

Electroweak radiative corrections in Z-boson decays

M I Vysotsky, V A Novikov, L B Okun, A N Rozanov

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Abstract. We present a detailed review of the electroweak radiative corrections to the Z-boson decay, in the framework of the Minimal Standard Model (MSM). After a short historical introduction we describe the optimal parametrization of the MSM, especially of the Born approximation, and derive expressions for the one-loop electroweak corrections. Finally a global fit of all relevant experimental data is performed, result-

ing in fitted values of the t-quark mass, m_t , and strong coupling constant α_s . Allowed range for the value of the Higgs boson mass, m_H , is discussed. Various details of calculations are described in 16 appendices.

1. Introduction

1.1 New theories, new symmetries, new particles, new phenomena

The creation of the unified electroweak theory at the end of the 1960s [1, 2] and of the quantum chromodynamics (QCD) at the beginning of the 1970s [3] has dramatically changed the entire picture of elementary particle physics. Its foundation changed to gauge symmetries: the electroweak symmetry $SU(2)_L \times U(1)$ and the colour symmetry $SU(3)_c$. It became clear that the gauge symmetries determine the dynamics of the fundamental physical processes in which the key players are the gauge vector bosons, i.e. the well-familiar photon and a host of new particles: W^+ , W^- , Z-bosons and eight gluons,

M I Vysotsky, V A Novikov, L B Okun Institute for Theoretical and Experimental Physics, ul. B. Cheremushkinskaya 25, 117259 Moscow, Russia
Tel. (7-095) 123 31 92. Fax (7-095) 123 65 84
E-mail: vysotsky@vxitep.itep.ru;
novikov@vxitep.itep.ru; okun@vxitep.itep.ru
A N Rozanov ITEP and CPPM, IN2P3-CNRS, Marseilles, France
E-mail: rozanov@cppm.in2p3.fr

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which differ from one another in colour charge. Even though the Higgs condensate, filling the entire space, remains enigmatic in the electroweak theory, while the problem of confinement is still unsolved in chromodynamics, the two theories are nevertheless so inseparable from modern physics that they were given the name of the Minimal Standard Model (MSM). We assume in this review that the reader is familiar with the basics of the MSM (see, for example, the monographs [4]).

The concept of quarks has undergone dramatic expansion in the process of creation of the MSM. In Ref. [1] the electroweak theory was suggested for leptons (electron and electron neutrino). The subsequent inclusion of quarks into the theory led to the hypothesis that in addition to the three quarks known at the time (u , d and s) there exists the fourth quark, c . According to Ref. [2], if d - and s -quarks are analogues, respectively, of e and μ , then the mutually orthogonal combinations $u \cos \theta_C + c \sin \theta_C$ and $-u \sin \theta_C + c \cos \theta_C$, where θ_C is the Cabibbo angle, must constitute the analogues of ν_e and ν_μ [5].

One of the important consequences of the electroweak theory was the prediction of the weak neutral currents. According to the theory, the neutral weak currents must be diagonal; in other words, neutral currents changing quark flavour (FCNC) are forbidden. This explained the absence of such decays as $K^0 \rightarrow \bar{e}e$, $K^0 \rightarrow \bar{\mu}\mu$, and $K^+ \rightarrow \pi^+ \bar{e}e$. Since there is no neutral current $\bar{s}d$ in the Lagrangian, these decays cannot occur in the tree approximation: they require loops with virtual W-bosons. This is also true for the transitions $K^0 \rightarrow \bar{K}^0$ that are responsible for the mass difference between the K_L^0 - and K_S^0 -mesons.

Diagonal neutral currents were discovered in reactions with neutrino beams [6], in rotation of the polarisation plane of a laser beam in bismuth vapor [7], and in the scattering of polarised electrons by deuterons [8].

The charmed quark c was discovered in 1974 [9]. Even before that, Kobayashi and Maskawa [10] conjectured that in addition to two generations of leptons and quarks, (ν_e, e, u, d) , (ν_μ, μ, c, s) , there must exist the third generation (ν_τ, τ, t, b) . The 2×2 Cabibbo matrix for two generations,

$$\begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix}$$

is replaced in the case of three generations by a 3×3 unitary matrix, that in its most general form contains three angles and one phase; the phase is nonzero if the CP-invariance is violated. This is how the mechanism of CP-violation at the level of quark currents was proposed.

The τ -lepton [11] and the b -quark [12] were discovered experimentally in mid-1970s. The heaviest fermion, the t -quark, was discovered only two decades later [13, 14]. As for the mechanism of CP-invariance violation, it remains unknown even now.

The renormalisability of the electroweak theory and QCD [15] and the property of asymptotic freedom in QCD [16] opened a wide field for reliable computations based on perturbation theory. On the basis of such computations in the tree-diagram approximation, it was possible to predict such qualitatively novel phenomena as quark and gluon jets; using the data on neutral currents, it proved possible to perform preliminary computations of the masses and partial widths of the W- and Z-bosons even before their actual discoveries.

1.2 W- and Z-boson ‘factories’

To test the predictions of the electroweak theory, proton–antiproton colliders were built at the beginning of the 1980s in Europe (at CERN) and then in the USA (at FNAL). The discovery of the W- and Z-bosons [17] provided spectacular confirmation of the tree-diagram calculations [4] and made feasible and urgent precision tests of the electroweak theory with loops included. (When speaking about the tree (Born) approximation and loops, we always mean the corresponding Feynman diagrams.)

A unique object for such tests was the Z-boson. To carry out a precision study of its properties, electron–positron colliders were built at the end of the 1980s: LEP1 at CERN and SLC at SLAC. Electron and positron in these colliders collide at the centre-of-mass energy equal to the Z-boson mass. This results in the resonance creation and decay of a Z-boson (see Fig. 1).

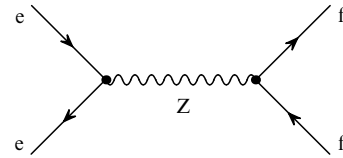


Figure 1. The Z-boson as a resonance in e^+e^- annihilations. A fermion–antifermion pair in the Z-boson decay is denoted by $\bar{f}f$, where f can be either a lepton ($e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$) or a quark (u, d, s, c, b). In this last case a quark–antiquark pair typically transforms, owing to interactions with gluons, to a multi-hadron state. The outgoing arrow in this and subsequent diagrams corresponds to the emission of the fermion (f) and to the absorption of the antifermion (e^+); an incoming arrow denotes the emission of the antifermion (\bar{f}) and the absorption of the fermion (e^-).

The LEP1 completed its operations in October 1995; about 2×10^7 Z-bosons were detected in the four detectors of this collider (ALEPH, DELPHI, L3, OPAL). The total number of Z-bosons recorded at the SLC by the sole SLD detector was approximately 10^5 ; however, since the colliding electrons are longitudinally polarised, it was possible to study the dependence of the annihilation cross section e^+e^- into the Z-boson on the sign of beam polarisation. As a result, the SLC proved its competitiveness even with substantially lower statistics. The statistical and systematic accuracy achieved in the study of the properties of the Z-boson are of the order of 10^{-5} for the Z-boson mass and of the order of several thousandths for the observables that characterise its decays.

1.3 What is the point of studying loop corrections?

A natural question is: why do we need to compare the experimental data and the loop corrections of the electroweak theory? We need it mostly to gather data on the not yet discovered particles. For instance, even before the t -quark was discovered on the Tevatron by CDF [13] and D0 [14] collaborations, its mass was predicted by analyzing the radiative loop corrections and the LEP1 and SLC data [18]. The main loop involving the virtual t - and \bar{t} -quarks is shown in Fig. 2a.

The scalar Higgs boson (or simply the higgs) had not been found yet. In the minimal version of the theory, the so-called MSM, there is a single higgs: a neutral particle whose mass is not fixed by the model. In the Minimal Supersymmetric Standard Model (MSSM) we have three neutral and two

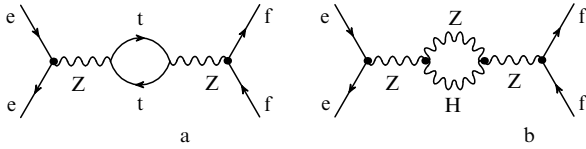


Figure 2. Contribution of the $t\bar{t}$ to the Z-boson propagator (a). Contribution of the loop with virtual Z-boson and a higgs to the Z-boson propagator (b).

charged higgses. The lightest of the neutral higgses must not be heavier than 135 GeV [19, 20]. The simplest diagram involving a virtual higgs is shown in Fig. 2b.

When planning experiments on LEP1 and SLC, people had great expectations that precision measurements would detect pronounced deviations from the predictions of the Standard Model and would thereby unambiguously point to the reality of some sort of ‘new physics’. In fact, even though some discrepancies with the MSM were found, they go beyond the three standard deviations only in a single case (that of the decay of a Z-boson to a $b\bar{b}$ -pair). If these discrepancies are not caused by some sort of systematic error, they may indicate (see Conclusions) the existence of the relatively light (~ 100 GeV) squarks and gluino: the

supersymmetric partners of quarks and gluons, respectively.

2. Brief history of electroweak radiative corrections

The pioneer calculations of electroweak corrections in MSM were performed in the 1970s, long before the discovery of the W- and Z-bosons. The calculations were devoted to the muon decay, and to the β -decays of the neutron and nuclei, and to deep inelastic processes. In connection with the construction of LEP and SLC, a number of teams of theorists carried out detailed calculations of the required radiative corrections. These calculations were discussed and compared at special workshops and meetings. The result of this work was the publication of two so-called ‘CERN yellow reports’ [21, 22], which, together with the ‘yellow report’ [18], became the ‘must’ books for experimentalists and theoreticians studying the Z-boson.

2.1 Muon and neutron decays

Sirlin [23] calculated the radiative corrections to the muon decay due to one-loop Feynman diagrams (see Fig. 3).

We must emphasise that the purely electromagnetic correction to the muon decay due to the exchange of virtual

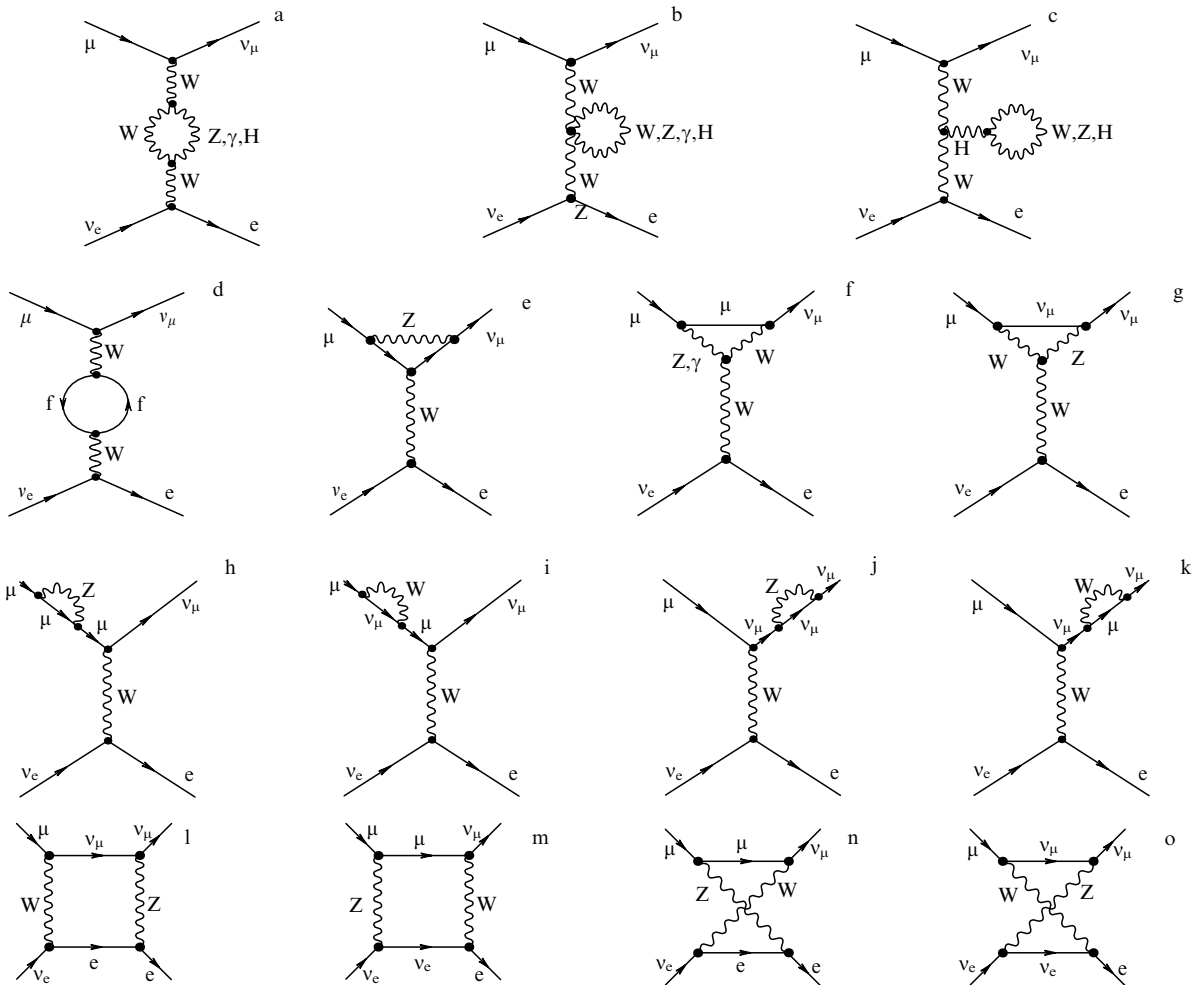


Figure 3. One-loop diagrams in muon decay: loops in the W-boson propagator (a) – (d), in the W-vertex (e), (f), (g) and in external fermion lines (h), (i), (j), (k) (similar diagrams for e and $\bar{\nu}_e$ are assumed), as well as box-type diagrams (l), (m), (n), (o).

photons and the emission of real photons was calculated even earlier [24], for the pointlike four-fermion interaction, i.e. without taking the W-boson into account (see Fig. 4). It was found that the correction is finite: it contained no divergences. The four-fermion interaction constant G_μ , extracted from the muon lifetime τ_μ ,

$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right) \left[1 - \frac{\alpha}{2\pi} \left(\pi^2 - \frac{25}{4}\right)\right], \quad (1)$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$, already includes this electromagnetic correction proportional to α ; $G_\mu = 1.16639(2) \times 10^{-5} \text{GeV}^{-2}$. The finiteness of the purely electromagnetic correction in the muon decay is caused by the V-A nature of the interacting charged currents $\bar{\nu}_\mu \mu$ and $\bar{\nu}_e e$.

In the neutron decay, the purely electromagnetic correction to the four-fermion interaction (Fig. 4i) diverges logarithmically. In view of the W-boson propagator (Fig. 4h), the logarithmic divergence is cut off at the W-boson mass.

This correction to the vector vertex in the leading logarithmic approximation, calculated in Ref. [25], is given by the factor

$$1 + \frac{3\alpha}{2\pi} \ln \frac{m_W}{m_p}, \quad (2)$$

where m_p is the proton mass. Numerically, its value is of the order of 1.7%. Only after this correction is taken into

account, does $\cos^2 \theta_C$ extracted from the nuclear β -decay become equal to $1 - \sin^2 \theta_C$, where $\sin^2 \theta_C$ is found from the decays of strange particles (θ_C is the Cabibbo angle). Although the correction we discuss now contains m_W in the logarithmic term, it is essentially electromagnetic and not electroweak, since it is insensitive to details of the electroweak theory at short distances, in contrast to, say, electroweak corrections to the muon decay (Fig. 3).

Calculations of electroweak corrections to the muon decay show that the main contribution, exceeding all others, is caused by the vacuum polarisation of the photon (Fig. 5a).

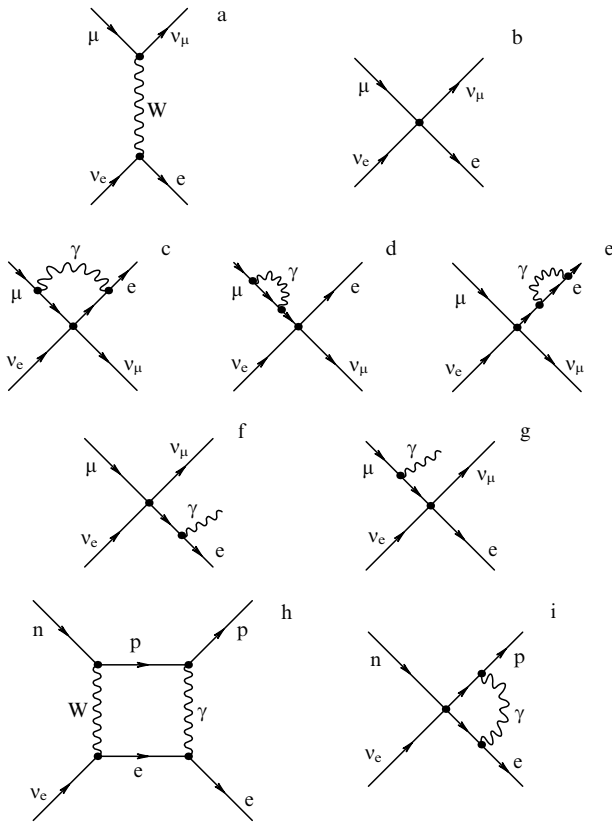


Figure 4. Muon decay in the tree approximation (a) and in the local four-fermion approximation (b). Electromagnetic corrections to muon decay in the local approximation (c) – (g). Electromagnetic corrections to the β -decay of the neutron: in the tree approximation (h) and in four-fermion approximation (i).

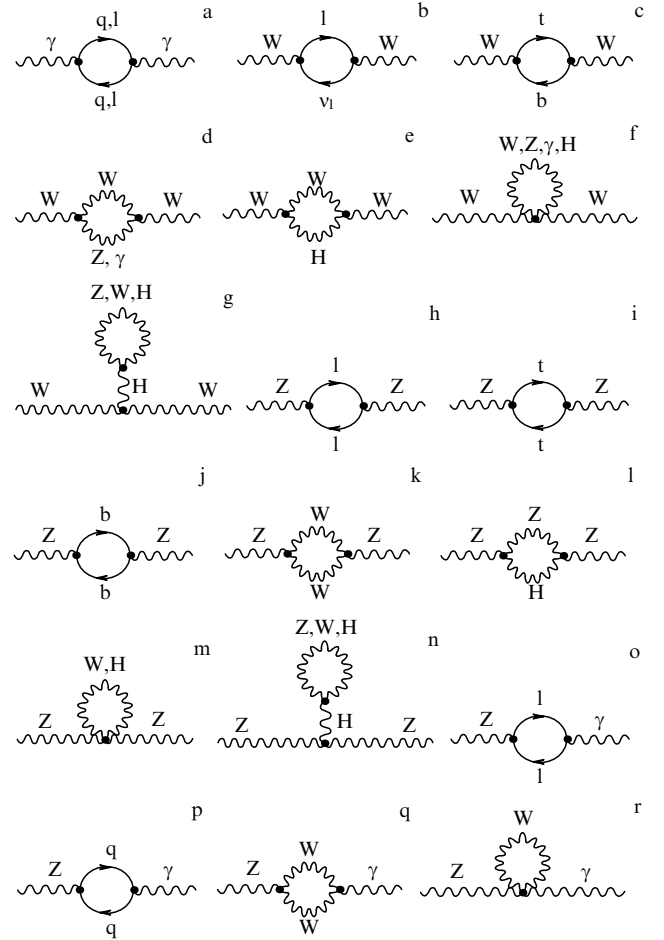


Figure 5. Photon polarisation of the vacuum, resulting in the logarithmic running of the electromagnetic charge e and the fine structure constant $\alpha \equiv e^2/4\pi$, as a function of q^2 , where q is the 4-momentum of the photon (a). Some of the diagrams that contribute to the self-energy of the W-boson (b) – (g), some of the diagrams that contribute to the self-energy of the Z-boson (h) – (n), and some of the diagrams that contribute to the $Z \leftrightarrow \gamma$ transition (o) – (r) are shown.

At the first glance, this correction should not have emerged in the μ -decay in the one-loop approximation: it is not there among the loops in Fig. 3. It does appear, however, when G_μ is expressed in terms of the fine structure constant α and the masses of the W- and Z-bosons.

2.2 Main relations of the electroweak theory

It is well known [4] that in the Born approximation we have (see Fig. 4a)

$$G_\mu = \frac{g^2}{4\sqrt{2}m_W^2}, \quad (3)$$

where m_W is the W-boson mass and g is its coupling constant to the charged current.

On the other hand, we have in the same approximation

$$m_W = \frac{1}{2} g\eta, \quad (4)$$

where η is the vacuum expectation value of the higgs field.

Likewise,

$$m_Z = \frac{1}{2} f\eta, \quad (5)$$

where m_Z is the Z-boson mass and f is its coupling constant to the neutral left-handed current.

Therefore,

$$\frac{m_W}{m_Z} = \frac{g}{f}. \quad (6)$$

If we introduce the famous Weinberg angle [1], it becomes obvious that the two definitions are equally valid in the Born approximation:

$$\cos \theta_W = \frac{m_W}{m_Z} \quad \text{and} \quad \cos \theta_W = \frac{g}{f}. \quad (7)$$

It is well known [1] that the angle θ_W in the electroweak theory defines the relation between the electric charge e and the weak charge g :

$$e = g \sin \theta_W. \quad (8)$$

In the Born approximation, therefore,

$$\begin{aligned} G_\mu &= \frac{g^2}{4\sqrt{2}m_W^2} = \frac{1}{\sqrt{2}\eta^2} = \frac{\pi\alpha}{\sqrt{2}m_W^2 \sin^2 \theta_W} \\ &= \frac{\pi\alpha}{\sqrt{2}m_Z^2 \sin^2 \theta_W \cos^2 \theta_W}. \end{aligned} \quad (9)$$

If we now take into account the electroweak corrections, we find

$$\frac{m_W}{m_Z} \neq \frac{g}{f}.$$

2.3 Traditional parametrization of corrections to the μ -decay and the running α

Sirlin's definitions [23] are widely used in the literature (see the review [35] and references therein); according to them

$$s_W^2 \equiv \sin^2 \theta_W \equiv 1 - c_W^2 \equiv 1 - \cos^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}, \quad (10)$$

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}m_W^2 s_W^2 (1 - \Delta r)}, \quad (11)$$

where

$$\Delta r = \Delta r_{\text{em}} + \Delta r_{\text{ew}}$$

includes both the truly electroweak correction Δr_{ew} for loops in Fig. 3 and the purely electromagnetic correction Δr_{em} due to α running from $q^2 = 0$ to $q^2 \sim m_W^2, m_Z^2$. This correction

arises because

$$\alpha \equiv \alpha(q^2 = 0) = [137.035985(61)]^{-1} \quad (12)$$

is defined for $q^2 = 0$, while typical momenta of virtual particles in the electroweak loop are of the order of the intermediate boson masses. It is convenient to denote

$$\bar{\alpha} \equiv \alpha(m_Z^2) = \frac{\alpha}{1 - \Delta r_{\text{em}}} \equiv \frac{\alpha}{1 - \delta\alpha}. \quad (13)$$

Δr_{em} was calculated in a number of papers [26]; the necessary formulas are given in Appendices II and III. The contribution of the lepton loops to Δr_{em} is described by the expression

$$\delta\alpha^l \equiv \Delta r_{\text{em}}^l = \frac{\alpha}{3\pi} \sum_l \left(\ln \frac{m_Z^2}{m_l^2} - \frac{5}{3} \right) = 0.03141, \quad (14)$$

where $l = e, \mu, \tau$. The contribution of quark loops cannot be calculated theoretically because the quark mass in the logarithm has no rigorous theoretical definition. This reflects our ignorance of strong interactions at large distances (small momenta). The quark (hadron) part Δr_{em}^h is therefore calculated by substituting into the dispersion relation the experimental data on the cross section of the e^+e^- -annihilation into hadrons:

$$\delta\alpha^h \equiv \Delta r_{\text{em}}^h = \frac{m_Z^2}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{ds}{m_Z^2 - s} \sigma_{e^+e^-}^h, \quad (15)$$

where $\sigma_{e^+e^-}^h$ is the cross section of the e^+e^- -annihilation into hadrons via one virtual photon.

In this review we make use of the recent result reported in Ref. [26]: $\Delta r_{\text{em}}^h = 0.02799(66)$, so that

$$\delta\alpha = \delta\alpha^l + \delta\alpha^h \equiv \Delta r_{\text{em}} = \Delta r_{\text{em}}^l + \Delta r_{\text{em}}^h = 0.05940(66). \quad (16)$$

As follows from Refs (13) and (16),

$$\bar{\alpha} = [128.896(90)]^{-1}. \quad (17)$$

A summary of results of various calculations of $\bar{\alpha}$ is given in Appendix III. Following tradition, the contributions of the t-quark loop and the W-boson loop are not included into $\alpha(m_Z)$. In the leading approximation in $1/m_t^2$, the contribution of the t-quark loop is

$$\Delta r_{\text{em}}^t = -\frac{4}{45} \frac{\alpha}{\pi} \left(\frac{m_Z}{m_t} \right)^2 = -0.00005 \quad \text{for } m_t = 180 \text{ GeV}, \quad (18)$$

and the exact formula [see Eqn (67)] corresponds to

$$\Delta r_{\text{em}}^t = -0.00006 \quad \text{for } m_t = 180 \text{ GeV}.$$

The contribution of the W-boson loop is gauge-dependent. In the 't Hooft–Feynman gauge it is $\Delta r_{\text{em}}^W = 0.00050$ [see Ref. (66)].

2.4 Deep inelastic neutrino scattering by nucleons

A predominant part of theoretical work on electroweak corrections prior to the discovery of the W- and Z-bosons was devoted to calculating the neutrino–electron [27] and especially nucleon–electron [28] interaction cross sections.

The reason for this is that after the discovery of neutral currents the quantity

$$s_W^2 \equiv 1 - \frac{m_W^2}{m_Z^2}$$

was extracted precisely from a comparison of the cross section of neutral currents (NC) and the charged currents (CC). While a W-boson interacts with a charged current of V–A type, for example,

$$\frac{g}{2\sqrt{2}} W_\alpha \bar{u} (\gamma_\alpha + \gamma_\alpha \gamma_5) d, \quad (19)$$

the Z-boson interacts with neutral currents that have a more complex form,

$$\frac{f}{2} Z_\alpha \bar{\psi}_f [T_3^f \gamma_\alpha \gamma_5 + (T_3^f - 2Q^f s_W^2) \gamma_\alpha] \psi_f, \quad (20)$$

where T_3^f is the third projection of the weak isotopic spin of the left-hand component of the fermion f (quark or lepton), Q^f is its charge, and ψ_f is the Dirac spinor describing it. In the tree approximation, the NC/CC cross section ratio for purely axial interactions (and isoscalar target) equals unity, since in this approximation the quantity

$$\rho = \frac{f^2}{g^2} \frac{m_W^2}{m_Z^2} \quad (21)$$

equals unity. With the vector current taken into account, the ratio of NC and CC is a function of s_W^2 . Measurements of this ratio gave $s_W^2 \approx 0.23$, which thus made it possible to predict the masses of the W- and Z-bosons [from the formula for the muon decay (9)]. More accurate measurements of NC/CC made it possible to improve the accuracy of s_W^2 to such an extent that it became necessary to take into account the electroweak radiative corrections both in s_W^2 and in ρ , which now, with the corrections taken into account, is not equal to unity any more.

Veltman was the first to point out [29] that if $m_t/m_Z \gg 1$, the main correction to ρ and s_W^2 is caused by the violation of the electroweak isotopic invariance by the masses of the t- and b-quarks in loops of self-energies of the Z- and W-bosons. To find ρ in the limit $m_t/m_Z \gg 1$, it is sufficient to consider these loops neglecting the momentum of W- and Z-bosons q in comparison with the masses of the W and Z, i.e. for $q^2 = 0$. Elementary calculation of the loops indicated above yields (see Appendix VIII):

$$\rho = 1 + \frac{3\alpha_Z}{16\pi} \left(\frac{m_t}{m_Z} \right)^2 = 1 + \frac{3\alpha_W}{16\pi} \left(\frac{m_t}{m_W} \right)^2 = 1 + \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2}. \quad (22)$$

Here and hereafter we denote

$$\alpha_Z = \frac{f^2}{4\pi}, \quad \alpha_W = \frac{g^2}{4\pi}. \quad (23)$$

Since in real life $m_t/m_Z \simeq 2$, the sum of the remaining, non-leading corrections is found to be comparable to the correction proportional to m_t^2 (see below).

At the present moment, the quantity s_W^2 extracted from the data on deep inelastic vN-scattering is determined as 0.2260(48) (the global fit of the data from the collaborations

CDHS [30] and CHARM [31]), or 0.2218(59) (collaboration CCFR) [32]. The accuracy of these data is poorer than found by the direct measurement of the W-boson mass m_W and of the ratio m_W/m_Z by the collaborations UA2 in CERN [33] and CDF at Tevatron [34]. According to PDG [35], the fitted quantity is $m_W = 80.22(26)$, which corresponds to $s_W^2 = 0.2264(25)$. Note that according to the most recent data [36], the measurement accuracy is even higher for m_W : $m_W = 80.26(16)$ GeV. For this reason we will not discuss further the deep inelastic scattering of the neutrino. Moreover, additional assumptions on the effective mass of the c-quark and on the accuracy of the QCD corrections are necessary for the interpretation of these experiments.

2.5 Other processes involving neutral currents

This is true to even greater degree for the parity violation in eD-scattering [8], which, in addition, provides a considerably less accurate $s_W^2 = 0.216(17)$.

The measurement accuracy of the $\nu_\mu e$ - and $\bar{\nu}_\mu e$ -scattering even in the highest-accuracy experiment CHARM II [37] is not sufficient for revealing the genuine electroweak corrections. However, after an analysis in the Born approximation, this experiment has demonstrated for the first time that the interaction constant of the current $\bar{\nu}_\mu \nu_\mu$ with the Z-boson is in satisfactory agreement with the theory (see Refs [38, 39]).

The experiment on measuring parity violation in cesium $^{133}\text{Cs}_{55}$ [40] is also insufficiently sensitive, at the accuracy achieved. Here the effect is produced by the interaction of the nucleon vector current with the electron axial current. Since the characteristic momentum of electron in an atom is small in comparison with nuclear dimensions, all nucleons of a nucleus ‘function’ coherently and the nucleus is characterised by an aggregate weak charge Q_W , which is experimentally found to be $Q_W^{\text{exp}} = -71.0 \pm 1.8$, while the theoretically anticipated value is -72.9 ± 0.1 . A spectacular property of Q_W is the fact that owing to an accidental cancellation of protons’ and neutrons’ contributions, it is practically independent of m_t . On the contrary, Q_W is very sensitive to the contribution of neutral bosons, Z' and Z'' , heavier than the Z-bosons (if they exist).

The best object for testing the electroweak theory at the loop level is therefore the Z-boson with which this review is predominantly connected.

3. On optimal parametrization of the theory and the choice of the Born approximation

The electroweak theory is in many ways different from electrodynamics, in particular in the diversity of particle interactions that must be taken into account when considering any effect in the loop approximation. The parametrization of QED is straightforward: fundamental quantities are the electron mass and charge, which are known with very high accuracy. It is therefore natural to express all theoretical predictions of QED in terms of α and m_e .

3.1 Traditional choice of the main parameters

The choice of the main parameters in electroweak theory is not equally obvious. Historically, those selected were G_μ as an experimentally measured with best accuracy weak decay constant, $s_W^2 \equiv 1 - m_W^2/m_Z^2$, since W- and Z-bosons were not yet directly observed at that time, while the value of s_W^2 was known from experiments with neutral currents, and finally, α . This parametrization of the 1970s proved to be

surprisingly long-lived; the loop parameters Δr and ρ connected with it are widely used in the literature and are very likely to survive beyond the end of this century.

In fact, this parametrization is far from being optimal because m_W (and thus s_W as well) is measured experimentally at much poorer accuracy than m_Z :

$$m_W = 80.26(16) \text{ GeV}, \quad (24)$$

$$m_Z = 91.1884(22) \text{ GeV}. \quad (25)$$

As a result, s_W^2 is extracted by fitting the loop formulas for various observables. This extraction inevitably requires that we fix the values of the t-quark mass and the higgs mass. Another drawback of this parametrization is that the quantity α , despite its superior accuracy, is not directly related to the electroweak loops, which are characterised by a quantity $\bar{\alpha}$, and this we know with much less impressive accuracy. As a result, the purely electromagnetic correction Δr_{em} is not separated from the genuinely electroweak corrections, thus blurring the interpretation of the experimental data.

3.2 Optimal choice of the main parameters

As follows from the above remarks, the currently optimal parametrization is one based on G_μ , m_Z and $\bar{\alpha}$. With this parametrization it is convenient to introduce the weak angle θ , defined [by analogy to Eqn (9)] by the relation

$$G_\mu = \frac{\pi \bar{\alpha}}{\sqrt{2} m_Z^2 s^2 c^2}, \quad (26)$$

where $s^2 \equiv \sin^2 \theta$, $c^2 \equiv \cos^2 \theta$.

As follows from Eqn (26),

$$\sin^2 2\theta = \frac{4\pi \bar{\alpha}}{\sqrt{2} G_\mu m_Z^2} = 0.71078(50),$$

$$s^2 = 0.23110(23), \quad c = 0.87687(13). \quad (27)$$

The angle θ was introduced in mid-1980s [41]. However, its consistent use began only after the publication of Ref. [42].

Using θ instead of θ_W automatically takes into account the running of α and makes it possible to concentrate on the genuinely electroweak corrections. Using m_Z instead of s_W allows one to explicitly single out the dependence on m_t and m_H for each electroweak observable.

Note that a different definition of the Z-boson mass \bar{m}_Z is known in the literature, related to a different parametrization of the shape of the Z-boson peak [43]. This mass \bar{m}_Z is smaller than m_Z by approximately 30 MeV. In this review, we consistently use only m_Z , following the summary reports of LEP collaborations [44].

Let us show how the parametrization in terms of G_μ , m_Z and $\bar{\alpha}$ is applied to the decay amplitudes of Z-boson and to the ratio m_W/m_Z .

3.3 Z-boson decays. Amplitudes and widths

In correspondence with equation (20), we rewrite the amplitude of the Z-boson decay into a fermion–antifermion pair $f\bar{f}$ in the form

$$M(Z \rightarrow f\bar{f}) = \frac{1}{2} \bar{f} Z_\alpha \bar{\psi}_f (\gamma_\alpha g_{Vf} + \gamma_\alpha \gamma_5 g_{Af}) \psi_f. \quad (28)$$

Here by definition \bar{f} is the value of the coupling constant f in the Born approximation,

$$\bar{f}^2 = 4\sqrt{2} G_\mu m_Z^2 = 0.54866(4). \quad (29)$$

(The use of the same letter f to denote both the fermion and the coupling constant cannot lead to confusion, to such an extent are these objects different.) The high accuracy, with which the numerical value of \bar{f} is known, comes from the fact that \bar{f} is independent of $\bar{\alpha}$. All electroweak radiative corrections are ‘hidden’ in dimensionless constants g_{Vf} and g_{Af} . These coefficients do not include the contribution of the final state interactions due to the exchange of gluons (for quarks) and photons (for quarks and leptons). The final state interactions have nothing in common with the electroweak corrections and must be taken into account as separate factors in the expressions for the decay rates. These factors are sometimes known as ‘radiators’, since they cover not only exchange of photons and gluons but also their emission.

Radiators are trivially unities in the case of decay to any of the neutrino pairs $\nu_e \bar{\nu}_e$, $\nu_\mu \bar{\nu}_\mu$, $\nu_\tau \bar{\nu}_\tau$ and therefore

$$\Gamma_\nu = \Gamma(Z \rightarrow \nu \bar{\nu}) = 4\Gamma_0 (g_{Av}^2 + g_{Vv}^2), \quad (30)$$

where Γ_0 is the so-called standard width:

$$\Gamma_0 = \frac{G_\mu m_Z^3}{24\sqrt{2}\pi} = 82.944(6) \text{ MeV}. \quad (31)$$

If neutrino masses are assumed to be negligible, then

$$g_{Av} = g_{Vv} \equiv g_\nu, \quad (32)$$

so that

$$\Gamma_\nu = 8\Gamma_0 g_\nu^2. \quad (33)$$

For decays to any of the pairs of charged leptons $\bar{l}l$ we have [18]

$$\Gamma_l \equiv \Gamma(Z \rightarrow \bar{l}l) = 4\Gamma_0 \left[g_{Vl}^2 \left(1 + \frac{3\bar{\alpha}}{4\pi} \right) + g_{Al}^2 \left(1 + \frac{3\bar{\alpha}}{4\pi} - 6 \frac{m_l^2}{m_Z^2} \right) \right], \quad (34)$$

where the QED correction is taken into account only to the lowest approximation in $\bar{\alpha}$; we neglect terms on the order of $(\bar{\alpha}/\pi)^2 \sim 10^{-6}$. The term proportional to m_l^2 is negligible for $l = e, \mu$ and must be included only for $l = \tau$ ($m_\tau^2/m_Z^2 = 3.8 \times 10^{-4}$).

For decays to any of the five pairs of quarks $q\bar{q}$ we have

$$\Gamma_q \equiv \Gamma(Z \rightarrow q\bar{q}) = 12\Gamma_0 [g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}]. \quad (35)$$

Here the factor of 3, additional in comparison with leptons, takes into account the three colours of each quark. The ‘radiators’ in the first approximation are identical for the vector and the axial-vector interactions,

$$R_{Vq} = R_{Aq} = 1 + \frac{\hat{\alpha}_s}{\pi}, \quad (36)$$

where $\hat{\alpha}_s$ is the constant of interaction of gluons with quarks at $q^2 = m_Z^2$. There are different conventions for the choice of $\hat{\alpha}_s$. In calculations of decays of the Z-boson, it is quite typical to determine $\hat{\alpha}_s$ using the so-called modified minimal subtraction scheme, $\overline{\text{MS}}$ (see at the end of Appendix I). The numerical value of $\hat{\alpha}_s$, found from Z-boson decays, is of the order of 0.12. For additional details on the value of $\hat{\alpha}_s$ and for more accurate expressions for radiators see Appendix VI. Here we only remark that vast literature is devoted to radiator

calculations; they are calculated using perturbation theory up to terms $(\hat{\alpha}_s/\pi)^3$, see Refs [45–49]. The full hadron width is (to the accuracy of very small corrections) the sum of widths of five quark channels:

$$\Gamma_h = \Gamma_u + \Gamma_d + \Gamma_s + \Gamma_c + \Gamma_b. \quad (37)$$

The full width of the Z-boson is given by the obvious expression:

$$\Gamma_Z = \Gamma_h + \Gamma_e + \Gamma_\mu + \Gamma_\tau + 3\Gamma_\nu. \quad (38)$$

The cross section of annihilation e^+e^- into hadrons at the Z-peak is given by the Breit–Wigner formula:

$$\sigma_h = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_h}{\Gamma_Z^2}. \quad (39)$$

Finally, the following notation for the ratio of partial widths is widely used:

$$R_b = \frac{\Gamma_b}{\Gamma_h}, \quad R_c = \frac{\Gamma_c}{\Gamma_h}, \quad R_l = \frac{\Gamma_l}{\Gamma_l}. \quad (40)$$

(Note that in contrast to R_b and R_c , Γ_h in R_l is in the numerator.)

3.4 Asymmetries

In addition to the total and partial widths of Z-boson decays, experimentalists also measure effects due to parity non-conservation, i.e. the interference of the vector and axial-vector currents. For pairs of light quarks (u, d, s, c) and leptons we determine the quantity

$$A_f = \frac{2g_{Af}g_{Vf}}{g_{Af}^2 + g_{Vf}^2}. \quad (41)$$

For the pair $b\bar{b}$ (see Appendix VII),

$$A_b = \frac{2g_{Ab}g_{Vb}}{v^2 g_{Ab}^2 + (3 - v^2) g_{Vb}^2/2} v, \quad (42)$$

where v is the velocity of the b-quark (in units of c):

$$v = \sqrt{1 - \frac{4\hat{m}_b^2}{m_Z^2}}. \quad (43)$$

Here \hat{m}_b is the value of the ‘running mass’ of the b-quark with momentum m_Z , calculated in the $\overline{\text{MS}}$ terms [50]. The forward–backward charge asymmetry in the decay to $f\bar{f}$ equals (see Appendix VII)

$$A_{\text{FB}}^f \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4} A_e A_f, \quad (44)$$

where A_e refers to the creation of a Z-boson in e^+e^- -annihilations, and A_f refers to its decay into $f\bar{f}$.

The longitudinal polarisation of the τ -lepton in the $Z \rightarrow \tau\tau$ decay is

$$P_\tau = -A_\tau. \quad (45)$$

If, however, we measure polarisation as a function of the angle θ between the momentum of a τ^- and the direction of

the electron beam, this permits the determination of not only A_τ but A_e as well:

$$P_\tau(\cos\theta) = -\frac{A_\tau(1 + \cos^2\theta) + 2A_e \cos\theta}{1 + \cos^2\theta + 2A_\tau A_e \cos\theta}. \quad (46)$$

The polarisation P_τ is found from $P_\tau(\cos\theta)$ by separately integrating the numerator and the denominator in Eqn (46) over the total solid angle.

The relative difference between total cross sections in the Z-peak for the left- and right-polarised electron, that collide with non-polarised positrons (this quantity is measured at the SLC collider), is

$$A_{\text{LR}} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e. \quad (47)$$

The measurement of the asymmetries outlined above allows one to determine experimentally the quantities g_{Vf}/g_{Af} , since these asymmetries are caused by the interference of the vector and axial-vector currents. In their turn, measurements of the widths Γ_f , Γ_h , Γ_Z mostly permit the experimental determination of g_{Af} , since $|g_{Vq}|^2 < |g_{Aq}|^2$ for quarks, and for leptons $|g_{Vl}|^2 \ll |g_{Al}|^2$. As for Γ_q and Γ_h , getting these quantities allows one to find $\hat{\alpha}_s$.

3.5 The Born approximation for hadronless observables

Before discussing the loop electroweak corrections, let us consider expressions for m_W/m_Z , g_{Af} and g_{Vf}/g_{Af} in the so-called $\bar{\alpha}$ -Born approximation. Using the angle θ introduced earlier ($\sin\theta \equiv s$ and $\cos\theta \equiv c$), we automatically take into account the purely electromagnetic correction due to the running of α . It is easily shown that in the $\bar{\alpha}$ -Born approximation

$$\left(\frac{m_W}{m_Z}\right)^B = c, \quad (48)$$

$$g_{Af}^B = T_{3f}, \quad (49)$$

$$\left(\frac{g_{Vf}}{g_{Af}}\right)^B = 1 - 4|Q_f|s^2. \quad (50)$$

It is of interest to compare the Born values with their experimental values. Table 1 presents this comparison for the so-called ‘hadronless’ observables (here and below the experimental results are taken from Ref. [44].)

Table 1.

Observable	Experiment	$\bar{\alpha}$ -Born
m_W/m_Z	0.8802(18)	0.8769(1)
m_W , GeV	80.26(16)	79.96(2)
s_W^2	0.2253(31)	0.2311(2)
g_{Af}	−0.5011(4)	−0.5000(0)
Γ_l , MeV	83.93(14)	83.57(2)
g_{Vl}/g_{Al}	0.0756(14)	0.0756(9)
s_l^2	0.2311(4)	0.2311(2)

For the reader’s convenience, the table lists different representations of the same observables known in the literature. Thus, according to widely used definitions,

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}, \quad (51)$$

$$s_1^2 \equiv s_{\text{eff}}^2 \equiv \sin^2 \theta_{\text{eff}}^{\text{lept}} \equiv \frac{1}{4} \left(1 - \frac{g_{V1}}{g_{A1}} \right). \quad (52)$$

The experimental value of s_1^2 in the table is the average of two numbers, 0.2316(5) (LEP) and 0.2305(5) (SLC). It is assumed in the table that the lepton universality holds, thus the lepton decay data have been averaged over a number of observables.

Table 1 shows that the $\bar{\alpha}$ -Born approximation provides good description of the experimental data. (An agreement is found for the hadron decays of Z-bosons as well). An especially (and unexpectedly!) good agreement, and the ensuing smallness of radiative corrections, is found for g_{V1}/g_{A1} . The anomalous smallness of true electroweak corrections was first pointed out in 1992 [42], when the $\bar{\alpha}$ -Born approximation was applied for the first time. (Before that it was hidden in the shadow of the large contribution of purely electromagnetic running of α , that was not separated from truly electroweak corrections). The LEP1 data of 1992 were not sufficiently accurate to allow detecting them. Even the data presented to the Marseilles conference in summer 1993 were not, as pointed out in a number of reports [51], sufficiently accurate for the detection of radiative corrections. At the Glasgow conference in summer 1994 corrections were detectable at the level of 2.3σ for g_{A1} , 1.5σ for m_W and 1σ for g_{V1}/g_{A1} [52]. Note that the difference between most recent experimental values $s_W^2 = 0.2253(31)$ and $s_1^2 = 0.2311(4)$ is a 2σ manifestation of electroweak radiative corrections, which is independent of either the choice of the Born approximation or of the choice of calculation scheme.

4. One-loop corrections to hadronless observables

The fact that the experimentally observed electroweak corrections are small suggests that one-loop approximation would be sufficient for describing them [42]. This idea is supported by a thorough evaluation [18] of theoretical uncertainties contributed by higher-order perturbation theory in electroweak interaction. There exists essentially a single two-loop diagram that should be taken into account. It was calculated in Ref. [53], and we will discuss its contribution and take it into account below [see Eqn (79)].

4.1 Four types of Feynman diagrams

Four types of Feynman diagrams contribute to electroweak corrections for the observables of interest to us here, m_W/m_Z , g_{A1} , g_{V1}/g_{A1} :

- (1) Self-energy loops for W and Z-bosons with virtual v, l, q, H, W and Z in loops. Examples of some of these diagrams are shown in Figs 5b–5n.
- (2) Loops of charged particles that result in $Z \leftrightarrow \gamma$ transitions (Figs 5o–5r).
- (3) Vertex triangles with virtual leptons and W- or Z-boson (Figs 6a–6c).
- (4) Electroweak corrections to lepton wavefunctions (Figs 6d, e).

It must be emphasised that loops shown in Figs 5h–5n contribute not only to the m_Z mass and, consequently, to the m_W/m_Z ratio but also to the Z-boson decay to $\bar{l}l$, to which $Z \leftrightarrow \gamma$ transitions also contribute (Figs 5o–5r). This occurs owing to the diagrams of the type of Figs 6f–6g, which give corrections to the Z-boson wavefunction.

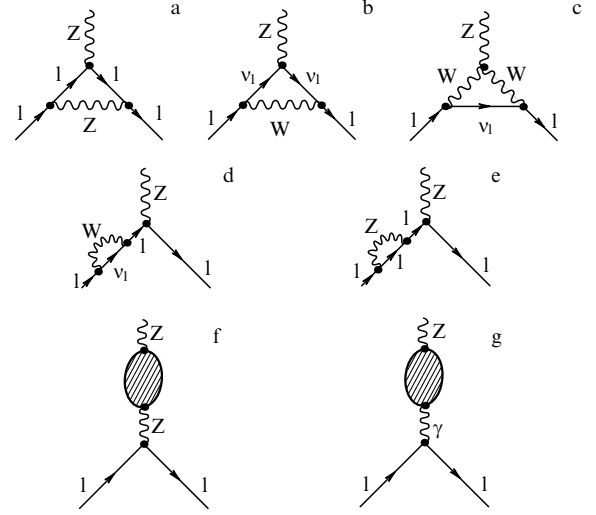


Figure 6. Vertex triangular diagrams in the $Z \rightarrow \bar{l}l$ decay (a) – (c). Loops that renormalised the antilepton wavefunctions in the $Z \rightarrow \bar{l}l$ decay. (Of course lepton has similar loops.) (d), (e). Types of diagrams that renormalised the Z-boson wavefunction in the $Z \rightarrow \bar{l}l$ decay (f), (g). Virtual particles in the loops are those presented in Fig. 5.

Obviously, electroweak corrections to m_W/m_Z , g_{A1} and g_{V1}/g_{A1} are dimensionless and thus can be expressed in terms of $\bar{\alpha}$, c , s and the dimensionless parameters

$$t = \left(\frac{m_t}{m_Z} \right)^2, \quad h = \left(\frac{m_H}{m_Z} \right)^2,$$

where m_t is the mass of the t-quark and m_H is the higgs mass. (We neglect the masses of leptons and all quarks except t.)

4.2 The asymptotic limit at $m_t^2 \gg m_Z^2$

It is convenient to split the calculation of corrections into a number of stages and begin with calculating the asymptotic limit for $t \gg 1$.

According to the reasons mentioned above [see Eqn (22)], the main contribution comes from diagrams that contain t- and b-quarks (Figs 5c, i, j). A simple calculation (see Appendix VIII) gives the following result for the sum of the Born and loop terms:

$$\frac{m_W}{m_Z} = c + \frac{3c}{32\pi s^2(c^2 - s^2)} \bar{\alpha} t, \quad (53)$$

$$g_{A1} = -\frac{1}{2} - \frac{3}{64\pi s^2 c^2} \bar{\alpha} t, \quad (54)$$

$$R \equiv \frac{g_{V1}}{g_{A1}} = 1 - 4s^2 + \frac{3}{4\pi(c^2 - s^2)} \bar{\alpha} t, \quad (55)$$

$$g_v = \frac{1}{2} + \frac{3}{64\pi s^2 c^2} \bar{\alpha} t. \quad (56)$$

4.3 The functions $V_m(t, h)$, $V_A(t, h)$ and $V_R(t, h)$

If we now switch from the asymptotic case of $t \gg 1$ to the case of $t \sim 1$, then, first, the change in the contribution of the diagrams 5c, i, j can be written in the form

$$t \rightarrow t + T_i(t), \quad (57)$$

where the index $i = m, A, R$, v for m_W/m_Z , g_{A1} , $R \equiv g_{V1}/g_{A1}$ and g_v , respectively. The functions T_i are relatively simple combinations of algebraic and logarithmic functions. They are listed in explicit form in Appendix IX. Their numerical values for a range of values of m_t are given in Table 2. The functions $T_i(t)$ thus describe the contribution of the quark doublet t, b to m_W/m_Z , g_A , $R = g_{V1}/g_{A1}$ and g_v . If, however, we now take into account the contributions of the remaining virtual particles, then the result can be given in the form

$$t \rightarrow V_i(t, h) = t + T_i(t) + H_i(h) + C_i + \delta V_i(t, h). \quad (58)$$

Here $H_i(h)$ contain the contribution of the virtual vector and Higgs bosons W, Z and H are functions of the higgs mass m_H . [The masses of the W - and Z -bosons enter $H_i(h)$ via the parameters c, s , defined by Eqn (26)]. The explicit form of the functions H_i is given in Appendix IX, and their numerical values for various values of m_H are given in Table 3.

Table 2.

m_t , GeV	t	T_m	T_A	T_R
0	0	-0.188	0.875	0.444
10	0.012	0.192	0.934	0.038
20	0.048	-0.256	0.955	-0.015
30	0.108	-0.430	0.812	-0.305
40	0.192	-0.753	0.403	-0.959
50	0.301	-0.985	0.111	-0.748
60	0.433	-0.931	0.327	-0.412
70	0.589	-0.688	0.390	-0.250
80	0.770	-0.317	0.421	-0.143
90	0.974	-0.080	0.440	-0.061
100	1.203	0.084	0.451	0.006
110	1.455	0.214	0.460	0.062
120	1.732	0.323	0.465	0.111
130	2.032	0.418	0.470	0.154
140	2.357	0.503	0.473	0.193
150	2.706	0.579	0.476	0.228
160	3.079	0.649	0.478	0.261
170	3.476	0.713	0.480	0.291
180	3.896	0.772	0.481	0.319
190	4.341	0.828	0.483	0.345
200	4.810	0.880	0.484	0.370
210	5.303	0.929	0.485	0.393
220	5.821	0.975	0.485	0.415
230	6.362	1.019	0.486	0.436
240	6.927	1.061	0.487	0.456
250	7.516	1.101	0.487	0.475
260	8.130	1.139	0.487	0.493
270	8.767	1.176	0.488	0.510
280	9.428	1.211	0.488	0.527
290	10.114	1.245	0.489	0.543
300	10.823	1.277	0.489	0.559

The constants C_i in Eqn (58) include the contributions of light fermions to the self-energy of the W - and Z -bosons, and also to the diagrams in Fig. 3, describing the muon decay, and in Figs 6a–6c, describing the Z -boson decay. The constants C_i are relatively complicated functions of s^2 (see Appendix XII). We list here their numerical values for $s^2 = 0.23110 - \delta s^2$:

$$C_m = -1.3497 + 4.13 \delta s^2, \quad (59)$$

$$C_A = -2.2621 - 2.63 \delta s^2, \quad (60)$$

$$C_R = -3.5045 - 5.72 \delta s^2, \quad (61)$$

$$C_v = -1.1641 - 4.88 \delta s^2. \quad (62)$$

Table 3.

m_H , GeV	h	H_m	H_A	H_R
0.01	0.000	1.120	-8.716	1.359
0.10	0.000	1.119	-5.654	1.354
1.00	0.000	1.103	-2.652	1.315
10.00	0.012	0.980	-0.133	1.016
50.00	0.301	0.661	0.645	0.360
100.00	1.203	0.433	0.653	-0.022
150.00	2.706	0.275	0.588	-0.258
200.00	4.810	0.151	0.518	-0.430
250.00	7.516	0.050	0.452	-0.566
300.00	10.823	-0.037	0.392	-0.679
350.00	14.732	-0.112	0.338	-0.776
400.00	19.241	-0.178	0.289	-0.860
450.00	24.352	-0.238	0.244	-0.936
500.00	30.065	-0.292	0.202	-1.004
550.00	36.378	-0.341	0.164	-1.065
600.00	43.293	-0.387	0.128	-1.122
650.00	50.809	-0.429	0.095	-1.175
700.00	58.927	-0.469	0.064	-1.223
750.00	67.646	-0.506	0.035	-1.269
800.00	76.966	-0.540	0.007	-1.311
850.00	86.887	-0.573	-0.019	-1.352
900.00	97.410	-0.604	-0.044	-1.390
950.00	108.534	-0.633	-0.067	-1.426
1000.00	120.259	-0.661	-0.090	-1.460

4.4 Corrections $\delta V_i(t, h)$

Finally, the last term in Eqn (57) includes the sum of corrections of five different types. Their common feature is that they are all quite small (except for $\delta_2^1 V_i$) and that they represent two loops (with the exception of a one-loop $\delta_1 V_i$ and a three-loop $\delta_3 V_i$).

(1) $\delta_1 V_i$ contains contributions of the W -boson and the t -quark to the polarisation of the electromagnetic vacuum $\delta_W \alpha \equiv \Delta r_{\text{em}}^W$ and $\delta_t \alpha \equiv \Delta r_{\text{em}}^t$ [see Figs 7a–7c and Eqn (16)], which traditionally are not included into the running of $\alpha(q^2)$, i.e. into $\bar{\alpha}$. It is reasonable to treat them as electroweak corrections. This is especially true for the W -loop that depends on the gauge chosen for the description of the W - and Z -bosons. Only after this loop is taken into account, the resultant electroweak corrections become gauge-invariant, as it should indeed be for physical observables. Here and hereafter in the calculations the 't Hooft–Feynman gauge is used (see Appendix I),

$$\delta_1 V_m(t, h) = -\frac{16}{3} \pi s^4 \frac{1}{\alpha} (\delta_W \alpha + \delta_t \alpha) = -0.055, \quad (63)$$

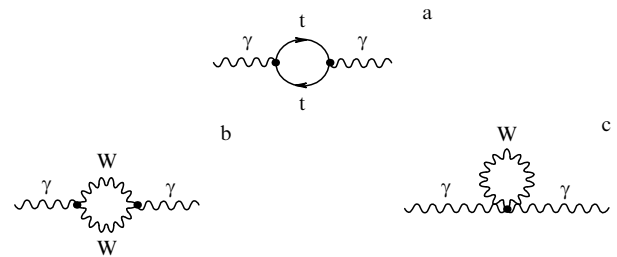


Figure 7. Virtual t -quarks (a) and W -bosons (b), (c) in the photon polarisation of the vacuum.

$$\delta_1 V_R(t, h) = -\frac{16}{3} \pi s^2 c^2 \frac{1}{\alpha} (\delta_W \alpha + \delta_t \alpha) = -0.181, \quad (64)$$

$$\delta_1 V_A(t, h) = \delta_1 V_V(t, h) = 0, \quad (65)$$

where

$$\frac{\delta_W \alpha}{\alpha} = \frac{1}{2\pi} \left[(3 + 4c^2) \times \left(1 - \sqrt{4c^2 - 1} \arcsin \frac{1}{2c} \right) - \frac{1}{3} \right] = 0.0686, \quad (66)$$

$$\frac{\delta_t \alpha}{\alpha} = -\frac{4}{9\pi} \left[(1 + 2t) F_t(t) - \frac{1}{3} \right] \approx -\frac{4}{45\pi} \frac{1}{t} + \dots \approx -0.00768. \quad (67)$$

(Unless specified otherwise, we use $m_t = 175$ GeV in numerical evaluations.)

(2) The corrections $\delta_2 V_i$ are caused by including virtual gluons in electroweak loops in the order $\bar{\alpha} \hat{\alpha}_s$ (see Figs 8a–8c); similar diagrams can, of course, be drawn for W-bosons. In addition to loops with light quarks $q = u, d, s, c$, there exist similar loops with third-generation quarks t and b :

$$\delta_2 V_i(t) = \delta_2^q V_i + \delta_2^t V_i(t).$$

The analytical expressions for corrections $\delta_2^q V_i$ and $\delta_2^t V_i(t)$ are given in Appendix XV. Here we only give numerical

estimates for them:

$$\delta_2^q V_m = -0.377 \frac{\hat{\alpha}_s}{\pi}, \quad (68)$$

$$\delta_2^q V_A = 1.750 \frac{\hat{\alpha}_s}{\pi}, \quad (69)$$

$$\delta_2^q V_R = 0, \quad (70)$$

$$\delta_2^t V_m(t) = -11.67 \frac{\hat{\alpha}_s(m_t)}{\pi} = -10.61 \frac{\hat{\alpha}_s}{\pi}, \quad (71)$$

$$\delta_2^t V_A(t) = -10.10 \frac{\hat{\alpha}_s(m_t)}{\pi} = -9.18 \frac{\hat{\alpha}_s}{\pi}, \quad (72)$$

$$\delta_2^t V_R(t) = -11.88 \frac{\hat{\alpha}_s(m_t)}{\pi} = -10.80 \frac{\hat{\alpha}_s}{\pi}, \quad (73)$$

since [16]

$$\hat{\alpha}_s(m_t) = \frac{\hat{\alpha}_s}{1 + (23/12\pi) \hat{\alpha}_s \ln t}. \quad (74)$$

(For numerical evaluation, we use $\hat{\alpha}_s \equiv \hat{\alpha}_s(m_Z) = 0.125$.) We have mentioned already that the corrections $\delta_2^t V_i(t)$, whose numerical values were given in Eqns (71) – (73), are much larger than all other terms included in δV_i . We emphasise that the term in $\delta_2^t V_i$ that is leading for high t is universal: it is independent of i . As shown in Reef. [54], this leading term is obtained by multiplying the Veltman asymptotics t by a factor

$$1 - \frac{2\pi^2 + 6}{9} \frac{\hat{\alpha}_s(m_t)}{\pi}, \quad (75)$$

or, numerically,

$$t \rightarrow t \left[1 - 2.86 \frac{\hat{\alpha}_s(m_t)}{\pi} \right]. \quad (76)$$

Qualitatively the factor (75) corresponds to the fact that the running mass of the t -quark at momenta $p^2 \sim m_t^2$, that circulate in the t -quark loop, is lower than ‘on the mass-shell’ mass of the t -quark. It is interesting to compare the correction (76) with the quantity

$$\tilde{m}_t^2 \equiv m_t^2(p_t^2 = -m_t^2) = m_t^2 \left[1 - 2.78 \frac{\hat{\alpha}_s(m_t)}{\pi} \right], \quad (77)$$

calculated in the Landau gauge in Ref. [55], p. 102. The agreement is overwhelming. There is, therefore, a simple mnemonic rule for evaluating the main gluon corrections for the t -loop.

(3) Corrections $\delta_3 V_i$ of the order of $\bar{\alpha} \hat{\alpha}_s^2$ were calculated in the literature [56] for the term leading in t (i.e. $\bar{\alpha} \hat{\alpha}_s^2 t$). They are independent of i (in numerical estimates we use for the number of quark flavours $N_f = 5$):

$$\delta_3 V_i(t) \approx -(2.38 - 0.18 N_f) \hat{\alpha}_s^2(m_t) t \approx -1.48 \hat{\alpha}_s^2(m_t) t = -0.07. \quad (78)$$

The corrections $\delta_1 V_i, \delta_2 V_i, \delta_3 V_i$, are independent of m_H , the corrections $\delta_4 V_i$ depend both on m_t , and on m_H , while the corrections $\delta_5 V_i$ are proportional to m_H^2 . In contrast to all previous corrections, they arise due to the electroweak interaction in two loops, not one.

(4) In the leading approximation in t the correction $\delta_4 V_i(t, h)$ produced by the diagrams of Figs 8d–8f is independent of i and takes the form

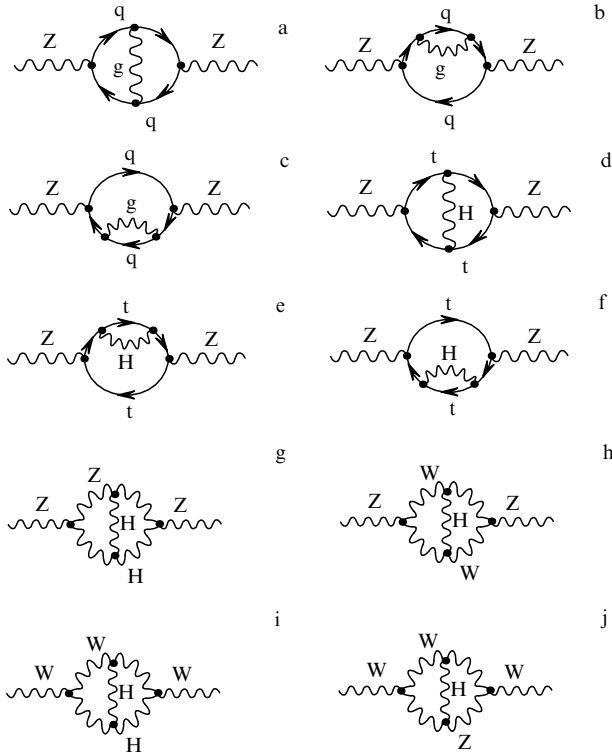


Figure 8. Gluon corrections to the electroweak quark loop of the self-energy of the Z-boson (a) – (c). Higgs corrections to the electroweak t -quark loop of the Z-boson self-energy (d) – (f). Two-loop higgs corrections (g) – (j).

$$\delta_4 V_i(t, h) = -\frac{\bar{\alpha}}{16\pi s^2 c^2} A\left(\frac{h}{t}\right) t^2, \quad (79)$$

where the function $A(h/t)$, calculated in Ref. [53], is given in Table 4 for $m_H/m_t < 4$. For $m_t = 175$ GeV and $m_H = 300$ GeV

$$\delta_4 V_i = -0.11. \quad (80)$$

Table 4.

m_H/m_t	$A(m_H/m_t)$	$\tau^{(2)}(m_H/m_t)^*$	m_H/m_t	$A(m_H/m_t)$	$\tau^{(2)}(m_H/m_t)^*$
0.00	0.739	5.710	2.10	9.655	1.373
0.10	1.821	4.671	2.20	9.815	1.421
0.20	2.704	3.901	2.30	9.964	1.475
0.30	3.462	3.304	2.40	10.104	1.533
0.40	4.127	2.834	2.50	10.235	1.595
0.50	4.720	2.461	2.60	10.358	1.661
0.60	5.254	2.163	2.70	10.473	1.730
0.70	5.737	1.924	2.80	10.581	1.801
0.80	6.179	1.735	2.90	10.683	1.875
0.90	6.583	1.586	3.00	10.777	1.951
1.00	6.956	1.470	3.10	10.866	2.029
1.10	7.299	1.382	3.20	10.949	2.109
1.20	7.617	1.317	3.30	11.026	2.190
1.30	7.912	1.272	3.40	11.098	2.272
1.40	8.186	1.245	3.50	11.165	2.356
1.50	8.441	1.232	3.60	11.228	2.441
1.60	8.679	1.232	3.70	11.286	2.526
1.70	8.902	1.243	3.80	11.340	2.613
1.80	9.109	1.264	3.90	11.390	2.700
1.90	9.303	1.293	4.00	11.436	2.788
2.00	9.485	1.330			

* See Appendix XV.

The following expansion holds for $m_H/m_t > 4$:

$$A\left(\frac{h}{t}\right) = -\frac{49}{4} - \pi^2 - \frac{27}{2} \ln r - \frac{3}{2} \ln^2 r - \frac{1}{3} r \times (2 - 12\pi^2 + 12 \ln r - 27 \ln^2 r) - \frac{r^2}{48} (1613 - 240\pi^2 - 1500 \ln r - 720 \ln^2 r), \quad (81)$$

where $r = t/h$. $\delta_4 V_i(t, h)$ is the greatest of the two-loop corrections in electroweak interaction; however, it is also several times smaller than the main gluon corrections $\delta_2^t V_i$.

(5) Corrections $\delta_5 V_i$ due to two-loop diagrams of the type of Figs 8g–8j are negligible, but for the sake of completeness of the presentation, we list them in Appendix XVII.

4.5 Accidental (?) compensation and the mass of the t-quark

Now that we have expressions for all terms in Eqn (58), it will be convenient to analyze their roles and the general behaviour of the functions $V_i(t, h)$. As functions of m_t at three fixed values of m_H , they are shown in Figs 9a, 10a, 11a. In all these figures, we see a cusp at $m_t = m_Z/2$. This is a typical threshold singularity that arises when the channel $Z \rightarrow t\bar{t}$ is opened. It is of no practical significance since experiments give $m_t \gg m_Z/2$. What really impresses is that the functions V_i are nearly zero for $m_t \sim 100 \div 200$ GeV. This happens because of the compensation of the leading term t and the rest of the terms,

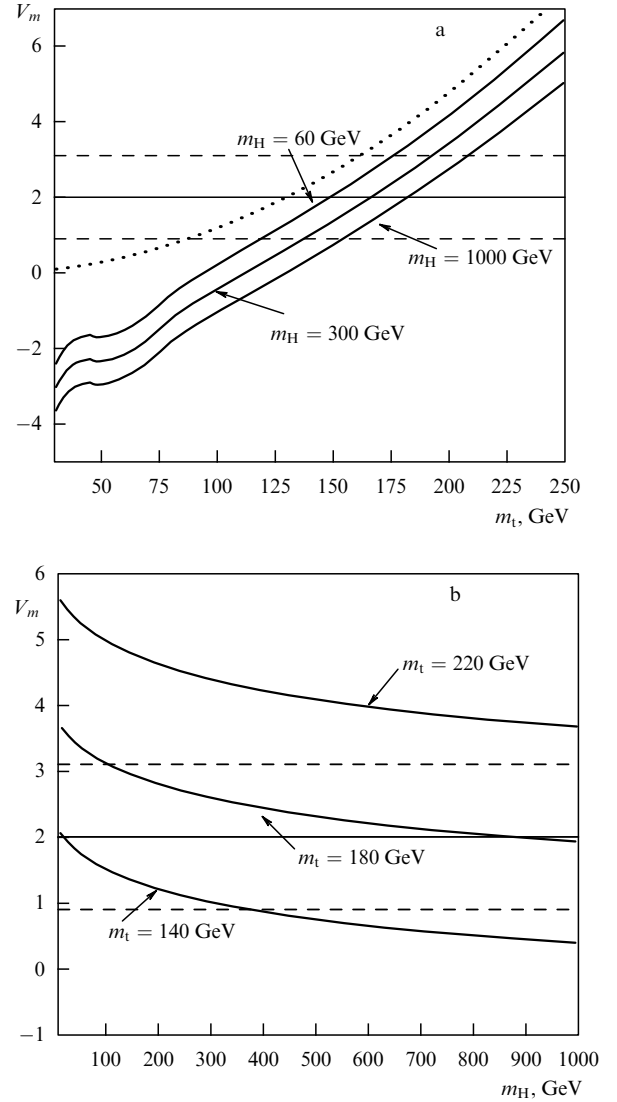


Figure 9. V_m as a function of m_t for three values of m_H : 60, 300 and 1000 GeV, according to Eqn (58). The dotted parabola $t = (m_t/m_Z)^2$ corresponds to the Veltman approximation. Solid horizontal line traces the experimental value of V_m^{exp} while the dashed horizontal lines give its upper and lower limits at the 1σ level (a). V_m as a function of m_H for three values of m_t : 140, 180 and 220 GeV, according to Eqn (58). (Horizontal lines the same as in Fig. 9a) (b).

which produce a negative aggregate contribution. This is especially well pronounced in the function V_R at $m_t \simeq 180$ GeV. Here the main negative contribution comes from the light fermions (the constant C_R).

If we neglect the small correction $\delta_4 V_i(t, h)$ which depends both on t and on h , then each function $V_i(t, h)$ is a sum of functions one of which is t -dependent but independent of h , while the second is h -dependent but independent of t (plus, of course, a constant, which is independent of both t and h). Therefore the curves for $m_H = 60$ and 1000 GeV in Figs 9a, 10a, 11a are (mostly) produced by the parallel transfer of the curve for $m_H = 300$ GeV.

We see in Figs 9a, 10a and 11a that if the t-quark was light, radiative corrections would be negative, and if it was very heavy, they would be much larger. This looks like a conspiracy of the observable mass of the t-quark and all other

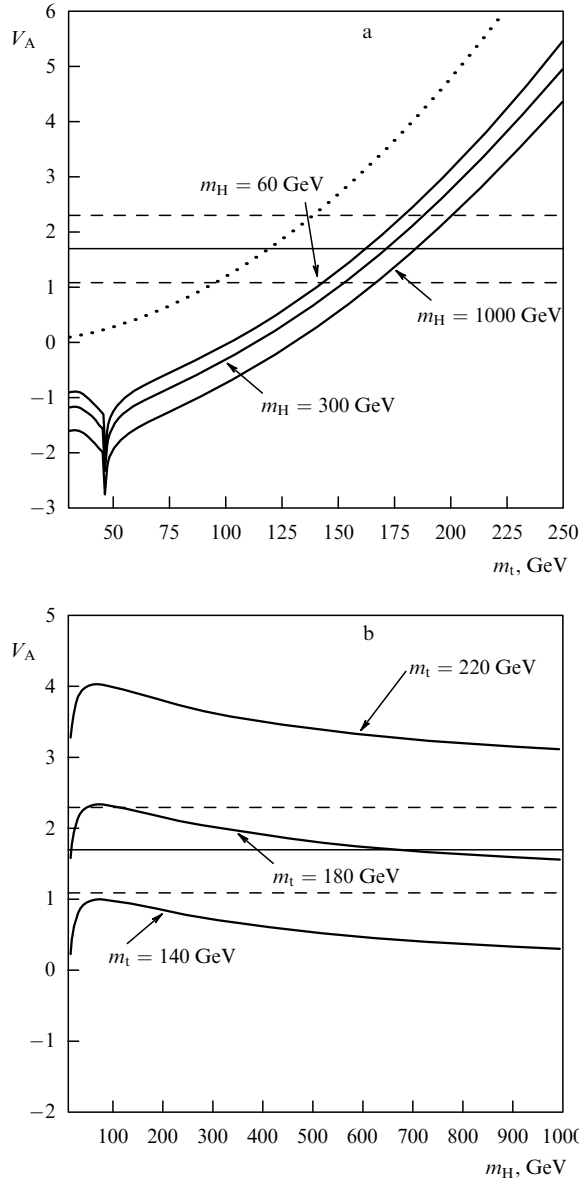


Figure 10. V_A as a function of m_t (a). V_A as a function m_H (b). (The remaining clarifications are similar to those given to Fig. 9).

parameters of the electroweak theory, as a result of which the electroweak correction V_R becomes anomalously small. Note that the corrections do not vanish simultaneously because, if we fix the value of m_H , then $V_m(t)$, $V_A(t)$ and $V_R(t)$ cross the horizontal lines $V_i = 0$ at different values of m_t . What happens is an approximate vanishing of the correction, which as if corresponds to some broken symmetry. The nature of this symmetry is not clear at all, and even its existence is very problematic.

One should specially note the dashed parabola in Figs 9a, 10a and 11a corresponding to the Veltman term t . We see that in the interval $0 < m_t < 250$ GeV it lies much higher than V_A and V_R and approaches V_m only in the right-hand side of Fig. 9a. Therefore, the so-called non-leading ‘small’ corrections that are typically replaced with ellipses in standard texts, are found to be comparable with the leading term t .

A glance at Figs 9a, 10a, 11a readily explains how the experimental analysis of electroweak corrections allows, despite their smallness, a prediction, within the framework

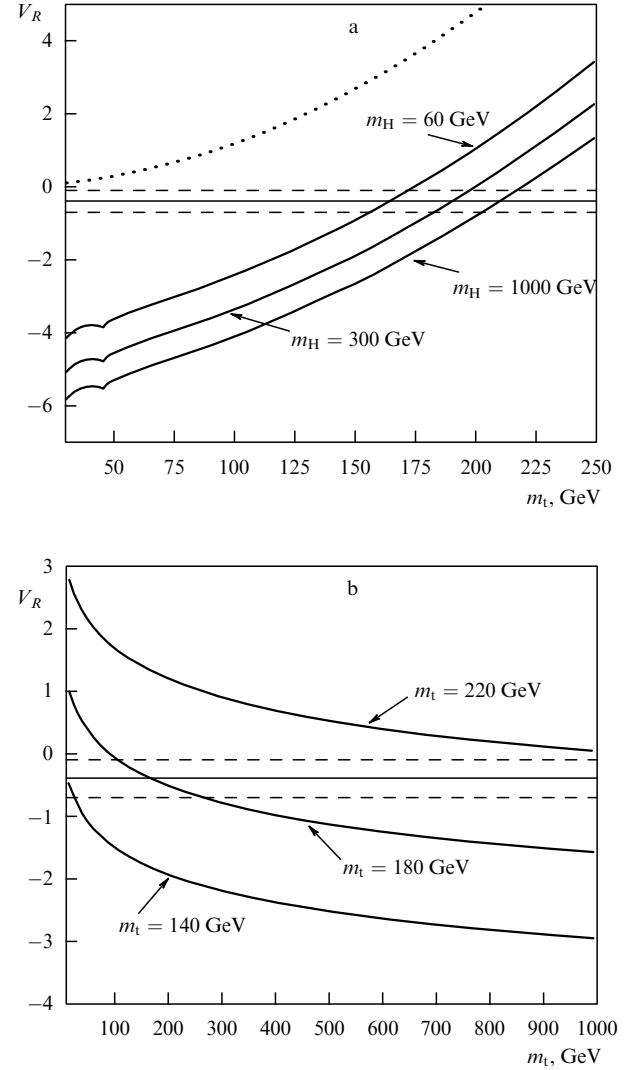


Figure 11. V_R as a function of m_t (a). V_R as a function of m_H (b). (The remaining clarifications are similar to those given to Fig. 9).

of the MSM, of the t-quark mass. Even when the experimental accuracy of LEP1 and SLC experiments was not sufficient for detecting electroweak corrections, it was sufficient for establishing the t-quark mass using the points at which the curves $V_i(m_t)$ intersect the horizontal lines corresponding to the experimental values of V_i and the parallel to them dashed lines that show the band of one standard deviation. The accuracy in determining m_t is imposed by the band width and the slope of $V_i(m_t)$ lines.

The dependence $V_i(m_H)$ for three fixed values of $m_t = 150, 175$ and 200 GeV (Figs 9b, 10b, 11b) can be presented in a similar manner. As follows from the explicit form of the terms $H_i(m_H)$, the dependence $V_i(m_H)$ is considerably less steep (logarithmic). This is the reason why the prediction of the higgs mass extracted from electroweak corrections has such a high uncertainty. We will see later (Figs 13 – 15) that the accuracy of prediction of m_H will greatly depend on what the t-quark’s mass is going to be. If $m_t = 150 \pm 5$ GeV, then $m_H < 200$ GeV at the 3σ level. If $m_t = 200 \pm 5$ GeV, then $m_H > 120$ GeV at the 3σ level. If, however, $m_t = 175 \pm 5$ GeV, we are hugely unlucky: there is practically no constraint on m_H .

Before starting a discussion of hadronic decays of the Z-boson, let us ‘go back to the roots’ and recall how the equations for $V_i(m_t, m_H)$ were derived.

4.6 How to calculate V_i ? ‘Five steps’

An attentive reader should have already come up with the question: what makes the amplitudes of the lepton decays of the Z-boson in the one-loop approximation depend on the self-energy of the W-boson? Indeed, the loops describing the self-energy of the W-boson appear in the decay diagrams of the Z-boson only beginning with the two-loop approximation. The answer to this question is as follows. We have already emphasised that we find expressions for radiative corrections to Z-boson decays in terms of $\bar{\alpha}$, m_Z and G_μ . However, the expression for G_μ includes the self-energy of the W-boson even in the one-loop approximation. The point is thus in expressing some observables (in this particular case, m_W/m_Z , g_A , g_V/g_A) in terms of other, more accurately measured observables ($\bar{\alpha}$, m_Z , G_μ).

Let us trace how this is achieved, step by step. There are altogether ‘five steps to happiness’, based on the one-loop approximation.

Step I. We begin with the electroweak Lagrangian after it had undergone the spontaneous violation of the $SU(2) \times U(1)$ -symmetry by the higgs vacuum condensate (vacuum expectation value – VEV) η and the W- and Z-bosons became massive. Let us consider the bare coupling constants (the bare charges e_0 of the photon, g_0 of the W-boson and f_0 of the Z-boson) and the bare masses of the vector bosons:

$$m_{Z0} = \frac{1}{2} f_0 \eta, \quad (82)$$

$$m_{W0} = \frac{1}{2} g_0 \eta, \quad (83)$$

and also bare masses: m_{t0} of the t-quark and m_{H0} of the higgs.

Step II. We express $\bar{\alpha}$, G_μ , m_Z in terms of f_0 , g_0 , e_0 , η , m_{t0} , m_{H0} and $1/\varepsilon$ (see Appendix V). Here, $1/\varepsilon$ appears because we use the dimensional regularization, calculating the Feynman integrals in the space of D dimensions (see Appendix I). These integrals diverge at $D = 4$ and are finite in the vicinity of $D = 4$. By definition,

$$2\varepsilon = 4 - D \rightarrow 0. \quad (84)$$

Note that in the one-loop approximation $m_{t0} = m_t$, $m_{H0} = m_H$, since we neglect the electroweak corrections to the masses of the t-quark and the higgs. For the higgs this approximation is quite legitimate, since the accuracy of extracting its mass from radiative corrections is very poor. As for the t-quark, this statement is also true at the current accuracy of the experimental measurement of electroweak corrections; however, this would become an unacceptably crude approximation if this accuracy could be improved by an order of magnitude at LEP and SLC. The situation here is analogous to that for G_μ and the self-energy of the W-boson. Step II is almost physics: we calculate the Feynman diagrams (we say “almost” to emphasise that observables are expressed in terms of nonobservable, ‘bare’, and generally infinite quantities).

Step III. Let us invert the expressions derived at Step II and write f_0 , g_0 , η in terms of $\bar{\alpha}$, G_μ , m_Z , m_t , m_H and $1/\varepsilon$. This step is pure algebra.

Step IV. Let us express V_m , V_A , V_R (or the electroweak one-loop correction to any other electroweak observable, all of them being treated on an equal basis) in terms of f_0 , g_0 , η , m_t , m_H and $1/\varepsilon$. (Like Step II, this step is again almost physics.)

Step V. Let us express V_m , V_A , V_R (or any other electroweak correction) in terms of $\bar{\alpha}$, G_μ , m_Z , m_t , m_H using the results of Steps III and IV. Formally this is pure algebra, but in fact pure physics, since now we have expressed certain physical observables in terms of other observables. If no errors were made on the way, the terms $1/\varepsilon$ cancel out. As a result, we arrive at Eqn (58), which gives V_i as elementary functions of t , h and s .

The five steps outlined above are very simple and visually clear. We obtain the main relations without using the ‘heavy artillery’ of quantum field theory with its counterterms in the Lagrangian and the renormalisation procedure. This simplicity and visual clarity became possible owing to the one-loop electroweak approximation (even though this approach to renormalisations is possible in multiloop calculations, it becomes more cumbersome than standard procedures). As for the QCD-corrections to quark electroweak loops and the two-loop higgs contribution hidden in the terms δV_i in Eqn (58), we take the relevant formulas from the calculations of other authors.

5. Hadronic decays of Z-boson

5.1 The leading quarks and hadrons

As discussed above [see Eqns (35) – (40) and the subsequent Section 3.4], an analysis of hadronic decays reduces to the calculation of decays to pairs of quarks: $Z \rightarrow q\bar{q}$. The key role is played by the concept of leading hadrons that carry away the predominant part of the energy. For example, the $Z \rightarrow c\bar{c}$ decay mostly produces two hadron jets flying in opposite directions, in one of which the leading hadron is the one containing the \bar{c} -quark, for example, $D^- = \bar{c}d$, and in the other the hadron with the c-quark, for example, $D^0 = c\bar{u}$ or $\Lambda_c^+ = udc$. Likewise, $Z \rightarrow b\bar{b}$ decays are identified by the presence of high-energy B or \bar{B} mesons. If we select only particles with energy close to $m_Z/2$, the identification of the initial quark channels is unambiguous. The total number of such cases will, however, be small. If we take into account as a signal less energetic B-mesons, we face the problem of their origin. Indeed, a pair $b\bar{b}$ can be created not only directly by a Z-boson but also by a virtual gluon in, say, a $Z \rightarrow c\bar{c}$ decay (Fig. 12a) or $Z \rightarrow u\bar{u}$, or $s\bar{s}$. This example shows the sort of difficulty encountered by experimentalists trying to identify a specific quark–antiquark channel. Furthermore, owing to secondary pairs, the total hadron width is not strictly equal to the sum of partial quark widths.

We remind the reader that for the partial width of the $Z \rightarrow q\bar{q}$ decay we had (35)

$$\Gamma_q \equiv \Gamma(Z \rightarrow q\bar{q}) = 12\Gamma_0 [g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}], \quad (85)$$

where the standard width Γ_0 is [according to Eqn (31)]

$$\Gamma_0 = \frac{G_\mu m_Z^3}{24\sqrt{2}\pi} = 82.944(6) \text{ MeV}, \quad (86)$$

and the radiators R_{Aq} and R_{Vq} are given in Appendix VI. As for the electroweak corrections, they are included in the

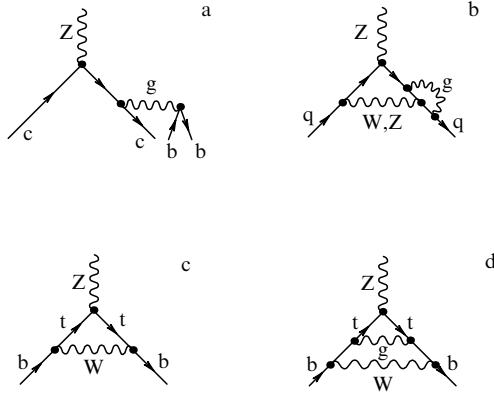


Figure 12. The $Z \rightarrow c\bar{c}$ decay producing a secondary $b\bar{b}$ pair created by a virtual gluon g (a). The $Z \rightarrow q\bar{q}$ decay with a virtual gluon g which connects a side of the quark triangle with an external quark line. Diagrams of this type have not been calculated yet (b). The vertex electroweak diagram involving t -quarks and contributing to the $Z \rightarrow b\bar{b}$ decay (c). One of the diagrams (d) describing gluon corrections to (c) diagram.

coefficients g_{Aq} and g_{Vq} . The sum of the Born and one-loop terms has the form

$$g_{Aq} = T_{3q} \left[1 + \frac{3\bar{\alpha}}{32\pi s^2 c^2} V_{Aq}(t, h) \right], \quad (87)$$

$$R_q \equiv \frac{g_{Vq}}{g_{Aq}} = 1 - 4|Q_q|s^2 + \frac{3|Q_q|}{4\pi(c^2 - s^2)} \bar{\alpha} V_{Rq}(t, h). \quad (88)$$

5.2 Decays to pairs of light quarks

Here, as in the case of hadronless observables, the quantities V that characterise corrections are normalised in the standard way: $V \rightarrow t$ as $t \gg 1$. Naturally, those terms in V that are due to the self-energies of vector bosons are identical for both leptons and quarks. The deviation of the differences $V_{Aq} - V_A$ and $V_{Rq} - V_R$ from zero are caused by the differences in radiative corrections to vertices $Z \rightarrow q\bar{q}$ and $Z \rightarrow l\bar{l}$. For four light quarks we have

$$V_{Au}(t, h) = V_{Ac}(t, h) = V_A(t, h) + \frac{128\pi s^3 c^3}{3\bar{\alpha}} (F_{Al} + F_{Au}), \quad (89)$$

$$V_{Ad}(t, h) = V_{As}(t, h) = V_A(t, h) + \frac{128\pi s^3 c^3}{3\bar{\alpha}} (F_{Al} - F_{Ad}), \quad (90)$$

$$\begin{aligned} V_{Ru}(t, h) &= V_{Rc}(t, h) \\ &= V_R(t, h) + \frac{16\pi s c(c^2 - s^2)}{3\bar{\alpha}} \left\{ F_{Vl} - (1 - 4s^2) F_{Al} \right. \\ &\quad \left. + \frac{3}{2} \left[-\left(1 - \frac{8}{3}s^2\right) F_{Au} + F_{Vu} \right] \right\}, \end{aligned} \quad (91)$$

$$\begin{aligned} V_{Rd}(t, h) &= V_{Rs}(t, h) \\ &= V_R(t, h) + \frac{16\pi s c(c^2 - s^2)}{3\bar{\alpha}} \left\{ F_{Vl} - (1 - 4s^2) F_{Al} \right. \\ &\quad \left. + 3 \left[\left(1 - \frac{4}{3}s^2\right) F_{Ad} - F_{Vd} \right] \right\}, \end{aligned} \quad (92)$$

where (see Appendix XIII):

$$F_{Al} = \frac{\bar{\alpha}}{4\pi} (3.0099 + 16.4\delta s^2), \quad (93)$$

$$F_{Vl} = \frac{\bar{\alpha}}{4\pi} (3.1878 + 14.9\delta s^2), \quad (94)$$

$$F_{Au} = -\frac{\bar{\alpha}}{4\pi} (2.6802 + 14.7\delta s^2), \quad (95)$$

$$F_{Vu} = -\frac{\bar{\alpha}}{4\pi} (2.7329 + 14.2\delta s^2), \quad (96)$$

$$F_{Ad} = \frac{\bar{\alpha}}{4\pi} (2.2221 + 13.5\delta s^2), \quad (97)$$

$$F_{Vd} = \frac{\bar{\alpha}}{4\pi} (2.2287 + 13.5\delta s^2). \quad (98)$$

The values of F are given here for $s^2 = 0.23110 - \delta s^2$. The accuracy to five decimal places is purely arithmetic. The physical uncertainties introduced by neglecting higher-order loops manifest themselves already in the third decimal place. It is necessary to point out that corrections of the type of that shown in Fig. 12b have not yet been calculated.

5.3 Decays to $b\bar{b}$ pair

In the $Z \rightarrow b\bar{b}$ decay it is necessary to take into account additional t -dependent vertex corrections:

$$V_{Ab}(t, h) = V_{Ad}(t, h) - \frac{8s^2 c^2}{3(3 - 2s^2)} [\phi(t) + \delta\phi(t)], \quad (99)$$

$$V_{Rb}(t, h) = V_{Rd}(t, h) - \frac{4s^2(c^2 - s^2)}{3(3 - 2s^2)} [\phi(t) + \delta\phi(t)]. \quad (100)$$

Here the term $\phi(t)$ calculated in Ref. [57] corresponds to Fig. 12c and the term $\delta\phi(t)$ calculated in Refs [58, 53] corresponds to the leading gluon and higgs corrections to the term $\phi(t)$ (see Fig. 12d). Expressions for $\phi(t)$ and $\delta\phi(t)$ are given in Appendix XIV. For $m_t = 175$ GeV, $\hat{\alpha}_s(m_Z) = 0.125$, $m_H = 300$ GeV,

$$\phi(t) = 29.9, \quad (101)$$

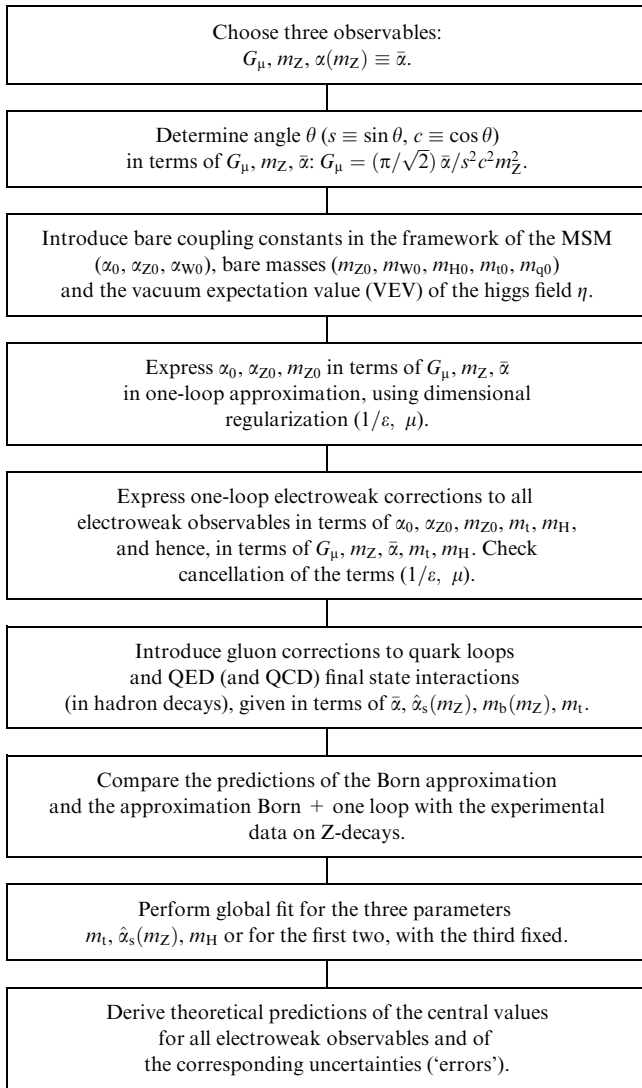
$$\delta\phi(t) = -3.0, \quad (102)$$

and correction terms in Eqns (99) and (100) are very large: they are equal to -5.0 and -1.8 , respectively.

6. Comparison of theoretical results and experimental LEP1 and SLC data

6.1 LEPTOP code

A number of computer programs (codes) were written for comparing high-precision data of LEP1 and SLC. The best known of these programs in Europe is ZFITTER [59], which takes into account not only electroweak radiative corrections but also all purely electromagnetic ones, including, among others, the emission of photons by colliding electrons and positrons. Some of the first publications in which the t quark mass was predicted on the basis of precision measurements [60], were based on the code ZFITTER. Other European codes, BHM [61], WOH, TOPAZO [62], somewhat differ from ZFITTER. The best known in the USA are the results generated by the code used by Langacker [35].

Flowchart.

The original idea of the authors of this review in 1991–1993 was to derive simple analytical formulas for electroweak radiative corrections, which would make it possible to predict the t-quark mass using no computer codes, just by analyzing experimental data on a sheet of paper. Alas, the diversity of hadron decays of Z-bosons, depending on the constants of strong gluon interaction $\hat{\alpha}_s$, was such that it was necessary to convert analytical formulas into a computer program which we jokingly dubbed LEPTOP [63]. The LEPTOP calculates the electroweak observables in the framework of the MSM and fits experimental data so as to determine the quantities m_t, m_H and $\hat{\alpha}_s(m_Z)$. The logical structure of LEPTOP is clear from the preceding sections of this review and is shown in the Flowchart. The code of LEPTOP can be downloaded from the Internet home page: <http://cppm.in2p3.fr/leptop/intro.leptop.html>.

A comparison of the codes ZFITTER, BHM, WOH, TOPAZO, and LEPTOP, carried out in 1994–95 [18], has demonstrated that their predictions for all electroweak observables coincide with accuracy that is much better than the accuracy of the experiment. The results of processing the experimental data using LEPTOP are shown below.

Table 5.

Observable	Experimental data	Fit standard model	Pull
(a) LEP			
shape of Z-peak and lepton asymmetries:			
m_Z , GeV	91.1884(22)	—	—
Γ_Z , GeV	2.4963(32)	2.4976(26) $^{+6}_{-16}$	−0.4
σ_h , nb	41.488(78)	41.450(20) $^{+3}_{-7}$	0.5
R_l	20.788(32)	20.770(24) $^{+5}_{-11}$	0.6
A_{FB}^l	0.0172(12)	0.0158(6) $^{+2}_{-3}$	1.2
τ -polarisation:			
A_τ	0.1418(75)	0.1450(26) $^{+7}_{-13}$	−0.4
A_e	0.1390(89)	0.1450(26) $^{+7}_{-13}$	−0.7
Results for b-and c-quarks:			
R_b	0.2219(17)	0.2155(3) $^{+7}_{-2}$	3.8
R_c	0.1543(74)	0.1724(1) $^{+2}_{-2}$	−2.5
A_{FB}^b	0.0999(31)	0.1017(19) $^{+6}_{-10}$	−0.6
A_{FB}^c	0.0725(58)	0.0726(14) $^{+4}_{-7}$	0.0
Charge asymmetry for pairs of light quarks $q\bar{q}$:			
$s_1^2((Q_{FB}))$	0.2325(13)	0.2318(3) $^{+1}_{-2}$	0.6
(b) SLC			
A_{LR}	0.1551(40)	0.1450(26) $^{+7}_{-13}$	2.5
$s_1^2(A_{LR})$	0.2305(5)	0.2318(3) $^{+1}_{-2}$	−2.5
R_b	0.2171(54)	0.2155(3) $^{+7}_{-2}$	0.3
A_b	0.8410(530)	0.9345(2) $^{+3}_{-4}$	−1.8
A_c	0.6060(900)	0.6670(12) $^{+3}_{-6}$	−0.7
(c) $p\bar{p}$ and νN			
m_W , GeV ($p\bar{p}$)	80.26(16)	80.35(5) $^{+1}_{-3}$	−0.5
	0.2253(31)		
$1 - m_W^2/m_Z^2$ (νN)	0.2257(47)	0.2237(9) $^{+2}_{-5}$	0.4
	80.24(24)		

6.2 General fit

Table 5 shows experimental values of the electroweak observables, obtained by averaging the results of four LEP detectors (part a), and also SLC data (part b), and the data on W-boson mass (part c). (The data on the W-boson mass from the $p\bar{p}$ -colliders are also shown, for the reader's convenience, in the form of s_W^2 , while the data on s_W^2 from νN -experiments are also shown in the form of m_W . These two numbers are given in italics, emphasising that they are not independent experimental data.) We take experimental data from Ref. [44].

Table 5 sums up the experimental data used for determining (fitting) the parameters of the Standard Model m_t and $\hat{\alpha}_s(m_Z)$ (see Table 7). The central values in the third column of Table 5 were calculated for $m_H = 300$ GeV. Shown in brackets is the uncertainty of the last significant decimal places due to the uncertainty of the fitted values m_t and $\hat{\alpha}_s$. Above and below we give the shifts in the last significant decimal places corresponding to $m_H = 1000$ GeV and $m_H = 60$ GeV, respectively. The last column shows the value of the 'pull'. By definition, the pull is the difference between the experimental and the theoretical values divided by experimental uncertainty. The pull values show that the maximum discrepancy between the experimental data and the MSM is found for R_b (3.8σ). Deviations at the level of 2.5σ are also found for R_c and $\sin^2 \theta_{\text{eff}}^{\text{lept}} \equiv s_1^2$ in A_{LR} (SLC). For most observables the discrepancy is less than 1σ . At the same

time, Table 6 shows that the value $s_1^2 = 0.23186(34)$ extracted from all asymmetries at LEP agrees quite well with the fitted MSM value $s_1^2 = 0.2321(4)$ from the LEP data, and for all sets of data.

Table 6 gives experimental values of s_1^2 . The third column was obtained by averaging of the second column, and the fourth by cumulative averaging of the third; it also lists the values of χ^2 over the number of degrees of freedom (d.o.f.).

Table 6.

Observable	s_1^2	Average over groups of observables	Cumulative average and $\chi^2/\text{d.o.f.}$
A_{FB}^l	0.23096(68)		
A_τ	0.23218(95)		
A_e	0.2325(11)	0.23160(49)	0.23160(49) 1.9/2
A_{FB}^b	0.23209(55)		
A_{FB}^c	0.2318(13)	0.23205(51)	0.23182(35) 2.4/4
$\langle Q_{\text{FB}} \rangle$	0.2325(13)	0.2325(13)	0.23186(34) 2.6/5
$A_{\text{LR}} (\text{SLD})$	0.23049(50)	0.23049(50)	0.23143(28) 7.8/6

Table 7 gives the fitted values m_t and $\hat{\alpha}_s \equiv \hat{\alpha}_s(m_Z^2)$, and also the values of χ^2 times the number of degrees of freedom for various sets of data, where m_W stands for both the data of m_W measurements in $p\bar{p}$ collisions and the values of s_W^2 , extracted from experiments with νN . The lower part of the table gives the values of

$$s_1^2 \equiv \sin^2 \theta_{\text{eff}}^{\text{lept}} \equiv \frac{1}{4} \left(1 - \frac{g_{Vl}}{g_{Al}} \right), \quad s_W^2 \equiv 1 - \frac{m_W^2}{m_Z^2},$$

calculated in one-loop electroweak approximation in the framework of the MSM using fitted values of m_t and $\hat{\alpha}_s$. Errors given in parentheses are due to uncertainties in m_t , $\hat{\alpha}_s$ and $\bar{\alpha}$, and were calculated by summation of squares, ignoring correlations. Note that the errors in the values of s_W^2 calculated using the fits are substantially lower than in the experimental values of this quantity (see Table 5); at the same time, the errors for s_1^2 are practically identical for the experimental (Table 6) and the theoretical (Table 7) values. Note that the first and second rows of the lower part of Table 7 carry identical information; the same is true for the third, fourth and fifth rows.

Table 7.

Physical quantities	LEP	LEP + SLC	LEP + m_W	LEP + SLC + m_W
m_t , GeV	171(9) $_{-21}^{+18}$	182(7) $_{-22}^{+18}$	170(8) $_{-21}^{+17}$	180(7) $_{-21}^{+18}$
$\hat{\alpha}_s$	0.125(4) $_{-2}^{+2}$	0.123(4) $_{-2}^{+2}$	0.125(4) $_{-2}^{+2}$	0.124(4) $_{-2}^{+2}$
$\chi^2/\text{d.o.f.}$	18/9	29/13	18/11	30/15
s_1^2	0.2321(4) $_{-2}^{+1}$	0.2317(3) $_{-2}^{+1}$	0.2321(4) $_{-2}^{+1}$	0.2318(3) $_{-2}^{+1}$
g_{Vl}/g_{Al}	0.07116(16) $_{+8}^{-4}$	0.0732(12) $_{+8}^{-4}$	0.0716(16) $_{+8}^{-4}$	0.0728(12) $_{+8}^{-4}$
s_W^2	0.2247(9) $_{+4}^{-2}$	0.2234(9) $_{+5}^{-2}$	0.2237(9) $_{+4}^{-2}$	0.2237(9) $_{+5}^{-4}$
m_W/m_Z	0.8805(5) $_{-2}^{+1}$	0.8813(5) $_{-3}^{+1}$	0.8804(5) $_{-2}^{+1}$	0.8811(5) $_{-3}^{+2}$
m_W , GeV	80.29(5) $_{-2}^{+1}$	80.36(5) $_{-3}^{+1}$	80.28(5) $_{-2}^{+1}$	80.35(5) $_{-3}^{+1}$

7. Conclusions

7.1 Achievements

What are the main results obtained with four detectors of the LEP1 collider and one detector of the SLC?

Judging by the criteria of accelerator and experimental techniques, the highest imaginable level has been achieved in the team creativity. The impossible became the reality owing to a never before dreamt-of sophistication of equipment of gigantic high-precision detectors. Twenty million decays of Z-bosons were measured with better accuracy than that admired in gemstone cutting.

From the physics standpoint, the main result is the experimental proof of the statement that there exist only three generations of quarks and leptons with light neutrinos. The number of light neutrinos, found from the sum width of the invisible Z-boson decays, is

$$N_\nu = 2.990(16). \quad (103)$$

The lower limit on the masses of the heavy neutrinos in the additional generations, provided they exist, is close to $m_Z/2$ and equals 44 GeV.

No new particles were found in Z-boson decays. In particular, the higgs was not found. From LEP1 data, the lower limit on the higgs mass is

$$m_H > 60 \text{ GeV}. \quad (104)$$

The high-precision measurement of the Z-boson mass, its total and partial decay widths and also of the P- and C-violating asymmetries made it possible to determine experimentally the electroweak radiative corrections. A comparison of these experimental values with the results of theoretical calculations based on the MSM led to prediction of the t-quark mass m_t and the constant of strong interaction for gluons $\hat{\alpha}_s$:

$$m_t = 180(7) $_{-21}^{+18}$ \text{ GeV}, \quad (105)$$

$$\hat{\alpha}_s = 0.124(4) $_{-2}^{+2}$. \quad (106)$$

Shown in parentheses here is the uncertainty (one standard deviation) due to the uncertainty of the experimental data. The central value corresponds to the assumption that $m_H = 300 \text{ GeV}$, and the upper and lower ‘shifts’ correspond to $m_H = 1000 \text{ GeV}$ and 60 GeV , respectively. Radiative corrections depend only weakly on m_H , so one cannot extract

the value of m_H from them. The t-quark mass predicted on the basis of the radiative corrections within the currently known experimental uncertainties is in excellent agreement with the results of direct measurements of m_t in CDF and D0 detectors at the Tevatron†:

$$m_t = 176(13) \text{ GeV} \quad (\text{CDF}[13]), \quad (107)$$

$$m_t = 199(30) \text{ GeV} \quad (\text{D0}[14]). \quad (108)$$

After the t-quark mass uncertainty is further reduced, radiative corrections can be used for determining the region in which the higgs mass can lie. Figs 13 – 15 show that the results will greatly depend on luck. If $m_t = 150(5) \text{ GeV}$, Fig. 13 demonstrates that $m_H < 150 \text{ GeV}$ at the 3σ level. If $m_t = 200(5) \text{ GeV}$, then $m_H > 120 \text{ GeV}$ at the 3σ level (Fig. 15). If, however, $m_t = 175(5)$, then the higgs can have any mass within 3σ .

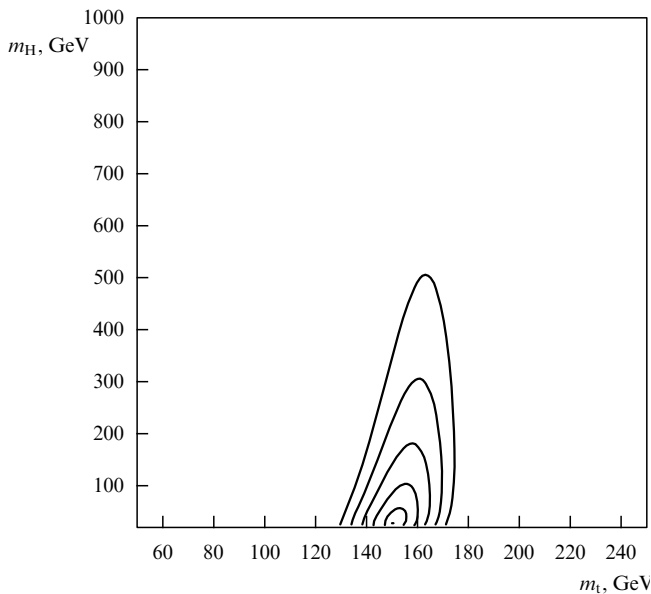


Figure 13. Isolines of χ^2 in the m_t, m_H plane, obtained by fitting the electroweak corrections under the assumption that direct measurements of the t-quark mass will give $m_t = 150 \pm 5 \text{ GeV}$.

7.2 Problems

A cursory glance at Table 5 is sufficient for identifying the main problem of the Z-boson physics: the discrepancy between the measured width of decay into a pair $b\bar{b}$ and its theoretical prediction.

The supersymmetrization of the Standard Model may help solving this problem [66] (see Fig. 16a, this diagram with superpartners increases R_b). The introduction of additional vector boson Z' may be of help as well.

Let us turn now to $\hat{\alpha}_s(m_Z)$. A number of papers [67, 68] pointed out that the value $0.124(4)^{+2}_{-2}$, shown in Table 7, is in contradiction with the measurements of $\hat{\alpha}_s(q^2)$ for $q^2 \lesssim (10 \text{ GeV})^2$ in deep inelastic scattering [69], in hadron decays of the Y-meson [70] and especially in the spectrum of upsilononium levels [71]. If the low-energy values $\hat{\alpha}_s(q^2 \lesssim (10 \text{ GeV})^2)$ are extrapolated in the framework of the

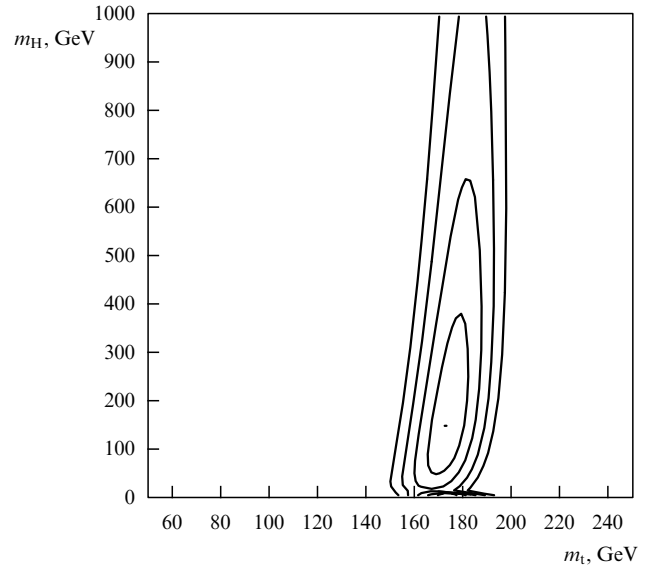


Figure 14. The same as in Fig. 13, for $m_t = 175 \pm 5 \text{ GeV}$.

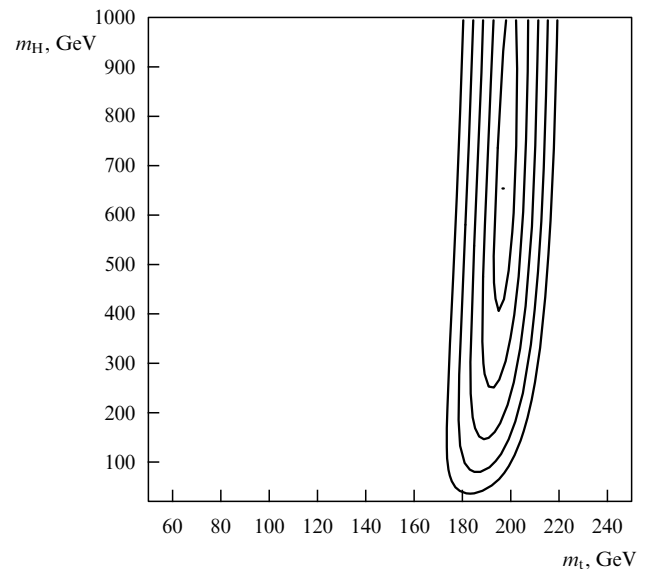


Figure 15. The same as in Fig. 13, for $m_t = 200 \pm 5 \text{ GeV}$.

standard QCD to $q^2 = m_Z^2$, we find $\hat{\alpha}_s(m_Z^2) = 0.110 \div 0.118$. As for the uncertainty in this range, the authors of Refs [68 – 71] do not come to a common opinion. The most cautious scientists evaluate it as ± 0.005 [69, 70]. The bravest one insists on ± 0.001 [68, 71]. In the last case, there is an obvious contradiction with the value derived by analyzing the Z-boson decay data. This contradiction served as a basis for hypotheses [67] that the MSM predictions for hadron widths are modified by the contribution of some new unknown particles to electroweak radiative corrections, for example, squarks and gluino, that is, the supersymmetric partners of quarks and gluons. For loops with these particles to result in sufficiently strong deviations from the MSM, it is necessary that squarks and gluino were sufficiently light, with masses of the order of 100 GeV. A deviation of the observable value of R_b by more than 3σ from the values predicted in the MSM on the basis of the global fit (see Table 5) is also considered as an

† At the spring 1996 conferences, more accurate data have been presented: $m_t = 175.6 \pm 5.7 \pm 7.4 \text{ GeV}$ (CDF, see Ref. [64]), $m_t = 170 \pm 15 \pm 10 \text{ GeV}$ (D0, see Ref. [65]).

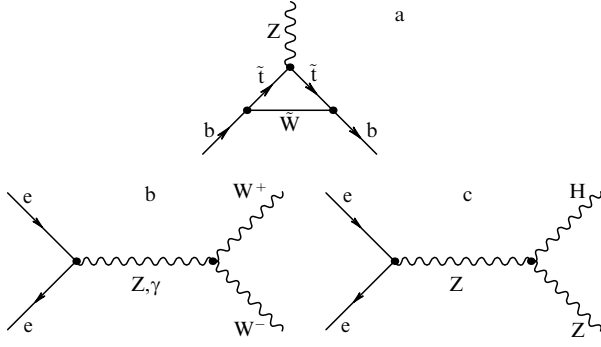


Figure 16. A vertex with virtual \tilde{t} squarks and a wino \tilde{W} (a). The reaction $e^+e^- \rightarrow W^+W^-$ with a virtual photon or Z-boson (b). The reaction $e^+e^- \rightarrow HZ$ (c).

independent argument in favour of the view that the new physics ‘lurks round the corner’. Two other deviations from the MSM in Table 5 are less serious: R_c and A_{LR} are off by 2.5 standard deviations. Note that in the latter case we witness a discrepancy not only with MSM but also with the measurements of the lepton asymmetries at LEP1, since, according to Eqn (47), $A_{LR} = A_l$.

Various parametrizations of the manifestations of the new physics can be found in the literature. The better known ones are the parameters S, T, U [72] and $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_b$ [73].

7.3 Prospects

The LEP collider completed its work in the LEP1 mode in autumn 1995 and began working in the LEP2 mode. It is expected that the total energy of the electron and positron collision will be raised to 192 GeV. What are the main goals of LEP2 [74]?

A careful measurement of the cross section of the reaction $e^+e^- \rightarrow W^+W^-$ needed to measure the W-boson mass with accuracy of the order of 50 MeV and to test whether the interaction of W-bosons with photons and with the Z-boson agrees with the Standard Model (see Fig. 16b).

A search for a higgs with a mass up to 92 GeV in the reaction $e^+e^- \rightarrow HZ$ (Fig. 16c).

A search for light superparticles (sleptons, squarks).

A search for the unanticipated.

Further prospects for testing the Standard Model and for searching for ‘new physics’ beyond its limits are rooted in the Large Hadron Collider (LHC) (a decision to build it has already been made at CERN) and in the so-called ‘Next Linear Collider’ for electrons and positrons, which is so far at the stage of discussion of competing projects.

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Appendices

I. Feynman rules in the electroweak theory

A consistent derivation of Feynman rules for theories with spontaneous violation of gauge symmetry can be found in textbooks (see, for example, Itzykson, Zuber; Ramond;

Slavnov, Faddeev in Ref. [4]). In this appendix we only give a summary of results, accompanied by brief comments.

I.1 Gauges and propagators

Quantization of gauge fields (in the MSM this is W_μ^\pm, Z_μ and A_μ) requires fixing gauge. The most popular is the so-called R_ξ gauge which corresponds to adding gauge fixing new terms $\delta\mathcal{L}_{GF}$ to the classical Lagrangian:

$$\delta\mathcal{L}_{GF} = -\frac{1}{2\xi_A} (\partial_\mu A_\mu)^2 - \frac{1}{2\xi_Z} (\partial_\mu Z_\mu - m_Z \xi_Z G^0)^2 - \frac{1}{\xi_W} (\partial_\mu W_\mu^+ - im_W \xi_W G^+) (\partial_\mu W_\mu^- + im_W \xi_W G^-), \quad (I.1)$$

where G^\pm, G^0 and H are the components of the higgs doublet Φ in the parametrization

$$\Phi = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}} [\eta + H(x) + iG^0(x)] \end{pmatrix}. \quad (I.2)$$

In what follows we use the particular case of R_ξ gauge, namely

$$\xi_A = \xi_W = \xi_Z = \xi.$$

With gauge fixed, it is possible to determine the propagators $D_{\mu\nu}^W(p)$, $D_{\mu\nu}^Z(p)$ and $D_{\mu\nu}^A(p)$ of the fields W_μ^\pm, Z_μ and A_μ :

$$\begin{aligned} D_{\mu\nu}^W(p) &= -\frac{i}{p^2 - m_W^2 + i\varepsilon} \left\{ g_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2 - m_W^2 \xi + i\varepsilon} \right\} \\ &\equiv -\frac{i}{p^2 - m_W^2 + i\varepsilon} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2} \right) - i \frac{p_\mu p_\nu}{m_W^2} \frac{1}{p^2 - m_W^2 \xi + i\varepsilon}, \end{aligned} \quad (I.3)$$

$$\begin{aligned} D_{\mu\nu}^Z(p) &= -\frac{i}{p^2 - m_Z^2 + i\varepsilon} \left\{ g_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2 - m_Z^2 \xi + i\varepsilon} \right\} \\ &= -\frac{i}{p^2 - m_Z^2 + i\varepsilon} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_Z^2} \right) - i \frac{p_\mu p_\nu}{m_Z^2} \frac{1}{p^2 - m_Z^2 \xi + i\varepsilon}, \end{aligned} \quad (I.4)$$

$$D_{\mu\nu}^A(p) = -\frac{i}{p^2 + i\varepsilon} \left[g_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2 + i\varepsilon} \right]. \quad (I.5)$$

The case $\xi = 1$ corresponds to the ‘t Hooft–Feynman gauge, $\xi = 0$ to the Landau gauge, $\xi \rightarrow \infty$ to the Proca gauge.

As follows from Eqns (I.3) and (I.4), the propagators of massive vector fields can be written as sums of a propagator in the Proca gauge that describes the propagation of physical degrees of freedom of a vector particle, and a scalar propagator with a gauge-dependent pole which corresponds to the propagation of non-physical degrees of freedom. As a result, diagrams with virtual W^\pm, Z -bosons contain non-physical threshold singularities whose positions depend on the gauge parameter ξ . These non-physical singularities partially cancel out after the appropriate diagrams with the virtual Goldstone bosons G^\pm, G^0 [arising from the higgs doublet Φ (I.2)] are added. The Goldstone boson propagators have the form:

$$D_{G^+}(p) = \frac{i}{p^2 - m_W^2 \xi + i\varepsilon}, \quad (I.6)$$

$$D_{G^0}(p) = \frac{i}{p^2 - m_Z^2 \xi + i\varepsilon}. \quad (I.7)$$

A complete restoration of unitarity (cancellation of non-physical singularities) and of gauge invariance (validity of the Ward identities) are achieved if one takes into account the diagrams with the Faddeev–Popov ghosts η^+ , η^Z and η^A , which interact only with gauge fields and the Goldstone fields and which do not correspond to any physical degrees of freedom.

Ghost propagators take the form

$$D_{\eta^+}(p) = \frac{i}{p^2 - m_W^2 \xi + i\epsilon}, \quad (I.8)$$

$$D_{\eta^Z}(p) = \frac{i}{p^2 - m_Z^2 \xi + i\epsilon}, \quad (I.9)$$

$$D_{\eta^A}(p) = \frac{i}{p^2 + i\epsilon}. \quad (I.10)$$

Ghosts obey the Fermi statistics, so an additional sign (-1) must be ascribed to ghost loops, as one does for fermion loops.

The propagators of other fields are written as for the higgs field:

$$D_H(p) = \frac{i}{p^2 - m_H^2 + i\epsilon}, \quad (I.11)$$

for fermion fields:

$$\hat{D}_f(p) = \frac{i}{\hat{p} - m_f + i\epsilon}. \quad (I.12)$$

In order to describe numerous three-particle vertices, it is convenient to unify the notations. Let us fix the momenta once and for all, as shown in Fig. 17, and let us denote a vertex by a set of fields in the following order: (ACB) . The Feynman rules for three-particle vertices are then written as follows.

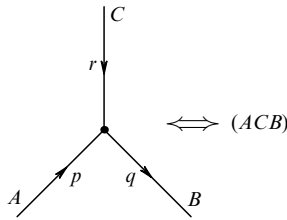


Figure 17. Three-particle vertex.

I.2 Interaction between gauge fields and fermions

$$\begin{aligned} (f A_\mu f): & -ieQ_f \gamma_\mu, \\ (f Z_\mu f): & -i \frac{f}{4} [g_V \gamma_\mu + g_A \gamma_\mu \gamma_5], \\ (v_e W_\mu^- l): & -i \frac{g}{2\sqrt{2}} \gamma_\mu (1 + \gamma_5), \\ (U W_\mu^- D): & -i \frac{g}{2\sqrt{2}} V_{DU} \gamma_\mu (1 + \gamma_5). \end{aligned} \quad (I.13)$$

where Q_f is the charge of the fermion f , T_3^f is the third projection of the isotopic spinor, describing the left-handed component of the fermion f , $g_A = 2T_3^f$, $g_V = 2T_3^f - 4Q_f \sin^2 \theta$; U and D denote any of the quarks with $T_3^f = 1/2$ and $T_3^f = -1/2$, respectively, and V_{DU} is an element of the Kobayashi–Maskawa matrix.

I.3 Interaction of scalar fields with fermions

$$\begin{aligned} (f H f): & -\frac{ig}{2m_W} m_f, \\ (f G^0 f): & -\frac{g}{4} \frac{m_f}{m_W} T_3^f, \\ (U G^- D): & -\frac{i}{2\sqrt{2}} \frac{g}{m_W} [(m_D - m_U) + \gamma_5(m_D + m_U)], \\ (D G^+ U): & -\frac{i}{2\sqrt{2}} \frac{g}{m_W} [(m_D - m_U) - \gamma_5(m_D + m_U)]. \end{aligned} \quad (I.14)$$

I.4 Three-boson interactions

Three gauge bosons:

$$\begin{aligned} (W_\lambda^+ A_\nu W_\mu^+): & ie[(r+q)_\lambda g_{\mu\nu} - (q+p)_\nu g_{\lambda\mu} + (p-r)_\mu g_{\nu\lambda}], \\ (W_\lambda^+ Z_\nu W_\mu^+): & ig \cos \theta [(r+q)_\lambda g_{\mu\nu} \\ & - (q+p)_\nu g_{\lambda\mu} + (p-r)_\mu g_{\nu\lambda}]. \end{aligned} \quad (I.15)$$

Two gauge bosons and one scalar boson (G or H):

$$\begin{aligned} (W_\mu^+ G^- A_\nu): & iem_W g_{\mu\nu}, \\ (W_\mu^+ G^- Z_\nu): & -igm_Z g_{\mu\nu} \sin^2 \theta, \\ (W_\mu^+ H W_\nu^+): & igm_W g_{\mu\nu}, \\ (Z_\mu H Z_\nu): & ig \frac{m_Z^2}{m_W} g_{\mu\nu}. \end{aligned} \quad (I.16)$$

One gauge boson and two scalar bosons (GG or GH):

$$\begin{aligned} (G^+ A_\mu G^+): & -ie(p+q)_\mu, \\ (G^+ Z_\mu G^+): & -ig \frac{\cos 2\theta}{2 \cos \theta} (p+q)_\mu, \\ (G^0 W_\mu^+ G^+): & \frac{1}{2} g(p+q)_\mu, \\ (H W_\mu^+ G^+): & -\frac{1}{2} ig(p+q)_\mu, \\ (G^+ W_\mu^- G^0): & -\frac{1}{2} g(p+q)_\mu, \\ (G^+ W_\mu^- H): & -\frac{1}{2} ig(p+q)_\mu, \\ (H Z_\mu G^0): & \frac{g}{\cos \theta} (p+q)_\mu. \end{aligned} \quad (I.17)$$

Interaction of higgses with goldstones and among themselves:

$$\begin{aligned} (G^+ H G^+): & \frac{i}{2} g \frac{m_H^2}{m_W}, \\ (G^- H G^-): & -\frac{i}{2} g \frac{m_H^2}{m_W}, \\ (G^0 H G^0): & -\frac{i}{2} g \frac{m_H^2}{m_W}, \\ (HHH): & -\frac{3i}{2} g \frac{m_H^2}{m_W}. \end{aligned} \quad (I.18)$$

Interaction between ghosts and gauge fields:

$$\begin{aligned} (\eta^+ A_\mu \eta^+): & -ieq_\mu, \\ (\eta^- A_\mu \eta^-): & ieq_\mu, \\ (\eta^- W_\mu^+ \eta^+): & -ieq_\mu, \\ (\eta^+ W_\mu^- \eta^-): & ieq_\mu, \end{aligned}$$

$$\begin{aligned}
(\eta^+ W_\mu^+ \eta^+) &: ieq_\mu, \\
(\eta^+ W_\mu^- \eta^-) &: -ieq_\mu, \\
(\eta^+ Z_\mu \eta^+) &: -ig \cos \theta q_\mu, \\
(\eta^- Z_\mu \eta^-) &: ig \cos \theta q_\mu, \\
(\eta^- W_\mu^+ \eta^Z) &: -ig \cos \theta q_\mu, \\
(\eta^+ W_\mu^- \eta^Z) &: ig \cos \theta q_\mu, \\
(\eta^Z W_\mu^+ \eta^+) &: ig \cos \theta q_\mu, \\
(\eta^Z W_\mu^- \eta^-) &: -ig \cos \theta q_\mu.
\end{aligned} \tag{I.19}$$

Interaction of ghosts with a higgs or a goldstone:

$$\begin{aligned}
(\eta^- H \eta^-) &: -\frac{i}{2} g \xi m_W, \\
(\eta^- G^0 \eta^-) &: -\frac{i}{2} g \xi m_W, \\
(\eta^+ G^0 \eta^+) &: \frac{i}{2} g \xi m_W, \\
(\eta^+ G^+ \eta^+) &: -ie \xi m_W, \\
(\eta^Z G^+ \eta^+) &: -\frac{i}{2} g \frac{\cos 2\theta}{\cos \theta} \xi m_W, \\
(\eta^- G^+ \eta^Z) &: \frac{i}{2} g \xi m_Z.
\end{aligned} \tag{I.20}$$

I.5 Four-boson interactions

To describe four-boson vertices, we introduce the notation $(ABCD)$, see Fig. 18.

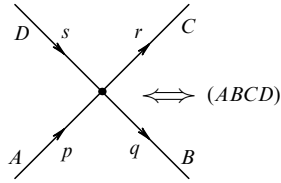


Figure 18. Four-particle vertex.

In this notation, the interactions of four vector bosons take the form

$$\begin{aligned}
(W_\lambda^+ W_\mu^- W_\nu^+ W_\rho^-) &: ig^2 [2g_{\lambda\nu} g_{\mu\rho} - g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho}], \\
(W_\lambda^+ W_\mu^- A_\nu A_\rho) &: -ie^2 [2g_{\nu\rho} g_{\mu\lambda} - g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}], \\
(W_\lambda^+ W_\mu^- Z_\nu Z_\rho) &: -ig^2 \cos^2 \theta [2g_{\nu\rho} g_{\mu\lambda} - g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}], \\
(W_\lambda^+ W_\mu^- A_\nu Z_\rho) &: -ieg \cos \theta [2g_{\nu\rho} g_{\mu\lambda} - g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}].
\end{aligned} \tag{I.21}$$

Interaction between two vector bosons and HH, GG or HG:

$$\begin{aligned}
(W_\mu^+ W_\nu^- HH) &: \frac{i}{2} g^2 g_{\mu\nu}, \\
(W_\mu^+ W_\nu^- G^0 G^0) &: \frac{i}{2} g^2 g_{\mu\nu}, \\
(W_\mu^+ W_\nu^- G^- G^+) &: \frac{i}{2} g^2 g_{\mu\nu}, \\
(A_\mu A_\nu G^- G^+) &: 2ie^2 g_{\mu\nu}, \\
(Z_\mu Z_\nu HH) &: \frac{i}{2} g^2 \sec^2 \theta g_{\mu\nu},
\end{aligned}$$

$$\begin{aligned}
(Z_\mu Z_\nu G^0 G^0) &: \frac{i}{2} g^2 \sec^2 \theta g_{\mu\nu}, \\
(Z_\mu Z_\nu G^- G^+) &: \frac{i}{2} g^2 \sec^2 \theta \cos^2 2\theta g_{\mu\nu}, \\
(A_\mu W_\nu^+ G^+ H) &: \frac{i}{2} eg g_{\mu\nu}, \\
(A_\mu W_\nu^+ G^- H) &: \frac{i}{2} eg g_{\mu\nu}, \\
(A_\mu W_\nu^+ G^+ G^0) &: -\frac{1}{2} eg g_{\mu\nu}, \\
(A_\mu W_\nu^- G^- G^0) &: \frac{1}{2} eg g_{\mu\nu}, \\
(A_\mu Z_\nu G^- G^+) &: i eg \sec \theta \cos 2\theta g_{\mu\nu}, \\
(Z_\mu W_\nu^+ G^+ H) &: \frac{i}{2} g^2 \sec \theta \left(\frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu}, \\
(Z_\mu W_\nu^- G^- H) &: \frac{i}{2} g^2 \sec \theta \left(\frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu}, \\
(Z_\mu W_\nu^+ G^+ G^0) &: -\frac{1}{2} g^2 \sec \theta \left(\frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu}, \\
(Z_\mu W_\nu^- G^- G^0) &: \frac{1}{2} g^2 \sec \theta \left(\frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu}.
\end{aligned} \tag{I.22}$$

Interactions of GGGG, HHHH or GGHH :

$$\begin{aligned}
(G^+ G^- G^- G^+) &: -\frac{i}{2} g^2 \frac{m_H^2}{m_W^2}, \\
(G^+ G^- G^0 G^0) &: -\frac{i}{4} g^2 \frac{m_H^2}{m_W^2}, \\
(G^+ G^- HH) &: -\frac{i}{4} g^2 \frac{m_H^2}{m_W^2}, \\
(HHHH) &: -\frac{3}{4} ig^2 \frac{m_H^2}{m_W^2}, \\
(G^0 G^0 G^0 G^0) &: -\frac{3}{4} ig^2 \frac{m_H^2}{m_W^2}, \\
(G^0 G^0 HH) &: -\frac{i}{4} g^2 \frac{m_H^2}{m_W^2}.
\end{aligned} \tag{I.23}$$

I.6 Regularization of Feynman integrals

Integrals corresponding to diagrams with loops formally diverge and thus need regularization. Note that there does not exist yet a consistent regularization of electroweak theory in all loops. A dimensional regularization can be used in the first several loops; this corresponds to a transition to a D -dimensional spacetime in which the following finite expression is assigned to the diverging integrals:

$$\int \frac{d^D p}{\mu^{D-4}} \frac{(p^2)^s}{(p^2 + m^2)^\alpha} = \frac{\pi^{D/2}}{\Gamma(D/2)} \frac{\Gamma(D/2 + s) \Gamma(\alpha - D/2 - s)}{\Gamma(\alpha)} \times \frac{(m^2)^{D/2 - \alpha + s}}{\mu^{D-4}}, \tag{I.24}$$

where μ is a parameter with mass dimension, introduced to conserve the dimension of the original integral.

This formula holds in the range of convergence of the integral. In the range of divergence, a formal expression (I.24) is interpreted as the analytical continuation. Obviously, the

integral allows a shift in integration variable in the convergence range as well. Therefore, a shift $p \rightarrow p + q$ for arbitrary D can also be done in (I.24). This factor is decisive in proving the gauge invariance of dimensional regularization.

At $D = 4$ the integrals in (I.24) contain a pole term

$$\Delta = \frac{2}{4-D} + \ln 4\pi - \gamma - \ln \frac{m^2}{\mu^2}, \quad (\text{I.25})$$

where $\gamma = 0.577\dots$ is the Euler constant. Choice of constant terms in (I.25) is a matter of convention.

The algebra of γ -matrices in the D -dimensional space is defined by the relations

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu} \times I, \quad (\text{I.26})$$

$$g_{\mu\mu} = D, \quad (\text{I.27})$$

$$\gamma_\mu \gamma_\nu \gamma_\mu = (2-D)\gamma_\nu, \quad (\text{I.28})$$

where I is the identity matrix.

As for the dimensionality of spinors, different approaches can be chosen in the continuation to the D -dimensional space. One possibility is to assume that the γ matrices are 4×4 matrices, so that

$$\text{tr } I = 4. \quad (\text{I.29})$$

The D -dimensional regularization creates difficulties when one has to define the absolutely antisymmetric tensor and (or) γ_5 matrix. For calculations in several first loops, a formal definition of γ_5 ,

$$\gamma_5 \gamma_\mu + \gamma_\mu \gamma_5 = 0, \quad (\text{I.30})$$

$$\gamma_5^2 = I, \quad (\text{I.31})$$

does not lead to contradictions.

Thus, the amplitudes of physical processes, once they are expressed in terms of bare charges and bare masses, contain pole terms of the order of $(D-4)^{-1}$.

If we eliminate bare quantities and express some physical observables in terms of other physical observables, then all pole terms cancel out. The general property of renormalisability guarantees this cancellation. (We have verified this cancellation directly in Ref. [42].) This renormalisation procedure is employed in this review.

In order to avoid divergences in intermediate expressions, one can agree to subtract from each Feynman integral the pole terms of the order of $(4-D)^{-1}$, since they will cancel out anyway in the final expressions. Depending on which constant terms (in addition to pole terms) are subtracted from the diagrams, different subtraction schemes arise: the $\overline{\text{MS}}$ scheme corresponds to subtracting the universal combination

$$\frac{2}{4-D} - \gamma + \ln 4\pi.$$

II. Relation between $\bar{\alpha}$ and $\alpha(0)$

We begin with the following famous relation of quantum electrodynamics [75]:

$$\alpha(q^2) = \frac{\alpha(0)}{1 + \Sigma_\gamma(q^2)/q^2 - \Sigma'_\gamma(0)}. \quad (\text{II.1})$$

Here the fine structure constant $\alpha \equiv \alpha(0)$ is a physical quantity. It can be measured as a residue of the Coulomb pole $1/q^2$ in the scattering amplitude of charged particles. As for the running coupling constant $\alpha(q^2)$, it can be measured from the scattering of particles with large masses m at low momentum transfer: $m \gg \sqrt{|q^2|}$. In the Standard Model we have the Z -boson, and the contribution of the photon cannot be identified unambiguously if $q^2 \neq 0$. Therefore, the definition of the running constant $\alpha(q^2)$ becomes dependent on convention and on details of calculations.

At $q^2 = m_Z^2$, the contribution of W -bosons to $\bar{\alpha} \equiv \alpha(m_Z^2)$ is not large, so it is convenient to make use of the definition accepted in QED:

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha}, \quad (\text{II.2})$$

where

$$\begin{aligned} \delta\alpha &= -\Pi_\gamma(m_Z^2) + \Sigma'_\gamma(0), \\ \Pi_\gamma(m_Z^2) &= \frac{1}{m_Z^2} \Sigma_\gamma(m_Z^2). \end{aligned} \quad (\text{II.3})$$

One-loop expression for the self-energy of the photon can be rewritten as [76]:

$$\begin{aligned} \Sigma_\gamma(s) &= \frac{\alpha}{3\pi} \sum_f N_c^f Q_f^2 \left[s\Delta_f + (s + 2m_f^2) F(s, m_f, m_f) - \frac{3}{2} \right] - \\ &\quad - \frac{\alpha}{4\pi} [3s\Delta_W + (3s + 4m_W^2) F(s, m_W, m_W)], \end{aligned} \quad (\text{II.4})$$

where $s \equiv q^2$, the subscript f denotes fermions, the sum \sum_f runs through lepton and quark flavours, and N_c^f is the number of colours. The contribution of fermions to $\Sigma_\gamma(q^2)$ is independent of gauge. The last term in Eqn (II.4) refers to the gauge-dependent contribution of W -bosons; the 't Hooft–Feynman gauge was used in Eqn (II.4).

The singular term Δ_i is:

$$\Delta_i = \frac{1}{\varepsilon} - \gamma + \ln 4\pi - \ln \frac{m_i^2}{\mu^2}, \quad (\text{II.5})$$

where $2\varepsilon = 4 - D$ (D is the variable dimension of spacetime, $\varepsilon \rightarrow 0$), $\gamma = -\Gamma'(1) = 0.577\dots$ is the Euler constant and μ is an arbitrary parameter. Both $1/\varepsilon$ and μ vanish in relations between observables.

The function $F(s, m_1, m_2)$ is defined by the contribution to self-energy of a scalar particle at $q^2 = s$, owing to a loop with two scalar particles (with masses m_1 and m_2) and with the coupling constant equal to unity:

$$\begin{aligned} F(s, m_1, m_2) &= -1 + \frac{m_1^2 - m_2^2}{m_1^2 - m_2^2} \log \frac{m_1}{m_2} \\ &\quad - \int_0^1 dx \log \frac{x^2 s - x(s + m_1^2 - m_2^2) + m_1^2 - i\varepsilon}{m_1 m_2}. \end{aligned} \quad (\text{II.6})$$

The function F is normalised in such a way that it vanishes at $q^2 = 0$, which corresponds to subtracting the self-energy at $q^2 = 0$:

$$F(0, m_1, m_2) = 0. \quad (\text{II.7})$$

The following formula holds for $m_1 = m_2 = m$:

$$F(s, m, m) \equiv F(\tau) = \begin{cases} 2 \left[1 - \sqrt{4\tau - 1} \arcsin \frac{1}{\sqrt{4\tau}} \right], & 4\tau > 1, \\ 2 \left[1 - \sqrt{1 - 4\tau} \ln \frac{1 + \sqrt{1 - 4\tau}}{\sqrt{4\tau}} \right], & 4\tau < 1, \end{cases}$$

where $\tau = m^2/s$.

To calculate the contributions of light fermions, the t-quark and the W-boson to $\delta\alpha$, we need the asymptotics $F(\tau)$ for small and large τ :

$$F(\tau) \approx \ln \tau + 2 + \dots, \quad |\tau| \ll 1, \quad (\text{II.8})$$

$$F(\tau) \approx \frac{1}{6\tau} + \frac{1}{60\tau^2} + \dots, \quad |\tau| \gg 1, \quad (\text{II.9})$$

$$F'(s, m, m) = \frac{d}{ds} F(s, m, m) \stackrel{s \rightarrow 0}{\approx} \frac{1}{m^2} \left[\frac{1}{6} + \frac{1}{30\tau} \right]. \quad (\text{II.10})$$

As a result we obtain

$$\begin{aligned} \Pi_\gamma(m_Z^2) &\equiv \frac{\Sigma_\gamma(m_Z^2)}{m_Z^2} = \frac{\alpha}{3\pi} \sum_8 N_c^f Q_f^2 \left(A_Z + \frac{5}{3} \right) \\ &\quad + \frac{\alpha}{\pi} Q_f^2 \left[A_t + (1 + 2t) F(t) - \frac{1}{3} \right] \\ &\quad - \frac{\alpha}{4\pi} [3A_W + (3 + 4c^2) F(c^2)], \end{aligned} \quad (\text{II.11})$$

where $t = m_t^2/m_Z^2$, and

$$\Sigma'_\gamma(0) = \frac{\alpha}{3\pi} \sum_8 N_c^f Q_f^2 A_f - \frac{\alpha}{4\pi} \left(3A_W + \frac{2}{3} \right), \quad (\text{II.12})$$

$$\begin{aligned} \delta\alpha &= \frac{\alpha}{\pi} \left\{ \sum_8 \frac{N_c^f Q_f^2}{3} \left(\ln \frac{m_Z^2}{m_f^2} - \frac{5}{3} \right) \right. \\ &\quad \left. - Q_t^2 \left[(1 + 2t) F(t) - \frac{1}{3} \right] + \left[\left(\frac{3}{4} + c^2 \right) F(c^2) - \frac{1}{6} \right] \right\}. \end{aligned} \quad (\text{II.13})$$

Therefore, $\delta\alpha$ is found as a sum of four terms,

$$\delta\alpha = \delta\alpha_1 + \delta\alpha_h + \delta\alpha_t + \delta\alpha_W, \quad (\text{II.14})$$

$$\delta\alpha_1 = \frac{\alpha}{3\pi} \sum_3 \left[\ln \frac{m_Z^2}{m_i^2} - \frac{5}{3} \right] = 0.03141, \quad (\text{II.15})$$

$$\delta\alpha_t \simeq -\frac{\alpha}{\pi} \frac{4}{45} \left(\frac{m_Z}{m_t} \right)^2 = -0.00005(1), \quad (\text{II.16})$$

where we have used $m_t = 175 \pm 10$ GeV. Note that $\delta\alpha_t$ is negligible and has the antiscreening sign (the screening of the t-quark loops in QED begins at $q^2 \gg m_t^2$, while in our case $q^2 = m_Z^2 < m_t^2$).

Finally, the W loop gives

$$\begin{aligned} \delta\alpha_W &= \frac{\alpha}{2\pi} \left[(3 + 4c^2) \left(1 - \sqrt{4c^2 - 1} \arcsin \frac{1}{2c} \right) - \frac{1}{3} \right] \\ &= 0.00050. \end{aligned} \quad (\text{II.17})$$

The value of $\delta\alpha_W$ depends on gauge [77]; here we give the result of calculations in the 't Hooft–Feynman gauge. Traditionally, the definition of $\bar{\alpha}$ takes into account the

contributions of leptons and five light quarks; the terms $\delta\alpha_l$ and $\delta\alpha_W$ are taken into account in the electroweak radiative corrections. In our approach, these terms give the corrections $\delta_1 V_i$.

III. Summary of the results for $\bar{\alpha}$

Among the three input parameters $\bar{\alpha}$, G_μ and m_Z , the first one has the maximum uncertainty; this uncertainty leads to the uncertainty ± 5 GeV in the value of the t-quark mass extracted from the measurements of m_W and the decay parameters of the Z-boson. According to Appendix II,

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha}, \quad \delta\alpha = \delta\alpha_l + \delta\alpha_h,$$

$$\delta\alpha_l = 0.0314.$$

In Ref. [26] (Burkhardt et al.) $\delta\alpha_h$ was calculated by substituting experimental data on $\sigma_{e^+e^- \rightarrow \text{hadrons}}$ into the dispersion integral at $\sqrt{s} < 40$ GeV and the parton model result at $\sqrt{s} > 40$ GeV:

$$\delta\alpha_h^{\text{[Burkh]}} = 0.0282(9), \quad \bar{\alpha}^{\text{[Burkh]}} = [128.87(12)]^{-1}. \quad (\text{III.1})$$

In Ref. [26] (Vysotsky et al.) it was pointed out that the simplest model (the lightest vector meson + the QCD-improved parton continuum in each flavour channel) produces a surprisingly close result:

$$\delta\alpha_h^{\text{[Vys]}} = 0.0280(4), \quad \bar{\alpha}^{\text{[Vys]}} = [128.90(6)]^{-1}. \quad (\text{III.2})$$

The model with infinite number of poles (see Ref. [26], Geshkenbein and co-auth.) yields the determination of $\delta\alpha_h$ with very high accuracy:

$$\delta\alpha_h^{\text{[Gesh]}} = 0.0275(2), \quad \bar{\alpha}^{\text{[Gesh]}} = [128.96(3)]^{-1}. \quad (\text{III.3})$$

A recent analysis of experimental data (see Ref. [26], Swartz and Ref. [26], Martin and co-auth.) yielded considerably lower values of $\delta\alpha_h$:

$$\delta\alpha_h^{\text{[Sw]}} = 0.0265(8), \quad \bar{\alpha}^{\text{[Sw]}} = [129.10(12)]^{-1}, \quad (\text{III.4})$$

$$\delta\alpha_h^{\text{[Ma]}} = 0.0273(4), \quad \bar{\alpha}^{\text{[Ma]}} = [128.99(6)]^{-1}. \quad (\text{III.5})$$

In this review we make use of the results of a recent analysis (see Ref. [26], Eidelman and co-auth.),

$$\delta\alpha_h^{\text{[Eid]}} = 0.0280(7), \quad \bar{\alpha}^{\text{[Eid]}} = [128.896(90)]^{-1}. \quad (\text{III.6})$$

IV. How $\alpha_W(q^2)$ and $\alpha_Z(q^2)$ ‘crawl’

The effect of ‘running’ of electromagnetic coupling constants $\alpha(q^2)$ (logarithmic dependence of the effective charge on momentum transfer q^2) is known for more than four decades [75]. In contrast to $\alpha(q^2)$, the effective constants of W- and Z-bosons $\alpha_W(q^2)$ and $\alpha_Z(q^2)$ in the region $0 < q^2 \lesssim m_Z^2$ ‘crawl’ rather than run [78].

If we define the effective gauge coupling constant $g^2(q^2)$ in terms of the bare charge g_0^2 and the bare mass m_0 , and sum up the geometric series with the self-energy $\Sigma(q^2)$ inserted in the gauge boson propagator, this gives the expression

$$g^2(q^2) = g_0^2 \left[1 + g_0^2 \frac{\Sigma(q^2) - \Sigma(m_Z^2)}{q^2 - m_Z^2} \right]^{-1}, \quad (\text{IV.1})$$

here m is the physical mass, and $\Sigma(q^2)$ contains the contribution of fermions only, since loops with W -, Z - and H -bosons do not contain large logarithms in the region $|q^2| \leq m_Z^2$.

The bare coupling constant in the difference $g^{-2}(q^2) - g^{-2}(0)$ is eliminated, which gives a finite expression. The result is

$$\frac{1}{\alpha_Z(q^2)} - \frac{1}{\alpha_Z(0)} = b_Z F(x), \quad x = \frac{q^2}{m_Z^2}, \quad (\text{IV.2})$$

$$\frac{1}{\alpha_W(q^2)} - \frac{1}{\alpha_W(0)} = b_W F(y), \quad y = \frac{q^2}{m_Z^2}, \quad (\text{IV.3})$$

$$F(x) = \frac{x}{1-x} \ln |x|. \quad (\text{IV.4})$$

If $x \gg 1$, Eqns (IV.2) and (IV.3) define the logarithmic running of charges owing to leptons and quarks, and b_Z and b_W represent the contribution of fermions to the first coefficient of the Gell-Mann–Low function:

$$b_Z = \frac{1}{48\pi} \left\{ N_u 3 \left[1 + \left(1 - \frac{8}{3} s^3 \right)^2 \right] + N_d 3 \left[1 + \left(-1 + \frac{4}{3} s^2 \right)^2 \right] + N_l \left[2 + (1 + (1 - 4s^2)^2) \right] \right\}, \quad (\text{IV.5})$$

$$b_W = \frac{1}{16\pi} (6N_q + 2N_l),$$

where $N_{u,d,q,l}$ are the numbers of quarks and leptons with masses that are considerably lower than $\sqrt{q^2}$.

For $q^2 \lesssim m_Z^2$, the numerical values of the coefficients $b_{Z,W}$ are [78]

$$b_Z \approx 0.195, \quad b_W \approx 0.239.$$

The massive propagator $(q^2 - m^2)^{-1}$ in Eqn (IV.1) greatly suppresses the running of $\alpha_W(q^2)$ and $\alpha_Z(q^2)$. Thus, according to Eqns (IV.2) and (IV.3), the constant $\alpha_Z(q^2)$ grows by 0.85% from $q^2 = 0$ to $q^2 = m_Z^2$,

$$[\alpha_Z(m_Z^2)]^{-1} = 22.905, \quad (\text{IV.6})$$

$$[\alpha_Z(m_Z^2)]^{-1} - [\alpha_Z(0)]^{-1} = -0.195,$$

and the constant $\alpha_W(q^2)$ grows by 0.95%,

$$[\alpha_W(m_Z^2)]^{-1} = 28.74, \quad (\text{IV.7})$$

$$[\alpha_W(m_Z^2)]^{-1} - [\alpha_W(0)]^{-1} = -0.272,$$

while the electromagnetic constant $\alpha(q^2)$ increases by 6.34%:

$$[\alpha(m_Z^2)]^{-1} - [\alpha(0)]^{-1} = 128.90 - 137.04 = -8.14. \quad (\text{IV.8})$$

With the accuracy indicated above, we can thus assume

$$\alpha_Z(m_Z^2) \approx \alpha_Z(0), \quad \alpha_W(m_Z^2) \approx \alpha_W(0). \quad (\text{IV.9})$$

At the same time, $\alpha(m_Z^2)$ differs greatly from $\alpha(0)$; therefore the latter has no connection to the electroweak physics but only to the purely electromagnetic physics.

V. Relation between $\bar{\alpha}$, G_μ , m_Z and the bare quantities

The bare quantities are marked by the subscript ‘0’. In the electroweak theory, three bare charges e_0 , f_0 and g_0 that describe the interactions of γ , Z and W are related by a single constraint:

$$\left(\frac{e_0}{g_0} \right)^2 + \left(\frac{g_0}{f_0} \right)^2 = 1. \quad (\text{V.1})$$

The bare masses of the vector bosons are defined by the bare vacuum expectation value of the higgs field η_0 :

$$m_{Z0} = \frac{1}{2} f_0 \eta_0, \quad m_{W0} = \frac{1}{2} g_0 \eta_0. \quad (\text{V.2})$$

The fine structure constant $\alpha = e^2/4\pi$ is related to the bare charge e_0 by the formula

$$\alpha \equiv \alpha(q^2 = 0) = \frac{e_0^2}{4\pi} \left[1 - \Sigma'_\gamma(0) - 2 \frac{s}{c} \frac{\Sigma_{\gamma Z}(0)}{m_Z^2} \right], \quad (\text{V.3})$$

where $\Sigma'(0) = \lim_{q^2 \rightarrow 0} \Sigma(q^2)/q^2$. In the Feynman gauge

$$\Sigma_{\gamma Z}(0) \approx -\frac{\alpha}{2\pi} \frac{m_W^2}{cs} \frac{1}{\varepsilon},$$

where the dimension of spacetime is $D = 4 - 2\varepsilon$. In the unitary gauge $\Sigma_{\gamma Z}(0) = 0$.

The simplest way to verify the presence of the term $2(s/c)\Sigma_{\gamma Z}(0)/m_Z^2$ is to consider the interaction of a photon with the right-handed electron e_R . Note that in this case there are no weak vertex corrections due to the W -boson exchange. (Note also that the left-handed neutrino remains neutral even when loop corrections are taken into account, since the diagram with the γ - Z - $\bar{\nu}_L \nu_L$ interaction is compensated for by the vertex diagram with the W -exchange).

Our first basic equation is the renormalisation group improved relation between $\bar{\alpha} = \alpha(q^2 = m_Z^2)$ and α_0 :

$$\bar{\alpha} = \alpha_0 \left[1 - \Pi_\gamma(m_Z^2) - 2 \frac{s}{c} \Pi_{\gamma Z}(0) \right], \quad (\text{V.4})$$

where $\Pi_\gamma(q^2) = \Sigma_\gamma(q^2)/m_Z^2$, $\Pi_{\gamma Z}(q^2) = \Sigma_{\gamma Z}(q^2)/m_Z^2$.

The second basic equation is:

$$m_Z^2 = m_{Z0}^2 [1 - \Pi_Z(m_Z^2)] = \frac{m_{W0}^2}{c^2} [1 - \Pi_Z(m_Z^2)]. \quad (\text{V.5})$$

A similar equation holds for m_W^2 :

$$m_W^2 = m_{W0}^2 [1 - \Pi_W(m_W^2)], \quad (\text{V.6})$$

where $\Pi_i(q^2) = \Sigma_i(q^2)/m_i^2$, $i = W, Z$.

Finally, the third basic equation is

$$G_\mu = \frac{g_0^2}{4\sqrt{2}m_{W0}^2} [1 + \Pi_W(0) + D], \quad (\text{V.7})$$

where $\Pi_W(0) = \Sigma_W(0)/m_W^2$ comes from the propagator of W, while D is the contribution of the box and the vertex diagrams (minus the electromagnetic corrections to the four-fermion interaction) to the muon decay amplitude. According to Sirlin (Ref. [23])

$$D = \frac{\bar{\alpha}}{4\pi s^2} \left(6 + \frac{7-4s^2}{2s^2} \ln c^2 + 4A_W \right), \quad (\text{V.8})$$

where

$$A_W \equiv A(m_W) = \frac{2}{4-D} + \ln 4\pi - \gamma - \ln \frac{m_W^2}{\mu^2}. \quad (\text{V.9})$$

VI. The radiators R_{Aq} and R_{Vq}

For decays to light quarks $q = u, d, s$, we neglect the quark masses and take into account the gluon exchanges in the final state up to terms of the order of α_s^3 [45–48], and also one-photon exchange in the final state, and the interference of the photon and the gluon exchanges [49]. These corrections are slightly different for the vector and the axial channels.

For decays to quarks we have

$$\Gamma_q = \Gamma(Z \rightarrow q\bar{q}) = 12[g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}] \Gamma_0, \quad (\text{VI.1})$$

where the factors $R_{A,V}$ are responsible for the interaction in the final state (in our previous papers, we used letter G instead of R). The results presented in Refs [45–48] are

$$\begin{aligned} R_{Vq} = & 1 + \frac{\hat{\alpha}_s}{\pi} + \frac{3}{4} Q_q^2 \frac{\bar{\alpha}}{\pi} - \frac{1}{4} Q_q^2 \frac{\bar{\alpha}}{\pi} \frac{\hat{\alpha}_s}{\pi} \\ & + \left[1.409 + (0.065 + 0.015 \ln t) \frac{1}{t} \right] \left(\frac{\hat{\alpha}_s}{\pi} \right)^2 \\ & - 12.77 \left(\frac{\hat{\alpha}_s}{\pi} \right)^3 + 12 \frac{\hat{m}_q^2}{m_Z^2} \frac{\hat{\alpha}_s}{\pi} \delta_{vm}, \end{aligned} \quad (\text{VI.2})$$

$$\begin{aligned} R_{Aq} = & R_{Vq} - 2T_{3q} \left[I_2(t) \left(\frac{\hat{\alpha}_s}{\pi} \right)^2 + I_3(t) \left(\frac{\hat{\alpha}_s}{\pi} \right)^3 \right] \\ & - 12 \frac{\hat{m}_q^2}{m_Z^2} \frac{\hat{\alpha}_s}{\pi} \delta_{vm} - 6 \frac{\hat{m}_q^2}{m_Z^2} \delta_{am}^1 - 10 \frac{\hat{m}_q^2}{m_t^2} \left(\frac{\hat{\alpha}_s}{\pi} \right)^2 \delta_{am}^2, \end{aligned} \quad (\text{VI.3})$$

where \hat{m}_q is the running quark mass (see below),

$$\delta_{vm} = 1 + 8.7 \frac{\hat{\alpha}_s}{\pi} + 45.15 \left(\frac{\hat{\alpha}_s}{\pi} \right)^2, \quad (\text{VI.4})$$

$$\delta_{am}^1 = 1 + 3.67 \frac{\hat{\alpha}_s}{\pi} + (11.29 - \ln t) \left(\frac{\hat{\alpha}_s}{\pi} \right)^2, \quad (\text{VI.5})$$

$$\delta_{am}^2 = \frac{8}{81} + \frac{\ln t}{54}, \quad (\text{VI.6})$$

$$I_2(t) = -3.083 - \ln t + \frac{0.086}{t} + \frac{0.013}{t^2}, \quad (\text{VI.7})$$

$$I_3(t) = -15.988 - 3.722 \ln t + 1.917 \ln^2 t, \quad (\text{VI.8})$$

$$t = \frac{m_t^2}{m_Z^2}.$$

Terms of the order of $(\hat{\alpha}_s/\pi)^3$ caused by the diagrams with three gluons in intermediate state were calculated in Ref. [79]. For R_{Vq} they are numerically very small ($\sim 10^{-5}$); for this reason, we dropped them from Eqn (VI.2).

For the $Z \rightarrow b\bar{b}$ decay, the b-quark mass is not negligible; it reduces Γ_b by about 1 MeV ($\sim 0.5\%$). The gluon corrections result in a replacement of the pole mass $m_b \approx 4.7$ GeV by the running mass, the virtuality being m_Z : $m_b \rightarrow \hat{m}_b(m_Z)$. We express $\hat{m}_b(m_Z)$ in terms of m_b , $\hat{\alpha}_s(m_Z)$ and $\hat{\alpha}_s(m_b)$ using standard two-loop equations in the $\overline{\text{MS}}$ scheme (see Ref. [50]).

For the $Z \rightarrow c\bar{c}$ decay, the running mass $\hat{m}_c(m_Z)$ is of the order of 0.5 GeV and the corresponding contribution to Γ_c is of the order of 0.05 MeV. We have included this infinitesimal term in the LEPTOP code, since it is taken into account in other codes (see, for example, Ref. [18]).

We need to remark in connection with Γ_c that the term $I_2(t)$, given by Eqn (VI.7), contains interference terms of the order of $(\hat{\alpha}_s/\pi)^2$. These terms are related to three types of final states: one quark pair, a quark pair and a gluon, two quark pairs. This last contribution comes to about 5% of I_2 and is infinitesimally small at the currently achievable experimental accuracy. Nevertheless, in principle these terms require special consideration, especially if these quark pairs are of different flavours, for example, $b\bar{b}c\bar{c}$. Such mixed quark pairs must be discussed separately.

Note that $\hat{\alpha}_s$ stands for the strong interaction constant in the $\overline{\text{MS}}$ subtraction scheme, with $\mu^2 = m_Z^2$.

VII. Derivation of formulas for the asymmetries

Asymmetry in the processes $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ is calculated with the masses of e and f neglected in comparison with the Z-boson mass. (Mass corrections for $f = b$ will be taken into account below). The amplitude (20) of the interactions of the Z-boson with massless fermions $f\bar{f}$ can be conveniently rewritten in the form

$$M(Z \rightarrow f\bar{f}) = \frac{1}{2} \bar{f} Z_\alpha [g_L^f j_\alpha^L + g_R^f j_\alpha^R], \quad (\text{VII.1})$$

where $\bar{f}^2 = 4\sqrt{2} G_\mu m_Z^2$,

$$j_\alpha^{L,R} = \bar{\psi}_{L,R} \gamma_\alpha \psi_{L,R},$$

$$\psi_{L,R} = \frac{1}{2} (1 \pm \gamma_5) \psi,$$

$$g_{L,R}^f = g_{Vf} \pm g_{Af}.$$

The chirality is a conserved quantum number for massless fermions (anomalies do not yet manifest themselves in the approximations we deal with here) and coincides with a fermion's helicity up to the sign.

Therefore, the pairs $e_L^- e_L^+$ and $e_R^- e_R^+$ do not transform into the Z-boson at all, and the pairs $e_L^- e_R^+$ and $e_R^- e_L^+$ create a Z-boson with the polarisation ± 1 , respectively (along the positron beam). The scattering amplitudes thus have the form

$$T(e_{L,R}^- e^+ \rightarrow f_{L,R} \bar{f}) = g_{L,R}^e g_{L,R}^f T_0 (1 + \cos \theta),$$

$$T(e_{L,R}^- e^+ \rightarrow f_{R,L} \bar{f}) = g_{L,R}^e g_{R,L}^f T_0 (1 - \cos \theta), \quad (\text{VII.2})$$

where coefficient T_0 is unimportant at the moment [it can be reconstructed from Eqn (VII.1)], and θ is the angle between the momenta of e^- and f . The sign in front of $\cos \theta$ is chosen for the helicity to be conserved in forward and backward scattering.

Once the form of the amplitude (VII.2) is known, all asymmetries are immediately found.

(a) Left–right asymmetry A_{LR} is defined as the ratio

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R},$$

where $\sigma_{L,R} = \sigma(e_{L,R}^- e^+ \rightarrow f\bar{f})$. Hence

$$A_{LR} = \frac{(g_L^e)^2 - (g_R^e)^2}{(g_L^e)^2 + (g_R^e)^2} \equiv A_e. \quad (\text{VII.3})$$

(b) Longitudinal polarisation $P_\tau(\cos \theta)$ is defined as the ratio of the difference to the sum of differential cross sections, $(d\sigma/d\theta)_{R,L} = (d\sigma/d\theta)(e\bar{e} \rightarrow \tau_{R,L} \bar{\tau})$:

$$P_\tau(\cos \theta) = \frac{(d\sigma/d\theta)_R - (d\sigma/d\theta)_L}{(d\sigma/d\theta)_R + (d\sigma/d\theta)_L}, \quad (\text{VII.4})$$

where

$$\left(\frac{d\sigma}{d\theta}\right)_R = \frac{1}{2m_Z^2} |T_0|^2 (g_R^e)^2 [(g_R^e)^2 (1 + \cos \theta)^2 + (g_L^e)^2 (1 - \cos \theta)^2],$$

$$\left(\frac{d\sigma}{d\theta}\right)_L = \frac{1}{2m_Z^2} |T_0|^2 (g_L^e)^2 [(g_L^e)^2 (1 + \cos \theta)^2 + (g_R^e)^2 (1 - \cos \theta)^2]. \quad (\text{VII.5})$$

Substituting Eqn (VII.5) into the definition (VII.4), we obtain

$$P_\tau(\cos \theta) = -\frac{A_\tau(1 + \cos^2 \theta) + 2A_e \cos \theta}{1 + \cos^2 \theta + 2A_e A_\tau \cos \theta}, \quad (\text{VII.6})$$

where A_e and A_τ are defined according to Eqn (VII.3). The longitudinal polarisation P_τ , averaged over directions of τ -leptons, is defined as the following ratio of the total cross sections

$$\sigma_{L,R} = \int_{-1}^1 d\cos \theta \left(\frac{d\sigma}{d\theta}\right)_{L,R}$$

and has the form:

$$P_\tau = \frac{\sigma_R^\tau - \sigma_L^\tau}{\sigma_R^\tau + \sigma_L^\tau} = -\frac{\int_{-1}^1 d\cos \theta [A_\tau(1 + \cos^2 \theta) + 2A_e \cos \theta]}{\int_{-1}^1 d\cos \theta [1 + \cos^2 \theta + 2A_e A_\tau \cos \theta]} = -A_\tau. \quad (\text{VII.7})$$

(c) Forward–backward asymmetry A_{FB}^f is calculated more simply in terms of $g_{A,V}$. The squared matrix element of the

process $e\bar{e} \rightarrow f\bar{f}$ is proportional to

$$|M|^2 \propto \left\{ (g_{Ae}^2 + g_{Ve}^2) [(g_{Af}^2 + g_{Vf}^2)(1 + v^2 \cos^2 \theta) + (g_{Af}^2 - g_{Vf}^2)(1 - v^2)] + \frac{1}{2} (g_{Ve} g_{Ae} g_{Vf} g_{Af}) v \cos \theta \right\}, \quad (\text{VII.8})$$

where θ is the scattering angle and $v = 1 - 4m_f^2/m_Z^2$ is the velocity of fermion f . This immediately implies that

$$A_{FB}^f = \frac{3}{4} A_e \left[\frac{2g_{Af} g_{Vf} v}{g_{Af}^2 v^2 + g_{Vf}^2 (3 - v^2)/2} \right]. \quad (\text{VII.9})$$

The mass m_f is negligible in all channels with the exception of $f = b$, where the nonzero mass produces effects of the order of 2×10^{-3} . Gluon corrections in the final state (see Appendix II) replace the pole mass $m_b \approx 4.7$ GeV in Eqns (VII.8), (VII.9) by the running mass at the m_Z scale: $m_b \rightarrow \hat{m}_b(m_Z)$.

Note that starting from gluon corrections of the order of $(\alpha_s/\pi)^2$, it is impossible to unambiguously separate different quark channels, since additional pairs of ‘alien’ quarks are created in this order. We do not consider corrections $(\alpha_s/\pi)^2$ in asymmetries. In our approximation the ratio g_{Vf}/g_{Af} is not renormalised by the gluonic interaction in the final state. Therefore, the expected accuracy of Eqn (VII.9) is $(\alpha_s/\pi)^2 \sim 2 \times 10^{-3}$, which is by an order of magnitude better than the experimental accuracy.

VIII. Corrections proportional to m_t^2

This appendix gives a simple mnemonic recipe for the derivation of corrections proportional to m_t^2 . A rigorous derivation requires careful regularization of Feynman integrals.

The terms proportional to m_t^2 contribute to radiative corrections to bare masses (squared) of the W- and Z-bosons, but not to the corrections to the bare coupling constants. This follows from dimensional arguments. Indeed, the dimension of self-energy Σ for the boson, equals m^2 ; therefore, the terms proportional to m_t^2 remain in $\Sigma(q^2)$ in the limit $q^2 \rightarrow 0$. On the other hand, the corrections to coupling constants are proportional to $d\Sigma/dq^2$ and do not contain terms of the order of m_t^2 . Therefore, it is easy to evaluate the contribution of the t-quark to the parameter $\rho = (\alpha_Z/\alpha_W)(m_W^2/m_Z^2)$ in the approximation of αm_t^2 order (the Veltman approximation [29]), neglecting the terms of the order of α :

$$\rho \approx \frac{\alpha_{Z0}}{\alpha_{W0}} \frac{m_{W0}^2 - \Sigma_W(m_{W0}^2)}{m_{Z0}^2 - \Sigma_Z(m_{Z0}^2)} \approx 1 + \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2} \equiv 1 + \Pi_Z(0) - \Pi_W(0). \quad (\text{VIII.1})$$

The evaluation of the difference $\Pi_Z(0) - \Pi_W(0)$ is elementary:

$$\Pi_W(0) = \frac{\Sigma_W(0)}{m_W^2} = \frac{3\alpha_W}{8\pi m_W^2} \int_0^\infty \frac{p^2 dp^2}{p^2 + m_t^2} = \frac{3\alpha_W}{8\pi m_W^2} \int_0^\infty dp^2 - m_t^2 \int_0^\infty \frac{dp^2}{p^2 + m_t^2}. \quad (\text{VIII.2})$$

(As we have neglected the mass of the b-quark, the propagator of the b-quark compensates the factor p^2 in the numerator.)

$$\begin{aligned} \Pi_Z(0) &= \frac{\Sigma_Z(0)}{m_Z^2} = \frac{3\alpha_Z}{8\pi m_Z^2} \left(\frac{1}{2} \int_0^\infty dp^2 + \frac{1}{2} \int_0^\infty \frac{p^4 dp^2}{(p^2 + m_t^2)^2} \right) \\ &= \frac{3\alpha_Z}{8\pi m_Z^2} \left(\frac{1}{2} \int_0^\infty dp^2 + \frac{1}{2} \int_0^\infty dp^2 - m_t^2 \int_0^\infty \frac{dp^2}{p^2 + m_t^2} + \frac{1}{2} m_t^2 \right). \end{aligned} \quad (\text{VIII.3})$$

Taking into account that in one-loop approximation we can set in Eqns (VIII.2) and (VIII.3)

$$\frac{\alpha_W}{m_W^2} = \frac{\alpha_Z}{m_Z^2}, \quad (\text{VIII.4})$$

we see that quadratic and logarithmic divergences cancel out and that finally

$$\rho \approx 1 + \Delta\rho_t = 1 + \frac{3\alpha_Z}{16\pi} \frac{m_t^2}{m_Z^2} = 1 + \frac{3\alpha_Z}{16\pi} t, \quad (\text{VIII.5})$$

$$\Delta\rho_t = \frac{3\bar{\alpha}}{16\pi s^2 c^2} t, \quad (\text{VIII.6})$$

where $t = m_t^2/m_Z^2$ and we assume that $t \gg 1$.

Let us express the leading in t corrections to the main quantities m_W/m_Z , g_{A1} and g_{V1} in terms of $\Delta\rho_t$.

We define

$$c_\alpha^2 = \frac{\alpha_W}{\alpha_Z}, \quad s_\alpha^2 = 1 - c_\alpha^2. \quad (\text{VIII.7})$$

Then

$$G_\mu = \frac{\pi\alpha_W}{\sqrt{2}m_W^2} = \frac{\pi}{\sqrt{2}\rho} \frac{\bar{\alpha}}{c_\alpha^2 s_\alpha^2 m_Z^2} \quad (\text{VIII.8})$$

and hence

$$s_\alpha^2 c_\alpha^2 \approx \frac{s^2 c^2}{1 + \Delta\rho_t}. \quad (\text{VIII.9})$$

Solving the last equation, we obtain

$$c_\alpha^2 \approx c^2 \left(1 + \frac{s^2}{c^2 - s^2} \Delta\rho_t \right), \quad (\text{VIII.10})$$

$$s_\alpha^2 \approx s^2 \left(1 - \frac{c^2}{c^2 - s^2} \Delta\rho_t \right), \quad (\text{VIII.11})$$

and therefore,

$$\frac{m_W^2}{m_Z^2} \approx c_\alpha^2 (1 + \Delta\rho_t) \simeq c^2 \left(1 + \frac{c^2}{c^2 - s^2} \Delta\rho_t \right) \quad (\text{VIII.12})$$

and

$$\frac{m_W}{m_Z} \approx c + \frac{3\bar{\alpha}}{32\pi} \frac{c}{(c^2 - s^2)s^2} t. \quad (\text{VIII.13})$$

Likewise,

$$\begin{aligned} g_V^2 &\approx g_{A1}^2 \approx \frac{1}{4} \frac{\bar{\alpha}}{c_\alpha^2 s_\alpha^2} \left(\frac{\bar{\alpha}}{c^2 s^2} \right)^{-1} \approx \frac{1}{4} (1 + \Delta\rho_t) \\ &\approx \frac{1}{4} \left(1 + \frac{3\bar{\alpha}}{16\pi s^2 c^2} t \right), \end{aligned} \quad (\text{VIII.14})$$

$$\begin{aligned} \frac{g_{V1}}{g_{A1}} &\approx 1 - 4s_\alpha^2 \approx 1 - 4s^2 + \frac{4c^2 s^2}{c^2 - s^2} \Delta\rho_t \\ &= 1 - 4s^2 + \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)} t. \end{aligned} \quad (\text{VIII.15})$$

The corrections proportional to m_t^2 were first pointed out by Veltman [29], who emphasised the appearance of such corrections for a large difference $m_t^2 - m_b^2$ which violates the isotopic symmetry. In this review, the coefficients in front of the factors t in Eqns (VIII.13) – (VIII.15) are used as coefficients for normalised radiative corrections V_i .

IX. Explicit form of the functions $T_i(t)$ and $H_i(h)$

The equations for $T_i(t)$ and $H_i(h)$ are [42, 63]:

(a) $i = m$:

$$\begin{aligned} T_m(t) &= \left(\frac{2}{3} - \frac{8}{9} s^2 \right) \ln t - \frac{4}{3} + \frac{32}{9} s^2 + \frac{2}{3} (c^2 - s^2) \\ &\quad \times \left(\frac{t^3}{c^6} - \frac{3t}{c^2} + 2 \right) \ln \left| 1 - \frac{c^2}{t} \right| + \frac{2}{3} \frac{c^2 - s^2}{c^4} t^2 \\ &\quad + \frac{1}{3} \frac{c^2 - s^2}{c^2} t + \left(\frac{2}{3} - \frac{16}{9} s^2 - \frac{2}{3} t - \frac{32}{9} s^2 t \right) F_t(t), \end{aligned}$$

$$\begin{aligned} H_m(h) &= -\frac{h}{h-1} \ln h + \frac{c^2 h}{h-c^2} \ln \frac{h}{c^2} - \frac{s^2}{18c^2} h - \frac{8}{3} s^2 \\ &\quad + \left(\frac{h^2}{9} - \frac{4h}{9} + \frac{4}{3} \right) F_h(h) - (c^2 - s^2) \\ &\quad \times \left(\frac{h^2}{9c^4} - \frac{4h}{9c^2} + \frac{4}{3} \right) F_h\left(\frac{h}{c^2}\right) \\ &\quad + 1.1203 - 2.59\delta s^2, \end{aligned} \quad (\text{IX.1})$$

where $\delta s^2 = 0.23110 - s^2$ (note the sign!).

(b) $i = A$:

$$\begin{aligned} T_A(t) &= \frac{2}{3} - \frac{8}{9} s^2 + \frac{16}{27} s^4 - \frac{1 - 2tF_t(t)}{4t-1} + \left(\frac{32}{9} s^4 - \frac{8}{3} s^2 - \frac{1}{2} \right) \\ &\quad \times \left[\frac{4}{3} tF_t(t) - \frac{2}{3} (1+2t) \frac{1 - 2tF_t(t)}{4t-1} \right], \end{aligned}$$

$$\begin{aligned} H_A(h) &= \frac{c^2}{1 - c^2/h} \ln \frac{h}{c^2} - \frac{8h}{9(h-1)} \ln h \\ &\quad + \left(\frac{4}{3} - \frac{2}{3} h + \frac{2}{9} h^2 \right) F_h(h) - \left(\frac{4}{3} - \frac{4}{9} h + \frac{1}{9} h^2 \right) F'_h(h) \\ &\quad - \frac{1}{18} h + 0.7752 + 1.07\delta s^2; \end{aligned} \quad (\text{IX.2})$$

(c) $i = R$:

$$T_R(t) = \frac{2}{9} \ln t + \frac{4}{9} - \frac{2}{9} (1 + 11t) F_t(t),$$

$$H_R(h) = -\frac{4}{3} - \frac{h}{18} + \frac{c^2}{1 - c^2/h} \ln \frac{h}{c^2} + \left(\frac{4}{3} - \frac{4}{9}h + \frac{1}{9}h^2 \right) F_h(h) + \frac{h}{1-h} \ln h + 1.3590 + 0.51\delta s^2; \quad (\text{IX.3})$$

(d) $i = v$:

$$T_v(t) = T_A(t),$$

$$H_v(h) = H_A(h). \quad (\text{IX.4})$$

The functions F_t and F_h are the limiting cases of the function $F(s, m_1, m_2)$, described in Appendix II. The explicit formulas for $F_t(t)$ and $F_h(h)$ are given in Appendix X [Eqn (X.3)] and Appendix XII [Eqn (XII.11)], respectively.

X. The contribution of heavy fermions to the self-energy of the vector bosons

Let us give the expressions for the contribution of third-generation quarks (t, b) to the polarisation operators (self-energies) of the vector bosons. We use the following notations: $t = m_t^2/m_Z^2$, $b = m_b^2/m_Z^2$, ($b \ll 1$), $h = m_H^2/m_Z^2$, $\Pi_\gamma(q^2) = \Sigma_{\gamma\gamma}(q^2)/m_Z^2$, $\Pi_{\gamma Z}(q^2) = \Sigma_{\gamma Z}(q^2)/m_Z^2$, $\Pi_W(q^2) = \Sigma_W(q^2)/m_W^2$.

The dimensional regularization yields the terms

$$A_i = \frac{2}{4-D} - \gamma + \ln 4\pi - \ln \frac{m_i^2}{\mu^2}, \quad (\text{X.1})$$

where $i = t, b, W, Z, \dots$, D is the variable dimension of space-time, $(4-D = 2\varepsilon, \varepsilon \rightarrow 0)$, $\gamma = -\Gamma'(1) = 0.577\dots$ (we follow Ref. [80], p. 53).

We begin with an auxiliary function $F_t(t)$, obtained as a limiting case of the function $F(s, m_1, m_2)$ (see Appendix II and Ref. [80], p. 54; Ref. [81], p. 88),

$$F_t(t) \equiv F(s = m_Z^2, m_t, m_t) = F(1, t, t), \quad (\text{X.2})$$

and get, using Ref. [80],

$$F_t(t) = \begin{cases} 2 \left[1 - \sqrt{4t-1} \arcsin \frac{1}{\sqrt{4t}} \right], & 4t > 1, \\ 2 \left[1 - \sqrt{1-4t} \ln \frac{1 + \sqrt{1-4t}}{\sqrt{4t}} \right], & 4t < 1. \end{cases} \quad (\text{X.3})$$

The asymptotics of F_t are

$$F_t \approx \ln t + 2, \quad t \rightarrow 0, \\ F_t \approx \frac{1}{6t} + \frac{1}{60t^2}, \quad t \rightarrow \infty. \quad (\text{X.4})$$

Differentiation gives

$$F'_t \equiv m_Z^2 \frac{dF}{dm_Z^2} = -t \frac{d}{dt} F_t = \frac{1-2tF_t}{4t-1}. \quad (\text{X.5})$$

In this Appendix, Π_i stands for the contribution of the doublet (t, b) to the corresponding polarisation operator:

$$\Pi_\gamma(0) = 0, \quad (\text{X.6})$$

$$\Pi_\gamma(m_Z^2) = \frac{\bar{\alpha}}{\pi} \left\{ Q_t^2 \left[A_t + (1+2t)F_t(t) - \frac{1}{3} \right] + Q_b^2 \left(A_b + \frac{5}{3} + \ln b \right) \right\}, \quad (\text{X.7})$$

$$\Pi_{\gamma Z}(0) = 0, \quad (\text{X.8})$$

$$\Pi_{\gamma Z}(m_Z^2) = \frac{\bar{\alpha}}{c s \pi} \left\{ \left(\frac{Q_t}{4} - s^2 Q_t^2 \right) \left[A_t + (1+2t)F_t(t) - \frac{1}{3} \right] - \left(\frac{Q_b}{4} + s^2 Q_b^2 \right) \left(A_b + \frac{5}{3} + \ln b \right) \right\}, \quad (\text{X.9})$$

$$\Pi_W(0) = -\frac{\bar{\alpha}}{4\pi s^2 c^2} \left(\frac{3}{2} t A_t + \frac{3}{4} t \right), \quad (\text{X.10})$$

$$\Pi_W(m_W^2) = \frac{\bar{\alpha}}{4\pi s^2} \left[\left(1 - \frac{3t}{2c^2} \right) A_t + \frac{5}{3} - \frac{t}{c^2} - \frac{t^2}{2c^4} - \left(1 - \frac{3t}{2c^2} + \frac{t^3}{2c^6} \right) \ln \left| 1 - \frac{c^2}{t} \right| \right], \quad (\text{X.11})$$

$$\begin{aligned} \Pi_Z(m_Z^2) = & \frac{\bar{\alpha} s^2}{\pi c^2} \left\{ Q_t^2 \left[A_t + (1+2t)F_t(t) - \frac{1}{3} \right] + Q_b^2 \left(A_b + \frac{5}{3} + \ln b \right) \right\} - \frac{\bar{\alpha}}{2\pi c^2} \\ & \times \left\{ Q_t \left[A_t + (1+2t)F_t(t) - \frac{1}{3} \right] - Q_b \left(A_b + \frac{5}{3} + \ln b \right) \right\} + \frac{\bar{\alpha}}{8\pi s^2 c^2} \\ & \times \left[(2-3t)A_t + (1-t)F_t(t) + \frac{4}{3} + \ln t \right], \end{aligned} \quad (\text{X.12})$$

$$\Pi_Z(0) = -\frac{\bar{\alpha}}{4\pi s^2 c^2} \left(\frac{3}{2} t A_t \right), \quad (\text{X.13})$$

$$\begin{aligned} \Sigma'(m_Z^2) = & \frac{\bar{\alpha} s^2}{\pi c^2} \left\{ Q_t^2 \left[A_t + F_t - \frac{1}{3} + (1+2t)F'_t \right] + Q_b^2 \left(A_b + \frac{5}{3} + \ln b \right) \right\} - \frac{\bar{\alpha}}{2\pi c^2} \\ & \times \left\{ Q_t \left[A_t + F_t - \frac{1}{3} + (1+2t)F'_t \right] - Q_b \left(A_b + \frac{5}{3} + \ln b \right) \right\} + \frac{\bar{\alpha}}{8\pi s^2 c^2} \\ & \times \left[2A_t + F_t + \frac{4}{3} + \ln t + (1-t)F'_t - 1 \right] \\ = & \frac{\bar{\alpha} s^2}{\pi s^2} \left[Q_t^2 \left(A_t + \frac{4+2t}{3(4t-1)} + \frac{1-2t+4t^2}{1-4t} F_t \right) + Q_b^2 \left(A_b + \frac{5}{3} + \ln b \right) \right] - \frac{\bar{\alpha}}{2\pi c^2} \left[Q_t \left(A_t + \frac{4+2t}{3(4t-1)} \right. \right. \\ & \left. \left. + \frac{1-2t+4t^2}{1-4t} F_t \right) - Q_b \left(A_b + \frac{5}{3} + \ln b \right) \right] \\ & + \frac{\bar{\alpha}}{8\pi s^2 c^2} \left[2A_t + \ln t + \frac{2+t}{3(4t-1)} + \frac{2t^2+2t-1}{4t-1} F_t \right], \end{aligned} \quad (\text{X.14})$$

$$\begin{aligned} \Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2) &= [2tF_t(t) - (1+2t)F'_t(t)] \\ &\times \left(\frac{\bar{\alpha}s^2}{\pi c^2} Q_t^2 - \frac{\bar{\alpha}}{2\pi c^2} Q_t - \frac{\bar{\alpha}}{16\pi s^2 c^2} \right) + \frac{3\bar{\alpha}}{8\pi s^2 c^2} t \Delta_t \\ &+ \frac{\bar{\alpha}}{16\pi s^2 c^2} [2 - 3F'_t(t)] + \frac{\bar{\alpha}}{2\pi c^2} Q_b + \frac{\bar{\alpha}s^2}{\pi c^2} Q_b^2. \end{aligned} \quad (\text{X.15})$$

Substituting the expressions for the polarisation operators into the formulas for physical observables, we verify the cancellation of the terms of the order of Δ_i , and also the terms proportional to $\ln b$, since the limit $m_b \rightarrow 0$ does not produce divergences. It is convenient to get rid of the terms of the order of Δ_b and of the order of $\ln b$ already in the expression for the polarisation operators using equation

$$\Delta_b + \ln b = \Delta_t + \ln t \quad (\text{X.16})$$

and then making sure that the terms of the order of Δ_t are indeed eliminated.

Our definition of the wavefunction of the Z-boson differs in sign from that assumed in Ref. [80]; hence the quantity $\Pi_{\gamma Z}$ we use also differs in sign from the expression given in Ref. [80]. With our definition, the interaction of Z-bosons with Weyl fermions is

$$-i f \bar{f} \gamma_\mu f (T_3 - Q s^2) Z_\mu,$$

and that of photons is

$$-ie Q \bar{f} \gamma_\mu f A_\mu.$$

The latter vertex coincides with the one given in Eqns (8), (9) of Ref. [80], while the former differs in sign.

Let us look at the formulas for physical observables.

The quantity $T_m(t)$ [see Eqn (IX.1)] is defined as the following combination of polarisation operators:

$$\begin{aligned} t + T_m(t) &= \frac{16\pi s^4}{3\bar{\alpha}} \left\{ \frac{c^2}{s^2} [\Pi_Z(m_Z^2) - \Pi_W(m_W^2)] \right. \\ &\quad \left. + \Pi_W(m_W^2) - \Pi_W(0) - \Pi_\gamma(m_Z^2) \right\}, \end{aligned} \quad (\text{X.17})$$

since $\Pi_{\gamma Z}(0) = 0$ for fermion loops.

Using Eqns (X.7), (X.10) – (X.12) we obtain

$$\begin{aligned} t + T_m(t) &= t + \left(\frac{2}{3} - \frac{8}{9} s^2 \right) \ln t - \frac{4}{3} + \frac{32}{9} s^2 + \frac{c^2 - s^2}{3c^2} t \\ &+ \frac{2}{3} (c^2 - s^2) \left(\frac{t^3}{c^6} - \frac{3t}{c^2} + 2 \right) \ln \left| 1 - \frac{c^2}{t} \right| + \frac{2(c^2 - s^2)}{3c^4} t^2 \\ &+ \left(\frac{2}{3} - \frac{16}{9} s^2 - \frac{2}{3} t - \frac{32}{9} s^2 t \right) F_t(t), \end{aligned} \quad (\text{X.18})$$

where we have taken into account that $\Pi_\gamma(m_Z^2)$ cancels out with the sum of terms proportional to Q_t^2 and Q_b^2 in $\Pi_Z(m_Z^2)$. The terms proportional to t arise from $\Pi_W(0)$ and $\Pi_W(m_Z^2)$; those proportional to t^2 arise from $\Pi_W(m_W^2)$; the terms proportional to $\ln t$ and the constants arise from $\Pi_W(m_W^2)$ and $\Pi_Z(m_Z^2)$; $\ln |1 - c^2/t|$ corresponds to the threshold (tb) in $\Pi_W(m_W^2)$, and finally, the only source of terms proportional to $F_t(t)$ is $\Pi_Z(m_Z^2)$.

The infinities in the contribution of the doublet (t, b) to the observables must cancel each other since the introduction of an additional fermion family into the electroweak theory does not violate its renormalisability.

Substituting the terms proportional to Δ_t in Eqn (X.17) and taking into account Eqn (X.16), we obtain zero:

$$\begin{aligned} \Delta_t \left\{ \frac{c^2}{s^2} \left[\frac{\bar{\alpha}}{8\pi s^2 c^2} (2 - 3t) - \frac{\bar{\alpha}}{2\pi c^2} - \frac{\bar{\alpha}}{4\pi s^2} \left(1 - \frac{3t}{2c^2} \right) \right] \right. \\ \left. + \frac{\bar{\alpha}}{4\pi s^2} \left(1 - \frac{3t}{2c^2} \right) + \frac{\bar{\alpha}}{4\pi s^2 c^2} \frac{3}{2} t \right\} = 0. \end{aligned} \quad (\text{X.19})$$

The expression for $T_A(t)$ is

$$t + T_A(t) = \frac{16\pi s^2 c^2}{3\bar{\alpha}} [\Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2) - \Pi_W(0)]. \quad (\text{X.20})$$

Using Eqns (X.10), (X.12), and (X.14), we have

$$\begin{aligned} t + T_A(t) &= t + \frac{2}{3} - \frac{8}{9} s^2 + \frac{16}{27} s^4 - F'_t \\ &+ \left(\frac{32}{9} s^4 - \frac{8}{3} s^2 - \frac{1}{2} \right) \left(\frac{4}{3} t F_t - \frac{2(1+2t)}{3} F'_t \right). \end{aligned} \quad (\text{X.21})$$

The terms proportional to Δ_t obviously cancel out. $\Pi_W(0)$ gives a contribution proportional to t , while all other terms arise from the difference $\Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2)$.

Finally, we look at $T_R(t)$:

$$\begin{aligned} t + T_R(t) &= -\frac{16\pi c^2 s^2}{3\bar{\alpha}} \left[\frac{(c^2 - s^2)}{cs} \Pi_{Z\gamma}(m_Z^2) + \Pi_\gamma(m_Z^2) \right. \\ &\quad \left. - \Pi_Z(m_Z^2) + \Pi_W(0) \right]. \end{aligned} \quad (\text{X.22})$$

The terms proportional to Q_t^2 and Q_b^2 cancel out, so that only $\Pi_W(0)$ remain, as well as terms that do not contain $Q_{t,b}^2$ coming from $\Pi_{Z\gamma}$ and Π_Z . Substituting Eqns (X.10), (X.9) and (X.12), we have

$$t + T_R(t) = t + \frac{4}{9} + \frac{2}{9} \ln t - \frac{2}{9} (1 + 11t) F(t). \quad (\text{X.23})$$

Here t comes from $\Pi_W(0)$, the term proportional to $F_t(t)$ is from Π_Z , and $4/9 + (2/9) \ln t$ comes from Π_Z and $\Pi_{Z\gamma}$.

Cancellation of infinities in Eqn (X.22) follows from

$$\Delta_t \left\{ (c^2 - s^2) \frac{\alpha}{4\pi cs} + cs \left[\frac{\alpha}{2\pi c^2} - \frac{(2-3t)\alpha}{8\pi c^2 s^2} - \frac{3t\alpha}{8\pi s^2 c^2} \right] \right\} = 0. \quad (\text{X.24})$$

XI. The contribution of light fermions to the self-energy of the vector bosons

The contribution of the doublet of light fermions to the polarisation operators is readily obtained using the formulas of the preceding Appendix.

To achieve this, $\Delta_{Z,W}$ must be substituted into the formulas of Appendix X instead of $\Delta_q + \ln(m_q/m_{Z,W})^2$. The physical reason for the absence of terms proportional to the logarithm of the mass of light quarks and leptons is the infrared stability of the quantities that are analyzed in this Appendix. In the Eqns (XI.4) and (XI.7) we use the equality $Q_u - Q_d = 1$. The subscripts u and d stand for the upper and lower components of the doublet:

$$\Pi_\gamma(0) = 0, \quad (\text{XI.1})$$

$$\Pi_\gamma(m_Z^2) = \frac{N_c \bar{\alpha}}{3\pi} (Q_u^2 + Q_d^2) \left(A_Z + \frac{5}{3} \right), \quad (\text{XI.2})$$

$$\Pi_{\gamma Z}(0) = 0, \quad (\text{XI.3})$$

$$\Pi_{\gamma Z}(m_Z^2) = \frac{N_c \bar{\alpha}}{3c\pi} \left(A_Z + \frac{5}{3} \right) \left[\frac{1}{4} - (Q_u^2 + Q_d^2) s^2 \right], \quad (\text{XI.4})$$

$$\Pi_W(0) = 0, \quad (\text{XI.5})$$

$$\Pi_W(m_W^2) = \frac{N_c \bar{\alpha}}{12\pi s^2} \left(A_W + \frac{5}{3} \right), \quad (\text{XI.6})$$

$$\Pi_Z(m_Z^2) = \frac{N_c \bar{\alpha}}{3\pi s^2 c^2} \left(A_Z + \frac{5}{3} \right) \left[\frac{1}{4} - \frac{s^2}{2} + s^4 (Q_u^2 + Q_d^2) \right], \quad (\text{XI.7})$$

$$\Pi_Z(0) = 0, \quad (\text{XI.8})$$

$$\Sigma'_Z(m_Z^2) = \frac{N_c \bar{\alpha}}{3\pi s^2 c^2} \left(A_Z + \frac{2}{3} \right) \left[\frac{1}{4} - \frac{s^2}{2} + s^4 (Q_u^2 + Q_d^2) \right]. \quad (\text{XI.9})$$

Eqns (XI.1) – (XI.9) must be used for three lepton doublets (ν_e, e), (ν_μ, μ) and (ν_τ, τ) with $N_c = 1$ and two quarks doublets (u, d) and (c, s) with $N_c = 3$.

Substituting Eqns (XI.1) – (XI.9) into expressions for physical observables in terms of polarisation operators (XII.16), (XII.20) and (XII.24), we arrive at the contributions to the constants C_i owing to the self-energies.

XII. The contribution of the vector and scalar bosons to the self-energy of the vector bosons

This appendix gives formulas for boson contributions to polarisation operators that we reproduced from Ref. [80], pp 53, 54. (There is a misprint in [80]: the term proportional to A_W in the expressions for $\Pi_W(q^2)$ must be multiplied by $1/3$).

The polarisation operators in Ref. [80] depend on c_W and s_W via coupling constants and depend dynamically on the ratio m_W/m_Z , which arises from Feynman integrals. We substitute everywhere c for c_W (and m_W/m_Z) and s for s_W . In the framework of the one-loop approximation, this substitution is justified. After this substitution, we find expressions for physical observables; ultraviolet divergences of polarisation operators cancel out in these expressions.

In the following formulas Π_i denotes only boson contributions to the corresponding polarisation operator (all calculations were performed in the 't Hooft–Feynman gauge):

$$\begin{aligned} \Pi_\gamma(m_Z^2) &= -\frac{\bar{\alpha}}{4\pi} \left[3A_W + 2(3 + 4c^2) \right. \\ &\quad \times \left. \left(1 - \sqrt{4c^2 - 1} \arcsin \frac{1}{2c} \right) \right] \\ &= -\frac{\bar{\alpha}}{4\pi} (3A_W + 1.53), \end{aligned} \quad (\text{XII.1})$$

$$\Pi_\gamma(0) = 0,$$

$$\Pi_{\gamma Z}(0) = -\frac{\bar{\alpha}}{4\pi cs} (2c^2 A_W) = -\frac{c\bar{\alpha}}{2\pi s} A_W, \quad (\text{XII.3})$$

$$\begin{aligned} \Pi_{\gamma Z}(m_Z^2) &= -\frac{\bar{\alpha}}{4\pi cs} \left[\left(5c^2 + \frac{1}{6} \right) A_W + 2 \left(\frac{1}{6} + \frac{13}{3} c^2 + 4c^4 \right) \right. \\ &\quad \times \left. \left(1 - \sqrt{4c^2 - 1} \arcsin \frac{1}{2c} \right) + \frac{1}{9} \right] \\ &= -\frac{\bar{\alpha}}{4\pi} \left(\frac{30c^2 + 1}{6cs} A_W + 3.76 \right), \end{aligned} \quad (\text{XII.4})$$

$$\begin{aligned} \Pi_W(0) &= \frac{\bar{\alpha}}{4\pi s^2} \left[\left(\frac{s^2}{c^2} - 1 \right) A_W + \frac{3}{4} \left(1 - \frac{c^2}{h} \right)^{-1} \ln \frac{c^2}{h} - \frac{h}{8c^2} \right. \\ &\quad + s^2 + \frac{s^4}{c^2} - \frac{1}{8c^2} - \frac{39}{12} + \left(\frac{s^2}{c^2} + 3 - \frac{17}{4s^2} \right) \ln c^2 \left. \right] = \frac{\bar{\alpha}}{4\pi s^2} \\ &\quad \times \left[\left(\frac{s^2}{c^2} - 1 \right) A_W + \frac{3}{4} \left(1 - \frac{c^2}{h} \right)^{-1} \ln \frac{c^2}{h} - \frac{h}{8c^2} + 0.85 \right], \end{aligned} \quad (\text{XII.5})$$

$$\begin{aligned} \Pi_W(m_W^2) &= \frac{\bar{\alpha}}{4\pi s^2} \left\{ -\left(\frac{25}{6} - \frac{s^2}{c^2} \right) A_W \right. \\ &\quad + \left[\frac{s^4}{c^2} - \frac{c^2}{3} \left(\frac{7}{c^2} + 17 - 2 \frac{s^4}{c^4} \right) - \frac{1}{6} \left(\frac{1}{2} + \frac{1}{c^2} - \frac{s^4}{2c^4} \right) \right] \\ &\quad \times F_h \left(\frac{1}{c^2} \right) + \left[\frac{c^2}{3} \left(\frac{3}{c^2} + 21 \right) - \frac{s^4}{c^2} + \frac{1}{4} \right] \frac{1}{s^2} \ln \frac{1}{c^2} - 3s^2 \\ &\quad - \frac{1}{6c^2} + \frac{s^4}{c^2} - \frac{113}{18} + \left(1 - \frac{h}{3c^2} + \frac{h^2}{12c^4} \right) F_h \left(\frac{h}{c^2} \right) - 1 \\ &\quad - \frac{h}{6c^2} + \frac{3h}{4(c^2 - h)} \ln \frac{h}{c^2} \left. \right\} = \frac{\bar{\alpha}}{4\pi s^2} \left[\left(\frac{s^2}{c^2} - \frac{25}{6} \right) A_W \right. \\ &\quad - 1.76 + \left(1 - \frac{h}{3c^2} + \frac{h^2}{12c^4} \right) F_h \left(\frac{h}{c^2} \right) - \frac{h}{6c^2} \\ &\quad \left. + \frac{3h}{4(c^2 - h)} \ln \frac{h}{c^2} \right], \end{aligned} \quad (\text{XII.6})$$

$$\begin{aligned} \Pi_Z(m_Z^2) &= \frac{\bar{\alpha}}{4\pi s^2} \left\{ \left(7s^2 - \frac{25}{6} + \frac{7}{6} \frac{s^2}{c^2} \right) A_W + \frac{73}{36c^2} - \frac{2}{9} \right. \\ &\quad + \frac{13}{12c^2} \ln c^2 + \left[2 + \frac{1 + 8c^2}{6c^2} (c^2 - s^2)^2 - \frac{20}{3} c^2 (1 + 2c^2) \right] \\ &\quad \times \left(1 - \sqrt{4c^2 - 1} \arcsin \frac{1}{2c} \right) - \frac{1}{c^2} - \frac{h}{6c^2} + \frac{3h}{4c^2(1 - h)} \ln h \\ &\quad + \left(\frac{1}{c^2} - \frac{h}{3c^2} + \frac{h^2}{12c^2} \right) F_h(h) \left. \right\} = \frac{\bar{\alpha}}{4\pi s^2} \left[\left(7s^2 - \frac{25}{6} + \frac{7}{6} \frac{s^2}{c^2} \right) \right. \\ &\quad \times A_W - 0.58 - \frac{h}{6c^2} + \frac{3h}{4c^2(1 - h)} \ln h \\ &\quad \left. + \left(\frac{1}{c^2} - \frac{h}{3c^2} + \frac{h^2}{12c^2} \right) F_h(h) \right], \end{aligned} \quad (\text{XII.7})$$

$$\begin{aligned} \Sigma'_Z(m_Z^2) &= \frac{d\Sigma_Z(s)}{ds} \Big|_{s=m_Z^2} = \frac{\bar{\alpha}}{4\pi} \left(3 - \frac{19}{6s^2} + \frac{1}{6c^2} \right) A_W \\ &\quad + \frac{\bar{\alpha}}{48\pi s^2 c^2} \left\{ [-40c^4 + (c^2 - s^2)^2] \right. \\ &\quad \times \left(2 - 2\sqrt{4c^2 - 1} \arcsin \frac{1}{2c} \right) \\ &\quad \left. + [12c^2 + (8c^2 + 1)(c^2 - s^2)^2 - 40c^4(1 + 2c^2)] \right\} \end{aligned}$$

$$\begin{aligned}
& \times \left(-1 + \frac{4c^2}{\sqrt{4c^2 - 1}} \arcsin \frac{1}{2c} \right) + [1 - (h-1)^2] \\
& \times F_h(h) + [11 - 2h + (1-h)^2] F'_h(h) \\
& + \left[1 - \frac{1+h}{2(h-1)} \ln h - \frac{1}{2} \ln \frac{h}{c^4} \right] \\
& + \frac{2}{3} [1 + (c^2 - s^2)^2 - 4c^4] \Big\} \\
& = \frac{\bar{\alpha}}{4\pi} \left(3 - \frac{19}{6s^2} + \frac{1}{6c^2} \right) \Delta_W + \frac{\bar{\alpha}}{4\pi s^2 c^2} \\
& \times \left[\left(1 - \frac{h}{3} + \frac{h^2}{12} \right) F'_h(h) + \left(\frac{h}{6} - \frac{h^2}{12} \right) F_h(h) \right. \\
& \left. + \frac{h}{12(1-h)} \ln h - 1.67 \right]. \quad (\text{XII.8})
\end{aligned}$$

The functions $F_h(h)$ and $F'_h(h)$ are defined as

$$F_h(h) \equiv F(s, m_Z, m_H) \Big|_{s=m_Z^2} \equiv F(1, 1, h), \quad (\text{XII.9})$$

$$F'_h(h) \equiv s \frac{dF(s, m_Z, m_H)}{ds} \Big|_{s=m_Z^2}. \quad (\text{XII.10})$$

Using Ref. [81], p. 88, we obtain

$$\begin{aligned}
F_h(h) &= 1 + \left(\frac{h}{h-1} - \frac{h}{2} \right) \ln h \\
&+ h \sqrt{1 - \frac{4}{h}} \ln \left(\sqrt{\frac{h}{4} - 1} + \sqrt{\frac{h}{4}} \right), \quad h > 4, \\
F_h(h) &= 1 + \left(\frac{h}{h-1} - \frac{h}{2} \right) \ln h \\
&- h \sqrt{\frac{4}{h} - 1} \arctan \sqrt{\frac{4}{h} - 1}, \quad h < 4. \quad (\text{XII.11})
\end{aligned}$$

If $h \rightarrow \infty$,

$$F_h(h) \approx \frac{1}{2h} - \frac{1}{h^2} \left(1 + \frac{4}{h^2} \right) \ln h + \frac{5}{3h^2} + \frac{59}{12h^3}. \quad (\text{XII.12})$$

If $h \rightarrow 0$,

$$F_h(h) \approx 1 - \pi\sqrt{h} + \left(1 - \frac{3}{2} \ln h \right) h. \quad (\text{XII.13})$$

Finally, for $F'_h(h)$ we have

$$\begin{aligned}
F'_h(h) &= -1 + \frac{h-1}{2} \ln h + (3-h) \\
&\times \sqrt{\frac{h}{h-4}} \ln \left(\sqrt{\frac{h-4}{4}} + \sqrt{\frac{h}{4}} \right), \quad h > 4,
\end{aligned}$$

$$\begin{aligned}
F'_h(h) &= -1 + \frac{h-1}{2} \ln h + (3-h) \\
&\times \sqrt{\frac{h}{4-h}} \arctan \sqrt{\frac{4-h}{h}}, \quad h < 4. \quad (\text{XII.14})
\end{aligned}$$

If $h \rightarrow \infty$,

$$F'_h(h) \approx \frac{1}{2h} - \frac{1}{h^2} \ln h. \quad (\text{XII.15})$$

All infinities in Eqns (XII.1) – (XII.8) are collected into the factors Δ_W by replacing the factors Δ_Z and Δ_H using the equation

$$\Delta_i = \Delta_j + \ln \frac{m_j^2}{m_i^2}.$$

The function $H_m(h)$ is

$$\begin{aligned}
H_m(h) &= \frac{16\pi s^4}{3\bar{\alpha}} \left\{ \frac{c^2}{s^2} [\Pi_Z(m_Z^2) - \Pi_W(m_W^2)] + \Pi_W(m_W^2) \right. \\
&\quad \left. - \Pi_W(0) - \Pi_\gamma(m_Z^2) - 2 \frac{s}{c} \Pi_{\gamma Z}(0) \right\} - \text{div } H_m, \quad (\text{XII.16})
\end{aligned}$$

where $\text{div } H_m$ denotes the sum of terms, proportional to Δ_W , in polarisation operators in Eqn (XII.16). Substituting the finite parts of the formulas for polarisation operators from this Appendix, we obtain

$$\begin{aligned}
H_m(h) &= -\frac{h}{h-1} \ln h + \frac{c^2 h}{h-c^2} \ln \frac{h}{c^2} - \frac{s^2}{18c^2} h \\
&+ \left(\frac{h^2}{9} - \frac{4h}{9} + \frac{4}{3} \right) F_h(h) - (c^2 - s^2) \\
&\times \left(\frac{h^2}{9c^4} - \frac{4h}{9c^2} + \frac{4}{3} \right) F_h \left(\frac{h}{c^2} \right) + 0.50, \quad (\text{XII.17})
\end{aligned}$$

where the term proportional to $F_h(h)$ arises from Π_Z , and the term proportional to $F_h(h/c^2)$ arises from $\Pi_W(m_W^2)$. The term proportional to $\ln h$ originates from Π_Z , while $\ln(h/c^2)$ arises both from $\Pi_W(m_W^2)$ and from $\Pi_W(0)$. The term proportional to h is contained in Π_Z , $\Pi_W(m_W^2)$ and $\Pi_W(0)$ and finally, all four polarisation operators make contribution to the constant.

Collecting the terms proportional to Δ_W in the polarisation operators in Eqn (XII.16), we get

$$\begin{aligned}
\text{div } H_m &= \frac{16\pi s^4}{3\bar{\alpha}} \Delta_W \left[\frac{\bar{\alpha}}{4\pi s^2} \frac{c^2}{s^2} \left(7s^2 - \frac{25}{6} + \frac{7s^2}{6c^2} + \frac{25}{6} - \frac{s^2}{c^2} \right) \right. \\
&\quad \left. + \frac{\bar{\alpha}}{4\pi s^2} \left(-\frac{19}{6} \right) + \frac{3\bar{\alpha}}{4\pi} + 2 \frac{s}{c} \frac{\bar{\alpha}}{s 2\pi} \right] = \frac{16\pi s^4}{3\bar{\alpha}} \Delta_W \frac{\bar{\alpha}}{\pi s^2}. \quad (\text{XII.18})
\end{aligned}$$

Note that the divergent term in D [see Eqns (V.8) and (XIII.26)] exactly compensates for the divergence in Eqn (XII.18), which justifies the subtraction of infinity in Eqn (XII.16).

The function $H_A(h)$ is expressed in terms of polarisation operators as follows:

$$H_A(h) = \frac{16\pi s^2 c^2}{3\bar{\alpha}} [\Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2) - \Pi_W(0)] - \text{div } H_A. \quad (\text{XII.19})$$

Substituting the finite parts of the polarisation operators, we obtain

$$\begin{aligned}
H_A(h) &= \frac{hc^2}{h-c^2} \ln \frac{h}{c^2} - \frac{8h}{9(h-1)} \ln h + \left(\frac{4}{3} - \frac{2}{3} h + \frac{2}{9} h^2 \right) \\
&\times F_h(h) - \left(\frac{4}{3} - \frac{4h}{9} + \frac{h^2}{9} \right) F'_h(h) - \frac{h}{18} + 0.78, \quad (\text{XII.20})
\end{aligned}$$

where $\ln(h/c^2)$ stems from Π_W and $\ln h$ stems from Π_Z . $F_h(h)$ arises both from Π_Z and from Σ'_Z , while the only source of $F'_h(h)$ is Σ'_Z . The term linear in h is contained in Π_Z and Π_W , while all three polarisation operators contribute to the constant.

Adding up the divergent terms, we have

$$\text{div } H_A = \frac{16}{3} c^2 s^2 A_W = -\frac{16\pi s^2 c^2}{3\bar{\alpha}} A_W \frac{\bar{\alpha}}{\pi}. \quad (\text{XII.21})$$

Note that the divergent part of D_A [see Eqns (XIII.18) and (XIII.14)] is:

$$\text{div } D_A = -\frac{16c^2}{3} A_W = -\frac{16\pi s^2 c^2}{3\bar{\alpha}} A_W \frac{\bar{\alpha}}{\pi s^2}. \quad (\text{XII.22})$$

Vertex parts also contain ultraviolet divergences [see Appendix XIII, Eqns (XIII.11), (XIII.15)]:

$$\text{div } \tilde{F}_A = -\left(\frac{16}{3} c^2 s^2 - \frac{16}{3} c^2\right) A_W = \frac{16\pi s^2 c^2}{3\bar{\alpha}} A_W \frac{c^2}{s^2} \frac{\bar{\alpha}}{\pi}. \quad (\text{XII.23})$$

The sum of terms (XII.21) — (XII.23) equals zero.

Finally, the expression for $H_R(h)$ is

$$\begin{aligned} H_R(h) = & -\frac{16\pi}{3\bar{\alpha}} c^2 s^2 \left[\frac{(c^2 - s^2)}{cs} \Pi_{Z\gamma}(m_Z^2) + \Pi_\gamma(m_Z^2) \right. \\ & \left. - \Pi_Z(m_Z^2) + \Pi_W(0) + 2\frac{s}{c} \Pi_{\gamma Z}(0) \right] - \text{div } H_R(h). \end{aligned} \quad (\text{XII.24})$$

Collecting the finite parts of the polarisation operators, we find

$$\begin{aligned} H_R(h) = & -\frac{h}{18} + \frac{c^2 h}{h - c^2} \ln \frac{h}{c^2} + \left(\frac{4}{3} - \frac{4}{9} h + \frac{1}{9} h^2 \right) F_h(h) \\ & + \frac{h}{1 - h} \ln h + 0.03. \end{aligned} \quad (\text{XII.25})$$

The term proportional to $F_h(h)$ stems from Π_Z , just as $\ln h$ does. Π_W generates the term of the order of $\ln(h/c^2)$. The term linear in h is contained both in Π_Z and in Π_W , and all polarisation operators with the exception of $\Pi_{\gamma Z}(0)$ contribute to the constant.

Adding up the divergent parts of the polarisation operators in Eqn (XII.24), we obtain

$$\begin{aligned} \text{div } H_R(h) = & A_W \left[\frac{2}{9} (c^2 - s^2) (1 + 30c^2) + 4c^2 s^2 + \frac{4}{3} c^2 \right. \\ & \times \left(7s^2 - \frac{25}{6} + \frac{7}{6} \frac{s^2}{c^2} \right) + \frac{4}{3} c^2 - \frac{4}{3} s^2 + \frac{16}{3} c^2 s^2 \left. \right] \\ = & A_W (8c^2 - \frac{16}{3} c^4). \end{aligned} \quad (\text{XII.26})$$

Taking into account the divergent term in D , yielding [see Eqns (V.8), (XIII.14), (XIII.18)]

$$\text{div } D_R = -\frac{16}{3} c^2 A_W, \quad (\text{XII.27})$$

and in \tilde{F}_R [see Appendix XIII, Eqns (XIII.11) and (XIII.22)], yielding

$$\text{div } \tilde{F}_R = \left(\frac{16}{3} c^4 - \frac{8}{3} c^2 \right) A_W, \quad (\text{XII.28})$$

we confirm that divergences cancel out in expressions for R .

XIII. The vertex parts of F_{Af} and F_{Vf} and the constants C_i

This Appendix collects the vertex functions that form a part of one-loop electroweak corrections to $Z \rightarrow \nu\bar{\nu}$, $Z \rightarrow l^{+1}l^{-}$, $Z \rightarrow u\bar{u}, c\bar{c}, d\bar{d}$ and $s\bar{s}$ decays. In the case of the $Z \rightarrow b\bar{b}$ decay, a t -quark can propagate in the loop, so vertex corrections are not reducible to numbers but are functions of m_t (see Appendix XIV).

The finite parts of vertex functions are given in Ref. [80], pp 29, 30. The corresponding expressions depend on $c_W(s_W)$ and m_W/m_Z . In the framework of the one-loop approximation, we replace c_W and m_W/m_Z with c , and s_W with s . For this reason, while vertex functions in Ref. [80] depend on m_t, m_H and the new physics, ours are numbers (see also Appendix XII).

This Appendix also gives the infinite parts absent from [80] and required for testing whether the infinities in physical observables do cancel out.

We begin with the $Z \rightarrow \nu\bar{\nu}$ decay:

$$\begin{aligned} F_V \equiv F_{V\nu} = F_{A\nu} = & \frac{\bar{\alpha}}{4\pi} \frac{1}{4cs} \left[\frac{1}{4c^2 s^2} A_2(m_Z^2, m_Z) \right. \\ & \left. + \frac{2s^2 - 1}{2s^2} A_2(m_Z^2, m_W) + \frac{3c^2}{s^2} A_3(m_Z^2, m_W) \right]. \end{aligned} \quad (\text{XIII.1})$$

For the decay to a pair of charged leptons or quarks we have

$$F_{Vf} = \frac{\bar{\alpha}}{4\pi} [v_f(v_f^2 + 3a_f^2) A_2(m_Z^2, m_Z) + F_L^f], \quad (\text{XIII.2})$$

$$F_{Af} = \frac{\bar{\alpha}}{4\pi} [a_f(3v_f^2 + a_f^2) A_2(m_Z^2, m_Z) + F_L^f], \quad (\text{XIII.3})$$

where $a_l = a_d = -1/4sc$, $a_u = 1/4sc$, $v_f = (T_3^f - 2Q^f s^2)/2sc$ ($T_3^l = T_3^d = -1/2$, $T_3^u = 1/2$, $Q^l = -1$, $Q^d = -1/3$, $Q^u = 2/3$). The functions F_L^f are

$$F_L^l = \frac{1}{8s^3 c} A_2(m_Z^2, m_W) - \frac{3c}{4s^3} A_3(m_Z^2, m_W), \quad (\text{XIII.4})$$

$$F_L^u = -\frac{1 - 2s^2/3}{8s^3 c} A_2(m_Z^2, m_W) + \frac{3c}{4s^3} A_3(m_Z^2, m_W), \quad (\text{XIII.5})$$

$$F_L^d = \frac{1 - 4s^2/3}{8s^3 c} A_2(m_Z^2, m_W) + \frac{3c}{4s^3} A_3(m_Z^2, m_W). \quad (\text{XIII.6})$$

For calculating F_{Vf} and F_{Af} we need to determine the values of three constants: $A_2(m_Z^2, m_W)$, $A_2(m_Z^2, m_Z)$, and $A_3(m_Z^2, m_W)$:

$$\begin{aligned} A_2(m_Z^2, m_W) = & -\frac{7}{2} - 2c^2 - (2c^2 + 3) \ln c^2 \\ & + 2(1 + c^2)^2 \left[\ln c^2 \ln \left(\frac{1 + c^2}{c^2} \right) - \text{Sp} \left(-\frac{1}{c^2} \right) \right], \end{aligned} \quad (\text{XIII.7})$$

where we have used $m_W/m_Z = c$; $\text{Sp}(x)$ is the Spence function:

$$\text{Sp}(x) = -\int_0^1 \frac{dt}{t} \ln(1 - xt), \quad \text{Sp}(-1) = -\frac{\pi^2}{12}. \quad (\text{XIII.8})$$

Using Eqns (XIII.7) and (XIII.8), we find

$$A_2(m_Z^2, m_Z) = -\frac{7}{2} - 2 - 8\text{Sp}(-1). \quad (\text{XIII.9})$$

Finally,

$$\begin{aligned} A_3(m_Z^2, m_W) &= \frac{5}{6} - \frac{2}{3}c^2 + \frac{2}{3}(2c^2 + 1) \\ &\quad \times \sqrt{4c^2 - 1} \arctan \frac{1}{\sqrt{4c^2 - 1}} \\ &\quad - \frac{8}{3}c^2(c^2 + 2) \left(\arctan \frac{1}{\sqrt{4c^2 - 1}} \right)^2. \end{aligned} \quad (\text{XIII.10})$$

The expressions for divergent parts of the vertex functions, describing the coupling of the Z-boson to the leptons, are

$$\text{div } F_v = \frac{\bar{\alpha}}{8\pi s^3} \Delta_W, \quad \text{div } F_{vI} = \text{div } F_{AI} = -\text{div } F_v. \quad (\text{XIII.11})$$

We switch to the calculation of the constants C_i . We begin with definitions. According to Ref. [42],

$$V_v(t, h) = t + T_v(t) + H_v(h) + L_v + D_v + \tilde{F}_v. \quad (\text{XIII.12})$$

The value of L_v represents the contribution of leptons and light quarks to the polarisation operators of the vector bosons and can be easily obtained from the formulas of Appendix XI for polarisation operators:

$$L_v = 4 - 8s^2 + \frac{304}{27}s^4. \quad (\text{XIII.13})$$

D_v originates from the box and vertex electroweak corrections to the μ -decay [23]. The expression for D see in Appendix V, Eqn (V.8). For D_v we have

$$D_v = -\frac{16\pi s^2 c^2}{3\bar{\alpha}} \left(D - \frac{\bar{\alpha}}{\pi s^2} \Delta_W \right). \quad (\text{XIII.14})$$

Finally,

$$\tilde{F}_v = \frac{128\pi s^3 c^3}{3\bar{\alpha}} F_v. \quad (\text{XIII.15})$$

Comparing Eqns (XIII.12) and (58), we arrive at the expressions for C_v whose ingredients are now all determined:

$$C_v = L_v + D_v + \tilde{F}_v. \quad (\text{XIII.16})$$

Let us switch to V_A :

$$V_A(t, h) = t + T_A(t) + H_A(h) + L_A + D_A + \tilde{F}_A. \quad (\text{XIII.17})$$

The expressions for L_A and D_A are already there:

$$L_A = L_v, \quad D_A = D_v; \quad (\text{XIII.18})$$

the formula for the vertex function is

$$\tilde{F}_A = -\frac{128\pi s^3 c^3}{3\bar{\alpha}} F_{AI}. \quad (\text{XIII.19})$$

Finally,

$$C_A = L_A + D_A + \tilde{F}_A. \quad (\text{XIII.20})$$

We now move to V_R :

$$V_R(t, h) = t + T_R(t) + H_R(h) + L_R + D_R + \tilde{F}_R, \quad (\text{XIII.21})$$

where $D_R = D_v$ and $L_R = 0$ since $\Pi_W(m_W^2)$ is absent from the ratio g_V/g_A .

For \tilde{F}_R we have

$$\tilde{F}_R = \frac{16\pi(c^2 - s^2)cs}{3\bar{\alpha}} [-F_{vI} + (1 - 4s^2)F_{AI}], \quad (\text{XIII.22})$$

and the formula for C_R is

$$C_R = L_R + D_R + \tilde{F}_R. \quad (\text{XIII.23})$$

We end this Appendix with formulas for C_m :

$$C_m = L_m + D_m, \quad (\text{XIII.24})$$

$$L_m = 4(c^2 - s^2) \ln c^2, \quad (\text{XIII.25})$$

$$D_m = -\frac{16\pi s^4}{3\bar{\alpha}} \left(D - \frac{\bar{\alpha}}{\pi s^2} \Delta_W \right). \quad (\text{XIII.26})$$

XIV. The functions $\phi(t)$ and $\delta\phi(t)$ in the $Z \rightarrow b\bar{b}$ decay

For the function $\phi(t)$ we use the expansion from Ref. [57]:

$$\begin{aligned} \phi(t) &= \frac{3 - 2s^2}{2s^2 c^2} \left\{ t + c^2 \left[2.88 \ln \frac{t}{c^2} - 6.716 \right. \right. \\ &\quad \left. \left. + \frac{1}{t} \left(8.368c^2 \ln \frac{t}{c^2} - 3.408c^2 \right) + \frac{1}{t^2} \left(9.126c^4 \ln \frac{t}{c^2} \right. \right. \right. \\ &\quad \left. \left. \left. + 2.26c^4 \right) + \frac{1}{t^3} \left(4.043c^6 \ln \frac{t}{c^2} + 7.41c^6 \right) + \dots \right] \right\}, \end{aligned} \quad (\text{XIV.1})$$

and for $\delta\phi(t)$ we use the leading approximation calculated in Refs [58] and [53]:

$$\delta\phi(t, h) = \frac{3 - 2s^2}{2s^2 c^2} \left\{ -\frac{\pi^2}{3} \frac{\hat{\alpha}_s(m_t)}{\pi} t + \frac{1}{16s^2 c^2} \frac{\bar{\alpha}}{\pi} t^2 \tau_b^{(2)} \left(\frac{h}{t} \right) \right\}, \quad (\text{XIV.2})$$

where the function $\tau_b^{(2)}$ is tabulated in Table 4 for $m_H/m_t < 4$. For $m_H/m_t > 4$ we use the expansion [53]:

$$\begin{aligned} \tau_b^{(2)} \left(\frac{h}{t} \right) &= \frac{1}{144} \left[311 + 24\pi^2 + 282 \ln r + 90 \ln^2 r \right. \\ &\quad \left. - 4r(40 + 6\pi^2 + 15 \ln r + 18 \ln^2 r) + \frac{3}{100} r^2 \right. \\ &\quad \left. \times (24209 - 6000\pi^2 - 45420 \ln r - 18000 \ln^2 r) \right], \end{aligned} \quad (\text{XIV.3})$$

where $r = t/h$. For $m_t = 175$ GeV and $m_H = 300$ GeV $\tau_b^{(2)} = 1.245$.

XV. The $\delta_2 V_i$ corrections

The corrections $\delta_2 V_i \sim \bar{\alpha} \hat{\alpha}_s$ arise from gluon exchanges in quark electroweak loops [54] (see also Ref. [82]). For two generations of light quarks ($q = u, d, s, c$) we have

$$\delta_2^q V_m(t, h) = 2 \left[\frac{4}{3} \frac{\hat{\alpha}_s(m_Z)}{\pi} (c^2 - s^2) \ln c^2 \right] \\ = \frac{\hat{\alpha}_s(m_Z)}{\pi} \times (-0.377), \quad (\text{XV.1})$$

$$\delta_2^q V_A(t, h) = \delta_2^q V_V(t, h) = 2 \left[\frac{4}{3} \frac{\hat{\alpha}_s(m_Z)}{\pi} \left(c^2 - s^2 + \frac{20}{9} s^4 \right) \right] \\ = \frac{\hat{\alpha}_s(m_Z)}{\pi} \times 1.750, \quad (\text{XV.2})$$

$$\delta_2^q V_R(t, h) = 0. \quad (\text{XV.3})$$

The result of calculations for the third generation is obtained in the form of fairly complicated functions of the t -quark mass:

$$\delta_2^t V_m(t, h) = \frac{4}{3} \frac{\hat{\alpha}_s(m_t)}{\pi} \left[t A_1 \left(\frac{1}{4t} \right) + \left(1 - \frac{16}{3} s^2 \right) t V_1 \left(\frac{1}{4t} \right) \right. \\ \left. + \left(\frac{1}{2} - \frac{2}{3} s^2 \right) \ln t - 4 \left(1 - \frac{s^2}{c^2} \right) t F_1 \left(\frac{c^2}{t} \right) - 4 \frac{s^2}{c^2} t F_1(0) \right], \quad (\text{XV.4})$$

$$\delta_2^t V_A(t, h) = \delta_2^t V_V(t, h) = \frac{4}{3} \frac{\hat{\alpha}_s(m_t)}{\pi} \left\{ t A_1 \left(\frac{1}{4t} \right) - \frac{1}{4} A_1' \left(\frac{1}{4t} \right) + \left(1 - \frac{8}{3} s^2 \right)^2 \left[t V_1 \left(\frac{1}{4t} \right) - \frac{1}{4} V_1' \left(\frac{1}{4t} \right) \right] \right. \\ \left. + \left(\frac{1}{2} - \frac{2}{3} s^2 + \frac{4}{9} s^4 \right) - 4 t F_1(0) \right\}, \quad (\text{XV.5})$$

$$\delta_2^t V_R(t, h) = \frac{4}{3} \frac{\hat{\alpha}_s(m_t)}{\pi} \left[t A_1 \left(\frac{1}{4t} \right) - \frac{5}{3} t V_1 \left(\frac{1}{4t} \right) - 4 t F_1(0) + \frac{1}{6} \ln t \right], \quad (\text{XV.6})$$

where

$$\hat{\alpha}_s(m_t) = \hat{\alpha}_s(m_Z) \left[1 + \frac{23}{12\pi} \hat{\alpha}_s(m_Z) \ln t \right]^{-1}. \quad (\text{XV.7})$$

Note that $\delta_2 V_i$ are independent of m_H . The functions $V_1(r)$, $A_1(r)$ and $F_1(x)$ have rather complex form and were calculated in Ref. [54]. Their expansions for sufficiently small values of arguments are (we have added cubic terms to the expansions from Ref. [54]):

$$V_1(r) = r \left[4\zeta(3) - \frac{5}{6} \right] + r^2 \frac{328}{81} + r^3 \frac{1796}{25 \times 27} + \dots, \quad (\text{XV.8})$$

$$A_1(r) = \left[-6\zeta(3) - 3\zeta(2) + \frac{21}{4} \right] + r \left[4\zeta(3) - \frac{49}{18} \right] + r^2 \frac{689}{405} + r^3 \frac{3382}{7 \times 25 \times 27} + \dots, \quad (\text{XV.9})$$

$$F_1(x) = \left[-\frac{3}{2} \zeta(3) - \frac{1}{2} \zeta(2) + \frac{23}{16} \right] + x \left[\zeta(3) - \frac{1}{9} \zeta(2) - \frac{25}{72} \right] + x^2 \left[\frac{1}{8} \zeta(2) + \frac{25}{3 \times 64} \right] + x^3 \left[\frac{1}{30} \zeta(2) + \frac{5}{72} \right] + \dots, \quad (\text{XV.10})$$

where $\zeta(2) = \pi^2/6$, $\zeta(3) = 1.2020569\dots$

Adding up the contributions (XV.4) – (XV.6) and using the expansions (XV.8) – (XV.10), we obtain (up to terms of the order of t^3):

$$\delta_2^t V_m(t, h) = \frac{\hat{\alpha}_s(m_t)}{\pi} \left[-2.86t + 0.46 \ln t - 1.540 - \frac{0.68}{t} - \frac{0.21}{t^2} \right] = \frac{\hat{\alpha}_s(m_t)}{\pi} \times (-11.67), \quad (\text{XV.11})$$

$$\delta_2^t V_A(t, h) = \delta_2^t V_V(t, h) = \frac{\hat{\alpha}_s(m_t)}{\pi} \left[-2.86t + 0.493 - \frac{0.19}{t} - \frac{0.05}{t^2} \right] = \frac{\hat{\alpha}_s(m_t)}{\pi} \times (-10.10), \quad (\text{XV.12})$$

$$\delta_2^t V_R(t, h) = \frac{\hat{\alpha}_s(m_t)}{\pi} \left[-2.86t + 0.22 \ln t - 1.513 - \frac{0.42}{t} - \frac{0.08}{t^2} \right] = \frac{\hat{\alpha}_s(m_t)}{\pi} \times (-11.88). \quad (\text{XV.13})$$

These formulas hold for $m_t > m_Z$. In the region $m_t < m_Z$ we either set $\delta_2^t V_i = 0$ or make use of the massless limit

$$\delta_2^t V_i = \frac{1}{2} \delta_2^q V_i.$$

In any case this region gives negligible contribution to the global fit.

XVI. The $\delta_5 V_i$ corrections

In the second order of weak interactions quadratic dependence on the mass of the Higgs boson is given by expressions [83]:

$$\delta_5 V_m = \frac{\bar{\alpha}}{24\pi} \frac{m_H^2}{m_Z^2} \times \frac{0.747}{c^2} = 0.0011, \quad (\text{XVI.1})$$

$$\delta_5 V_A = \delta_5 V_V = \frac{\bar{\alpha}}{24\pi} \frac{m_H^2}{m_Z^2} \times \frac{1.199}{s^2} = 0.0057, \quad (\text{XVI.2})$$

$$\delta_5 V_R = -\frac{\bar{\alpha}}{24\pi} \frac{m_H^2}{m_Z^2} \frac{c^2 - s^2}{s^2 c^2} \times 0.973 = -0.0032. \quad (\text{XVI.3})$$

The numerical evaluations above were made with $m_H = 300$ GeV.

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