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The collapses of wave functions

B B Kadomtsev, M B Kadomtsev

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Abstract. The collapse of the wave function has recently reemerged as a subject of extensive discussion in the quantum mechanical literature. In the present paper, wave function collapses occurring during the irreversible evolution of complex quantum systems, including those involved in measurement procedures, are described.

1. Introduction

The following simple example is often used (see, e.g., Ref. [1]) to demonstrate the principles and statistical nature of quantum theory. Suppose we have a silvered glass plate which, upon incidence of a beam of light on it, transmits and reflects exactly the half of the original beam intensity. Now let a single photon be incident on the plate. The photon wave function naturally splits into a reflected and transmitted waves, but if we place a photodetector in the path of these, only one photodetector will respond, meaning that the photon finds itself either to the right or to the left of the plate or, in other words, the photon either is reflected from or passes through the plate. The detection of the photon is a random process which is registered in either detector with probability 1/2.

Here we encounter a typical example of wave function collapse. At the time of photon registration, or more precisely within a very short photodetector operation interval, the photon wave function is destroyed everywhere outside of the detector, whereas inside the detector the photon is absorbed and hence also disappears.

It is readily seen that there is actually no need to take a semitransparent plate to have the wave function collapse. If a monochromatic photon described by a highly extended wave function is incident on the photodetector, it can be absorbed at a small portion of the wave packet, and the packet will be destroyed over the entire space all at once.

Exactly the same kind of collapse occurs for any quantum particle incident on an irreversible medium such as a photographic plate, a Wilson chamber, or simply a gas at room temperature. A particle in this case is 'detected ' within the medium, whereas everywhere outside the registration region its wave function is destroyed. We again have a typical example of wave function collapse. The term 'wave function shrinking' sometimes used here is unsuitable as carrying the connotation of a certain physical process in which the wave function 'sinks' to the collapse region. In actual fact, the wave function has no physical sink, and what really happens is simply that it is destroyed outside the region of 'registration'. We will take this statement as the basic postulate following from experimental data. In doing so we ascribe to the wave function a purely information-carrying meaning in the sense that it is different from zero only where the particle can be found and is zero where it cannot. This approach is fully consistent with the basic principles of atomism, whose fundamental assertion is the conservation of the indivisible atom (particle) as an entity.

The collapse of the wave function cannot be described by means of the Schrödinger equation relating the change in the density $|\psi|^2$ to certain fluxes. It does not involve any such fluxes, and what happens is simply that the wave function viewed as a certain potential source of information is destroyed outside the region where the particle has proved to be involved in an irreversible process. Note that such a process, in which the wave function is destroyed in a large region of space, may correspond to negligibly small changes in physical quantities l_i defined by the relations $l_i = \langle\psi|L_i|\psi\rangle/\langle\psi|\psi\rangle$, where L_i are the corresponding operators.

2. Irreversibility

As the simplest example of an irreversible system, let us consider a dilute atomic gas at a high enough temperature to make it not important whether the particle statistics is of Bose or Fermi type. Let the gas be inside a closed vessel whose

B B Kadomtsev, M B Kadomtsev Russian Research Centre 'Kurchatov Institute', Nuclear Fusion Institute
46 Kurchatov Square, Moscow, 123182, Russia
Tel. (095) 196-9814
Fax (095) 943-0073
E-mail: kadomtse@ufn.msk.su

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linear dimensions are very large compared to the mean free path of the atoms. Suppose further that the gas, together with the vessel, are in a thermostat in full thermodynamic equilibrium. Now let us perform certain thought experiments with the gas. Suppose at some time $t = 0$ the walls of the vessel become specularly reflective and, correspondingly, thermally insulating. Simultaneously, let one of the gas atoms be replaced by a test particle of the same mass, velocity, and scattering cross section. Such a replacement leads to a very small change in the state of the gas: its thermal energy remains the same, and its entropy increases by $k \ln(V/\Delta V)$ since the test particle is not identical with the gas atoms and occupies a small portion ΔV of the total volume V (k is Boltzmann's constant). This done, one would expect that an irreversible relaxation of the gas will begin. From the classical mechanics point of view, the test particle must diffuse in space, its mean distribution function tending to occupy the entire volume of the vessel. Quantum mechanically, the test particle wave function is expected to become more and more extended due to subsequent rescattering on gas atoms. After a sufficiently long time, it is expected that full thermodynamic equilibrium will be attained.

Although the above picture appears quite plausible, it involves a paradox. Our system is a closed one and as such evolves in a deterministic way, following the laws of conventional mechanics in the classical case and the Schrödinger equation in the quantum mechanical case.

Such an evolution is fully reversible, so that if at any time $t = t_0$ one reverses all the particle velocities of a classical system or, for a quantum system, replaces t by $-t$ in the argument of the wave function ψ, the evolution process will reverse its direction, returning the system to its initial state at $t = -t_0$. The test particle, in particular, will return to its initial state with the entropy it possessed at $t = 0$.

Clearly, this picture is too idealistic, and the reversibility will disappear as soon as we turn to a more practical case of an ordinary vessel in thermal equilibrium with a thermostat. The probability for the test particle to return to its initial state then becomes negligible since, under the natural conditions of thermal equilibrium, it is not that simple to reduce the entropy of the system.

In the classical case, irreversibility relates to the strong instability of the system, i.e., to the divergence of its phase space trajectories, whereas in the quantum case, the reversal of time results in transforming scattered waves from divergent into convergent. Clearly, a small external perturbation easily destroys the coherence of the converging waves, so that, for a gas in contact with a thermostat, the evolution of the function $\psi(-t)$ must be similar to that of $\psi(t)$: in either case scattered waves must be present.

Now let us return to our thought experiments with a test particle and a gas in thermal equilibrium with a thermostat. Suppose that at some time $t = t_0$ the vessel with the gas is instantly divided in two halves by an impenetrable partition. Clearly, the test particle will then find itself in only one of the halves and will be there as long as the system's subsequent irreversible evolution lasts. Under these conditions the test particle wave function in the empty half may be taken to be zero, at least after a few collisions following $t = t_0$, when a return to the previous state is definitely ruled out.

Now one can introduce many partitions rather than one, thus dividing the gas-containing vessel into many little volumes each a few mean free paths in size. Again, a few collisions render all long-range correlations obsolete, and the test particle and its wave function will find themselves in only one of the little volumes available.

Quite obviously, the wave function of the test particle must be localised in this little volume even if no partitions have been introduced. Indeed, the time of the order of a few collision times is too short for the particle to displace for more than a few mean free paths. In other words, the collisions themselves act as 'partitions' separating little gas volumes (assuming the gas is in a stationary state).

Thus, the test particle wave function at any time can be considered localised in a little volume a few mean free paths in cross section. Now let us go mentally to the past, starting from $t = t_0$. In doing so, all the divergent waves become convergent. This means that as $t_0 - t$ increases, the test particle wave function must be steadily shrinking into a tiny lump whose ultimate size is determined by the competition between the quasioptical beam focusing and the wave packet diffraction spread. As will be seen later, in a dilute gas the size of such a wave packet is just a fraction of the mean free path. Therefore, the past evolution of the test particle wave function can be described in terms of a random walking of a compact wave packet being successively scattered by the gas atoms.

A similar behaviour is expected for the wave functions of the gas atoms.

3. Gas atom wave functions

The wave functions of the atoms of a dilute gas are usually taken to be plane waves. This assumption comes from the standard two-particle scattering theory, where the $|\text{in}\rangle$ and $|\text{out}\rangle$ states can always be considered to be outside of the interaction region, and appears to be quite natural in the present context. Indeed, in a dilute gas the mean free path exceeds the average atom separation. Scattered waves therefore have enough time to travel far away from the scattering point and have an approximately plane wave structure locally. This point needs to be studied in more detail, however, because, in contrast to ordinary two-particle scattering, gas atoms interact continuously with each other.

The point is that, over the mean free path, the wave of a given atom has enough time to be scattered by many other atoms, giving rise to a complicated pattern of many scattered waves. It can be argued that a highly complicated coherent structure of many scattered waves thus emerges. Obviously enough, such a structure cannot survive in a gas with chaotically moving atoms. In the scattering events that follow, the gaseous medium can 'feel' only one of the possible values of the momentum of the scattered particle. One can therefore speak of a certain decoherence (i.e., 'self-measurement') mechanism continuously operating within the gas, which picks up, randomly, only one of the available scattered waves just leaving all the rest of them to be destroyed. In other words, even the simplest — plane wave — representation of the wave functions suggests the presence of a continuously operating collapse mechanism supposed to 'purge' the wave functions of 'empty' waves.

The real situation must be even more complicated. Suppose the particle j does indeed find itself in one of the scattered waves with a certain momentum $\hbar\mathbf{k}_j$, $\mathbf{k}_j$ being the wave vector. Going back in time along the direction of the momentum, one can find the scattering point and learn the number of the particle, say q, at which the scattering event had taken place (needless to say, one must also go back along the momentum $\hbar\mathbf{k}_q$ of the scattering particle). On the average, the time interval time one must go back for this purpose is $\tau = \lambda/v_T$, where $\lambda = 1/n\sigma$ is the mean free path, n is the mass density of gas atoms, σ is the scattering cross section, $v_T = \sqrt{kT/m}$ the average thermal velocity, T the temperature, and m the atom mass. Now suppose that, in this travel to the past, the wave function of the jth atom is represented by the wave packet $\exp[\mathrm{i}\mathbf{k}_j\mathbf{r}_j - (\mathbf{r}_j - \mathbf{r}_{0j})^2/2\Lambda^2]$. Here the factor $\exp(\mathrm{i}\mathbf{k}_j\mathbf{r}_j)$ corresponds to the short-wavelength plane wave 'filling', while $\exp[-(\mathbf{r}_j - \mathbf{r}_{0j})^2/2\Lambda^2]$ is (as yet unknown) envelope localised around $\mathbf{r}_{0j}$. Suppose that at the instant of wave packet origination the parameter $\Lambda^2 = b^2$, where b^2 is the constant determining the width of the newly born packet. A relatively simple argument allows one to determine the constant b^2.

Suppose a wave packet initially of the form $\exp[-(\mathbf{r} - \mathbf{r}_0)^2/2b^2]$ evolves in accordance with the Schrödinger equation. Then the quantity Λ^2 must equal $b^2 + \mathrm{i}\hbar t/m$, where the time t is measured from the instant the packet comes to existence. The average packet lifetime until the next scattering event is evidently $\tau = \lambda/v_T$, and the expectation value of Λ^2 is $\Lambda^2 = b^2 + \mathrm{i}\hbar\tau/m$. Roughly speaking, all wave packets have, on the average, a standard Gaussian form with $\Lambda^2 = b^2 + \mathrm{i}\hbar\tau/m$ provided at the instant of creation they were Gaussian with $\Lambda^2 = b^2$.

Accordingly, viewing its evolution in retrospect, the wave packet can be thought of as a 'pulsing' entity with an initial value $\Lambda^2 = b^2$ and a final value $\Lambda^2 = b^2 + \mathrm{i}\hbar\tau/m$.

Now let us consider the wave packet collapse per se, a process in which Λ^2 changes rapidly from $\Lambda^2 = b^2 + \mathrm{i}\hbar\tau/m$ to $\Lambda^2 = b^2$. It can be argued that this collapse is caused by the collapse of the wave function of particle q, the second partner in the scattering process. If the post-collapse wave function of the particle q looks like a Gaussian packet with $\Lambda^2 = b^2$, we can go back in time to the scattering event, and the particle q will then have Λ^2 equal to $\Lambda^2 = b^2 - \mathrm{i}\hbar\tau/m$, which is the average conjugate $(\Lambda^2)^*$ of the parameter Λ^2 for the jth particle. But the scattered particles j and q have a joint wave function, so that the collapse of the qth particle produces automatically a form factor $\exp[-(\mathbf{r}_j - \mathbf{r}_{0j})^2/2\Lambda_0^{*2}]$ in the wave function of the j particle. In other words, we have $\exp[-(\mathbf{r}_j - \mathbf{r}_{0j})^2/2\Lambda^{*2})]\exp[-(\mathbf{r}_j - \mathbf{r}_{0j})^2/2\Lambda^2] = \exp[-(\mathbf{r}_j - \mathbf{r}_{0j})^2/2b^2]$. This yields $\Lambda^{-2} + \Lambda^{*-2} = b^{-2}$, i.e., $b^2 = \hbar\tau/m$. Since

$$|\psi|^2 = \exp\left[-\frac{b^2(\mathbf{r}_j - \mathbf{r}_{0j})^2}{|\Lambda|^4}\right],$$

the size of $|\psi|^2$ pulsates from the initial value $b^2 = \hbar\tau/m$ to the final $|\Lambda|^4/b^2 = 2\hbar\tau/m$ [2, 3].

The neglect of these pulses results in the continuous collapse model. The outline of this model is as follows. Since every wave packet has a finite lifetime τ, its energy has an uncertainty of the order of $\hbar/2\tau$. The corresponding uncertainty in the space of wave numbers $\varkappa$ follows from the relation $\hbar^2\varkappa^2/2m = \hbar/2\tau$. When two wave packets collide (scatter), the particles exchange their momenta and, due to the two wave number uncertainties of order $\varkappa$ being added, a regular broadening of their packets in k also occurs. The continuous collapse model can include this effect approximately by using the one-dimensional equation for diffusion in k space,

$$\frac{\partial\psi}{\partial t} = D\,\frac{\partial^2\psi}{\partial k^2} + \gamma\psi\,. \tag{1}$$

Here ψ is the wave packet wave function, $D = \varkappa^2/2\tau$ is the diffusion coefficient in k, and the constant γ takes account of the normalisation of $|\psi|^2$. When changing to the configuration space, the operator $\partial^2/\partial k^2$ should be replaced by $-(x - x_0)^2$, where x is the coordinate along the packet motion and x_0 is the centre of the packet. In changing to three-dimensional space, the collapse in all three coordinates must be taken into account. Adding the kinetic energy operator yields the generalised Schrödinger equation for this model,

$$\mathrm{i}\hbar\,\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\,\Delta\psi - \mathrm{i}\,\frac{\hbar(\mathbf{r} - \mathbf{r}_0)^2}{2\Lambda_0^2\tau}\,\psi + \mathrm{i}\hbar\gamma\psi\,, \tag{2}$$

where $\Lambda_0^2 = 2\hbar\tau/m$.

The steady-state solution to this equation is

$$\psi = \exp\left[-\mathrm{i}\omega t - \frac{(\mathbf{r} - \mathbf{r}_0)^2}{2\Lambda^2}\right], \tag{3}$$

where $\omega = \gamma = 3/4\tau$, $\Lambda^{-2} = (1 - \mathrm{i})\,\Lambda_0^{-2}$, $\Lambda_0^2 = 2\hbar\tau/m$. The quantity γ is selected to ensure that the frequency ω is real. As one can see, the width of the wave packet is determined by the quantity $\Lambda_0^2 = 2\hbar\tau/m$. In order of magnitude, $\Lambda_0 \cong \sqrt{\lambda\lambda_B}$, where $\lambda_B = \hbar/mv_T$ is the average de Broglie wavelength. The geometrical mean of the macroscopic λ and microscopic λ_B parameters is of course a hallmark of the mesoscopic region. Since usually $\lambda_B \ll \lambda$, wave packets look like compact entities reminiscent of classical particles. Their centre trajectories are broken lines with straight-line segments between and sharp kinks at the collisions. The collisions are described by conventional quantum mechanics, and the general behaviour of wave packets, with account for random collapses, may be described by the kinetic equation.

The above approach includes wave function collapses explicitly. This means that we attribute to the wave function an information-carrying meaning and introduce 'self-measurement' processes, in which the particle wave function is completely destroyed in the regions where the particle is absent. It is therefore not surprising that an equation of the type (2) can be used in modelling continuous measurement processes [4].

Now the question naturally arises, why not to employ the standard apparatus of quantum mechanics to describe a gas? Since the interatomic interaction in a dilute gas is small, perturbation theory would seem to be the most suitable tool for this purpose. This question received a detailed treatment in the works by Prigogine and Petrosky [5 – 7], who showed that when applied directly perturbation theory leads to divergences. The reason is that a classical gas is a typical example of Poincare's Grand System, which is inherently stochastic. Accordingly, quantum theory involves the self-scattering paradox similar to the problem of small denominators in classical theory. To obviate the quantum divergence difficulties, Prigogine and Petrosky develop a complicated formalism for describing quantum systems in terms of the Liouville representation [7]. A more preferable approach, in our view, is to include the wave function collapse explicitly as discussed in Refs [2, 3, 8].

4. Gas as a measurement apparatus

There are three stages in any quantum measurement, the spectral decomposition of the wave function, the collapse of the wave function, and the registration of the event (see, e.g., Ref. [9]). The first stage is a purely preparatory one, which does not perform a measurement but just prepares the spectral decomposition of the wave function for the subsequent measurement. In the second, the most extraordinary and delicate step, the wave function projects itself, as a result of an interaction with a macroscopic body, onto one of the possible states, following the law of random events in doing so. This is where all the specificity of quantum measurement is contained. As regards the third stage, this is nothing else than an archive record of the result produced by the collapse.

Being able to describe the collapse of the wave function of a gas atom we may analyse this most intriguing stage of a quantum measurement event. To do this, it suffices to take a dilute gas as a measuring device, i.e., a system which implements the collapse of the wave function.

Consider a vertically oriented gas layer of thickness $L \gg \lambda$. Let the x axis of a rectangular coordinate system be coincident with the normal to the layer, and suppose that along this axis the wave function $\psi(\mathbf{r}, t)$ of a certain particle is incident on the layer, the particle mass m being equal to that of the gas atom. The velocity of the particle is v_0, and its cross section for scattering from the gas atom is σ_0, yielding the mean free path $\lambda_0 = 1/n\sigma_0$. The walls confining the gas are assumed to be transparent for the incident particle.

The incident wave function is scattered by the gas atoms, so that its unscattered part decreases with x as $\exp(-x/2\lambda_0)$, and the square of the wave function as $\exp(-x/\lambda_0)$ (assuming $v_0 \gg v_\mathrm{T}$ for the sake of simplicity). Because the incident particle undergoes many scatterings over a length of the order of λ_0, a large number of scattered waves form. To each one of these there corresponds the second interaction partner, the atom at which the scattering event occurred.

During the time $\tau = \lambda/v_\mathrm{T}$ the atoms of the gas experience collisions, and correspondingly wave function collapses occur. Of all the atoms of the gas, only one will be able to undergo a collapse jointly with the wave function of the incident particle; for all the others, the incident particle has nothing to do with their collapses. In the joint collapse, all the remaining — noncollapsing — part of the incident wave function is eliminated instantly.

Now let the incident wave function depends on one of the transverse coordinates, say y, so that $\psi = \psi(y)$. Since $\psi(y)$ prior to the collision enters as a factor in the total wave function of the system as a whole, it follows that the probability of a joint collapse for the wave function and one of gas atoms will be proportional to $|\psi(y)|^2$, for the simple reason that the probability for the gas atom wave function to collapse is proportional to its absolute value squared. We thus see that the incident particle 'is measured' with a probability proportional to $|\psi|^2$.

This example clearly demonstrates that the wave function collapse is a global process related to the total multiparticle wave function of the problem; it involves not only the wave function of the particle 'being measured' but also the wave functions of the gas atoms.

If $L \gg \lambda_0$, the wave function of the incident particle 'sticks' in the gas layer, i.e., it is certainly 'measured' by this layer. After the collapse of the incident wave function, a sufficiently compact ($\sim \sqrt{\lambda\lambda_\mathrm{B}}$ in size) wave packet of the incident particle forms. The probability of formation of such a packet is distributed as $|\psi(y)|^2$ in the transverse direction and as $\exp(-x/\lambda_0)$ with respect to depth. After the first 'measuring' collapse, the wave packet will diffuse in the gas experiencing, in doing so, a series of successive Brownian-type collisions with gas atoms. If the wave packet incident on the gas layer is much wider than $\sqrt{\lambda\lambda_\mathrm{B}}$, the post-collapse packet width, then the size of the wave packet is greatly reduced during the collapse. It may then be argued that the 'small spots' of the collapsed packets are approximately orthogonal to one another or, in other words, such collapses resemble what von Neumann's projection operator would do to the original wave packet.

We will next consider a more sophisticated version of the above thought experiment. Suppose that there are two slots cut in the gas layer under consideration and that at a large enough distance L_0 a second gas layer is placed as a screen. Clearly the waves passing through the slots produce an interference pattern on the screen layer. As before, collapses will produce there small spots of size $\sqrt{\lambda\lambda_\mathrm{B}}$, their probability being proportional to $|\psi(y)|^2$. Performed repeatedly, these 'measurements' will 'map' the intensity $|\psi(y)|^2$ in the second layer. The number of such spots will be the number of particles incident on the first layer times the probability of passing the slots. In other words, only those particles not 'measured' by the first layer, will go through. For the particles that have collapsed there, the waves passing through the slots correspond to the so-called 'empty waves', which naturally will no longer be able to collapse in the second layer.

At first sight, the collapse processes in the first and second gas layers appear to be totally independent: the second layer will register only those particles having passed through the slots in the first. The actual situation is somewhat more complex than that. Let τ_0 denote the quantity λ_0/v_0, where v_0 is the velocity of the incident particle, and λ_0 is its mean free path in the gas. Clearly, the incident particle will take no less than τ_0 or τ to collapse. If $\tau_0 \ll \tau$, it will travel a distance $\lambda_0\tau/\tau_0$ during the collapse time. If this exceeds L_0, the incident particle has enough time to produce scattered waves both in the first and the second gas layer, meaning that it suffers a collapse only in one of the layers, with a simultaneous destruction of all waves in the other. But this means, further, than the wave function of the incident particle has a correlating effect on the collapses occurring in two seemingly independent gas layers. Wave function collapses thus appear as a collective effect involving the total wave function of the entire system consisting of the particle and two gas layers.

This last point is even more effectively made if instead of a single particle a correlated pair of the Einstein – Podolsky – Rosen paradox [10] is considered. Suppose that two particles emerge from a common centre with total momentum zero, and that two gas layers are placed on the opposite sides of this point at the same distance L_0 away. If one of the particles suffers a collapse in one layer, clearly the wave function of the second collapses in the other, the latter event occurring at the exactly symmetric (to within $\sqrt{\lambda\lambda_\mathrm{B}}$) point. If both layers are distant equally from the emission source, one cannot indicate the exact layer in which a collapse occurs first. This means that the very presence of a correlated pair automatically leads to correlated collapses in two well-separated gas layers. Again, the collapse involves the entire system, the EPR pair and two gas layers.

It appears that the irreversibility property of either layer relates to its irreversibly interacting with the environment. But this interaction is of such a nature as to preserve the quantum correlations prior to the collapse. One can thus say that the collapse events are under sufficiently tight constraints, with the consequence that the collapse probability strictly follows the law $|\psi|^2$ and that the wave function ψ prior to the collapse obeys the Schrödinger equation.

The correlation of wave function collapses in a 'measurement' event is established instantly, i.e., superluminally. The idea that collapses in different regions in space must necessarily be linked in this way was discussed by Stapp [11] and is a subject of considerable current interest.

5. Quantum communication

The presence of nonlocal correlation links in quantum mechanics was first demonstrated in the work of Einstein, Podolsky, and Rosen [10]. Although originally viewed as something of a paradox, its existence was established beyond any doubt in later work, notably owing to Bell's theorem [12] according to which the presence of hidden parameters prior to a quantum measurement would result in certain inequalities acting to restrict experimental observation results [13 – 15]. While lending support to orthodox quantum mechanics, this is suggestive of nonlocal correlation links coming to existence at the instant of a quantum measurement. An experiment by Aspect, Dalibard, and Roger [15] clearly demonstrates that these links are established superluminally. An appropriate question to ask is, can quantum correlations be used for superluminal information exchange?

To employ correlated EPR quantum pairs would appear the simplest answer. One might think, for example, of Bohm's scheme in which correlated pairs of particles of spin $\pm 1/2$ and total momentum zero emerge from the common centre at $x = 0$. Registering the spin of one of these at a distance $x = L$ from the source would be accompanied by the registration of the second, opposite-spin particle at a distance $x = -L$. Correlation between the spins occurs at the measurement instantly, implying that some kind of information link is established.

It should be emphasised that the quantum measurement process differs considerably from its classical counterpart in that the spin of the particle has no definite value prior to the measurement and assumes a specific value only at the time of the measurement, due to the wave function collapse to one of the states available. This collapse is 'transferred' instantly to the second particle, so that the measured total moment of the pair turns out to be zero.

Thus, the instant nonlocal link does indeed exist between particles. It turns out, however, that it cannot be used for transmitting information. In order to do this one must average the signal over many particles. But both the right and the left particles have spin measurement probabilities of exactly $1/2$ for spin values $\pm 1/2$, and the measurement on one of them has no effect whatsoever on the statistics of the outcomes ($+1/2$ or $-1/2$) for the second. Thus, the simplest EPR pairs are of no use for information transfer purposes.

The possibility or otherwise of quantum-correlation superluminal information transfer can be analysed in more general terms and by considering more complicated quantum systems [16 – 19]. The answer is again no. A brief sketch of the arguments used follows.

The simplest proof is due to Bussey [18]. Suppose two quantum systems A and B interact for some time before, and cease to interact after, $t = 0$. If their common wave function ψ_{AB} does not factor into the product $\psi_{\mathrm{A}}\psi_{\mathrm{B}}$ of wave functions of the two systems, these form a so-called entangled state. A measurement on one of them then causes a collapse of the wave function of the second, which gives us grounds for speculating on the possibility of instant information transfer. That such transfer turns out to be prohibited, however, is due to the reversible evolution of quantum systems prior to the measurement. Let ρ_{AB} be the density matrix of the combined system. It is the density matrix which describes the signal which accumulates over a large number of measurements and might be employed for transmitting information. According to the Schrödinger equation, the density matrix evolves in an irreversible and causal way like

$$\rho_{\mathrm{AB}}(t) = \exp\left(-\frac{\mathrm{i}Ht}{\hbar}\right)\rho_{\mathrm{AB}}(0)\exp\left(\frac{\mathrm{i}Ht}{\hbar}\right). \tag{4}$$

Here $H = H_{\mathrm{A}} + H_{\mathrm{B}}$ is the Hamiltonian of the combined system in the absence of interaction. For any operator U_{A} in system A, the result of the measurement is given by

$$\langle U_{\mathrm{A}}\rangle = \mathrm{Tr}\,U_{\mathrm{A}}\rho_{\mathrm{AB}} = \mathrm{Tr}\,U_{\mathrm{A}}\bar{\rho}_{\mathrm{A}}\,, \tag{5}$$

where Tr denotes the trace of a matrix, and $\bar{\rho}_{\mathrm{A}}$ denotes the reduced density matrix, i.e., one resulting from taking the trace ρ_{AB} with respect to the variables of system B, $\bar{\rho}_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{B}}\,\rho_{\mathrm{AB}}$. In other words, in accordance with the general principles of quantum mechanics the description of system A can be given by taking the (partial) trace of the matrix ρ_{AB} with respect to the variables of system B. Since $H = H_{\mathrm{A}} + H_{\mathrm{B}}$,

$$\bar{\rho}_{\mathrm{A}} = \exp\left(-\frac{\mathrm{i}H_{\mathrm{A}}t}{\hbar}\right)\mathrm{Tr}_{\mathrm{B}}\left[\exp\left(-\frac{\mathrm{i}H_{\mathrm{B}}t}{\hbar}\right)\rho_{\mathrm{AB}}(0) \times \exp\left(\frac{\mathrm{i}H_{\mathrm{B}}t}{\hbar}\right)\right]\exp\left(\frac{\mathrm{i}H_{\mathrm{A}}t}{\hbar}\right). \tag{6}$$

But the trace of the bracketed expression with respect to the variables of system B is simply $\mathrm{Tr}_{\mathrm{B}}\,\rho_{\mathrm{AB}}(0)$ and does not depend on time. Therefore, no manipulations on physical quantities in system B can have any effect on system A, and accordingly it is absolutely impossible to transmit information from A to B after these systems have ceased to interact.

The elegant proof given by Bussey appears to rule out quite rigorously any possibility of signaling by means of quantum correlations. Ultimately, this conclusion relies on the fundamental quantum mechanical principles, according to which the evolution of a system prior to the measurement obeys the Schrödinger equation, and in the measurement process itself the probability of a given outcome is proportional to $|\psi|^2$ for the corresponding wave function components.

There is, however, one inaccuracy in Bussey's proof, and this was removed in a paper by Shimony [19], whose reasoning is very much in the spirit of Ghirardi, Rimini, and Weber [17]. The point is that Bussey ignores the presence of the measuring apparatus, which is generally incorrect. According to [19], however, if one replaces ρ_{AB} in (6) by ρ_{ABM} and H_{B} by $H_{\mathrm{B}} + H_{\mathrm{M}}$, where index M denotes the measuring apparatus, and then averages over the variables of systems B and M, the same final conclusion will be reached, that transmitting information by means of quantum correlations is prohibited by the fundamental principles of quantum mechanics.

Although apparently quite convincing and categorical, this conclusion is in fact valid only for a specific model. It is assumed, namely, that first two correlated particles or quantum systems are prepared, that they then fly apart —

obeying the Schrödinger equation in their evolution — and it is only then that they are subjected to a measuring procedure. The time t in brackets in expression (6) drops out for the only reason that the quantum system B is assumed to evolve causally and totally reversibly. Thus, the above proof does not give an answer to the question of whether or not information can be transmitted by using quantum correlations in irreversible quantum systems. In other words, the question remains, Is it possible, presumably by using a more sophisticated scheme or by introducing some elements of irreversibility into the instrument, to manufacture a quantum system in which transmitting information by means of quantum correlations would become possible? The answer is far from trivial.

One scheme of this type [20] employs EPR pairs of S-state photons, their detectors being provided with appropriate polarisers and with half wave plates at points A and B. Suppose the polarisation of the photons is such that upon the measurement of a definite polarisation at point A, the photon at point B is seen to be in the orthogonal polarisation state. If the photon measured at A is a plane polarised one, so too will be the photon detected at B. If, on the other hand, the detection scheme at point A is devised to register photons with a circular polarisation, then at point B photons will collapse into a circular polarisation state. In the former case, the light beam which is on the average unpolarised at B will consist of plane polarised photons, in the second, of circularly polarised photons. Were it possible to detect the difference between plane unpolarised and circularly unpolarised light at point B, the observer B would be able to know the exact state of the measuring system at point A. But then, since this information is created by collapses, superluminal telecommunication would be realised.

In order to find out the exact unpolarised state of the beam of photons in the detection region B, Herbert [20] proposes to place a laser gain tube before the detection system B. His idea is that such a tube will 'clone' photons, and each incident photon must accordingly give rise to a light burst consisting of many similar photons. Since it is a simple matter to establish the polarisation of such a burst, one would expect that this kind of irreversible device will help to understand what happens to photons in the detection region A.

This 'quantum telegraph' proves to be unworkable, though [21, 22]. The most convincing proof is due to Wootters and Zurek, who demonstrate that a single photon cannot be cloned because of the linearity of quantum mechanics. Their argument is as follows. A perfect amplifying device performs the following operation upon the photon:

$$|A_0\rangle|s\rangle \to |A_s|ss\rangle. \qquad (7)$$

Here $|s\rangle$ corresponds to the incident photon in the state s, $|A_0\rangle$ and $|A_s\rangle$ are respectively the initial and the final states of the apparatus, and the symbol $|ss\rangle$ refers to the state in which two photons have the same polarisation s. Suppose the amplification (7) can in fact be performed for the vertical polarisation $|\updownarrow\rangle$ and for the horizontal polarisation $|\leftrightarrow\rangle$, so that

$$A_0|\updownarrow\rangle \to |A_{\rm vert}\rangle|\updownarrow\updownarrow\rangle\,, \qquad (8)$$

$$A_0|\leftrightarrow\rangle \to |A_{\rm hor}\rangle|\leftrightarrows\rangle\,. \qquad (9)$$

In the spirit of quantum mechanics, such amplification can be represented as a linear transformation. Therefore, if the incoming polarisation is given by the superposition $\alpha|\updownarrow\rangle + \beta|\leftrightarrow\rangle$ (which for $\alpha = \beta = 1/\sqrt{2}$ corresponds to the linear polarisation at an angle of 45°) then, from (8) and (9), the result of the interaction with the apparatus must be

$$A_0\big(\alpha|\updownarrow\rangle + \beta|\leftrightarrow\rangle\big) \to \alpha|A_{\rm vert}\rangle|\updownarrow\updownarrow\rangle + \beta|A_{\rm hor}\rangle|\leftrightarrows\rangle\,. \qquad (10)$$

If the apparatus states $A_{\rm vert}$ and $A_{\rm hor}$ are not identical, the two photons emerging from the apparatus will be in a mixed state. Otherwise they will be in the pure state

$$\alpha|\updownarrow\updownarrow\rangle + \beta|\leftrightarrows\rangle\,. \qquad (11)$$

In neither case will two photons be in a polarisation state $\alpha|\updownarrow\rangle + \beta|\leftrightarrow\rangle$. For a perfect amplifier, such a state would have to be written as

$$2^{-1/2}(\alpha a^+_{\rm vert} + \beta a^+_{\rm hor})^2|0\rangle = \alpha^2|\updownarrow\updownarrow\rangle + 2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle + \beta^2|\leftrightarrows\rangle\,, \qquad (12)$$

where the $a^+_{\rm vert}$ and $a^+_{\rm hor}$ are photon production operators and $|0\rangle$ is the vacuum state. It is readily seen that the state (12) is not identical to the superposition (11), meaning that no apparatus exists which is capable to amplify an arbitrary polarisation. Needless to say, this does not exclude the possibility of a device capable to amplify either vertical or horizontal polarisation separately, but such an amplifier will be of no use for superluminal communication purposes.

Another aspect of the above device, brought out by Glauber [22], is photon noise. Using a simple upside-down pendulum amplifier model, Glauber shows that noise amplification is so large that the Herbert scheme is not viable for this reason either.

One further quantum communication scheme [22, 24] employs the irreversible Sokolov effect. The effect itself was discovered by Sokolov and associates [25] in atomic interference experiments, in which a beam of excited hydrogen atoms in the metastable state $2s$ is passed through a small slot with a longitudinal electric field. Due to the linear Stark effect, an atom in the field is polarised to form a time-decaying $2p$-component. With two slots, the interference of $2p$-amplitude can be observed. An unexpected result (dubbed the Sokolov effect) was that even in the absence of an electric field $2s$-atoms are polarised in their motion near a metallic sample.

A possible explanation of the effect [26] invokes the collapse of the wave functions of conduction electrons in the metal. As a $2s$-atom flies near the metal surface, conduction electrons interact with the excited atom. The effect of this interaction is on the average very small because there is virtually no macroscopic field outside the metal (the thermal fluctuations of the electric field are small, and the image field is even smaller). However, in their motion back into the metal after the interaction, conduction electrons may suffer collapses when scattered by other electrons, phonons, or impurity atoms. Although electron wave functions typically follow the $p \sim |\psi|^2$ law in such collapses, however, owing to the energy conservation law a very small deviation from this law occurs. The reason is that the post-collapse wave packet is more localised, and on the average the energy conservation law favours collapses into the slower part of the wave packet. A consequence of this asymmetry is that each electron, due to its entanglement with the moving atom, adds something to the shift of the atom's $2p$-amplitude. The effect of one single electron is very small as being proportional to the interaction matrix element times the small collapse asymmetry parameter $\alpha \approx \sqrt{\lambda_B/\lambda}$, where λ_B and λ are the conduction electron de Broglie wavelength and mean free path, respectively. However, since the number of interaction electrons is very large, the total effect is not small.

Thus, the Sokolov effect may be said [26] to be produced by quantum correlations between the excited atom and the collapsing wave functions of the conduction electrons. If the electron scattering rate is controlled one might, in principle, expect for a corresponding response on the part of the $2p$-state amplitude, i.e., the excited atom radiation intensity. It is here where quantum telecommunication on the basis of the Sokolov effect looks to be possible.

Recent experimental verifications [27] of the above mechanism are in good agreement with theory, in particular as far as the roles of the sample geometry and the beam-metal sample distance are concerned. It can be stated that the theory is confirmed by experiment. If the quantum communication scheme of Refs [23, 24] is indeed realisable, one can develop a better understanding of when it is possible and when not to transmit information by means of quantum correlations with the use of the wave function collapse. It should be noted that the signal transfer we are discussing here has nothing to do with either electromagnetic waves or modulated particle beams. Rather, by quantum communication we mean the possibility of an instant transfer of information via the collapse of correlated nonlocal wave functions.

All schemes involving very widely separated correlated quantum systems prove invalid due to the basic quantum mechanical principle that the probability of a given outcome of the contact between the wave function and outer devices (which contact is 'measurement') very accurately follows the law $|\psi|^2$. And since, as quantum systems fly apart, they evolve reversibly in accordance with the Schrödinger equation, it follows from (6) that measurements in one of them have no effect on the statistics of measurement results in the second.

Using irreversible systems with a continuum spectrum, such as an ordinary or a free electron gas, may lead to a small systematic departure from the $p \sim |\psi|^2$ law, which can conceivably be employed for transferring information. In this case, however, the time it takes to prepare the system for the information transfer act is not more than the relaxation time τ, due to, say, electron collisions in the metal. We must therefore limit our expectations to short information transfer distances, i.e., to those within one combined complex system which undergoes irreversible nonlocal relaxation. Quantum communication is most likely to be limited to complex irreversible systems. It might play some role in biological systems, which exemplify most vividly the complex evolution of irreversible self-organising systems.

6. Conclusions

First raised in the early years of quantum theory, the question of the wave function collapse has since been discussed continuously from a variety of viewpoints.

Mathematically, von Neumann [28] distinguishes here two dissimilar processes, the continuous evolution of the quantum system according to the Schrödinger equation between measurements, and random projections onto one of the possible states during the measurement. The latter process cannot be described by the Schrödinger equation and is random but, von Neumann argues, this random process cannot be described in terms of hidden variables either. Since the division of evolution processes into only two groups without a physically clear description of collapse is unsatisfactory for those inclined to a visual way of thinking, the problem of collapse has always been a subject of lively discussion and has been approached from many and often widely different directions.

There is a view [29, 30] that this problem does not exist at all since "the state vector cannot be ascribed to an individual system but only to an ensemble of systems." Accordingly the wave function, rather than being a property of the system, becomes only a 'procedure' for calculating probabilities — a difficult approach to accept. "Those readers", Peres adds [30], "who adopt a 'realistic' attitude will disagree with my approach. However, it is their problem then to explain the miraculous events" occurring during a measurement. In direct opposition to this view, a dynamical description of the collapse [31] and even spontaneous collapses of a free particle [32] are suggested. In order to describe such collapses, the introduction into the Schrödinger equation of phenomenological term with stochasticity is suggested. This, however, changes the dynamics of a free particle and indeed implies a cardinal revision of the very foundations of quantum mechanics — an endeavour which is difficult to justify at present.

The more natural approach is perhaps to relate the collapse phenomenon to the influence of a complex environment [33 – 35]. In fact, taking an oscillator bath to model the interaction with the environment, Unruh and Zurek [36] discover the possibility of a collapse of the wave function of the upside-down pendulum. The same type of wave function collapse might be expected in systems whose classical analogues have divergent trajectories in phase space. The simplest example is an ordinary gas, and it is therefore natural to assume that a gas acts as stochasticity amplifier, whether classical or quantum [8]. If this is indeed the case, we naturally arrive at the molecular chaos picture [2], in which the amplification of external stochasticity manifests itself in the collapse of gas atom wave functions as they are scattered and subsequently undergo a complicated decoherence process.

Assuming the effect of this decoherence to be the elimination of wave function in regions where no atoms are present, we are naturally led to a description of a gas in terms of wave packets. Each one of these first interacts with other atoms, creating scattered waves, and then collapses into one of possible scattered packets. In this approach the wave functions of gas atoms appear as information-carrying entities, which makes it quite natural to speak of the elimination of waves in the regions where no particles are present.

The collapses of the wave functions of gas atoms lead to the wave function collapses for other particles interacting with the gas. Thus, the gas can be thought of as a measuring instrument: it performs easily this most delicate — wave function collapse — stage in the measurement process.

Wave function collapses do not proceed in an arbitrary way but rather obey a universal outside constraint that their probabilities be proportional to $|\psi|^2$ for the corresponding state. This universal law prevents superluminal communication over arbitrarily large distances. However in a gas and particularly in a free electron gas, small departures of the universal $|\psi|^2$ law are possible which, although generally of minor importance, provide a clue to explaining the Sokolov effect. Based on this latter, it is then conceivable that information can be transmitted across relatively short distances by means of quantum correlations, the irreversible relaxation of conduction electrons being instrumental in the process.

Thus, if one adopts the 'realistic' viewpoint, the wave function collapse should be treated as a quite real process. Wave function collapses may occur within a physical system as a kind of an 'inner measurement' or a 'self-measurement'. It is these processes which are involved in the evolution of the wave function of atoms or Brownian particles in a gas. Wave function collapses manifest themselves even more vividly in conventional 'external' measurements, in which the wave function of the microobject under measurement and that of the measuring instrument collapse simultaneously. Such a collapse clearly demonstrates the quantum correlation of these two systems.

During the collapse of a correlated system an exchange of information occurs. The question is, whether this exchange is a purely random event or it has the potential for the controllable transfer of information which can be accumulated by many microobjects. Since collapses of correlated systems may occur within sufficiently short time intervals, the possibility of quantum correlation information transfer is readily associated with the superluminal signaling idea. Clearly, faster-than-light signaling across large distances is at odds with the principle of relativity, so that instant signalling across large distances is forbidden. This follows [16 – 19] from the general quantum mechanical principle that the probability of an event is proportional to $|\psi|^2$. Conversely, the principle of relativity implies the random nature of quantum events as well as the $p \sim |\psi|^2$ law (see Ref. [37] for more on that). However, in complex irreversible systems there appears to be no ban on internal exchange of information via quantum correlations. Hopefully future research will clarify this point.

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