Big)\,, \\
-\mathrm{i}\omega \mathbf{m}_2 &= g\Big(\mathbf{M}_{20} \times \big[\mathbf{h} + A_\uparrow n_\uparrow' \mathbf{l}_z + A_\downarrow n_\downarrow' \mathbf{l}_z \\
&\quad - (\delta_0 + \alpha k^2)\mathbf{m}_2 - (\delta_0 + \alpha_{12} k^2)\mathbf{m}_1\big]\Big)\,; \qquad (151) \\
\mathrm{i}k\delta_\uparrow &\left(E_\sim - \frac{A_{s\uparrow}}{|e|}(m_1 + m_2)\,\mathrm{i}k\right) - \mathrm{i}\omega |e| n_\uparrow' = 0\,, \\
\mathrm{i}k\delta_\downarrow &\left(E_\sim - \frac{A_{s\downarrow}}{|e|}(m_1 + m_2)\,\mathrm{i}k\right) - \mathrm{i}\omega |e| n_\downarrow' = 0\,; \qquad (152) \\
A_{s\uparrow,\downarrow} &= A_{\uparrow,\downarrow}\,\frac{a^3}{4\mu_0}\,, \qquad \mathrm{i}k\varepsilon E_\sim = -|e| n_\uparrow' - |e| n_\downarrow'\,. \qquad (153)
\end{aligned}$$

Solving the system (151) – (153), we obtain the dispersion equation (42) in which F is equal to

$$\begin{aligned}
F = {} & g M_0 \sqrt{1 - \frac{H^2}{H_c^2}}\,\frac{k^2}{e^2} \\
& \times \left[-\frac{\sigma_\uparrow \sigma_\downarrow}{\varepsilon}\left(|A_{s\downarrow}|^2 - |A_{s\uparrow}|^2\right) + \mathrm{i}\omega\left(\sigma_\uparrow |A_{s\uparrow}|^2 - \sigma_\downarrow |A_{s\downarrow}|^2\right)\right] \\
& \times \left[\frac{\mathrm{i}\omega(\sigma_\uparrow + \sigma_\downarrow)}{\varepsilon_0} + \omega^2\right]^{-1}. \qquad (154)
\end{aligned}$$

Now putting $A_\uparrow = -A_\downarrow = A$, we obtain, in place of (154), the following expression:

$$F = \mathrm{i} g M_0 \sqrt{1 - \frac{H^2}{H_c^2}}\,\frac{k^2}{e^2}\,\frac{\varepsilon_0 A_s^2}{\omega \varepsilon(\omega, k)}(\sigma_\uparrow - \sigma_\downarrow)\,. \tag{155}$$

Thus, we obtain the spin wave amplification coefficient containing, in place of $\tilde{A}$ in (48), the factor $\tilde{\tilde{A}}$ which is related to $\tilde{A}$ as follows:

$$\tilde{\tilde{A}} = \tilde{A}\,\frac{n_\uparrow^0 - n_\downarrow^0}{n_\uparrow^0 + n_\downarrow^0}\,, \tag{156}$$

where $n_\uparrow^0$ and $n_\downarrow^0$ are the equilibrium concentrations of current carriers with a spin along and against the magnetic field, respectively. In the weak magnetic field limit, the ratio (156) can be represented as

$$\tilde{\tilde{A}} = \tilde{A}\tanh\frac{ASH}{2H_c T} \tag{157}$$

for a nondegenerate semiconductor and as

$$\tilde{\tilde{A}} = \tilde{A}\,\frac{AS}{\varepsilon_F}\,\frac{H}{H_c} \qquad \left(\frac{ASH}{H_c} \ll \varepsilon_F\right) \tag{158}$$

for a degenerate semiconductor.

From (157) and (158) it follows that the amplification coefficient is zero when there is no external magnetic field and, consequently, electrons make zero contribution to the magnetic susceptibility (36) and (38).

7. Amplification of spin waves in disordered magnets

Amorphous ferromagnets (Fig. 1e), spin glasses (Fig. 1f), disordered ferromagnets and antiferromagnets all come under disordered magnets. Amorphous magnet is a stochastic system whose parameters are random coordinate functions. Typically, an amorphous magnet has structural and chemical disorders [40].

7.1 General remarks

Several conclusion concerning the contribution of s – f exchange to spin wave attenuation in disordered magnets directly follow from the law of conservation of momentum (Section 3). Thus, the requirement of conservation of total momentum implies that the contribution of s – f exchange to spin wave attenuation (amplification) in amorphous ferromagnet (Fig. 1e) vanishes identically as a consequence of total oriented orderedness of magnetic moments. A similar conclusion can also be drawn for the case in which spin waves are generated by the variations in the modulus of magnetisation of an amorphous ferromagnet due to the fluctuations in the density of the magnet material.

The picture is entirely different for disordered ferromagnets or antiferromagnets. The law of conservation of total momentum does not forbid the excitation of spin density waves in these magnets as a result of s – f exchange. The basic difficulty in the way of resolving these equations is rather pronounced in the case of spin glass (Fig. 1f). When there is no magnetic field, the total magnetic moment in this system is zero. Contrary to the case of isotropic or anisotropic antiferromagnets, the procedure developed here for calculating the s – f exchange induced effects (applicable to any system with a finite number of sublattices) is altogether unsuitable for spin glass which cannot be described with the help of a finite number of sublattices.

However, we can take recourse to the general phenomenological theory which does not need the use of model

representations for sublattices whose motion is described by the Landau – Lifshits equations. Such a consistent exposition based only on the general concepts of symmetry of the state of a magnet is presented in detail in Ref. [41]. Here the dynamic equations describing the oscillations of magnetisation are represented in the form of the Lagrange equations with a Lagrangian density derived from the invariance of the energy of a magnet to symmetry transformations. We shall apply such an approach to calculate the spectrum of spin waves in a spin glass in the presence of s – f exchange.

7.2 Spin glass in the absence of current carriers

According to Ref. [41], the dynamics of a spin glass can be described by the Lagrangian:

$$L_0 = \frac{a}{2}\, g_{\alpha\beta}(\phi)\left(\frac{1}{c^2}\,\dot{\phi}^{\alpha}\dot{\phi}^{\beta} - \frac{\partial\phi^{\alpha}}{\partial x_i}\,\frac{\partial\phi^{\beta}}{\partial x_i}\right), \tag{159}$$

where ϕ^{α} ($\alpha = 1, 2, 3$) are the parameters which parametrise the group of spin rotations of a spin glass and $g_{\alpha\beta}$ is a metric tensor of the type:

$$g_{\alpha\beta} = \frac{\delta_{\alpha\beta}}{1+\phi^2} - \frac{\phi^{\alpha}\phi^{\beta}}{(1+\phi^2)^2}\,, \qquad \phi_{\alpha} = n_{\alpha}\phi\,; \tag{160}$$

n_{α} are the components of the unit vector along the rotation axis and a and c are phenomenological constants.

The dynamic equations which correspond to (159) are

$$\ddot{\phi}^{\alpha} - c^2\Delta\phi^{\alpha} + \Gamma^{\alpha}_{\beta\gamma}\left(\dot{\phi}^{\beta}\dot{\phi}^{\gamma} - c^2\,\frac{\partial\phi^{\beta}}{\partial x_i}\,\frac{\partial\phi^{\gamma}}{\partial x_i}\right) = 0\,, \tag{161}$$

where $\Gamma^{\alpha}_{\beta\gamma}$ is the Christoffel symbol corresponding to the metric $g_{\alpha\beta}$. Linearised equations (161) give the spectrum of spin waves in the absence of conduction electrons:

$$\omega = ck\,. \tag{162}$$

7.3 The s – f exchange mechanism for the amplification of spin waves in spin glass

In a most general form the Lagrangian for the system of conduction electrons in an isotropic spin glass is

$$L = L_0 + L_1\,, \tag{163}$$

$$L_1 = M_{\alpha}H_{\alpha}\,, \tag{164}$$

$$M_{\alpha} = \frac{a\gamma}{2c^2}\,\frac{\dot{\phi}^{\alpha} + e_{\alpha\beta\gamma}\phi_{\beta}\dot{\phi}_{\gamma}}{1+\phi^2}\,, \tag{165}$$

$$H_{\alpha} = A_1\int \mathrm{tr}\,(\hat{\sigma}_{\alpha}\hat{\rho})\,\mathrm{d}\mathbf{P}\,, \tag{166}$$

where M_{α} is the magnetisation of the spin glass, $e_{\alpha\beta\gamma}$ is the antisymmetric unit tensor, $\hat{\sigma}_{\alpha}$ is the Pauli matrix, $\hat{\rho}$ is the matrix of electron distribution function for the conduction band and γ is the gyromagnetic ratio. The quantity H_{α} given by (166) has the meaning of effective magnetic field induced by the conduction electrons (A_1 is the effective constant of s – f exchange). In (166) integration is taken over electron pulses. The relations (159), (160) and (163) – (166) define the dynamics of spin glass in the presence of conduction electrons. To make the system closed, we have to add the following Maxwell's equations:

$$e_{\alpha\beta\gamma}\,\frac{\partial H_{\alpha}}{\partial x_{\beta}} = 0\,, \tag{167}$$

$$\frac{\partial H_{\alpha}}{\partial x_{\alpha}} = -4\pi\,\frac{\partial M_{\alpha}}{\partial x_{\alpha}}\,, \tag{168}$$

$$\frac{\partial D_{\alpha}}{\partial x_{\alpha}} = -e\,\mathrm{tr}\int \mathrm{d}\mathbf{P}\,\hat{\rho}\,, \tag{169}$$

and the equation of motion for the density matrix:

$$\frac{\partial\hat{\rho}}{\partial t} + \frac{1}{\hbar}\left\{\frac{\partial\hat{\rho}}{\partial r}\,\frac{\partial\hat{\varepsilon}}{\partial k}\right\} - \frac{1}{\hbar}\left\{\frac{\partial\hat{\rho}}{\partial k}\,\frac{\partial\hat{\varepsilon}}{\partial r}\right\} + \frac{eE}{\hbar}\,\frac{\partial\hat{\rho}}{\partial k} - \frac{\mathrm{i}}{\hbar}\,[\hat{\varepsilon},\hat{\rho}] = I\,, \tag{170}$$

where $\hat{\varepsilon}(k, r)$ is the electron energy operator and I is the collision integral. Equations (167) – (170) determine the electrodynamics of a Fermi liquid containing spinning particles.

In applying the macroscopic description to the s – f exchange interaction of conduction electrons, we shall assume that when a spin wave travels, electrons are acted upon by a force comparable with the strength of the field generated:

$$\varepsilon_{\alpha} = \frac{1}{e}\,\frac{\partial}{\partial x_{\alpha}}\,H_{\mathrm{int}}\,, \tag{171}$$

$$D_{\alpha} = \varepsilon E_{\alpha} + \frac{\varepsilon}{e}\,\frac{\partial}{\partial x_{\alpha}}\,H_{\mathrm{int}}\,, \tag{172}$$

$$H_{\mathrm{int}} = A_1 n_{\alpha} M_{\alpha}\,. \tag{173}$$

The systems (167) – (170) and (171) – (173) wholly determine the dynamics of electrons and of magnetic moments related to s – f exchange in hydrodynamic approximation.

It is a simple matter to evaluate the spin wave attenuation coefficient if we assume that the matrix $\hat{\rho}$ contains only one nonzero component corresponding to the 'upward' spin. In this case (170) yields the following equations of hydrodynamics:

$$\dot{\mathbf{v}}^{\alpha} = -\frac{e}{m}\,E^{\alpha} - \mathbf{v}^{\alpha}\nu\,, \tag{174}$$

$$\mathbf{v}^{\alpha} = \mathrm{tr}\int \mathrm{d}\mathbf{P}\,\hat{\rho}v^{\alpha}\,, \tag{175}$$

$$\frac{\partial j^{\alpha}}{\partial x_{\alpha}} + \mathrm{tr}\int \mathrm{d}\mathbf{P}\,\hat{\dot{\rho}} = 0\,. \tag{176}$$

For the sake of simplicity, the contribution of the internal pressure tensor is omitted in (174). For small deviations from the equilibrium values of the quantities

$$\delta M,\ \delta\hat{\rho},\ \delta\phi,\ \delta\mathbf{v} \sim \exp(\mathrm{i}kr - \mathrm{i}\omega t)$$

contained in (167) – (170) and (174) – (176), we obtain the dispersion equation:

$$(\omega^2 - k^2c^2)\left[\nu\omega_{\mathrm{R}} + (\omega - kv_0)(kv_0 - \omega - \mathrm{i}\nu)\right] = k^2\omega^2\Gamma_0\,, \tag{177}$$

$$\Gamma_0 = \gamma^2 A_1^2\,\frac{a}{2c^2}\,\frac{n_0}{m}\,. \tag{178}$$

From (177) and (178) we obtain the spin wave attenuation coefficient:

$$\alpha(\omega) \sim \frac{\Gamma_0 \omega (v_0/c - 1)}{vc^2 \left[\omega_R^2/\omega^2 + (v_0/c - 1)^2\right]} . \tag{179}$$

This expression shows that for a velocity v_0 greater than the phase velocity c of the spin waves in a spin glass, the spin waves experience amplification rather than suffer attenuation.

The foregoing results demonstrate that the spin wave amplification coefficient (179) for the spin glass is of the same order of magnitude as for isotropic antiferromagnets. Consequently, we can list a whole class of materials promising from the viewpoint of amplification of spin waves in them — the so-called semimagnetic semiconductors of the general formula $A_xB_{1-x}C$, where A stands for Mn, Fe or Ni, B for Hg, Cd or Zn and C is Te, Se or S.

Semimagnetic semiconductors are superior to ordinary magnetic semiconductors in that they have a high current carrier mobility and, consequently, Cherenkov's radiation may be realised in them. For instance, in $Mn_xHg_{1-x}Te$ or $Mn_xCd_{1-x}Te$ the mobility u is as high as 10^5 cm^2 V^{-1} s^{-1} [42, 43]. What is important is that in the region $x \sim 0.1$ these semiconductors have as a rule the magnetic structure of spin glass and their electron properties are controllable [42, 43]. Consequently, they are expected to find application in spin wave amplification.

From the application viewpoint, one- and two-dimensional magnets, many of which are either antiferromagnets or spin glasses, have a bright future. By way of example, we may mention here quasi-one-dimensional antiferromagnet $(CH_3)_4NMnCl_3$ [44].

We may anticipate that these systems will exhibit strong anisotropic properties (for the phenomena associated with radioelectric effect, see Section 8).

8. Radioelectric effect in antiferromagnets

In Sections 3 and 4 we examined the amplification and drag of bulk spin waves by electron drift in antiferromagnets caused by the s–f exchange interaction between electron stream and magnetisation. One of the stringent constraint essential for amplification is that the Cherenkov condition should be satisfied. Therefore, of special interest are the effects related to the identification of the interaction of electrons with spin waves not governed by the Cherenkov condition. An example of such an effect is the opposite of what was studied in Sections 3 and 4, i.e., the drag of current carriers by a spin wave.

For the drag of electrons by photons (radioelectric effect) to be observed in ordinary nonmagnetic crystals, the effect being weak, the radiation source is to be sufficiently powerful [45]. But the situation is quite different in magnetically ordered crystals where the major contribution to radioelectric effect comes from the drag of current carriers by a spin wave excited by an external magnetic field. The effect should be conspicuous particularly in antiferromagnets. In Sections 3 and 4 we have demonstrated that the coefficient of spin wave amplification due to s–f exchange interaction is several orders of magnitude greater in an antiferromagnet than in a ferromagnet.

In an antiferromagnet the magnitude of the effect can be evaluated from the balance between the momentum transferred to current carriers by the spin wave and the corresponding electric field E generated:

$$E = \frac{I}{nev_s} \operatorname{Im} k , \tag{180}$$

where I is the density of the energy stream entrained by the spin wave and n is the current carrier concentration.

Relation (180) resembles the expression derived for sound [46]. It also holds for anisotropic antiferromagnets; however, the phase velocity of the spin wave here depends on the angle θ between the magnetic field and the anisotropy axis: $v_s = v_s(\theta)$ (for uniaxial antiferromagnets).

What is more important from the viewpoint of practical realisation of the effect is the dependence of the phase velocity on the wave vector $v_s = v_s(\theta, \mathbf{k})$. Owing to the energy gap in the spectrum of an anisotropic antiferromagnet, the phase velocity v_s tends to infinity as the wave vector k tends to zero. Hence, under uniform antiferromagnetic resonance ($k \to 0$), the s–f exchange induced radioelectric effect (180) vanishes. In reality, under uniform resonance, electromagnetic waves excite spin waves with a $k < 10^2$ cm^{-1}. According to Section 4.1, under this condition, radioelectric effect should easily be observed under uniform resonance if the wave vector is properly chosen. For an easy plane antiferromagnet of the type EuTe, this direction is along the anisotropy axis (because of the anisotropy in the basal plane, the phase velocity v_s is of the order of 10^7 cm s^{-1} for a wave vector of $k \sim 10^2$ cm^{-1}).

To observe the radioelectric effect in general, spin waves with a large wave vector k have to be excited. Various methods may be used for this purpose, say, parametric excitation under longitudinal or transverse pumping [47–50], generation of spin waves in a nonuniform constant magnetic field [51–53], creation of various types of magnetic 'pockets' and many other methods.

Let us now evaluate the potential difference U arising in a crystal when a spin wave passes:

$$U = \int E(z)\, \mathrm{d}z = \frac{uI}{\sigma_0 v_s} \frac{\operatorname{Im} k}{\operatorname{Im} k + Y v_s^{-1}} . \tag{181}$$

For instance, in EuTe for a wave vector $k > 10^5$ cm^{-1}, the phase velocity v_s is $\sim 10^5 - 10^6$ cm s^{-1}. For a $u \sim 10^2$ cm^2 V^{-1} s^{-1}, $\sigma \sim 10^{-4}$ Ω^{-1} cm^{-1}, $\operatorname{Im} k \sim Y/v_s$, $I = 1$ W cm^{-2}, the spin wave generates a potential difference of $U \sim 1 - 10$ V. These values are in agreement with those obtained in an experiment on acoustoelectric effect in cadmium sulfide [46] with a current carrier concentration stimulated by light (as a rule, $n \sim 10^{13} - 10^{14}$ cm^{-3}). Just like in the case of acoustoelectric effect, transverse radioelectric effect is also possible in a transverse external magnetic field. If the strength of the external electric field is close to its threshold value (when $v_s = v_d$), the sign of the radioelectric effect is reversed. This phenomenon may be utilised to detect spin wave amplification.

If a crystal contains carriers of unlike signs, then the radioelectric effect is moderated. If there is no additional scattering mechanism (other than the scattering of magnons by electrons), with electrons and holes present in equal concentrations, the radioelectric effect altogether vanishes due to total compensation. In this case, nonzero effect can nonetheless be stimulated by directing the external magnetic field normal to the direction of spin wave propagation.

To conclude we may note that the effects associated with the drag of current carriers by spin waves were observed

repeatedly in ferromagnets [54–58] (magnetoelectric or resonance galvanomagnetic effect). This phenomenon was also observed in hybrid systems like ferrite semiconductors [59]. In all the cases listed above radioelectric effect owes its origin to induced (relativistic) interaction. As applied to antiferromagnets, this question is examined theoretically in Ref. [60].

9. Quantum theory of spin wave amplification

In the previous sections we used the macroscopic approach to describe the interaction of spin waves with the current carriers in magnetically ordered crystals.

In this section we shall apply the quantum mechanical approach to study the spins of magnetic atoms. By way of example we shall discuss the Cherenkov amplification of spin waves in an antiferromagnet with easy axis anisotropy.

9.1 Hamiltonian of s–f exchange in an antiferromagnet

In the most general form the Hamiltonian describing the motion of conduction electrons in an anisotropic antiferromagnet placed in a magnetic field is [7]:

$$H = H_{\mathrm{e}} + H_{\mathrm{int}} + H_{\mathrm{M}}\,, \tag{182}$$

$$H_{\mathrm{e}} = \frac{1}{2m}\left(\hat{\mathbf{P}} + \frac{e}{c}\,\mathbf{A}\right)^2,$$

$$H_{\mathrm{int}} = -\sum_{m,m'} A(R_m - R_{m'})(S_m, \sigma_{m'})\,,$$

$$H_{\mathrm{M}} = -\frac{1}{2}\sum_{i,m_1\neq m_1'} I^i_{m_1m_1'} S^i_{m_1} S^i_{m_1'} - \frac{1}{2}\sum_{i,m_2\neq m_2'} I^i_{m_2m_2'} S^i_{m_2} S^i_{m_2'} - \sum_{i,m_1,m_2} I^i_{m_1m_2} S^i_{m_1} S^i_{m_2} - \sum_{i,m_1} S^i_{m_1} H_i - \sum_{i,m_2} S^i_{m_2} H_i\,,$$

where H_{e} describes the motion of an electron in a magnetic field with a vector potential $\mathbf{A}$, H_{int} corresponds to the interaction of a conduction electron with the magnetic subsystem of a crystal having an s–f exchange constant which, in the sequel, is always assumed equal to $A_{mm'} = A\delta_{mm'}$, and H_{M} is the exchange Hamiltonian of the anisotropic antiferromagnet placed in a magnetic field H (H is expressed in energy units). For a uniaxial antiferromagnet with anisotropy axis along the y-axis, the exchange integral takes the form

$$I^x_{mm'} = I^z_{mm'} = I_{mm'}\,, \qquad I^y_{mm'} = I_{mm'} + \Delta I_{mm'}\,. \tag{183}$$

Confining ourselves to a case where interaction takes place only between two equivalent sublattices, let us introduce the notation:

$$\sum_{m_1,m_2} I_{m_1m_2} S^2 = NJ_{12}\,, \qquad \sum_{m_1,m_2} \Delta I_{m_1m_2} S^2 = N\Delta J_{12}\,, \tag{184}$$

where N is the number of magnetic atoms in the sublattice.

First consider the case when the external magnetic field $\mathbf{H}$ is directed along the z-axis, i.e., perpendicular to the anisotropy axis. So the magnetisation vector of the antiferromagnet is directed along the field ($\Delta J_{12} < 0$) everywhere in the range of variation of $\mathbf{H}$. Assuming the field be strong enough so that the electron spin is completely polarised in the field direction, and using the magnon production and annihilation operators ξ^+ and ξ, and electron production and annihilation operators a^+ and a in H_{M} and H_{int}, we obtain in place of (182) the following

$$\tilde{H} = \tilde{H}_{\mathrm{e}} + \tilde{H}^{(1)}_{\mathrm{int}} + \tilde{H}^{(2)}_{\mathrm{int}} + \tilde{H}_{\mathrm{M}}\,; \tag{185}$$

$$\tilde{H}_{\mathrm{e}} = \sum_{\alpha} \varepsilon_\alpha a^+_\alpha a_\alpha\,, \qquad \tilde{H}_{\mathrm{M}} = \sum_{j,q} \hbar\omega_{qj}\xi^+_{qj}\xi_{qj}\,,$$

$$\tilde{H}^{(1)}_{\mathrm{int}} = \sum_{\alpha\alpha' q} A^{(j)}(\alpha\alpha' q) a^+_\alpha a_\alpha (\xi^+_{-qj}\xi_{qj})\,,$$

$$\tilde{H}^{(2)}_{\mathrm{int}} = \sum_{\alpha\alpha' qq' j} B^{(j)}(\alpha\alpha' qq') a^+_\alpha a_{\alpha'} \xi^+_{-qj}\xi_{q'j} + \sum_{\alpha\alpha' qq' j} C^{(j)}(\alpha\alpha' qq') a^+_\alpha a_{\alpha'} (\xi^+_{-qj}\xi^+_{-q'j} + \xi_{qj}\xi_{q'j})\,,$$

where the subscript $j = 1, 2$ corresponds to the two branches of magnons in the anisotropic antiferromagnet. The matrix elements A, B and C are

$$A^{(j)}(\alpha, \alpha', q) = Q_q \langle \alpha | \exp(\mathrm{i}qr) | \alpha' \rangle \delta_{j,2}\,,$$

$$B^{(j)}(\alpha, \alpha', q, q') = \Gamma^{(j)}_{qq'} \langle \alpha | \exp[\mathrm{i}(q+q')r] | \alpha' \rangle\,,$$

$$C^{(j)}(\alpha, \alpha', q, q') = Z^{(j)}_{qq'} \langle \alpha | \exp[\mathrm{i}(q+q')r] | \alpha' \rangle\,. \tag{186}$$

The index α in (185) and (186) enumerates the eigenfunctions of the kinetic energy operator of an electron in the magnetic field $\tilde{H}_{\mathrm{e}}$ with eigenvalues

$$\varepsilon_{P_z,n} = \frac{P_z^2}{2m} + \hbar\omega_{\mathrm{c}}\left(n + \frac{1}{2}\right)$$

and includes the quantum numbers n, P_z and P_y, where n is the number of the Landau level and P_z and P_y are the components of electron momentum along z- and y-axis, respectively (calibration: $A_x = 0$, $A_y = Hx$ and $A_z = 0$). For this case the matrix elements Q, Γ, and Z in (186) ($j = 2$) are of the form:

$$Q_q = A(SH_{11})^{1/2}\left(1 - \frac{H^2}{H_{\mathrm{E}}^2}\right)^{3/4} \times \left\{4N\left[H_{\mathrm{E}}\left(1 - \frac{H^2}{H_{\mathrm{E}}^2}\right) + 2H_{\mathrm{a}}\right]^{1/2}\right\}^{-1},$$

$$\Gamma_{qq'} = AH\left[H_{\mathrm{E}}\left(1 - \frac{H^2}{H_{\mathrm{E}}^2}\right) - H_{\mathrm{a}}\right] \times \left[8NH_{\mathrm{E}}H_{11}\left(1 - \frac{H^2}{H_{\mathrm{E}}^2}\right)\right]^{-1/2},$$

$$Z_{qq'} = -AH\left[H_{\mathrm{E}}\left(1 - \frac{H^2}{H_{\mathrm{E}}^2}\right) + H_{\mathrm{a}}\right] \times \left[16NH_{\mathrm{E}}H_{11}\left(1 - \frac{H^2}{H_{\mathrm{E}}^2}\right)\right]^{-1/2},$$

$$H_{11} = (H_{\mathrm{E}}H_{\mathrm{a}})^{1/2}\,, \tag{187}$$

where H_{a} is the anisotropy field and H_{E} is the exchange field for sublattice collapse. Using the longwave approximation, in

(187) we have written only the main terms of the expansion over the wave vector of magnetisation oscillations. In this approximation, for the framework of our geometry, the magnon frequencies are expressed as follows:

$$\omega_{q1} = g\sqrt{H^2 + H_{\mathrm{EA}}^2}\,, \tag{188}$$

$$\omega_{q2} = gH_{\mathrm{EA}}\sqrt{1 - \frac{H^2}{H_{\mathrm{E}}^2}}\,, \tag{189}$$

$$H_{\mathrm{EA}} \sim H_{11}\,.$$

The fields H_{a} and H_{E} contained in (187) and (189) are expressed through exchange integrals (184) and if $|\Delta J_{12}| \ll |J_{12}|$, they take the form

$$H_{\mathrm{E}} \approx 4|J_{12}|\,, \qquad H_{\mathrm{a}} \approx 2|J_{12}|\,.$$

9.2 Spin wave amplification coefficient in a magnetic field

The kinetic equation for the magnon distribution function describing one- and two-magnon emission and absorption processes, which correspond to the Hamiltonian (185), has the form:

$$\begin{aligned}\frac{\partial m_q^{(j)}}{\partial t} = {} & \frac{2\pi}{\hbar}\sum_{\alpha,\alpha'}\left|A^{(j)}(\alpha,\alpha',q)\right|^2\left[(m_q^{(j)}+1)f_{\alpha'}(1-f_\alpha)\right. \\ & \left. - m_q^{(j)}f_\alpha(1-f_{\alpha'})\right]\delta(\varepsilon_{\alpha'}-\varepsilon_\alpha-\omega_{qj}) \\ & - \frac{2\pi}{\hbar}\sum_{\alpha\alpha' q'}\left|B^{(j)}(\alpha',\alpha,-q',q)\right|^2\left[f_\alpha m_q^{(j)} - m_{q'}^{(j)}f_{\alpha'}\right. \\ & \left. - f_{\alpha'}f_\alpha(m_q^{(j)} - m_{q'}^{(j)}) + (f_\alpha - f_{\alpha'})m_{q'}^{(j)}m_q^{(j)}\right] \\ & \times \delta(\varepsilon_\alpha - \varepsilon_{\alpha'} + \omega_{qj} - \omega_{q'j}) \\ & + \frac{8\pi}{\hbar}\sum_{\alpha\alpha' q'}\left|C^{(j)}(\alpha,\alpha',-q,-q')\right|^2 \\ & \times \left[(m_q^{(j)} + m_{q'}^{(j)} + 1)f_\alpha(1-f_{\alpha'}) + (f_\alpha - f_{\alpha'})m_q^{(j)}m_{q'}^{(j)}\right] \\ & \times \delta(\varepsilon_\alpha - \varepsilon_{\alpha'} - \omega_{qj} - \omega_{q'j}) + I_{\mathrm{d}}\{m_q^{(j)}\}\,, \end{aligned} \tag{190}$$

where $m_q^{(j)}$ and f_α are the magnon and electron distribution functions; $I_{\mathrm{d}}\{m_q^{(j)}\}$ is the collision integral which includes all other (except the electron – magnon) scattering mechanisms.

Let us consider the deviation of $m_{q_z j}^{(1)}$ from the stationary magnon distribution $\tilde{m}_q^j$ established as a result of the drift of electrons having the distribution function

$$\tilde{f}_\alpha = \left\{\exp\beta\left[\varepsilon_n + \frac{1}{2m}(P_z - mv_{\mathrm{D}})^2 - \mu\right] + 1\right\}^{-1}, \tag{191}$$

where v_{D} is the electron drift velocity along the z-axis and μ is the chemical potential. Taking $m_q^{(j)} = \tilde{m}_q^{(j)} + m_{qj}^{(1)}$ and $f_\alpha = \tilde{f}_\alpha$, from (191) we obtain the following equation for $m_{q_z j}^{(1)}$:

$$\frac{\partial m_{q_z j}^{(1)}}{\partial t} = R_1^{(j)}(q_z)m_{q_z j}^{(1)} + R_2^{(j)}(q_z)m_{q_z j}^{(1)} + I_{\mathrm{d}}\{m_q^{(j)}\}\,, \tag{192}$$

$$R_1^{(j)}(q) = \frac{2\pi}{\hbar}\sum_{\alpha,\alpha'}\left|A^{(j)}(\alpha,\alpha',q)\right|^2(\tilde{f}_{\alpha'} - \tilde{f}_\alpha)\delta(\varepsilon_{\alpha'} - \varepsilon_\alpha - \omega_{qj})\,, \tag{193}$$

$$\begin{aligned}R_2^{(j)}(q) = {} & -\frac{8\pi}{\hbar}\sum_{\alpha\alpha' q'}\left|C^{(j)}(\alpha'\alpha,-q,-q')\right|^2\left[\tilde{f}_\alpha(1-\tilde{f}_{\alpha'})\right. \\ & \left. + \tilde{m}_{q'}^{(j)}(\tilde{f}_\alpha - \tilde{f}_{\alpha'})\right]\delta(\varepsilon_\alpha - \varepsilon_{\alpha'} - \omega_{qj} - \omega_{q'j})\,. \end{aligned} \tag{194}$$

In deriving (192) and (193) due consideration is paid to the fact that, since we are using the longwave approximation $\omega_{qj} = \omega_{0j}$, the term $\sim |B|^2$ on the right-hand side of (190) makes zero contribution to the collision integral in the quantizing magnetic field approximation (considered below) because simultaneous production and annihilation of magnons with an energy ω_{0j} do not alter the stationary distribution. The quantity $R^{(j)}(q) = R_1^{(j)} + R_2^{(j)}$ introduced in (192) determines the spin wave amplification coefficient for the j-th branch. For an external magnetic field of arbitrary strength, the matrix elements $\langle\alpha|\exp(\mathrm{i}qr)|\alpha'\rangle$ in (186) and (193) are of the form:

$$\begin{aligned}\langle\alpha|\exp(\mathrm{i}qr)|\alpha'\rangle = {} & \delta_{p_y\alpha',\,p_y\alpha+q_y}\delta_{p_z\alpha',\,p_z\alpha+q_z}\sqrt{n!n'!} \\ & \times \exp\left(-\frac{q_\perp^2\varrho_0^2}{4}\right)\left(\frac{q_\perp^2\varrho_0^2}{2}\right)^{|n'-n|/2} L_n^{|n'-n|}\left(\frac{q_\perp^2\varrho_0^2}{2}\right), \end{aligned} \tag{195}$$

where $q_\perp^2 = q_x^2 + q_y^2$, $L_n^{|n'-n|}$ is the generalised Laguerre polynomial and $\varrho_0 = (2\mu_0 c\hbar/eH)^{1/2}$ is the magnetic length.

9.3 Spin wave amplification coefficient in ultraquantum limit

We shall now study the amplification of the low-frequency branch ($j = 2$). In the quantum limit, where the cyclotron frequency $\hbar\omega_{\mathrm{c}}$ is greater than $\hbar\omega_0$ and T, we can restrict ourselves to the first Landau level approximation in (192) and (195). Here putting $n = n' = 0$, from (186) and (195) we obtain

$$\left|A^{(2)}(\alpha\alpha' q_z)\right|^2 = Q^2\delta_{q_z,\,p_z'-p_z}\delta_{p_x',\,p_x}\,, \tag{196}$$

$$\begin{aligned}&\left|C^{(2)}(\alpha\alpha',-q-q_z)\right|^2 \\ &\qquad = Z^2\exp\left(\frac{q_\perp^2\varrho_0^2}{2}\right)\delta_{p_z'-p_z,\,-q_z-q_z'}\delta_{p_x'-p_x,\,-q_x'}\,, \end{aligned} \tag{197}$$

where $Q_q = Q$ and $Z_{qq'}^{(2)} = Z$ do not depend on either q or q'. Substituting (196) into (193) and taking into account that summation over p_x gives rise to a factor $L_x L_y m\omega_{\mathrm{c}}/2\pi\hbar$, we obtain

$$R_1(q_z) = \frac{Q^2 V m\omega_{\mathrm{c}}}{2\pi\hbar^4|q_z|}\,\Delta\,, \tag{198}$$

$$\begin{aligned}\Delta = {} & f^0\left(p_z = \frac{m\omega_{02}}{q_z} - p_{\mathrm{D}} + \frac{\hbar q_z}{2}\right) \\ & - f^0\left(p_z = \frac{m\omega_{02}}{q_z} - p_{\mathrm{D}} - \frac{\hbar q_z}{2}\right), \end{aligned} \tag{199}$$

where f^0 is the equilibrium electron distribution function and V is the volume of the system. If the wave vector obeys the condition $\hbar^2 a_z^2/8mT \ll 1$, we obtain, in place of (199), the following expression for nondegenerate electron gas:

$$\begin{aligned}\Delta \approx {} & \exp\left(\frac{\mu - h\omega_{\mathrm{c}}}{T}\right)\frac{h\omega_{02}}{T}\left(1 - \frac{q_z v_{\mathrm{D}}}{\omega_{02}}\right) \\ & \times \exp\left[-\frac{\omega_{02}^2 m}{2q_z^2 T}\left(1 - \frac{q_z v_{\mathrm{D}}}{\omega_{02}}\right)^2\right]. \end{aligned} \tag{200}$$

As regards R_2 we may note the following: owing to the term $I_d\{m_q\}$ contained in (194), the stationary magnon distribution function is not known in a general form. It can therefore be assessed only approximately. Assuming that under electron drift the quantity $\tilde{m}_q$ differs little from its equilibrium value, i.e., taking $\tilde{m}_q \approx m_q^0$ and $mv_D^2/2 \ll T$, by virtue of (197), from (194) we obtain

$$R_2(q_z) = -\frac{\omega_c}{\pi^3}\frac{mV^2Z^2}{\varrho_0^2\hbar^4}\exp\left(\mu - \frac{h\omega_c}{2}\right)K_0\left(\frac{h\omega_{02}}{T}\right), \quad (201)$$

where $K_0(x)$ is the McDonald function. From (198)–(201), we find that $R_1(q_z) < 0$ if $\omega_{02} - q_z v_D > 0$ and $R_1(q_z) > 0$ if $\omega_{02} - q_z v_D < 0$. On the contrary, for the case under consideration $R_2(q_z)$ is always negative. Hence the high-frequency branch is a damped one under our assumptions. Therefore for the amplification of spin waves it is necessary that

$$\Lambda = \frac{|R_1(q_z)|}{|R_2(q_z)|} > 1 \quad (202)$$

for $\omega_{02} - q_z v_D < 0$.

In a particular case of practical importance, $H^2/H_E^2 \ll 1$ (the typical value of H_E in an antiferromagnet is $\sim 10^6$–10^7 Oe), assuming $q_z v_D > \omega_{02}$ and using (198)–(201) and Eqns (187) for Q and Z, we obtain

$$\Lambda \approx \frac{4\pi^2 S}{K_0(\omega_{02}/T)}\frac{\hbar v_D}{aT}\left(\frac{\varrho_0}{a}\right)^2\frac{H_\parallel^2}{H^3}, \quad (203)$$

where a is the lattice constant. The strength of the magnetic field in (202) and (203) is bounded below by the condition $H \gg H_\parallel$ $(\omega_c \gg \omega_0)$. For instance, for $T = 20$ K, $H_\parallel = 5\times 10^4$ Oe, $H = 5\times 10^5$ Oe, $v \approx 10^6$ cm s^{-1}, $a = 3\times 10^{-8}$ cm, $\omega_0 = 10^{-3}$ eV, and $S = 2$ we have $\Lambda \sim 20$, i.e. the criterion of amplification (202) is easily satisfied. In practice, a more strong restriction on spin wave amplification is posed by the condition

$$\eta > \gamma, \quad (204)$$

where $\eta = (R_1 + R_2)/\omega_{02}$ is the increment of spin wave amplification, and γ is the attenuation decrement determined by the collision integral $I_d\{m_q\} \sim \gamma m_q$. Using the same values for the parameters we chose in finding Λ, and taking $H_E = 10^6$ Oe, $A = 0.5$ eV and a current carrier concentration $n \sim 5\times 10^{17}$ cm^{-3}, we find that $\eta \approx R_1/\omega_{02} \sim 10^{-1}$, i.e., it is one order of magnitude greater than the attenuation decrement used in estimating the spin wave attenuation in yttrium ferrite, viz., $\gamma \sim 10^{-2} - 10^{-3}$ [4].

9.4 Amplification of spin waves in a nonquantizing field

For a nonquantizing magnetic field the kinetic equation (190) for the magnon distribution function $R_1(q)$ yields

$$R_1 = \frac{2\pi}{\hbar}\sum_{k,k'}|A(k,k',q)|^2(\tilde{f}_k - \tilde{f}_{k'})\delta(\varepsilon_k - \varepsilon_{k'} - \omega_q),$$
$$A(k,k',q) = Q_q\langle k|\exp(iqr)|k'\rangle. \quad (205)$$

The matrix elements $A(k,k',q)$ for a wave function in the nonquantizing magnetic field limit are

$$A(k,k',q) = Q_q\,\delta_{k',k-q}. \quad (206)$$

Now from (205) and (206) it follows that

$$R_1 = \frac{Q}{\pi}\left(\frac{ma^2}{\hbar^2}\right)^2\frac{1}{qa}(qv_D - \omega_0)\exp\left[-\left(\frac{\hbar q}{2} + \frac{m\omega_0}{q}\right)^2\frac{1}{2mT}\right]. \quad (207)$$

To estimate the contribution of the quadratic part of the Hamiltonian (185) to the total amplification coefficient, let us consider the contribution from the term $\sim |c|^2$ to the kinetic equation (190):

$$R_2 = \frac{8\pi}{\hbar}\sum_{k,k',q}|C(k',k,-q,-q')|^2[\tilde{f}_k(1-\tilde{f}_{k'}) + \tilde{m}_{q'}(\tilde{f}_k - \tilde{f}_{k'})]\delta(\varepsilon_k - \varepsilon_{k'} - \omega_q - \omega_{q'}). \quad (208)$$

The matrix elements $C(k,k',q,q')$ for a wave function in the nonquantizing magnetic field limit are

$$C(k,k',q,q') = Z\delta_{k',k-q-q'}. \quad (209)$$

Applying the same assumptions as in Section 9.3 for the magnon distribution function and using the relations (208) and (209), we find that

$$R_2 = \frac{32\Gamma(3/2)}{\pi^3\sqrt{\pi}}\frac{Z^2T^2}{\hbar}\left(\frac{ma^2}{\hbar^2}\right)^3\frac{\omega_0}{T}K_1\left(\frac{\omega_0}{T}\right). \quad (210)$$

where $K_1(x)$ is the modified Bessel function and $\Gamma(x)$ is the gamma-function. The spin wave amplification condition requires that $R_1 > R_2$ and that (204) be satisfied. Just as in the case of a quantizing magnetic field, comparing the contributions (207) and (210), we find that $R_1 > R_2$ for a wide range of variations of the parameters contained in (207)–(210).

The results derived in Section 9.3 and in this section state the condition for the amplification of spin waves in an easy axis antiferromagnet. Here we underline that in the collisionless limit $ql \gg 1$ studied in this section, an antiferromagnet is supposed to have a sufficiently high mobility for the current carriers (say, $Hg_{1-x}Mn_xTe$ in the antiferromagnetic phase). If there is a sufficiently slow electromagnetic wave in the crystal (for example, in an antiferromagnet with high dielectric permittivity $\varepsilon \sim 10^4$ as in a segnetomagnet), these result also determine the condition for the amplification of electromagnetic waves.

An amplified spin wave should be converted into an electromagnetic wave near the antiferromagnetic resonance which, for the case under consideration, takes places on both the branches of magnetisation oscillations of the antiferromagnet. In this case the electromagnetic wave with a component along the magnetic field experiences amplification. And the electromagnetic wave with a component normal to the magnetic field interacts with the first resonance branch, which results in the usual wave absorption phenomenon under antiferromagnetic resonance. The same situation holds also for the amplification in all other cases when the antiferromagnetism vector is normal to the magnetic field: only the wave with a longitudinal magnetic component experiences amplification.

Let us also note the qualitative differences between the conditions for spin wave amplification in collision and collisionless limits. In the collision limit $ql \ll 1$ when the spin waves are generated in a finite specimen, they might be excited during multiple reflections from the specimen boundary, in

other words, by the multi-path mechanism. On the other hand, in the collisionless limit, only the one-path mechanism is possible, i.e., spin wave is amplified only as it passes through the specimen. If this condition is not satisfied, the only consequence is the absorption of spin waves. In the collisionless limit the multi-path amplification mechanism can be realised only with the help of a pulsating drifting electric field which should be reversed at each reflection of the spin wave from the specimen boundary.

10. Concluding remarks

The macroscopic formalism developed to describe the s – f exchange interaction between the electron and magnetic subsystems of a crystal is a convenient tool for evaluating various effects accompanying this interaction. An interesting application of this interaction seems to be the spin wave amplification by electron drift. The theoretical principles derived in this review may form a basis for practical realisation of spin wave amplification in antiferromagnets and ferrimagnets [61]. An important application of spin wave amplification is the creation of electromagnetic wave amplifiers. This seems to have a promising future as the characteristic spin wave frequencies in many antiferromagnets and ferromagnets lie in the longwave range $10^{13} - 10^{14}$ Hz — still unexplored area in modern radio engineering. The advantage of this amplifier is that the frequency can easily be tuned by simple variation of the external magnetic field.

The reverse effect (Section 9) seems to have a wide range of promising potentialities. This effect may be used for the development of various information processing systems, delay lines, new microwave circuits etc. Aside from practical implications, the study of different effects related to spin wave amplification is of general theoretical interest. This would promote the development of new methods of spin wave generation (since the exchange amplification is strong in the high frequency range, a key role would be played by short-wave magnon generation techniques) and methods of the detection of spin wave amplification. Detection of spin wave amplification and magnon heating under electron drift may be based on the following:

(a) scattering of neutrons by spin waves;

(b) Mandelstam – Brillouin scattering of electromagnetic waves by spin waves;

(c) oscillations and non-Ohmic behaviour of current during spin wave generation;

(d) relaxation of nuclear spin waves by electron beams;

(e) second sound in magnon gas, and many other phenomena.

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