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Ultracold neutrons — discovery and research

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Contents

Abstract. A review of the progress in the physics of ultracold neutrons (UCN), starting with the first experimental studies, is given. Problems dealt with by the author under the supervision of Fyodor L'vovich Shapiro are presented. It is shown how these problems were gradually solved and how their relation to other issues in physics was revealed. A review of the present status of UCN physics is given, and prospects are discussed.

1. Introduction

This review is dedicated to the memory of Fyodor L'vovich Shapiro, who would have been 80 years old on May 6, 1995. His untimely departure on January 30, 1973 was perceived as a bad loss by everyone who had come to know this person closely or to work near him. In connection with the 80-th anniversary of his birthday, all those who knew him, who remembered him and wished to share their recollections decided to publish a jubilee collected works as a tribute to his memory. The present review was initially prepared as part of this collection. Ultimately, the work has not turned out to be successful. On the one hand, it was conceived as a scientific presentation, typical of such reviews, while on the other, it contained much personal information related to F L Shapiro himself. Both aspects were actually too broad to be dealt with in one manifold. So the works belonging to two genres were separated. Everything that was personal (needless to say, from a viewpoint of scientific contacts, not an everyday occurrence) was published in the above-mentioned collection [1], while the actual review is presented here. Whatever the case may be, at the insistence of the editing board and under the rules of *Physics-Uspekhi*, all 'lyrical digressions' were excluded from the review. And unquestionably an account of the development of UCN physics could not have appeared without the ideas, errors and disappointments of F L Shapiro. But such was the role of this personality that not only his ideas, but his errors and delusions also resulted in the establishment of a creative atmosphere and served as the driving force behind research work.

In choosing material for this review preference was mainly given to work performed directly under the supervision of F L Shapiro and to papers published after 1990, i.e. after publication of the monographs [2, 3].

2. The beginning of experimental research with UCN

It is just time now to turn to the early history of experimental investigation of UCN,† the honour of pursuing which is to be attributed, besides F L Shapiro, also to V I Lushchikov, Yu N Pokotilovskiĭ, and A V Strelkov [5].

2.1 Neutrons in between crystals

Before proceeding with the first experiment, the success of which seemed dubious owing to the low number of UCN

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† We remind those who may be unfamiliar that neutrons are said to be ultracold when their energies are on the order of 10^{-7} eV. They experience total reflection from most substances at all angles of incidence, so they can be kept in hermetically sealed vessels. The author of the first publication on UCN [4] was Ya B Zel'dovich.

expected,† F L Shapiro considered the possibility of confining a thermal neutron in between perfect crystals.‡

His idea was to place two monocrystals opposite each other or four monocrystals at the angles of a rectangular region so that a neutron undergoing multiple Bragg scattering would travel from one crystal to another, thus accumulating a long trajectory inside a limited region of space. Theoretical studies along these lines, which were carried out in 1967 and remained unpublished, yielded rather a pessimistic result. Essentially, it stated that a beam spreads out linearly with time in a direction perpendicular to the trajectory delineated between the crystals. To reduce this spread, a strong collimation of the beam is required. But, then, the amount of neutrons in the beam drops catastrophically down to a level equivalent to the number of UCN.

It must be said that 22 years later this idea was actually realised [7, 8].

The layout of the experiment is shown in Fig. 1. To overcome the spread, the neutron beam was limited by the walls of the glass neutron guide. But for the glass neutron guide to reflect all the neutrons their velocities perpendicular to the walls have to be smaller than the boundary velocity§ of the walls. These conditions determine the total fraction Φ of the neutron flux that can be confined in this way.

If the primary spectrum is Maxwellian, then the fraction of confined neutrons differs insignificantly from the fraction of UCN in this spectrum. Indeed, the spectral density of the neutron flux departing from the surface of the moderator in the direction, for instance, of the z axis is represented by the Maxwellian expression

$$\mathrm{d}\Phi = 2v_z\,\mathrm{d}^3v\,\frac{\Phi_0}{\pi v_T^4}\exp\left(-\frac{v^2}{v_T^2}\right) \equiv \mathrm{d}v_z^2\,\mathrm{d}^2v_\perp\rho\exp\left(-\frac{v^2}{v_T^2}\right), \quad (1)$$

where Φ_0 is the total flux density of thermal neutrons, $\rho = \Phi_0/(\pi v_T^4)$ is the differential flux density, which is constant within the whole range of velocities, $v_T = \sqrt{2mk_\mathrm{B}T}$ is the thermal neutron velocity at temperature T, v_z is the velocity component along the chosen z axis, and, finally, $v_\perp$ is the velocity component perpendicular to the z axis. The differential $\mathrm{d}v_z^2\,\mathrm{d}^2v_\perp$ determines the velocity interval of the confined neutrons. In the case of UCN, for which $v < v_l \ll v_T$, this interval is v_l^4 and the flux density of confined neutrons is $\int \mathrm{d}\Phi = \rho v_l^4$.

In the experiment presented in Refs [7, 8], the interval $\mathrm{d}v_z^2$ is determined by the width of the Darwin table and amounts to $2v_l^2$.¶ Since in the direction perpendicular to the crystals the neutron confinement is due to total reflection from the walls of the neutron guide, $\mathrm{d}^2v_\perp$ can also be set equal to v_l^2.

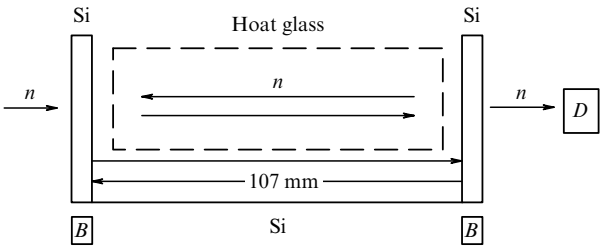

Figure 1. Neutrons with a velocity of 650.8 m s^{-1} were transmitted through two crystalline Si plates placed at a distance of 107 mm from each other and which were cut out of a sole monolith together with the base. For the beam, when spreading, not to leave the boundaries of the system (the end crystals were $52 \times 30 \times 3.9$ mm^3), a segment of the neutron guide with perfectly smooth glass walls was placed in between the crystals. Injection of the neutrons into the system and their extraction from it were implemented with the aid of a short-time (1.2 ms) switch-on of a 1.25 T magnetic field furnished close to one of the crystals. Experiments were carried out [7, 8] with the powerful pulsed proton source of neutrons at the Rutherford – Appleton laboratory (England). The source burst was 120 μm. Neutrons with the above indicated velocity formed a cloud 10 cm in size, and their time of flight between the crystals amounted to 1.7 ms. In a single filling 0.5 neutrons were accumulated in between the crystals. The total number of neutrons counted after 12 reflections from the crystals (6 transits there and back, the exposure time $t_\mathrm{exp} = 20.2$ ms) was 438 neutrons per 1000 fillings, after 96 reflections ($t_\mathrm{exp} = 161.9$ ms) it was 154 neutrons, and after 156 reflections ($t_\mathrm{exp} = 263.1$ ms) it amounted to about 80 neutrons. The mean reflection coefficient from the monocrystal turned out to be 0.9978.

Thus, the number of neutrons confined owing to Bragg reflection in between the crystals may exceed the fraction of UCN by a value determined by the factor $2\exp(-v_\mathrm{B}^2/v_T^2) < 2$, and in the case of experiments in search of the neutron EDM this excess is far from compensating for the loss in confinement time. This shows that the estimates made in 1967 were correct. Nonetheless, the experiment looks very attractive and merits being carried out, but it requires sufficiently high neutron intensities and good background conditions.

Apropos of mechanical UCN generators. The preceding arguments concerning the spectral distribution of neutrons confined by a crystal also apply to mechanical UCN generators. In mechanical generators a neutron beam of velocity v impinges on a mirror escaping at a velocity u. The neutrons reflected from the mirror have a velocity $v' = v - 2u$. If $|v'| < v_l$, the reflected neutrons happen to be ultracold. For a long time it was believed that in this way one can obtain significantly more UCN, than in the Maxwellian spectrum. The argument in favour of this was that, since the neutron flux is proportional to $v\Delta v$, a greater v leads to a higher flux within a fixed reflection interval Δv. Since neutrons are not lost in the case of specular reflection, all the neutrons in a more rapid flux are transformed into UCN. The resulting UCN density† happens to seem v/v_l times greater, than when the UCN are extracted directly from the Maxwellian spectrum.

Actually, no gain exists, because the reflection does not take place within a fixed interval of velocities Δv, but within a fixed range of energies

$$\Delta E \propto \Delta v^2 = 2v\Delta v = v_l^2\,,$$

† In this connection, it is useful to quote the well-known book by I I Gurevich and A V Tarasov [6], where the following is written word for word: even it were possible to single out UCN from the Maxwellian spectrum, "the intensity of such a neutron beam would be so small that it would most likely be insufficient for utilisation in experiments".

‡ He was interested in this issue because of his intention to perform an experiment in search of the electric dipole moment (EDM). If the neutron has an EDM $\mathbf{d}$, then it should interact with an external electric field $\mathbf{E}$: $U = \mathbf{dE}$, and the longer the time a neutron is in a field, the smaller the value of $|\mathbf{d}|$ that can be revealed during a single measurement cycle.

§ The boundary velocity v_l is determined as $v_l = \sqrt{2mU}$, where U is the potential of the walls, and m is the neutron mass. In the case of glass it amounts to 4.26 m s^{-1} [2, 3].

¶ Total Bragg reflection occurs in the velocity interval $v_\mathrm{B} \leqslant v_\perp \leqslant v_\mathrm{B} + \Delta$, where v_B is the Bragg velocity, and Δ determines the width of the total reflection interval and is called the Darwin table, having been first calculated by Ch Darwin. For each substance Δ turns out to be equal to, say, v_l^2/v_B, where v_l is the UCN boundary velocity.

† It was assumed that $\Delta v = v_l$.

and the higher the velocity v, the smaller the interval $\Delta v = v_l^2/2v$. Thus, mechanical generators do not, essentially, lead to a gain in the amount of UCN generated, as compared with an ordinary moderator.

Naturally, this does not mean that mechanical generators are useless. They may have technological advantages in comparison with converters,† if the experimenter only has at hand a given extracted beam, or the losses during the transfer of UCN from the converter along a lengthy neutron guide are too significant. Incidentally, at present one of the most powerful UCN sources is precisely a mechanical generator — namely, the Steyerl turbine installed in Grenoble [9].

Magnetic shutters. The system serving for injection and extraction of neutrons in experiments presented in Refs [7, 8] is also interesting. It operates as follows. Since the neutrons possess a magnetic moment μ, they interact with the magnetic field B: $U = \pm\mu B$, which results in neutrons with one spin projection experiencing acceleration in the magnetic field, while those with another projection are slowed down and lose their capability of undergoing total reflection from the crystals, because their velocity happens to be outside the boundaries of the Darwin table ($v_B, v_B + v_l^2/v_B$).

This approach was proposed earlier for pulsed accumulation of UCN using, for example, a beryllium converter [10]. If a magnetic field pulse is applied to the beryllium converter during the burst of the reactor, then the total interaction energy in the converter is significantly reduced for one of the spin orientations, and the neutrons leave the converter, like a sluice, without being accelerated. Then the magnetic field is removed, and the Be potential barrier is restored, thus hindering the leakage of UCN back through the converter. The difficulty here is that the field for the UCN must be completely concentrated within the Be material (as in the case of the sluice, when the water level may vary only in between the gates). But in experiments [8] this is certainly not obligatory.

2.2 The first experiment with UCN

According to accounts by those who participated in the discovery (it so happened that it was prestigious for us to use the word 'discovery', instead of 'observation'), an attempt was first made to detect UCN in the very hall of the reactor. The idea was to observe neutrons with a certain time delay after the burst of the reactor. But the background in the hall of the reactor was so high, that it turned out to be absolutely impossible to pick out signals directly from the UCN. Thus, the decision was taken to construct a lengthy bent copper neutron guide (a pipe 10.5 m long and 96 mm in diameter) along which the UCN were guided away from the moderator to the experimental hall. They were filtered then from the direct beam of fast neutrons and γ-quanta, and detected by two scintillation counters alternately shut out by a reflecting copper shutter [5]. The mean UCN counting rate‡ in the first experiment, where the reactor generated one pulse every 5 s for a given average power of 6 kW, merely amounted to 1 neutron per 200 s, but the background was even lower: 0.001 neutron s^{-1}. This intensity was sufficient to form an idea of the properties of UCN and to perform certain experiments with them, for example, transmission experiments with the neutron guide filled with helium. But we shall return to this below.

2.3 A dramatic moment

The first attempt at observing UCN was unsuccessful for a purely technical reason. Although the neutron guide was hermetically sealed at both ends, it turned out impossible to achieve a good vacuum. With a poor vacuum, the UCN were heated in collisions with molecules of the gas and could not reach the detectors. There could be two reasons for a poor vacuum: one, very simple, could be a leak, through which atmospheric air penetrated into the neutron guide. Another, more serious, reason was related to possible radiative disintegration of the UCN source itself. The point is that UCN cannot penetrate into a pipe from outside for the same reason that they cannot leave it. Therefore it is necessary to load the neutron guide with some additional material in which the faster neutrons, freely penetrating through the walls, can experience inelastic scattering, lose energy and transform into UCN. Such material is a source of UCN and is called a converter.

In the first experiment, the part of the converter was assigned to a thick§ piece of polyethylene loaded at the end of the neutron guide, closest to the reactor, and the suspicion arose that the reactor radiation knocked protons out of the polyethylene. In this case hydrogen accumulated inside the neutron guide, and a good vacuum could not be achieved in a crucial respect without changing the construction of the converter.

Further developments of events turned out to really dramatic. The great desire to carry out the experiment encountered a serious obstacle. Moreover, the time had arrived for a scheduled shut-down of the reactor for reconstruction. Actually, the reactor was already stopped, and F L Shapiro, who was Vice-Director, had to exert enormous efforts in order to shift the beginning of the reconstruction. At a meeting of the Institute Directorate, I M Frank (as Director of the Laboratory) raised doubts concerning the expedience of delaying work for the sake of UCN. His arguments were irrefutable: the blitz had been unsuccessful, and it was necessary to calmly understand the reasons for the defeat and to prepare subsequent experiments thoroughly. For them it may be required to change the construction of the converter completely, if the faulty vacuum was due to irradiation of the polyethylene from the reactor resulting in its disintegration.

The only way to overcome these doubts was to assure themselves and prove to others that the poor vacuum was not due to disintegration of the converter, but to atmospheric air penetrating into the neutron guide. F L Shapiro left the meeting and asked A Strelkov to perform the appropriate tests in half an hour. A detailed description of how Strelkov rapidly mounted a bicycle, took an empty retort, found sensitive scales, broke them (because there was not sufficient room for the retort), assembled the scales again with the external hanging retort, pumped out the air from the retort, weighed it, filled it with the gas which could not be evacuated from the neutron guide and which impeded performance of the experiment, weighed the retort with the gas, verified that

† Inside a closed volume, impenetrable to UCN from the outside, there is established an additional moderator called a converter. This issue will be dealt with further in greater detail.

‡ The difference between the respective counting rates of an open and closed detector in between bursts of the reactor.

§ Subsequently, it became clear that only thin layers of material are required for generating UCN.

the molecular weight of the gas was precisely in accordance with that of air, rushed back on the bicycle, and put a note under the door of the director's office (since the secretary would just not let him in) with the sole word 'AIR' written in large letters, all can only be given by the actual participant of the events. But it was important that all doubts were finally scattered, the experiments continued, the leak in the neutron guide found and removed, and that in spite of everything UCN were actually found.

The experiments were carried out rapidly not for fear of competition, but because of the passion which usually possesses experimenters: the desire to see in the short run what actually evolves. Therefore, it was a surprise for F L Shapiro when, only two months after the article published in *Pis'ma v JETP* [5], an article appeared in *Phys. Lett.* [11], reporting A Steyerl in Germany to have also observed very slow neutrons with a spectrum extremely close to the UCN region. This news confirmed the timeliness of the experiments and initiated a close collaboration with A Steyerl, who subsequently became one of the principal investigators of UCN [12].

2.4 Episode with roughnesses

As soon as UCN were first observed, the first experiments with them were performed. In these experiments the transmission of neutron guides was studied. Diffuse reflection from the walls was considered to take place, so the flux J along the neutron guide was described by the diffusion formula $J = -D \operatorname{grad} n$, where $D = 2rv/3$ is the diffusion coefficient, r is the radius of the neutron guide, v is the neutron velocity, $n = n(z)$ is the neutron density depending on the z coordinate along the neutron guide. Applying the continuity equation

$$\operatorname{div} J = -\frac{n}{\tau}\,, \tag{2}$$

where $\tau = 2r/(\mu v)$ is the neutron lifetime in the pipe, μ is the loss probability in one neutron collision with the wall, and substituting here the expression for J, we obtain the diffusion equation

$$\frac{\mathrm{d}^2 n}{\mathrm{d}z^2} = \frac{n}{L_\mathrm{D}^2}\,, \tag{3}$$

the solution of which, without taking into account the boundary conditions, has the form $n = n_0 \exp(-z/L_\mathrm{D})$. Hence, it follows that the flux $J(L)$ at the exit from a neutron guide of length L is

$$J(L) = J(0) \exp\left(-\frac{L}{L_\mathrm{D}}\right), \tag{4}$$

where $L_\mathrm{D} = \sqrt{D\tau}$, n_0 and $J(0)$ are the neutron density and flux, respectively, at the inlet.

The formulae are absolutely clear and leave no room for doubt. Measuring the dependence $J(L)$ we find L_D, from which it is not yet possible to derive the lifetime and the diffusion coefficient separately.

To determine them separately, the neutron guide was filled with gaseous ^{4}He, which scatters neutrons inelastically, and measurement was made of the dependence of the UCN counting rate on the gas pressure.

The lifetime of neutrons in the pipe was derived from this dependence as follows. Since the neutron velocity v_UCN is significantly smaller than the velocities of atoms (v_He), a neutron was considered to be at rest in the pipe and to be bombarded with helium atoms from all possible directions. The number of atoms interacting with the neutron per unit time equals

$$\nu = N_\mathrm{He} v_\mathrm{He} \sigma(v_\mathrm{He}) = \frac{p_\mathrm{He} v_\mathrm{He} \sigma(v_\mathrm{He})}{k_\mathrm{B} T}\,,$$

where the subscript He indicates a helium atom, k_B is the Boltzmann constant, N is the number of atoms per unit volume at temperature T and pressure p. Accordingly, the lifetime of a neutron before it undergoes a collision with some atom of the gas is

$$\tau_\mathrm{He} = \frac{1}{\nu} = \frac{k_\mathrm{B} T}{p_\mathrm{He} v_\mathrm{He} \sigma(v_\mathrm{He})} = \frac{250}{p_\mathrm{He}}\ \mathrm{s}\,, \tag{5}$$

if the pressure p_He is measured in mm Hg, and the temperature is considered room temperature. The very first experiment [5] revealed that an enhancement of the pressure from 0 up to 1.25 mm Hg results in the counting rate of the detectors at the exit from the channel being reduced by a factor of 2, from which the conclusion was made that the lifetime of a neutron in the channel is 200 s.

The results of subsequent experiments were quite paradoxical: neutron guides with approximately identical L_D yielded differing τ and, since $L_\mathrm{D} = \sqrt{D\tau}$, the greater was τ (i.e. the smaller the losses), the smaller was D, indicating that the roughnesses should be greater.† In this connection, the question arose as to whether roughnesses could reduce the loss coefficient, i.e. reduce the loss probability in a single collision with the wall.

Theoretical studies led to a totally contrary result: enhancement of the roughnesses increases the loss coefficient, so ultimately a critical inspection of the formula (5) and of the processing of experimental data turned out to be necessary.

Indeed, we are interested in the lifetime of a neutron inside the neutron guide before its loss in the walls, but not in the time required for it to be lost owing to the influence of helium atoms. The quantity τ_He (5) is only one of the components of the neutron loss, to be taken into account in estimating the total lifetime τ_t and the diffusion length $L_\mathrm{D} = \sqrt{D\tau_\mathrm{t}}$. The influence of helium becomes greater, the longer the neutron remains in the neutron guide. The latter circumstance depends not only on the lifetime of the neutron without helium, but also on the time it travels along the neutron guide, i.e. on the length L.

If τ_He (5) increases, i.e. the flux at the end of the neutron guide becomes two times smaller at a lower pressure of the He, this may only point to the neutron being present in the neutron guide for a longer time, for instance, when the neutron guide is made longer, but not to the losses in the walls being reduced.

For a correct interpretation of the experiment it is necessary to substitute $1/\tau + 1/\tau_\mathrm{He}$ for $1/\tau$ in equation (2). Accordingly, L_D in equation (3) and solution (4) is

$$L_\mathrm{D}(p) = \sqrt{D\frac{\tau\tau_\mathrm{He}}{\tau + \tau_\mathrm{He}}} = L_\mathrm{D}(0)\sqrt{\frac{\tau_\mathrm{He}}{\tau + \tau_\mathrm{He}}}\,,$$

where $L_\mathrm{D}(0) = \sqrt{D\tau}$ is the diffusion length in the absence of helium. From these formulae it immediately follows that in

† The relation between roughnesses and the diffusion coefficient will be dealt with below.

experiments one measures, instead of the quantity τ_{He} (5), the diffusion length which varies with the pressure of helium in the neutron guide. If, at a certain pressure p, the counting rate at the exit of the neutron guide becomes e times lower, this indicates that $L/L_D(p) = L/L_D(0) + 1$, i.e.

$$\frac{L}{L_D(0)}\left(\sqrt{\frac{\tau+\tau_{He}}{\tau_{He}}}-1\right)=1\,. \tag{6}$$

Equation (6) is readily solved, and we obtain

$$\tau=\tau_{He}\left[\left(\frac{L_D(0)}{L}+1\right)^2-1\right]=\tau_{He}\left(\frac{L_D^2(0)}{L^2}+2\,\frac{L_D(0)}{L}\right), \tag{7}$$

where τ_{He} is the neutron lifetime before collision with helium atoms *calculated* by formula (5) at a gas pressure p_{He}.

Thus, knowing $L_D(0)$ and τ_{He} we readily find τ, and then D. It is also possible, subsequently, to determine the time of neutron stay inside the neutron guide. For this purpose it is necessary to take advantage of the fact that, according to the laws of random walk, in a time T the neutron covers a distance s determined by the relation $s^2 = 2DT$. Thus, for example, the time required for the neutron to travel from the neutron guide entrance to its exit ($s = L$) is $T = L^2/(2D)$. Making use of this relation, we rewrite expression (7) as

$$\tau_{He}=\frac{2T}{1+2\sqrt{2T/\tau}}\,. \tag{8}$$

This expression explicitly shows the relationship between τ_{He}, τ and T.

Thus, although τ can be derived from τ_{He}, the interplay is not really direct. Indeed, if the expression for $L_D(0)$ is substituted into (7), then we obtain

$$\tau=\tau_{He}\,\frac{4D\tau_{He}L^2}{(L^2-D\tau_{He})^2}\,. \tag{9}$$

This, however, is not the full story. The point is that to determine the transmission of the neutron guide we made use of the simplest formula (4), which, generally speaking, is insufficient, since it does not take into account the boundary conditions at the entrance and the exit openings. If all the neutrons departing from the neutron guide are assumed not to come back, then the transmission of the neutron guide $T(L) = J(L)/J(0)$ and its reflection (the latter is defined as $R(L) = J_L(0)/J(0)$, where $J_L(0)$ is the flux of neutrons departing from the entrance of the neutron guide) are as follows:†

$$T(L)=\mathrm{e}\,\frac{1-r^2}{1-r^2\mathrm{e}^2}\,,\qquad R(L)=r\,\frac{1-\mathrm{e}^2}{1-r^2\mathrm{e}^2}\,,$$
$$\mathrm{e}\equiv\exp\left(-\frac{L}{L_D}\right), \tag{10}$$

where

$$r=\frac{1-q}{1+q} \tag{11}$$

is the reflection from the entrance opening of a semi-infinite neutron guide, and $q = 2D/(vL_D)$.

Taking into account that helium pressure occurs in L_D, we obtain that the relation between τ and τ_{He} turns out to be even more complex than it follows from (9). At any rate, it is quite available for analysis using the simplest computer means.

Applying the aforecited formulae it is possible to establish whether the flow of the UCN gas is indeed described by a constant diffusion coefficient D, or whether D depends on the length of the neutron guide. If it turns out that D varies with L, the question arises as to which processes regulate the flow of a rarefied neutron gas? Precisely this issue will be dealt with somewhat later.

And this is still not the full story. The helium curves may also yield information on the time the neutron is to be found inside the neutron guide, when the derivative of the transmission, $\mathrm{d}T(L)/\mathrm{d}p_{He}$, is measured for $p_{He} \to 0$. In this connection it is useful to make a small digression towards modern studies in fundamental physics, in which the same issue of losses described by expression (5) is raised.

2.4.1 A modest proposal concerning tunnelling times. Such is the title of Ref. [13], the content of which is related to the issue indicated. In this work a recipe is proposed for determining the time a quantum particle is to be found in the region of a scattering potential. Generally speaking, there exists a great number of such recipes (see, for instance, a review article [14]), but so as not to let our reasoning spread out too much, we shall dwell upon one only, the one dealt with in Ref. [13].

So, imagine having a purely real rectangular potential step of height U (for example, an infinitely thick layer of a nonabsorbing monatomic‡ substance with its atoms arranged in perfect order§ at zero temperature¶) and measuring the corresponding UCN reflection coefficient $R = |r|^2$.† How can one find out how long the particle stays inside the potential well?

The following approach is proposed: we add to the potential a small imaginary part

$$-\mathrm{i}W \ll U\,.$$

For example, this may be a uniform solution of absorbing atoms with low concentration (just like helium inside the neutron guide):

$$W=\frac{\hbar}{2}\,N_a v\,\sigma_a(v)=\frac{\hbar}{2\tau_a}\,,$$

where N_a is the concentration of atoms, $\sigma_a(v)$ is the absorption cross section for a neutron velocity v, $\tau_a = 1/[N_a v \sigma_a(v)]$ is the characteristic absorption time defined by the second equality in (5).

Now consider the total reflection of UCN from an infinitely wide potential step. The time the neutron stays within the step can be expected beforehand to be characterised by a distribution and not just a single value. If we denote the stay-time distribution density by $f(\tau)$, the follow-

† We shall apply formulae (10) here without giving any derivation (it should be only recorded that they are widely applied in the most varying fields of physics), since such a derivation will be presented below in the course of studying the reasons for the total reflection of UCN from the walls.

‡ To avoid elastic incoherent scattering.
§ To avoid diffuse elastic scattering.
¶ To exclude inelastic scattering.
† Evidently, in these conditions $R = |r|^2 = 1$.

ing holds valid in the case of total reflection:

$$\int_0^\infty f(\tau)\,\mathrm{d}\tau = 1\,.$$

Addition of absorbing atoms, which do not affect the reflection law, but do absorb neutrons with a characteristic time τ_a, results in the number of departing neutrons equal to

$$R = \int_0^\infty \exp\left(-\frac{\tau}{\tau_\mathrm{a}}\right) f(\tau)\,\mathrm{d}\tau < 1\,. \tag{12}$$

From this expression it is easy to find the mean value $\langle\tau\rangle$:

$$\langle\tau\rangle = \int_0^\infty \tau f(\tau)\,\mathrm{d}\tau = \lim_{\tau_\mathrm{a}\to\infty} \tau_\mathrm{a}^2 \frac{\mathrm{d}}{\mathrm{d}\tau_\mathrm{a}} \int_0^\infty \exp\left(-\frac{\tau}{\tau_\mathrm{a}}\right) f(\tau)\,\mathrm{d}\tau\,.$$

Since the last integral is related to the reflection coefficient (12), then

$$\langle\tau\rangle = \lim_{\tau_\mathrm{a}\to\infty} \tau_\mathrm{a}^2 \frac{\mathrm{d}}{\mathrm{d}\tau_\mathrm{a}} R = \lim_{W\to 0} \frac{\hbar}{2} \frac{\mathrm{d}}{\mathrm{d}W} R\,. \tag{13}$$

Precisely this time was determined in Ref. [13] to be the stay time inside the potential.†

To get a feel for the physical meaning of the result obtained, it is useful to consider a similar case in diffusion. The analogy with UCN reflection will be nearly complete if one considers a semi-infinite neutron guide with nonabsorbing walls and then adds helium into the neutron guide for determining the stay time of the neutron.

The limit value of $R(L)$ in (10) as $L\to\infty$ is $R = r(L_\mathrm{D})$, where the dependence on L_D is indicated explicitly. In the absence of helium, the diffusion length $L_\mathrm{D} = \infty$ and $r(L_\mathrm{D}) \to 1$.

When a small amount of helium is added, we shall make use of the first equality in expression (13):

$$\begin{aligned}\langle\tau\rangle &= \lim_{\tau_\mathrm{He}\to\infty} \tau_\mathrm{He}^2 \frac{\mathrm{d}}{\mathrm{d}\tau_\mathrm{He}} r \\ &= \lim_{\tau_\mathrm{He}\to\infty} \tau_\mathrm{He}^2 \frac{\mathrm{d}L_\mathrm{D}}{\mathrm{d}\tau_\mathrm{He}} \frac{\mathrm{d}q}{\mathrm{d}L_\mathrm{D}} \frac{\mathrm{d}}{\mathrm{d}q} r \propto \frac{\tau_\mathrm{He}^2}{\tau_\mathrm{He}^{3/2}} \to \infty\,.\end{aligned} \tag{14}$$

Thus, we have obtained the absolutely natural result that the time a neutron stays in an infinitely long neutron guide before its departure is infinity.

Hitherto we have only dealt with the stay time for semi-infinite systems, but the same reasoning can also be applied to the case of finite neutron guides and finite potentials, without limiting the consideration to subbarrier energies.

Thus, quite earthly problems related to UCN lead to interesting results in fundamental physics. Now we shall move on to one more episode involving roughnesses, which also happens to be related to 'high poetry'.

2.4.2 One more episode involving roughnesses. Meanwhile, the issue of roughnesses had acquired new aspects. For quite a long time F L Shapiro was convinced of the diffuse propagation of a neutron along a neutron guide. But at the 1st school in neutron physics held in 1971 discussions with its participants compelled him to change his mind. The idea of diffuse propagation stemmed from information on the dynamics of a rarefied gas. From vacuum physics it is known that the flow of molecules along pipes is accompanied by their almost totally diffuse reflection from the walls. But physicists who have already dealt with neutron guides for thermal neutrons are fully aware that the probability of neutrons undergoing specular reflection from a surface is rather high. It was necessary to learn how to calculate the diffusion coefficient D for most diverse indicatrices of neutron reflection from the walls. To this end one can consider an infinitely long neutron guide, imagine marking a neutron at an arbitrary point a and examining the time dependence of its square separation $\left\langle(z-a)^2\right\rangle$ from this point in the case of random walk. In a long time this dependence should become linear: $\left\langle(z-a)^2\right\rangle = At$, so equating it to $2Dt$ we readily obtain the diffusion coefficient $D = A/2$.

If in each collision with the wall, totally diffuse reflection occurs, then $D = 2rv/3$. While, if the probability of diffuse reflection is only $g < 1$ and the probability of purely specular reflection is $1-g$, where g is independent of the incidence angle, then the diffusion coefficient becomes $D = (2rv/3)\times(2-g)/g$, i.e. when g decreases, it increases. This should be expected, since taking advantage of specular reflection the particle can cover a greater distance from point a in time t.

But such a reflection law is, generally speaking, not realistic. In practice, everyone knows that the more grazing the angle at which one looks at a surface, the more specular the surface seems. This means that the coefficient g must be dependent of the angle. Though, what does it matter? If the method of calculation is known, one sees that there exists a mathematical algorithm, there is a reflection law, so one should just apply the algorithm to the law and obtain the diffusion coefficient. But, alas, the algorithm yields a meaningless result in this case, namely, infinity. The reason is that the neutron, having experienced reflection at a small angle to the axis of the neutron guide even only once, subsequently covers (between two successive collisions with the walls) enormous distances comparable to the total length of the neutron guide and, with a predominant probability, undergoes purely specular reflection. Such neutron propagation can surely not be considered random walk, and therefore it is not described by diffusion formulae.

Physicists, however, have accumulated experience in dealing with infinities. They can be overcome by introducing cut-offs. An appropriate cut-off may also be introduced here. It results, however, in the diffusion coefficient becoming dependent of the length of the neutron guide L, which should affect, for instance, the value of τ derived from the helium curves for various lengths, in accordance with formula (9). Subsequently, it was taken into account that owing to the existence of the gravity force the neutron mean free path along the axis of a horizontal neutron guide of radius r cannot exceed $L_g = v\sqrt{4r/g}$, where g is the acceleration of free fall, and precisely this quantity was adopted as the cut-off parameter. At this point one could stop. True, it was also interesting to examine propagation in vertical or inclined neutron guides taking gravity into account, and this was actually done, but we shall deal with this issue below. The main thing is that the problem could be considered solved.

However, if one looks at the problem of UCN diffusion in a neutron guide from a general point of view, and not only considering calculation of the diffusion coefficient, it turns

† Note that, although $|r|^2 = 1$, the derivative $\mathrm{d}R/\mathrm{d}W$ in (13) is not zero, since in the case of departure from the real axis ($\mathrm{Im}\,U = 0$) the reflection coefficient decreases.

out that such a simple everyday problem is directly adjacent to the high poetry of mathematics — to the issue of the foundation of statistics, the classical St.-Petersburg paradox pertaining to games of chance [15], the dynamics of chaos, self-similar processes, Levi statistics, fractals, etc. (the list could go on).

2.4.3 Concluding remarks on diffusion and neutron guides

Overview of experimental research. There have been only a few detailed experimental studies performed with neutron guides. Besides measuring the dependence of the transmission of a neutron guide on its length, carried out from time to time in various institutes with the aim of extracting qualitative information, two nonstationary experiments and one further experiment were performed, the latter aimed at investigating the angular distribution of neutrons at the output of a neutron guide.

One of the nonstationary experiments was carried out in Canada [16]. Here, the thermal neutron beam from a stationary reactor could be shut off by a slide shutter. When this was done, UCN generation stopped and the UCN counting rate at the end of the neutron guide (5.3 m long) started to fall rapidly. The time required for the counting rate to become e times lower determines the time of a neutron leakage from the neutron guide. It can be estimated by the usual thermodynamic formula for a gas, $\tau_e = 4V/(vS)$, where V is the volume containing the gas, S is the area of the surface through which it leaks out, v is the mean velocity of the gaseous molecules. For a cylindrical neutron guide of length L and two openings at the entrance and exit (neutrons can return to the converter and be lost there owing to inelastic scattering) we obtain $\tau_e = 2L/v$, which for $v \approx 5$ m s^{-1} amounts to approximately 2–3 s. The experiment was purely qualitative and showed that $\tau \approx 4$ s. No conclusions can be made, here, naturally. Similar experiments relevant to leakage, not from the neutron guide, but from vessels used for UCN storage, were subsequently carried out repeatedly and served as a source of information for estimating the mean velocity of the neutrons kept in the vessels.

In Gatchina [17], another, more interesting, experiment was performed for studying nonstationary diffusion in a neutron guide and yielded a result, both effective and comprehensive from the viewpoint of physics. The experiment ran as follows. Not far from its output opening the neutron guide was closed by a shutter. At a certain moment the shutter was opened and the dependence of the counting rate J on the time t was measured at the output opening. The striking feature of the result consisted in that a peak was observed in $J(t)$. By investigating the peak amplitude, its width and symmetry versus the distance of the shutter from the output and versus other parameters of the neutron guide, it is possible to obtain reliable information relevant to the character of the neutron propagation. But, regretfully, these studies were no longer carried out, and the result remained, to a significant degree, qualitative.

Investigations of angular distributions were mainly carried out in Dimitrovgrad [18] (see, also, Ref. [19]). The result of these studies can be formulated as follows: the angular distribution at the exit of the neutron guide is not isotropic. This finding was predicted earlier [20] on the basis of Monte Carlo calculations.

On theoretical calculations. Theoretical studies took the path of Monte Carlo calculations [21, 22]. Even when analytical computations were performed, they were limited to specular reflection, including nonspecularity in the form of a parameter characterising losses. Such an approach is justified for neutrons of higher energies and was applied, for instance, in Ref. [23] for computing the transportation of thermal neutrons within fibre focusing systems.

It must be said that Monte Carlo calculations sometimes help to understand the physics of various processes. For example, it seemed evident that in the case of isotropic reflection from the walls and of isotropically distributed neutrons entering the neutron guide, the angular distribution at the exit of the neutron guide should be also isotropic. However, calculations [20] revealed a forward elongation of the angular neutron distribution at the output. This result had to be analyzed, and then it became clear that the neutrons travelling at small angles to the axis are those neutrons that were reflected from the walls closer to the entrance opening, where the particle density is greater, so here the wall reflects (or, in other words, emits) more neutrons.

Having understood this, it was possible to predict the results of calculations with various reflection indicatrices. For example, if the indicatrix is such that a neutron incident upon the wall at a grazing angle most likely undergoes specular reflection, then the angular distribution at the output will be even more elongated. The results of calculations confirmed these predictions.

The second surprise was the result for totally specular reflection from the walls. It had always seemed that, in the case of specular reflection, bending a neutron guide should result in its transmission being reduced. However, calculations [21] revealed this not always to be true. Even rotation through 180° may not increase the resistance of the neutron guide. An analysis of this result helped to understand the point, here. If one imagines a neutron guide of rectangular cross section bent in the horizontal plane, so that the normal to one of its vertical walls at the same time serves as the normal to another one, then in the case of such a rotation and specular reflection no neutron would be able to change its motion and return to the entrance opening. Therefore the transmission of the neutron guide turns out to be independent of the bending angle and the number of rotations in it.

3. The main problem of UCN

The main problem, which we shall now proceed to consider, results from the storage of UCN in closed vessels and consists in that the losses due to the collisions of neutrons with the walls of the trap may significantly exceed ones predicted by theory.

The problem occurred at the very beginning of experiments with UCN, but all the time it seemed to be most likely due to the experiments not being perfect, owing to the experimenters not being experienced, rather than to a real contradiction requiring re-examination of certain concepts for its resolution.

3.1 The layout of an experiment

The experimental layout for UCN storage is presented in Fig. 2.

The result of measurements is represented by the curve $N(t) = N(0)\exp(-t/\tau)$ known as the 'storage curve' (Fig. 3), where τ is the characteristic lifetime in a trap termed the 'storage time', $1/\tau = 1/\tau_\beta + 1/\tau_l$.

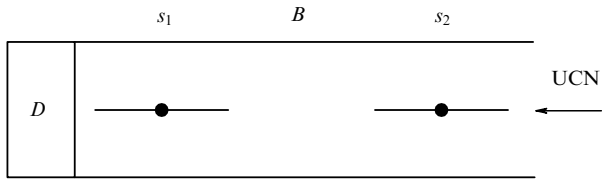

Figure 2. Layout of the UCN storage experiment. When shutter s_1 is closed, shutter s_2 opens, and the neutrons fill up vessel B. Then shutter s_2 is closed, and the neutrons are confined within the vessel for a time $t_{\rm exp}$ called the exposure time. Further, shutter s_1 is opened, and the neutrons remaining in the vessel are counted by detector D.

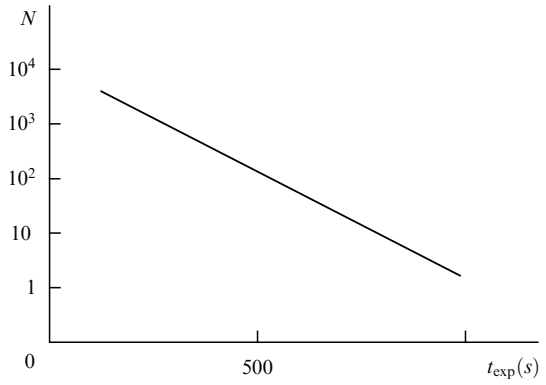

Figure 3. The storage curve gives an indication of the number of neutrons left in the vessel after the exposure time $t_{\rm exp}$. In the ideal case, this curve exhibits an exponential form, $N(t_{\rm exp}) = N_0 \exp(-t_{\rm exp}/\tau)$, where τ characterises the storage time.

The amount of neutrons in the trap decreases with time for two reasons: owing to its own β-decay (τ_β) and owing to absorption and inelastic scattering on the walls ($\tau_{\rm l}$). Here we are interested precisely in the interaction with the walls.

The storage of neutrons in traps is possible because from the point of view of a neutron many materials serve as a potential step of height

$$U = \frac{\hbar^2}{2m}\, u_0\,, \qquad \text{where} \qquad u_0 = 4\pi N_0 b\,, \tag{15}$$

m is the neutron mass (henceforth we shall omit the factor $\hbar^2/m$), N_0 is the number of nuclei per unit volume of the substance, b is the coherent neutron scattering length on the nuclei. The height of the step, U, varies from substance to substance, but it is on the order of 10^{-7} eV. Neutrons with energies $E < U$ are reflected from a wall with such a potential, whatever their angles of incidence, and the reflection amplitude is

$$R = \frac{k_\perp - {\rm i}\sqrt{u_0 - k_\perp^2}}{k_\perp + {\rm i}\sqrt{u_0 - k_\perp^2}}\,, \tag{16}$$

where $k_\perp$ is the wave vector component normal to the wall. In the case of real u_0 we have $|R| = 1$ and a total reflection occurs.

However, the quantity u_0 is complex: $u_0 = u_0' - {\rm i}u_0''$, since the scattering amplitude contains an imaginary part: $b = b' - {\rm i}b''$, where $b'' = \sigma_{\rm l}/2\lambda$, λ is the neutron wavelength, $\sigma_{\rm l} = \sigma_{\rm a} + \sigma_{\rm ie}$ is the cross section defining UCN losses due to a single collision with the wall ($\sigma_{\rm a}$ is the absorption cross section, and $\sigma_{\rm ie}$ is the inelastic scattering cross section).

When u_0 is complex, the reflection coefficient $|R|^2$ is less than unity,† and it is possible to introduce the loss coefficient $\mu = 1 - |R|^2$:

$$\mu = \frac{2k_\perp}{\sqrt{u_0 - k_\perp^2}}\,\eta\,, \quad \text{where} \quad \eta = \frac{b''}{b'} = k\,\frac{\sigma_{\rm l}}{4\pi b'} = \frac{\sigma_{\rm l}}{2\lambda b'}\,, \tag{17}$$

$\sigma_{\rm l}$ is the loss cross section equal to the sum of the absorption cross section $\sigma_{\rm a}$ and the inelastic scattering cross section $\sigma_{\rm ie}$.

The storage time $\tau_{\rm l}$ due only to losses in the walls is determined by the expression

$$\tau_{\rm l} = \frac{\tau_{\rm f}}{\mu}\,,$$

where $\tau_{\rm f}$ is the time of free flight between two collisions with the walls ($\tau_{\rm f} = 4V/(vS)$, V is the volume of the vessel, S is the surface area of its walls, $v = \hbar k/m$ is the neutron velocity‡).

Note that the loss coefficient μ and the storage time $\tau_{\rm l}$ depend on the neutron velocity, then, if the UCN spectrum in the vessel is not strictly monochromatic, the storage curve $N(t) = N(0)\exp\bigl[-t/\tau_{\rm l}(v)\bigr]$, averaged over the spectrum, has a nonexponential form. Therefore, the experimental data are used for deriving the reduced coefficient η (17), which is independent of the neutron velocity.§

3.2 Some recent experiments

The history of experiments, in which the loss coefficient was measured, is presented in some reviews [24–26] and monographs [2, 27], but to make our exposition more coherent it will be useful to repeat it briefly. Further we shall continue our story in detail, mainly taking advantage of experimental studies performed after 1990.

Originally, the experimenters just accumulated experience, and the first results were quite discouraging. The loss coefficient used to turn out as quite large,¶ it amounted to 3×10^{-4} and seemed to be independent of both the material of the walls and the temperature. In attempts at measuring the temperature dependence, the experimenters heated only part of the vessel containing UCN, since they thought that, if there existed any temperature effect, it should be also manifested somehow in such conditions. But all in vain. To continue searching for the whereabouts of the disappearing neutrons, it was necessary to clarify whether they were heated owing to inelastic scattering, or they were absorbed.

It was easier to find out whether they were heated. For such a test, the trap was surrounded with counters of thermal neutrons, and the main part of the loss coefficient was indeed revealed to result from the inelastic scattering cross section. Since the cross section of inelastic scattering on the wall material was clearly insufficient for such heating of the UCN,

† The amplitude b may also contain an imaginary part in the absence of absorption and inelastic scattering processes. In accordance with the optical theorem, this imaginary part characterises the total scattering cross section and allows for the elastic scattering, as well. But the elastic scattering, although it reduces $|R|$, does not result in losses. The drop in $|R|$, in this case, signifies a reduction of specular reflection due to the onset of nonspecular reflection. We shall not deal with this issue.

‡ We neglect here the influence of gravity.

§ The explicit presence of k or λ in (17) should not give rise to confusion. These quantities cancel out owing to the dependence $\sigma_{\rm l} \propto 1/k$.

¶ We shall always deal with the reduced coefficient η.

a conclusion inevitably raised in the mind that it was due to hydrogen-bearing impurities on the walls. Hydrogen being ubiquitous, the presence of such contaminations is no surprise.

But if the presence of hydrogen and inelastic scattering on its atoms cause losses, the loss coefficient should depend on the temperature. However we recall that no such dependence was observed earlier. This means the experiments were not sufficiently sensitive. Attempts were made, first, to clean the surface of the walls better (this may be done either by prolonged heating in a good vacuum under oil-free evacuation, or by ion bombardment, or by covering the surface with freshly sprayed substances) and, second, to control the hydrogen content on the surface, its depth distribution in the subsurface layer, and the time variation of the hydrogen concentration after or during the cleaning of the surface.†

The results of these studies revealed hydrogen to be indeed present on the surface, its content could be reduced by some methods or others, but storage experiments continued to yield confusing results. On cleaning the trap (mainly by prolonged annealing at a high temperature) they managed to reduce the loss coefficient, but it still remained far from the theoretical value, and its temperature dependence turned out to be weak. True, one could not exclude the possibility of all this resulted from the experimental imperfections.

3.2.1 Crucis experiment that proved the opposite. Finally, the crucial experiment was performed [28], in which the trap was a beryllium pipe (with small absorption) and its temperature varied between 300 and 16.5 K (at a low temperature the inelastic scattering cross section could be totally excluded). The results of the experiment were such that the problem of anomalous losses was declared solved.

The idea of the authors was the following: first we measure the loss coefficient η_{exp} at 300 K. It should exceed the theoretical prediction $\eta^{\text{Be}}_{\text{theor}}$ by the value

$$\Delta\eta^{\text{H}}_{\text{a+ie}}(300) \propto c_{\text{H}}\left[\sigma^{\text{H}}_{\text{a}} + \sigma^{\text{H}}_{\text{ie}}(300)\right],$$

due to the admixture of hydrogen on the surface, where c_{H} is the hydrogen concentration, $\sigma^{\text{H}}_{\text{a, ie}}$ is the absorption and inelastic scattering cross section for hydrogen. From the quantity $\Delta\eta^{\text{H}}_{\text{a+ie}}(300)$ we estimate the hydrogen concentration c_{H} at the surface. Further, we reduce the temperature and again measure the loss coefficient $\eta(16)$. There should be hydrogen left on the walls, inelastic scattering on it should disappear, and the excess coefficient should become $\Delta\eta^{\text{H}}_{\text{a+ie}}(16) \propto c_{\text{H}}\sigma^{\text{H}}_{\text{a}}$.

The authors made a check, obtained what they expected to and declared everything to be clear.

However, to obtain $\Delta\eta^{\text{H}}_{\text{a+ie}}(16) \propto c_{\text{H}}\sigma^{\text{H}}_{\text{a}}$, it was necessary from the value of $\eta_{\text{exp}}(16)$ measured at 16 K to subtract $\eta^{\text{Be}}_{\text{theor}}(16)$, while they subtracted $\eta^{\text{Be}}_{\text{theor}}(300) \gg \eta^{\text{Be}}_{\text{theor}}(16)$, which was wrong. If they had made the correct subtraction, they would have seen that, for explaining the excess loss coefficient at 16 K, a hydrogen concentration 100 times greater, than the one determined from measurements at 300 K, was required.

Thus, the above experiment [28] did not solve the problem, but only aggravated the situation.

† For all these purposes, the ion beams were used together with their resonance nuclear interaction with protons.

We have now arrived at the point, from which continuation of the history proceeds.

3.2.2 The experiment performed by the Morozov group. The presence of hydrogen on the surface was checked in the other experiment [29]. Here, plates of stainless steel were introduced into the neutron guide, at the end of which a UCN detector was established for recording the total neutron flux (i.e. hydrogen was not investigated on the walls of the neutron guide, but on the plates, the surfaces of which could be chosen sufficiently large for enhancing the efficiency of the experiment). While some outer part of the neutron guide was coated with a layer of boron carbide 1 cm thick, and a detector of γ-rays was established for detecting quanta with energies of 2.23 MeV and 477 keV. The former were produced in the reaction $\text{n(p, d)}\gamma$, i.e. in neutron capture on hydrogen atoms at the surface of the plates, while the latter were produced in the reaction $^{10}\text{B(n, }\alpha + \gamma)^{7}\text{Li}$, involving neutrons heated by inelastic scattering on the plates. They penetrated through the walls of the neutron guide and were absorbed by boron atoms. Various methods were applied for cleaning the steel plates, and the dependence upon a processing method was studied for the amount of hydrogen on their surfaces: the plates were washed in acetone, twice electro-polished and annealed in vacuum (what vacuum was not reported) for 6 hours at a temperature of 350 °C.

The results of the experiment were qualitative and formulated as follows: after cleaning with acetone, the density of hydrogen atoms on the plates was 6×10^{17} cm^{-2}, and after annealing $< 6 \times 10^{15}$ cm^{-2}, which is not contradictory to earlier experiments. No information can be extracted form this experiment on the anomaly of interest. Also, no detailed report, from which it could be possible to draw out quantitative conclusions, was published.

3.2.3 Experimental observation of heated neutrons. In the experiment [29] an attempt was made to control simultaneously two UCN loss channels: absorption and inelastic scattering. It is easier to study only inelastic scattering, independently of whether it is caused by hydrogen atoms or atoms of other elements. Here it is possible to estimate the total intensity of inelastic scattering and the spectrum of heated neutrons after applying various methods for cleaning the surface. Along these lines, an experiment [30, 31] was performed, which yielded a result contradicting those of the similar experiment [32] carried out earlier by the group of Morozov.

Approximately identical schemes were adopted in performing both the experiments. In Ref. [30], a copper foil was placed inside the neutron guide for UCN, and detectors of thermal neutrons established outside the neutron guide recorded the neutron flux arising from the inelastic scattering of UCN by the surface of the foil. In the experiment [32], a stack of copper plates was used instead of the foil.

The results presented in Ref. [30] show that warming up the foil at a temperature of 800 K during 6 h reduces the probability of heating approximately by a factor of 8. If all the heating is attributed to hydrogen, then in the case of a warm-up the amount of hydrogen in the subsurface layer should, accordingly, decrease by the same factor.

A significant divergence between two experiments is due to the change in the mean energy of heated neutrons observed after surface degassing. Once in Ref. [32], the energy of heated neutrons after degasification (data are given for room

temperature) rises nearly up to 60 meV, which is significantly higher than the mean thermal energy of the wall atoms in Ref. [30], the energy after degasification falls down to about 10 meV.

In both works the arguments are given for explaining the observed change in energy. In Ref. [32], weakly bound hydrogen with low vibrational frequencies is considered to be removed during annealing, while strongly bound hydrogen with higher vibrational frequencies remains. The arguments presented in Ref. [30] are different: indeed, weakly bound hydrogen is removed in the course of annealing, while the remaining strongly bound hydrogen vibrates together with the heavy atoms, to which it is bonded by chemical forces. The atomic vibrational frequency decreases as the mass increases, and precisely this frequency determines the energy of the scattered neutron. The natural high-frequency hydrogen vibrations turn out to be unexcited at room temperature.

Thus, before degasification the neutrons are scattered on the light atoms of adsorbed hydrogen, while after degasification, on the heavy molecules of the substance with hydrogen atoms rigidly bonded to them. The arguments presented in Ref. [30] seem more convincing, but for final conclusions additional investigations are required.

3.2.4 Experiments in Grenoble. Since, according to the authors of Ref. [28], the large losses of UCN in traps are due to the presence of hydrogen, for reducing the losses it is necessary to carry out storage experiments with traps on the surface of which no hydrogen can be present. Therefore, experiments were performed in Grenoble [33] with a trap having walls covered with a hydrogen-free phombline oil. The dependence of the loss coefficient for the walls upon the neutron energy and temperature was studied. The neutron energy varied between 5 and 106 neV, and the temperature ranged from 283 up to 308 K.

The measurements demonstrated good agreement with the theoretically predicted dependence of the loss coefficient $\mu(v)$ on the neutron velocity and permitted determination of the reduced loss coefficient $2\eta = (4.7 \pm 0.2) \times 10^{-5}$ at 294 K. The observed small deviation from the theoretical dependence $\mu(v)$ in the vicinity of the boundary velocity towards higher values was interpreted as the result of a small enhancement in the neutron energy (on the order of $\Delta E \approx 2 \times 10^{-11}$ eV) in each collision with the wall.† The authors consider that a certain nonexponentiality of the storage curves for monoenergetic neutrons‡ confirms the above interpretation.

Moreover, in another experiment [35] a strong temperature dependence of the inelastic scattering cross section was observed in phombline oil. This was seen both in the UCN storage experiments and in transmission experiments with neutrons of a 60 A wavelength. Regretfully, no detailed report on this issue was published, so all the results are perceived only as qualitative.

3.2.5 Powder experiments with UCN. The experiments reported in Refs [36, 37], in which the loss coefficient was measured, stand somewhat apart, differing in performance from the storage experiments (Fig. 4).

† Observation of a similar effect is also reported in Refs [34] and [27].

‡ The nonexponentiality of the storage curves may also point to a high probability of specular reflection from the walls. In this case, neutrons incident upon the wall at an angle close to 90° are the fastest to die out.

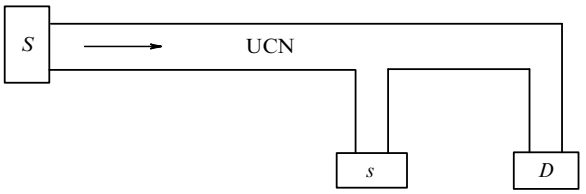

Figure 4. Layout of the experiment for measuring the loss coefficient for powders: *S* is the UCN source; *s* is the sample (cuvette with the powder); *D* is an UCN detector.

The experiment makes use of a branched neutron-guide system. A cuvette with a powder is placed in one of its arms, and in the other there is an UCN detector. Such an arrangement has an advantage consisting in that the cuvette is easily warmed up, and its height relative to the main neutron guide can be varied so as to change the spectrum of neutrons interacting with it, and, finally, it is also possible to vary the thickness of the powder and the substrate material.

True, there do exist difficulties. The main one concerns interpretation of the data obtained. The interpretation depends on the model adopted for describing the diffusion of a neutron inside such a complex branched neutron-guide system [2], and on the model applied in describing the interaction of a neutron with the powder, i.e. a highly disperse system, in which interaction with an individual grain is far from small. From the viewpoint of science, both these aspects are of independent interest, since this field has not been really elaborated yet.

The first experiments carried out with Cu, CuO, and Be powders revealed that, if the coefficient R of reflection from an infinitely thick layer is described by the albedo formula:

$$R = \frac{1 - 2\sqrt{\mu/3}}{1 + 2\sqrt{\mu/3}} \approx 1 - 4\sqrt{\frac{\mu}{3}},$$

where μ is the loss coefficient for a neutron collision with a single grain of the powder averaged over the angles, then after the powder annealing its loss coefficient η (see its definition in (17)) turns out to be in agreement with theory.

The experiments themselves and their interpretation were subjected to severe criticism, and the main objection to them consisted in that for a given system it is necessary to account for the packing density of the powder.

And, indeed, after taking into account the powder packing [38, 39], it turned out [37] that the formula for the albedo should be transformed into the following:

$$R = \frac{1 - 2\sqrt{K\mu/3}}{1 + 2\sqrt{K\mu/3}},$$

where K is a quite complex function depending on the packing density $C_V = Vn_V$, here V is the volume of a single grain, n_V is the number of grains per unit volume. In so doing, the loss coefficient inferred from experiments has become the same as in other experiments, i.e. it diverges from a theory.

3.2.6 The most important experiment relevant to the UCN anomaly. The most important result was recently obtained in joint experiments conducted by JINR (Dubna) and LINP (Gatchina) [40]. It is revealed that the loss probability in a single collision in the best, on this point, traps is on the order

of 3×10^{-5}, which, although small, exceeds the theoretical value by two orders of magnitude.

Losses are also related to neutron absorption on nuclei and to inelastic scattering involving the heating of neutrons (heated neutrons have an energy higher than U, and pass freely through the walls of the trap). However, the cross section σ_{ie} depends on the temperature; by cooling it can be excluded, and this can be subjected to experimental tests.

Experiments [40] indicated that the ratio $\sigma_l^{obs}/\sigma_l^{th}$ between the observed and theoretical loss cross sections reaches values on the order of 100. In this case, however, the portion of the observed cross section which depends on the temperature, σ_{ie}^{obs}, is in good agreement with the theoretical inelastic cross section σ_{ie}^{th} both in magnitude and in the character of the temperature dependence. As to the gradual heating of UCN, in the experiments described in Ref. [40] it was demonstrated that, even if a neutron acquires some energy in each collision with the wall, it is less than 10^{-12} eV.

3.2.7 The results of experimental studies on the UCN anomaly. Such are the experimental facts. Now we shall list all the presently conceivable factors that may lead to an enhancement in the loss coefficient [2, 27]:

(1) absorbing admixtures on the surface. This possibility is rejected by the fact that the surface coating with weakly absorbing substances, for example, oxygen, like in the experiment reported in Ref. [40], does not lead to an anomaly reduction. Attempts to explain the anomaly by the enhancement of the admixtures' influence, owing to their some special distribution in the vicinity of the surface [41], are also not consistent, since the absorption coefficient is little sensitive to variations of such a distribution;

(2) leakage of neutrons in the slits of the trap. In the experiment of Ref. [40], there were no slits;

(3) low-energy heating under sonic vibrations of the trap walls. This factor has not been thoroughly investigated yet. But estimates show [2, 3] that for acoustic vibrations to contribute significantly to the UCN loss from the trap, the sonic vibrations must give rise to considerable noise in the immediate surrounding, which was not observed in the experiments;

(4) admixtures of hydrogen or hydrogen-bearing compounds. This possibility is rejected by the fact that hydrogen presence leads to a significant enhancement of inelastic scattering. On the contrary, experiments reveal that the inelastic portion of the loss cross section is well explained by thermal atomic vibrations in the main substance without admixtures;

(5) roughnesses on the surface. This factor may, on the whole, lead to increase in the surface area of the wall, which is equivalent to enhancement in the loss cross section. However, in this case both the absorptive and inelastic parts of the cross section should increase proportionally, which is not consistent with results of the experiment [40];

(6) the cluster structure of matter. This factor, however, leads only to enhancement of the inelastic portion of the scattering cross section, which contradicts the results of the known experiment [40].

3.3 Attempts at an explanation

3.3.1 An encroachment upon quantum mechanics. Thus, none of the above factors provides a satisfactory explanation of the experimental results. Such a situation compels one to seek the reason in the domain of fundamental concepts, which, on the one hand, is always desirable and, on the other, quite dangerous. It must be emphasised that the first step toward re-examination of quantum mechanics was made by F L Shapiro himself. He put forward the assumption that a neutron makes a wave packet, part of whose components has energies above the barrier, and due to these components the neutron possesses a certain probability to penetrate through the walls. But this assumption was immediately rejected considering that quantum mechanics refers to a linear theory, thus, in the first collision with the wall, the high-energy components should disappear, while the low-energy ones should be reflected. Precisely the latter represent the wave function of the ultracold neutrons. Counter-arguments can be found against these objections, but since this review article is not the place to indulge in detailed discussions, we shall only present the result of the development of this idea voiced by F L Shapiro.

It is anticipated that the neutron can be represented by a wave packet

$$\psi(s, \mathbf{v}, \mathbf{r}, t) = c \exp(\mathrm{i}\mathbf{v}\mathbf{r} - \mathrm{i}\omega t) \frac{\exp(-s|\mathbf{r} - \mathbf{v}t|)}{|\mathbf{r} - \mathbf{v}t|}, \qquad (18)$$

where c is a normalising factor. This function, first introduced by de Broglie [42], satisfies the inhomogeneous Schrödinger equation

$$\left(\mathrm{i}\frac{\partial}{\partial t} + \frac{\Delta}{2}\right)\psi(\mathbf{r}, t) = -2\pi C(t)\delta\big(\mathbf{r} - \mathbf{r}(t)\big), \qquad (19)$$

where $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}t$ represents coordinates of a particle moving with a velocity $\mathbf{v}$, and

$$C(t) = c \exp \frac{\mathrm{i}(v^2 + s^2)t}{2}.$$

This means that the wave packet (18) satisfies the homogeneous Schrödinger equations at nearly all (but one) points in space. When de Broglie applied this function, he considered it as a singular solution to the homogeneous equation.

We shall define the probability for the wave packet to be reflected from a potential step as the ratio of the reflected neutron flux to the incident flux, taking advantage of the Fourier transform and the superposition principle:

$$\psi(x, t) = \int_{-\infty}^{+\infty} \mathrm{d}k\, \Psi(k) \exp(\mathrm{i}kx - \mathrm{i}\omega t),$$

$$|R|^2 = \frac{\int_{-\infty}^{+\infty} \mathrm{d}k\, k\big|\Psi(k)R(k)\big|^2}{\int_{-\infty}^{+\infty} \mathrm{d}k\, k\big|\Psi(k)\big|^2}. \qquad (20)$$

Expression (20) represents the recipe for calculating the reflection coefficient, similar to that used for plane waves. Here, the reflected packet is assumed to reduce to its initial form. How this happens cannot be described by quantum mechanics, just as it cannot determine in which direction each individual particle will propagate upon being scattered.†

Calculation of the reflection coefficient by formula (20) results in part of the neutrons being lost with a probability $W = s/v_l$ owing to their penetration into the medium.

† To a certain extent, this circumstance points to the incompleteness of quantum mechanics. Concerning this issue, see also Ref. [43].

Comparison of the derived variable with the UCN loss coefficient [3]

$$\mu = \frac{2\eta v}{v_l} \equiv W,$$

where $\eta \approx 3 \times 10^{-5}$ is the reduced anomalous loss coefficient observed in the experiment [40], we find

$$s = 2\eta v \approx 6v \times 10^{-5}.$$

This value can be adopted as an estimate of the hidden parameter s, i.e. as the proper width of the neutron wave packet.

The question arises as to what is the further destiny of a neutron having penetrated into a substance above the barrier? Clearly, the neutron has no other channels, except absorption, inelastic scattering and departure from the substance through the interface boundary. The neutron loss cross section inside the medium is readily assessed. To estimate these losses we shall assume that the loss cross section depends on the wave vector of the incident plane wave in accordance with the common $1/p'$ law, where $p' = \sqrt{p^2 - u}$ is the wave vector within the medium, i.e. it can be represented as $\sigma_l(p) = \sigma_l(k_T)k_T/p'$, where k_T is the momentum of a thermal neutron. Averaging the cross section over the above-the-barrier part of the spectrum of the neutron wave packet yields

$$\langle\sigma_l\rangle \approx \frac{\sigma_l(k_T)k_T}{v_l}.$$

Thus, above-the-barrier transmission is accompanied by absorption and heating of the neutrons with their path length in the medium, l_l, determined by the quantity $l_l = 1/N_0\langle\sigma_l\rangle$, where N_0 is the number of nuclei per unit volume.

Let us take a look at the experimental consequences such an approach should lead to. In the experiment of Ref. [40], the walls were made of beryllium. Its loss cross section in the thermal region amounts to 8 mb [6], therefore the path length due to absorption in the substance is $l_a = 1/N_0\sigma_a \approx 1$ cm.

At room temperature, the inelastic cross section is 30 times greater than σ_a, so the path length due to inelastic scattering, $l_{ie} = 1/N_0\sigma_{ie} \approx l_l$, amounts to fractions of a millimetre. This means that at room temperature all above-the-barrier transmission ends up in inelastic scattering. As the temperature decreases, inelastic scattering plays a lesser significant role, and above-the-barrier penetration is terminated either by the absorption or its departure from the trap, if the walls of the latter have a thickness smaller than l_a.

Consider a modification of the storage experiment, the layout of which is shown in Fig. 5. Here, the UCN trap is surrounded with counters of thermal neutrons, and both the storage curve and the counting rates of the thermal neutron detectors are measured during storage versus the temperature of the trap walls.

If the temperature is changed, and variations in the lifetime of a neutron inside the trap are traced together with the number of heated neutrons, then a paradoxical phenomenon is observed: at room temperature the whole loss coefficient, measured by the storage of UCN, is explained by inelastic neutron scattering, as shown in Fig. 6. Hence, if above-the-barrier penetration is not invoked, the conclusion must be made that all the losses are described by the inelastic

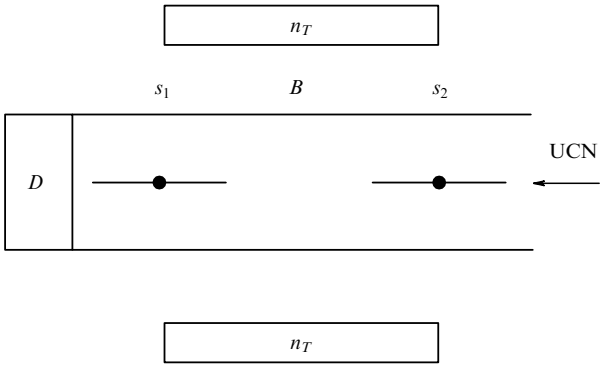

Figure 5. The layout of storage experiments (see Fig. 2) may be modified by surrounding the UCN trap with counters of thermal neutrons for detecting the UCN heated up to the energy of thermal neutrons in collisions with the walls.

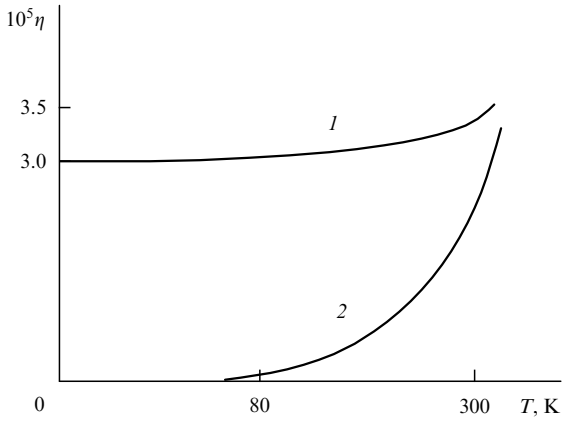

Figure 6. Temperature dependence of η determined from the storage curve (curve *1*) and from the counting rate of the thermal neutron counters surrounding the trap (curve *2*).

scattering cross section. But since the substance cannot provide the required cross section, this means a presence of hydrogen admixture, and the hydrogen concentration can ultimately be calculated.

When the temperature is lowered, the number of heated neutrons decreases sharply. It would result in an enhancement of the storage time, however, the storage time remains nearly the same. How can such a fact be explained? The calculated hydrogen concentration clearly turns out to be insufficient to provide the absorption of those neutrons which avoid inelastic scattering. Therefore, hydrogen has nothing to do with the issue.

If the above hypothesis is valid [44], the experimental result at hand has a simple explanation. When the temperature is lowered, the probability of above-the-barrier penetration of neutrons in a substance does not change, but their path length in the substance does. Here, once the neutrons earlier underwent heating, now they should either be absorbed or pass through the walls.

Naturally, the above-presented approach is far from completion. Quantum mechanics describes scattering processes, as well as the energies of a bound states. The new scheme should combine all the merits exhibited by the apparatus of quantum mechanics, and there exist possibilities for this to occur. Recently performed calculations [45] show

that it is also possible to define a bound state for the inhomogeneous Schrödinger equation, with the nonrelativistic spectrum of the hydrogen atom being identical to the conventionally adopted spectrum.

3.3.2 The lazy neutron. One of the alternative approaches was described in Ref. [46]. Therein, the neutron is assumed to possess a complex structure and to have excited states. The reflection of a neutron from a wall is accompanied by its internal excitation and loss of kinetic energy. As a result, the neutron slows down (the lazy neutron) and special measures are required for its detection. For example, since the lazy neutrons gather at the bottom of the trap, for their detection it is necessary to remove the bottom. The detector must be established at a sufficiently low point for the neutrons experiencing acceleration in the gravitational field to acquire sufficient velocities for penetration through the detector window.

In these conditions, the leakage curve† should exhibit a peak at the moment of arrival of the accelerated lazy neutrons.

In the above presented scheme no losses actually exist, which can be verified on the basis of a more thorough measurements of the number of neutrons remaining after a given storage time. True, apparent losses of this type should increase with the neutron storage time in the vessel. Accordingly, the storage curve in Fig. 3 for experiments with monochromatic neutrons should become more and more steep as the exposure time increases, which has never been observed.

In the scheme with the lazy neutron it remains to understand how the excitation of an excited neutron is removed. If it is again quenched in collisions with the wall, then in this case the neutron should be accelerated. If, contrariwise, the excitation is removed in a free flight, then there should be emitted, for example, quanta of the electromagnetic field, which in principle can be detected.

Thus, like in the previous case, there remain many unclear issues, which must be clarified before such an explanation of the anomaly is adopted.

3.3.3 Re-examination of the refraction coefficient. The next idea consists in that the refraction coefficient must be altered [47 – 50]. If the reflection of a neutron from a wall is computed by some method differing from the usual one, the refraction coefficient may acquire an additional imaginary part, related neither with absorption nor with inelastic scattering processes.

However, such a formulation is essentially contradictory. The appearance of an imaginary part in the refraction coefficient means presence of losses, but the problem does not consist in describing losses with the aid of an imaginary part, but in understanding: to what these losses are related. Simply introducing an imaginary part in the refraction coefficient does not permit solving this problem.

Multiple scattering of waves (MSW). In Refs [47 – 50], the assertion was made that reflection from a substance, generally speaking, has a potential character, and the wave vector inside the medium, if the problem of multiple scattering of waves in a medium is solved rigorously, will always contain an imaginary part and a real part, independently of whether losses or any imperfections are present in the substance. This result seems unacceptable on the basis of general physical principles. For example, if a thermal neutron within an ideal medium is described by a wave function exponentially decreasing with the depth away from the input surface, it should undergo total reflection. In the opposite case, accumulation of neutrons near the surface should take place, which leads to a decrease in entropy and contradicts the laws of thermodynamics.

In reality, when the theory of multiple scattering of waves (MSW) is applied correctly, nothing of the sort happens. This follows from Refs [51, 52], in which the reflection from an ordered monatomic medium is calculated rigorously by applying the theory of MSW.

Actually, in MSW theory no refraction coefficient or interaction potential of a neutron with the medium is introduced. Only neutron scattering on individual atoms and multiple re-scattering of scattered waves are assumed. The problem is formulated as follows.

1. A wave scattered on an individual nonabsorbing nucleus fixed at rest at a point $\mathbf{r}_1$ is described by the expression

$$\Psi = \psi_0(\mathbf{r}) - \psi_0(\mathbf{r}_1)\,\frac{b}{|\mathbf{r}-\mathbf{r}_1|}\,\exp\bigl(\mathrm{i}k|\mathbf{r}-\mathbf{r}_1|\bigr)\,, \tag{21}$$

where $\psi_0(\mathbf{r}) = \exp(\mathrm{i}\mathbf{k}\mathbf{r})$ is the wave function of the incident neutron, $\mathbf{k}$ is its wave vector and $k = |\mathbf{k}|$.

2. The scattering amplitude b in the absence of absorption is a complex number of the form

$$b = \frac{b_0}{1 + \mathrm{i}kb_0}\,, \tag{22}$$

where b_0 is the real scattering length. The amplitude of such a form satisfies the requirement of unitarity (the optical theorem):

$$\mathrm{Im}\,b = \frac{k\sigma}{4\pi}\,, \qquad \text{where} \qquad \sigma = 4\pi|b|^2\,. \tag{23}$$

In the presence of absorption, the quantity b_0 itself becomes complex: $b_0 = b_0' - \mathrm{i}b_0''$, and $b_0'' = k\sigma_\mathrm{a}/(4\pi)$, where σ_a is the absorption cross section. Here $\mathrm{Im}\,b = k\sigma_\mathrm{t}/(4\pi)$, where $\sigma_\mathrm{t} = \sigma_\mathrm{a} + 4\pi|b|^2$ is the total cross section describing the neutron interaction with a nucleus.

3. If there are several scatterers, then the wave function $\psi(\mathbf{r})$ resulting from multiple re-scatterings is given by

$$\psi(\mathbf{r}) = \exp(\mathrm{i}\mathbf{k}\mathbf{r}) - \sum_n \psi(\mathbf{r}_n)\,\frac{b_n}{|\mathbf{r}-\mathbf{r}_n|}\,\exp\bigl(\mathrm{i}k|\mathbf{r}-\mathbf{r}_n|\bigr)\,, \tag{24}$$

where $\psi(\mathbf{r}_n)$ is the effective local field illuminating the nucleus at the point $\mathbf{r}_n$. Notice that each nucleus gives rise to a spherical wave with a wave vector k, since for the neutron field there exists no concept of a near zone (contrary to the case of electromagnetic waves). Thus, writing the scattered field in the form of a spherical wave is correct for any distances from the nucleus, at least, if these distances exceed the size of the nucleus itself.

4. The local field on the nucleus with number n is determined by the equation

$$\psi(\mathbf{r}_n) = \exp(\mathrm{i}\mathbf{k}\mathbf{r}_n) - \sum_{j\neq n} \psi(\mathbf{r}_j)\,\frac{b_j}{|\mathbf{r}_n-\mathbf{r}_j|}\,\exp\bigl(\mathrm{i}k|\mathbf{r}_n-\mathbf{r}_j|\bigr) \tag{25}$$

† That is the curve describing the dependence of the counting rate of the UCN detector on the time that passes after the output shutter is opened.

Equations (24) and (25) form the basis for all subsequent reasoning. For example, for a crystal plane with a square monatomic unit cell, the local fields $\psi(\mathbf{r}_n)$ are written, for symmetry reasons, in the simple form:

$$\psi(\mathbf{r}_i) = C\exp(\mathrm{i}\mathbf{k}_\parallel\mathbf{r})\,, \tag{26}$$

where $\mathbf{k}_\parallel$ represents components (parallel to the plane) of the wave vector of the incident wave, C is a constant identical for all nuclei, and the following equation holds for this constant:

$$C = 1 - Cb\sum_{j\neq 0}\exp(\mathrm{i}\mathbf{k}\mathbf{r}_j)\,\frac{1}{|\mathbf{r}_j|}\exp\bigl(\mathrm{i}k|\mathbf{r}_j|\bigr)\,, \tag{27}$$

whence $C = 1/(1+bS)$, where the sum S is indicated in (27). Its calculation leans upon the technique of lattice sums and is possible with any degree of accuracy.

Upon determining the local fields $\psi(\mathbf{r}_i)$, we can find the analytical form of the wave function of a neutron scattered on a crystal plane:

$$\Psi(\mathbf{r}) = \exp(\mathrm{i}\mathbf{k}\mathbf{r}) - C\sum_{\mathbf{n}}\frac{2\pi\mathrm{i}b}{a^2k_{\mathbf{n}\perp}}\exp\bigl(\mathrm{i}k_{\mathbf{n}\perp}|x| + \mathrm{i}\mathbf{k}_{\mathbf{n}\parallel}\mathbf{r}_\parallel\bigr), \tag{28}$$

where $\mathbf{r}_\parallel = (y,z)$ are the coordinates on the plane, $\mathbf{k}_\mathbf{n} = (k_{\mathbf{n}\perp}, \mathbf{k}_{\mathbf{n}\parallel})$ are the wave vectors of the waves having undergone diffraction and produced as a result of the transformation $\mathbf{k}_\parallel \to \mathbf{k}_{\mathbf{n}\parallel} = \mathbf{k}_\parallel + \boldsymbol{\tau}_\mathbf{n}$ by addition of the inverse lattice vector $\boldsymbol{\tau}_\mathbf{n} = (2\pi/a)(n_y, n_z)$ with the integer numbers n_y and n_z. The normal components of the diffracted waves are $k_{\mathbf{n}\perp} = (k^2 - \mathbf{k}^2_{\mathbf{n}\parallel})^{1/2}$ owing to the energy conservation law.

The constant C behaves so as if it transforms the scattering amplitude b of one nucleus, i.e. within a group of nuclei each nucleus scatters not with the amplitude b, but with an amplitude $b' = bC$, and it can be represented in the form

$$b' \equiv bC = \frac{b_1}{1+\mathrm{i}b_1\gamma}\,, \qquad \text{where} \qquad b_1 = \frac{b_0}{1+b_0\delta}\,,$$
$$\gamma = {\sum_{\mathbf{n}}}'\frac{(2\pi)^2}{a^2k_{\mathbf{n}\perp}}\,, \qquad \delta = {\sum_{\mathbf{n}}}''\frac{(2\pi)^2}{a^2|k_{\mathbf{n}\perp}|}\,. \tag{29}$$

The prime signifies that summation is extended over all $\mathbf{n}$ for which $k^2_{\mathbf{n}\perp} > 0$, and two primes mean that the summation is performed over all $\mathbf{n}$ for which $k^2_{\mathbf{n}\perp} < 0$. Thus, the local field renormalises the real part of the scattering amplitude: $b_0 \to b_1$, and changes its imaginary part: $\mathrm{i}kb_0 \to \mathrm{i}b_1\gamma$.

The scattered field is presented graphically in Fig. 7.

In the case of small k, diffraction only leads to exponentially decaying waves, and there turn out to be only two waves departing from the plane: one specularly reflected with a wave vector $\mathbf{k}_\mathrm{r} = (-k_\perp, \mathbf{k}_\parallel)$, and the other transmitted (not scattered) with the wave vector $\mathbf{k}_\mathrm{t} \equiv \mathbf{k}$. Therefore, expression (29) is represented in the form

$$b' = \frac{b_1}{1 + 2\pi\mathrm{i}b_1/(a^2k_\perp)}\,, \tag{30}$$

and, if one neglects the exponentially decaying waves, the wave function (28) equals

$$\Psi(\mathbf{r}) = \exp(\mathrm{i}\mathbf{k}\mathbf{r}) - \frac{2\pi\mathrm{i}b'}{a^2k_\perp}\exp\bigl(\mathrm{i}k_\perp|x| + \mathrm{i}\mathbf{k}_\parallel\mathbf{r}_\parallel\bigr)\,. \tag{31}$$

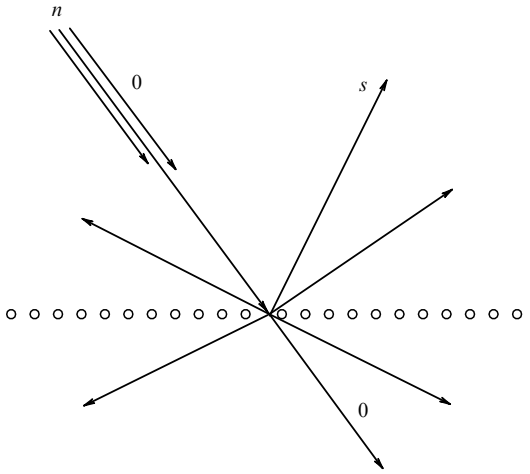

Figure 7. Diffraction of neutron wave on the crystal plane. The directions indicated by index 0 correspond to the initial one. The direction denoted by s corresponds to specular reflection. The remaining directions correspond to neutron diffraction.

In this case, the crystal may be considered as a one-dimensional potential, for which reflection and transmission of a sole period are determined, on the basis of (31), by expressions

$$r = -\frac{2\pi\mathrm{i}b'}{a^2k_\perp}\,, \qquad t = 1 - \frac{2\pi\mathrm{i}b'}{a^2k_\perp}\,. \tag{32}$$

For an arbitrary periodic potential shown in Fig. 8, it is possible, taking advantage of recurrent expressions, to find the equation for the reflection amplitude [53]:

$$R = r + tR(1-rR)^{-1}t \quad \text{or} \quad R^2 - \frac{R(1+r^2-t^2)}{r} + 1 = 0, \tag{33}$$

and its solution, which can be written in the form

$$R = \frac{\sqrt{(1+r)^2 - t^2} - \sqrt{(1-r)^2 - t^2}}{\sqrt{(1+r)^2 - t^2} + \sqrt{(1-r)^2 - t^2}}\,. \tag{34}$$

Substituting here the amplitudes r and t from (32), we obtain

$$R = \frac{\sqrt{k_\perp + p\tan(k_\perp a/2)} - \sqrt{k_\perp - p\cot(k_\perp a/2)}}{\sqrt{k_\perp + p\tan(k_\perp a/2)} + \sqrt{k_\perp - p\cot(k_\perp a/2)}}\,. \tag{35}$$

This formula describes both the Bragg reflections and total reflection for small $k_\perp$. The latter is reduced to the form

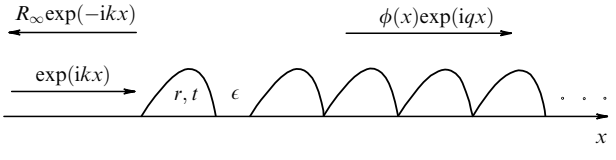

Figure 8. Principle of calculating the reflection from an arbitrary semi-infinite periodic potential. An infinitesimal slit of width ϵ is introduced in between the first period and the remaining potential, and multiple re-scattering at this slit is taken into account.

$$R = \frac{k_\perp - \sqrt{k_\perp^2 - u_0}}{k_\perp + \sqrt{k_\perp^2 - u_0}}, \tag{36}$$

where $u_0 = 2p/a \approx 4\pi N_0 b$ and the corrections to $u_0 = 4\pi N_0 b$ can be found with any degree of accuracy relative to the parameters $k_\perp^2 a^2$ and $u_0 a^2$. If exponentially decaying waves in (28) are taken into account, there arise additional corrections having a factor of smallness on the order of $\exp(-2\pi) \approx 10^{-2}$.

Thus, consistent application of the MSW theory does not lead to those changes in the behaviour of a neutron in a medium, which are dealt with in Refs [47–50].

4. Application of UCN for fundamental research

All research with UCN was initiated mainly with the purpose of making use of the possibility of their prolonged storage within a bounded region of a space for correction of the upper limit imposed on the neutron EDM and for more accurate measurement of the neutron lifetime. At present the situation, here, is the following.

4.1 The lifetime of the neutron

Three groups in the world are involved in measurement of the neutron lifetime in UCN storage experiments with the solid-state traps.

The following value was obtained in experiments presented in Ref. [35] (see Fig. 9a):

$$\tau_\beta = 887.6 \pm 3 \text{ s}. \tag{37}$$

The uncertainty is mainly related to the systematic error due to the necessity of introducing a gravitational correction. To reduce the systematic errors, it is necessary to provide an isotropic distribution of the neutrons inside the vessel independently of its volume [54, 55]. The gravitational correction also depends on the neutron spectrum [56] in the trap and can be reduced either by choosing the appropriate part of the spectrum of confined neutrons or by changing the experimental arrangement [57], for instance, by storing the neutrons in a container of the bellows type. Then the volume of the container can be varied without changing the area of the inner surface of the walls.

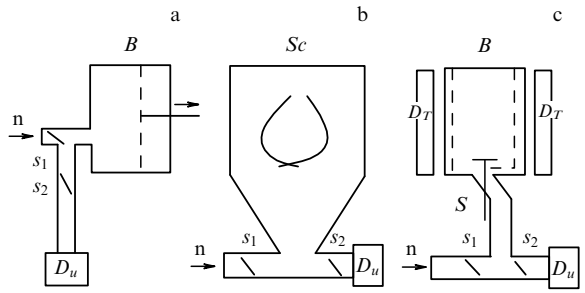

Figure 9. The layouts of the three main experiments for determining the neutron lifetime with the aid of UCN storage are the following: when shutter s_2 is closed, the neutrons n enter the apparatus through the open shutter s_1, after which shutter s_1 is closed and the neutrons are stored for a fixed time in vessel B. (In the case of layout (b) the neutrons fill up the vacuum casing, but are stored in a bucket Sc.) For exact measurement of the neutron lifetime, it is necessary to exclude losses at the walls. This is done in different ways in the three experiments. In scheme (a) the volume of the vessel can be varied by moving the wall indicated by the dashed line. In scheme (b) the change of the neutron spectrum in the bucket is controlled in the process of storage. In scheme (c) there can be introduced an additional surface in the vessel, indicated by the dashed line. After the given exposure time in the vessel, the neutrons are let out in all three experiments through shutter s_2 onto a UCN detector D_u. In scheme (c) there are established around the storage vessel the counters recording the number of UCN being heated during storage up to the thermal energies in collisions with the walls (S is an additional plate-like shutter, locking the neutrons in the vessel for storage).

The following result was reported in Ref. [58] (see Fig. 9b), in which, besides the storage time, measurements were also performed on the spectrum of confined neutrons and on the change in the spectrum during storage:

$$\tau_\beta = 888.4 \pm 2.9 \text{ s}. \tag{38}$$

This result was corrected in a detailed report [59] and at present has become

$$\tau_\beta = 888.4 \pm 3.3 \text{ s}, \tag{39}$$

with the systematic error shown to be approximately 1s, while the main uncertainty is due to the statistical error.

Finally, the following result was reported in Ref. [60] (see Fig. 9c):

$$\tau_\beta = 882.56 \pm 2.7 \text{ s}. \tag{40}$$

The last result, however, is not reliable. In processing the experimental data, the authors introduced a correction of the wrong sign for a leakage through the shutter S in Fig. 9b. If the sign of this correction is changed, the result will be

$$\tau_\beta = 887.6 \pm 2.3 \text{ s}, \tag{41}$$

i.e., it will totally coincide with (37) (and even has a smaller uncertainty) and be in a better agreement with the officially adopted value [61]:

$$\tau_\beta = 889.1 \pm 2.1 \text{ s}. \tag{42}$$

However, the scatter of results obtained under measurements at different temperatures (experiments were carried out at temperatures of +20, −12, and −55 °C) amounts to about 20 s, which exceeds, by two orders of magnitude, the systematic measurement error estimated by the authors.

In the nearest future, a new version of experimentation on the neutron lifetime is planned in Gatchina by the group of Serebrov [40, 62], and in Grenoble by the group of Morozov. A precision of the order of 1 s is to be achieved. A D Stoika [63] has proposed a modification of the experiment for measuring the neutron lifetime, in which the detection of decay electrons and heated neutrons is also planned to be performed during the storage of UCN.

4.1.1 Lifetime measurements based on storage in a magnetic trap. Besides storage experiments performed with the solid-state traps, the research continues of neutron storage in a magnetic toroidal trap. After publication of the result [64]

$$\tau_\beta = 876.7 \pm 10 \text{ s}$$

no new experimental data have appeared. But the calculations were performed [65]. Doyle and Lamoreaux [66] proposed generating UCN in superfluid helium in a magnetic trap for measurement of the neutron lifetime. Estimations showed it is

possible to reduce the error down to 0.004% in such an experiment.

4.2 The electric dipole moment of the neutron

No special progress has occurred in this direction during the past five years. We refer the reader to Refs [67, 68] for a review on this issue. Therefore, we shall only remind the results of the most recent experiments conducted in Grenoble and in Gatchina. Taking into account previous measurements, the last result of the Grenoble group [69] can be presented as

$$d_{\rm n} = (-3.3 \pm 4.3) \times 10^{-26}\ {\rm cm}\,,$$

or at a 90% confidence level

$$d_{\rm n} \leqslant 12 \times 10^{-26}\ {\rm cm}\,.$$

The result obtained by the Gatchina group [70] is given as

$$d_{\rm n} = (2.6 \pm 4.0 \pm 1.6) \times 10^{-26}\ {\rm cm}\,,$$

or, at the same confidence level

$$d_{\rm n} \leqslant 1.1 \times 10^{-25}\ {\rm cm}\,.$$

A comprehensive review of the development of methods aimed at searching the neutron EDM is presented in Ref. [67]. Therein, the possibilities are considered also for enhancement of the sensitivity of EDM searches by three orders of magnitude with the aid of UCN generation and storage in liquid helium with dissolved polarised ^{3}He. Here, ^{3}He plays the roles of the polariser and of the polarisation analyzer, of the magnetometer, and of the neutron detector. We refer the interested reader to this review. We only note that therein the reader will find a clear exposition of such issues as 'dressing the neutron' with the aid of an external alternating field, in which the magnetic moment of a particle seems to be effectively reduced.

4.3 Neutron-antineutron oscillations

The problems of oscillations arose in connection with the development of certain field-theoretical models, such as, for example, the Grand Unification model, in which the violation of the baryon number conservation law with $\Delta B = 2$ is foreseen [71]. In this case, transition may take place between a neutron and an antineutron. Naturally, this process proceeds very slowly, and when some neutrons have been chosen, it is desirable to follow them for a long time, so as to accumulate sufficient amounts of antineutrons for their detection.

If the process describing such transitions is characterised by an energy ϵ, then the transition probability in a time $t \ll t_{\rm c} \equiv \hbar/\epsilon$ is

$$W(n \to \overline{n}) = \left(\frac{\epsilon t}{\hbar}\right)^2 .$$

If experiments are carried out in a beam, the time the neutron can be followed is limited by the time of flight through the given experimental base. If, on the contrary, experiments are performed with UCN, then the question arises whether antineutrons are accumulated during the entire storage time $t_{\rm exp}$ or only during the free flight $t_{\rm f}$ between two successive collisions with the walls.

It is felt that since an antineutron just produced undergoes interaction with the walls, which differs essentially from that of a neutron, and since it may annihilate with nuclei of the wall, the accumulation actually occurs only in between two collisions with the wall. In other words, collisions with the walls hinder transition between the neutron and the antineutron. Here arises the well-known phenomenon of the wave function reduction in quantum mechanics: if one checks too often whether an α-particle has left the nucleus or not, the nucleus will never decay with emission of an α-particle. In everyday life this is equivalent to the effect of an impatient housewife: if she often lifts the lid of a pot to see whether the water has started boiling, it may never even do so. In this relation, the interaction of a neutron with the wall serves as a check: has an antineutron been produced or not.

Hence, the result follows directly that the probability of finding an antineutron with the aid of UCN in a trap is

$$W(n \to \overline{n}) = Nn\left(\frac{\epsilon t_{\rm f}}{\hbar}\right)^2 ,$$

where $n = t_{\rm exp}/t_{\rm f}$ is the number of bombardments against the walls during the neutron confinement in the trap, $N = N_0 t_{\rm exp}/\tau$ is the mean number of neutrons in the trap through the whole storage time, N_0 is the total number of neutrons accumulated in the trap. A possible method for enhancing this value is considered in Ref. [72] (see, also, Refs [73 – 75]).

4.4 The Berry phase

In recent years much irrelevant talk is to be heard on the Berry phase. In neutron physics, where everything concerning the interaction of spin with the magnetic field has long been known, neither general understanding nor measurements benefit from the introduction of the concept of Berry phase [76].

5. Application and academic research

Applied and fundamental research are conveniently combined in a single section, since in the field of UCN physics it is very difficult to draw the line separating real applied findings (or rather findings giving rise to intentions to make use of them in application problems) and purely academic results. The author, however, does not wish to be the judge and the highest instance, so he leaves it to the reader to decide for himself what is what.

5.1 Inhomogeneities inside the material

The work closest to applications makes the one performed by the group from the Lebedev Institute of Physics (Moscow), which is mostly reported in their Short Communications in Physics [77 – 84]. The work was carried out with a unique gravitational spectrometer and pertained to studies of inhomogeneities in condensed matter. These studies are essentially similar to small-angle scattering, but instead of measuring the angular distributions of scattered neutrons, here, measurement is performed of total scattering as a function of the incident neutron energy, since the neutron wavelength is large. In so doing, the part of the neutron spectrum utilised is somewhat higher than the region related directly to UCN. The character of concrete investigations is best judged from the titles of relevant publications, and owing to little space here we refer the reader directly to the original publications.

5.2 Investigation of surfaces

In surface investigations the dependence is studied of the reflection coefficient upon energy (all sorts of admixtures and inhomogeneities most strongly affect the reflection coefficient at energies just above the boundary energy) and the reflection indicatrix in the presence of roughnesses [85, 86].

As an example we can indicate Ref. [87]. Here, specular reflection of neutrons with a wavelength $\lambda = 80$ A from a film of liquid helium 5000 A thick is studied in the vicinity of the liquid – vapor transition. If the thickness of the film is finite, it is possible to observe the interference minimum in the reflection coefficient. The position of the interference minimum depends both on the thickness of the film and on its atomic density, as well as on the spreading of the boundary. The density and the spreading depend on the temperature. By measuring the variation in position of the minimum as the temperature changes in the vicinity of the critical point, we gain information on the phase diagram in the liquid – vapor transition region.

5.3 Super-ultracold neutrons

Neutrons forming a two-dimensional quantum gas over a plane surface in the gravitational field, or a one-dimensional gas in narrow channels, can be termed superultracold [88, 89]. The energy of these neutrons is quantised along the normal to the plane, and the ground level amounts to 1.4×10^{-12} eV. Accordingly, the height reached by such neutrons is about 10 μm, and their velocity amounts approximately to 1 cm s^{-1}. If the motion along the plane is characterised by significantly higher velocities, then a neutron scattered on roughnesses may acquire a greater velocity along the normal and no longer be two-dimensional. To avoid this effect, the neutron velocity in any direction should not essentially exceed a value on the order of 1 cm s^{-1}.

In Refs [88, 89], the storage time is calculated for two-dimensional and one-dimensional neutrons (the result should not differ essentially from the quantity $v_l/g\eta$ indicated in Refs [2, 3]), and the possibility is considered of the production of a bound state due to magnetic attraction between neutrons with opposite oriented spins. The assertion is made that two-dimensional neutrons do not form a bound state.† On the other hand, one-dimensional neutrons form a bound state with an energy on the order of 10^{-20} eV. The size of such a pair amounts to about 6 – 20 km [88].

To what extent this work pertains to applied research can be judged from the fact that it is published in the journal *Pis'ma Zh. Eksp. Teor. Fiz.* [89], which only accepts material requiring urgent publication. The urgency in this case indeed existed, since owing to a delay in the publication by *Yadernaya Fizika* "the size of a one-dimensional pair increased from 1 – 10 m [89] up to 6 – 20 km [88]".

Less happy was the fate of publication [90], in which a bound state of a neutron in superconductor fluxoids was considered. It remained in the form of a preprint. This could have happened because of its too academic character. Anyhow, its solution is available to anyone familiar with quantum mechanics and who is capable of computing bound states in a given potential.

5.4 Bound states of the neutron in the magnetic field of a straight wire

There exist two types of neutron magnetic storage: (1) the neutron is repelled from a strong field and is kept in the weak field region, and (2) the neutron is sucked into the strong field region. In the English-language literature these are called 'low field seekers' and 'high field seekers'. In the first case, the neutron is kept so long as its spin is parallel to the strong magnetic field and the quantity $-\boldsymbol{\mu}\mathbf{B} > 0$. If spin-flip occurs, then the potential energy $-\boldsymbol{\mu}\mathbf{B}$ in the strong field region becomes negative, the field starts sucking in the neutron, and the latter leaves the trap.

In the case of a second type storage, a neutron is confined, if its total energy is negative. This happens, for instance, when the neutron is stored in the magnetic field of a straight wire. The quantum-mechanical problem of neutron bound states was considered back in 1976 [91] and was solved analytically by Pron'ko and Stroganov in 1977 [92], which was mentioned in Ref. [2]. However, Russian literature is not read much abroad, so this problem was once again considered in 1989 [93, 94].‡ In the latter, unlike the earlier works, storage is considered in the magnetic field of a wire of finite diameter. Here, the neutron in the low-lying states turns out to be on orbits that are completely inside the material of the conductor, leading to numerous negative consequences.

5.5 The neutron microscope

The neutron microscope could be also considered to pertain to applied studies, but it is still far from the stage of being applied for practical purposes [95 – 99]. Calculation of its characteristics related to magnification and aberrations is performed on the basis of the ballistic principle, instead of the wave principle. Here, experiments are rare and mainly carried out for the sake of demonstration. The resolution, as pointed out in Ref. [96], amounts to 17 μm and is over three times worse than the computed value of 5 μm. But, as the authors themselves say: "Most likely it would be premature to draw any conclusions from the divergence between the observed and computed resolutions".

5.6 Wave optics

Here we shall only list the latest publications on this issue [100 – 111]. Not all of them are strictly related to the field of UCN, but they are quite applicable in the case of UCN. In most cases the titles of the works reveal their essence.

One of the lines of work in this field is the construction and utilisation of supermirrors. If we cover a substrate that has a reflecting potential u_1, with a set of layers so the sign of the potential alternates, we will obtain a periodic potential. Now, if we choose the thickness of the layers so that the periodic set results in Bragg reflection at the energy above u_1, then, since Bragg reflection is total within the range u_2 called the Darwin table, the whole range of total internal reflection becomes equal to $u_1 + u_2 > u_1$. If the distance between the layers is varied in accordance with some rule, an analog will be obtained of an imperfect crystal with a complex mosaic structure, and the range of reflection will be increased even more. True, reflection above u_1 will, in this case, no longer be total. All these systems are called supermirrors. They were studied in Refs [112, 113] and are applied in many fields.

† The author considers that the reason for this is the negative amplitude of neutron-neutron scattering, although a negative amplitude actually points to attraction between the neutrons.

‡ It should be noted that the authors of Ref. [94] considered themselves obliged to quote their Russian precursors.

For example, a neutron Fabry – Perot interferometer was described in Ref. [111]. It consists of two supermirrors separated by a layer of titanium 90 A thick. The mirrors, in turn, involve three bilayers of nickel (90 A) and titanium (90 A). The interferometer was tested using neutrons with wavelengths of 2 – 6 A, and with its aid diffuse scattering on roughnesses of the outer surface was studied, and interference effects were found, which revealed correlations between the roughnesses on different interlayer boundaries.

Supermirrors are also applied for constructing the mechanical generators of UCN — turbines [114, 115]. In this case, the supermirrors are vacuum deposited on the surfaces of the blades, allowing extension of the range of reflected neutrons.

5.7 The equivalence principle

A test of the equivalence principle using neutrons consists in measuring the acceleration of the neutron free fall with the aim of revealing its difference from that of macroscopic bodies [116, 117]. One of the methods for measuring the acceleration consists in deducing the scattering amplitude of the medium [116]: once with thermal neutron interferometer, and another time by measuring the boundary energy of the medium, i.e. by measuring the dependence of reflection from the wall on the energy of the incident neutron, while the energy is varied through handling of acceleration in the gravitational field (the sample is shifted vertically with respect to the primary horizontal beam).

The ratio of the two amplitudes thus measured contains the ratio between the masses, inertial and gravitational. The equality of the mass ratio to unity may be tested in a neutron experiment with a precision onthe order of 10^{-3}.

In Ref. [117], direct measurement is proposed of the gravitational acceleration of the neutron in a time-of-flight experiment with a good monochromatization using an interference filter and rapid interruption of the beam with the aid of an electromagnetic shutter.

6. Technical issues. Optimistic prospects

Technical issues include UCN detection, their polarisation, and, most important, their production. Precisely the latter was intended in relation to the words on optimistic prospects.

6.1 UCN detectors

In recent years, no special progress was made in this direction. The proportional UCN counter with ^{3}He constructed by A V Strelkov and described in detail only in his candidate (master) thesis† obviously turned out to be quite close to perfect. A similar detector constructed recently in Japan [119] also does not exceed it.

The detector with ^{3}He, however, turns out to be not very convenient for low-temperature investigations, such as studies of UCN generation and storage in superfluid helium. For such experiments a solid-state detector has been developed [120], which represents a stratified system of thin layers of ^{6}Li and Ti (35 double layers of 50 A Li and 30 A Ti plus 25 double layers of 50 A Li and 40 A Ti) deposited on the surface-barrier silicon detector. The system of vacuum-deposited layers served in this case for enhancing the efficiency of neutron capture and reducing reflection. The amplitude spectrum of this detector shows well-identified peaks from the triton and the α-particle, produced in neutron capture by the Li nucleus. Measurement of the capture efficiency at a temperature of 4.2 K using neutrons with a wavelength of 4 A yielded a value of 0.28% with a calculated value of 0.29%. Extrapolation to the UCN region shows that the efficiency of the detector should exceed there 60%.

A UCN detector that deserves mentioning is a gas scintillation detector with xenon [121], in which scintillations are caused by the decay products of uranium after the capture of a neutron in a foil representing an alloy of ^{235}U with Ti. This detector is insensitive to the γ- and β-background and detects UCN with a 40% efficiency.

6.2 Polarisation and analysis of polarisation

Experiments with polarised neutrons always involve three fundamental parts: a polariser, a spin flipper, and a polarisation analyzer. For processing the experimental data, one must aware of the polarisation of the beam incident upon the sample placed in the gap between the spin flipper and the analyzer. Therefore it is desirable to know the polarising power of the polariser P. In experiments, on the other hand, one succeeds only in measuring the product PA, where A is the analyzing power of the analyzer. In Ref. [122], a method is proposed for measuring only P. To analyse this method, we shall consider two approaches to the description of polarisation: (1) the formalism of the two-dimensional vector, and (2) the formalism of the density matrix.

6.2.1 The formalism of the two-dimensional vector. The neutron flux is described by the vector

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix},$$

where the parameter a determines how many neutrons in the beam are polarised along a chosen axis, while parameter b determines the number of neutrons polarised in the opposite direction. A polarised beam of polarisation P can be written in the form

$$\psi = \frac{I}{2}(1 + P\sigma_z)\psi_0 ,$$

where I is the total intensity of a polarised beam, σ_z is the Pauli matrix, ψ_0 is an auxiliary two-dimensional vector, which corresponds to a nonpolarised beam:

$$I = \frac{\psi_0(1 + P\sigma_z)\psi_0}{2}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \psi_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

and multiplication is performed according to the common rules of a matrix algebra.

From the relations presented it is clear that the beam is totally polarised along the chosen axis, if $P = 1$ or $b = 0$, and that it is polarised in the opposite direction, if $P = -1$ or $a = 0$. In the general case $|P| < 1$.

A beam of nonpolarised neutrons of intensity $2I$ is polarised by transmitting it through a polariser. The transmission of the polariser is characterised by the transmission matrix $T_0 = t(1 + P\sigma_z)$, where t is a constant factor determining the attenuation of the beam, P is the polarising power. If $P = 1$ and no reflection or absorption occur, then $t = 1/2$, since the polariser transmits only half of a nonpolarised neutron beam. The transmission of the analyzer, which is essentially identical to the polariser, is described in a similar manner.

† Some of its characteristics and construction parameters were noted in Refs [118] and [2, 3].

Rotation of the polarisation with the aid of a spin flipper is described by the operator

$$Q = 1 - f + f\sigma_x\,, \qquad \text{where} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

here f is a parameter characterising the efficiency of the spin-flipper. If the beam initially had a polarisation P along a chosen axis z, then after the spin flipper it is described by the vector

$$\psi = (1 - f + f\sigma_x)(1 + P\sigma_z)\psi_0 = \big[1 + P(1 - 2f)\sigma_z\big]\psi_0\,,$$

and possesses a polarisation $P(1 - 2f)$. In calculations we have taken advantage of the relations $\sigma_x\sigma_z = -\sigma_z\sigma_x$ and $\sigma_x\psi_0 = \psi_0$. When $f = 0$ no rotation takes place, and when $f = 1$ the spin flipper rotates the neutron spin through 180°, i.e. changes the sign of polarisation. Notice that in the formalism of the two-dimensional vector, the value $f = 1/2$ is not appropriate to the rotation through 90° about the x axis. But it is defined by a total depolarisation of the beam.

Upon transmission through a polariser of polarising power P_P and beam attenuation t_P, through the spin flipper with parameter f and the second polariser (analyzer) of analyzing power P_A (this is precisely the polarising power of the analyzer) and attenuation t_A, the initially nonpolarised beam of intensity $2I$ transforms to the form

$$\begin{aligned}\psi &= t_\mathrm{A}t_\mathrm{P}I(1 + P_\mathrm{A}\sigma_z)(1 - f + f\sigma_x)(1 + P_\mathrm{P}\sigma_z)\psi_0 \\ &= t_\mathrm{A}t_\mathrm{P}I(1 + P_\mathrm{A}\sigma_z)\big[1 + P_\mathrm{P}\sigma_z(1 - 2f)\big]\psi_0 \\ &= t_\mathrm{A}t_\mathrm{P}I\Big\{1 + P_\mathrm{A}P_\mathrm{P}(1 - 2f) + \big[P_\mathrm{A} + P_\mathrm{P}(1 - 2f)\sigma_z\big]\Big\}\psi_0\,.\end{aligned}$$

Accordingly, the intensity measured by the detector is $I_\mathrm{d} = t_\mathrm{A}t_\mathrm{P}I\big[1 + P_\mathrm{A}P_\mathrm{P}(1 - 2f)\big]$. On measuring I_d, when the spin flipper is switched on ($f = 1$) and then switched off ($f = 0$), we find the product $P_\mathrm{P}P_\mathrm{A}$. Evidently, P_A and P_P are inseparable in such a formulation.

But even for determining this product it is necessary to know the parameter f of the spin flipper well. It can be found, however, with the aid of an additional spin flipper [123, 124] in series with the first. Indeed, if we have two spin flippers with parameters f and f', then the intensity measured by the detector after transmission through the polariser, two spin flippers, and the analyzer will be as follows

$$I_\mathrm{d} = t_\mathrm{A}t_\mathrm{P}I\big[1 + P_\mathrm{A}P_\mathrm{P}(1 - 2f)(1 - 2f')\big]\,,$$

from which it follows that one can find both f, f' and $P_\mathrm{P}P_\mathrm{A}$ separately [123, 124] by switching the spin flippers† on and off, together or individually.

When determining the polarising power of the polariser separately, a scheme of three successive polarisers with spin flippers in between them was proposed in Ref. [122]. If one denotes the polarising power of these polarisers by P_i, where $i = 1, 2, 3$ (from left to right) and the parameters of the spin flippers by f_j, where $j = 1, 2$ (in the same direction), then transmission of the beam through the entire system from left to right transforms the initially nonpolarised beam into the following:

$$\begin{aligned}\psi = C(1 + P_3\sigma_z)(1 - f_2 + f_2\sigma_x)(1 + P_2\sigma_z) \\ \times (1 - f_1 + f_1\sigma_x)(1 + P_1\sigma_z)\psi_0\,.\end{aligned}$$

Accordingly, the intensity measured by the detector is

$$\begin{aligned}I_\mathrm{d} &= C\psi_0\big[1 + P_3(1 - 2f_2)\sigma_z\big] \\ &\quad \times (1 + P_2\sigma_z)\big[1 + P_1(1 - 2f_1)\sigma_z\big]\psi_0 \\ &= 2C\big[1 + P_3P_1(1 - 2f_2)(1 - 2f_1) \\ &\quad + P_3P_2(1 - 2f_2) + P_2P_1(1 - 2f_1)\big]\,.\end{aligned}$$

Hence, it is seen that manipulation of the flippers allows determination of the quantities P_1P_2, P_1P_3, P_3P_2 separately, and then finding $P_1^2 = P_1P_2 \times P_1P_3/(P_2P_3)$. All the other P_i can be determined in the same way.

In Ref. [122], this scheme was complicated by the polarisers 2 and 3, composing the analyzer,‡ together with the spin flipper 2 in between them being established on a platform capable of rotating through 180°, thus changing the sequence of polarisers 2 and 3 in the path of the neutron. Rotation of the analyzers allow additional tests of the technique, or to be more precise, testing the symmetry of the system of 2–3 polarisers.

On the whole, measurement of the polarising power, which turned out to be at the level of 70%, required carrying out about 20 different experiments (4 measurements of the spin-flipper parameter f_1, 8 measurements of the spin-flipper parameter f_2 for the rotated and nonrotated system of polarisers 2–3, and 8 measurements of the transmission of the entire system with rotated and nonrotated polarisers 2–3). It seems that for measurement of the quantity P_P it is, most likely, sufficient to perform 6 experiments for recording separately the transmission of polariser t_i and of the pairs $t_it_j(1 + P_iP_j)$, upon which for checking the reliability of the technique it would be sufficient to measure the transmission of the whole system, $t_1t_2t_3(1 + P_1P_2 + P_2P_3 + P_1P_3)$.

6.2.2 The density matrix. Actually, the description of polarisation problems on the basis of two-dimensional vectors is incorrect. Indeed, if the polarisation along a certain chosen axis is zero, i.e. the numbers of neutrons with spins along and opposite the chosen direction coincide, this does not mean that the beam is nonpolarised. It may be polarised along another axis [125]. If such a beam is transmitted through a bad spin flipper (in this case a nonpolarised beam should remain nonpolarised), resulting in the polarisation being rotated through an angle not quite equal to 180°, it becomes possible to notice the appearance of beam polarisation along the chosen direction, i.e. it is as if the spin flipper becomes a polariser.

Therefore, it is more correct to describe the neutron beam with the aid of the density matrix

$$\rho = \frac{I}{2}(1 + \mathbf{P}\boldsymbol{\sigma})\,,$$

where the polarisation $\mathbf{P}$ represents a vector, the length of which characterises the polarisation, while its orientation characterises the direction of the polarisation. Here, the beam intensity is $\mathrm{Sp}\,\rho = I$, and the polarisation along a certain chosen unit vector $\mathbf{e}$ is $\mathrm{Sp}\,(\mathbf{e}\boldsymbol{\sigma}\rho)/I = \mathbf{P}\mathbf{e}$.

† Notice that for determining $P_\mathrm{P}P_\mathrm{A}$ it is possible to do without any spin flippers, if the transmissions t_P, t_A, and $t_\mathrm{P}t_\mathrm{A}(1 + P_\mathrm{P}P_\mathrm{A})$ are measured separately.

‡ In Ref. [122], reflecting mirrors served as the polarisers.

The transmission of the polariser and the spin flipper are determined by matrices of the general form $\widehat{M} = \exp(\boldsymbol{\mu}\boldsymbol{\sigma})$, where $\boldsymbol{\mu}$ is the vector with complex parameters (in the general case these comprise 6 independent real parameters). These matrices transform the neutron beam density matrix as follows:

$$\rho \to \widehat{M}^{+}\rho\widehat{M}\,,$$

and for determination of these parameters a three-dimensional polarisation analysis is required.

6.2.3 Other studies of polarisation. To conclude we shall mention Refs [126] and [127]. The first work is devoted to computation of a model of an adiabatic spin flipper, and in the second the proposal is made to modulate the neutron beam intensity with the aid of ferromagnetic films. If the film is not magnetised, it scatters neutrons owing to refraction on the magnetic domains. If, contrariwise, it is magnetised up to saturation, there remains a single domain, and the film becomes transparent. On magnetising in turn several films put in the path of the neutron, it is also possible, besides modulation, to perform monochromatization of the beam, like in a system of choppers.

Finally, we shall also mention Ref. [128] in a popular journal, where a rigorous analysis is presented of the resonance rotation of polarisation with regard to the variation of the neutron kinetic motion in an alternating field.

6.3 The production of UCN

We shall now pass to the last section, to which the words on optimistic prospects are related. Clearly, the future of all experiments with UCN depends on the power of their source. The maximum density hitherto achieved did not exceed 100 neutrons cm^{-3} (see reviews [129 – 131]), and it seemed the limit was reached. The information on new sources [132] did not inspire, since their intensity was significantly lower than the record. One of the ideas for obtaining a higher intensity consisted in generating UCN in a vessel filled with parahydrogen. During the burst of the reactor (300 μs) the neutron beam generates UCN in the vessel, and, when the burst comes to an end, the vessel is opened and the gaseous molecules, being faster than the neutrons, are the first to leave the vessel. Then the vessel is closed again, and nothing prevents the neutrons from being stored. Calculation demonstrated that the gas leaving the vessel will not pull the neutrons away with it [133]. However, no attempts at realisation of this project have been made yet.

6.3.1 Superfluid helium. Another long-discussed project, consists in the production of neutrons in superfluid helium [134 – 136]. In Japan, a three-metre container with ^{4}He has already been made [137] for generating UCN. Helium is interesting in that, first, it does not absorb neutrons, second, in a superfluid state it has nearly no excitations that could heat the neutrons, and, third, its interaction with neutrons is exclusively coherent. Thus, for example, the production of UCN should be due only to neutrons of wavelength near 9 A, because just these neutrons are capable of giving away their energy and at the same time their momentum for excitation of phonons. The conservation laws: $p = q$ and $p^2/(2m) = cq$, where c is the speed of sound in helium, are satisfied solely for $p = 2mc$. Calculations show that the UCN production intensity should be quite high. However, experiments carried out [138] have not yet confirmed these predictions. True, in these experiments the container with helium was not closed during UCN generation. It was open all the time, and all the neutrons produced there were recorded. In this case, the neutron lifetime in the vessel was not determined, because the leakage time was not well known. The total time of UCN confinement inside the vessel, τ, was defined as

$$\frac{1}{\tau} = \frac{1}{\tau_0} + \frac{1}{\tau_1(T)}\,,$$

where τ_0 is a constant component, $\tau_1(T)$ is the portion of the storage time depending on the temperature (measurements were performed within the range $0.5\,\mathrm{K} < T < 1.5\,\mathrm{K}$). The total time τ proved to be on the order of 14 s, and its temperature dependence, in the opinion of the authors, did not correspond to theoretical predictions. In this connection, a discussion arose [139, 140], which may be resolved in subsequent experiments.

6.3.2 The dynamic converter. The background of one of the experiments currently under preparation for obtaining UCN from the pulsed reactor BIGR in Arzamas-16, is illustrated in Fig. 10. It was discussed earlier in Refs [141, 142].

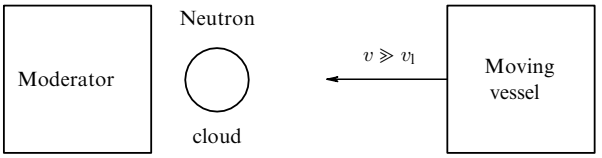

Figure 10. A rapidly moving hermetically sealed vessel impinges upon the UCN cloud produced during the burst of the powerful reactor. If the velocity of the container significantly exceeds $10\,\mathrm{m\,s^{-1}}$, the cloud will easily penetrate into the vessel through its walls. When the cloud turns out to be completely inside, then the vessel stops abruptly, and the neutron cloud appears to be confined. After this, the vessel filled up with neutrons, is slowly transported to the experimental hall.

The density of accumulated UCN is expected to reach 10^5 neutrons cm^{-3}. These experiments turn out to be very difficult, from a technical point of view. They require precise synchronisation of the reactor burst and the motion of the container, along with particularly thorough preparation of the reverse transportation of the container to the experimental hall for minimising losses during the displacement.

Earlier, experiments were carried out by the group of A B Antonov in the Lebedev Institute of Physics (Moscow) for obtaining UCN in the neighbourhood of the reactor [143 – 146] and their subsequent transportation in the trap itself [147, 148]. However, in that experimental arrangement, the converter was inside the storage vessel and was closed after the reactor burst by a mechanical shutter, through the slits of which a neutron could be absorbed in the converter during transportation. Therefore, by this method it appeared possible to bring only several neutrons per cycle to the experimental hall. In our arrangement, the vessel will be initially hermetically sealed, so no losses other than that in the walls should occur, and therefore the number of accumulated neutrons will be significantly higher.

6.3.3 Solid deuterium. Optimistic prospects are related to obtaining UCN at the stationary [149] or pulsed [150] reactor

from solid deuterium† at a temperature of 4 K. These hopes are based on experiments previously performed in Gatchina, when the rate of UCN generation in deuterium at 10 K was found to exceed the rate of their generation in liquid hydrogen, and this rise was not seen to stop.‡

If theoretical expectations prove to be correct,§ then in the case of a thermal flux density equal to $\Phi = 2 \times 10^{14}$ neutrons cm^{-2} s^{-1} it will be possible with the aid of such a converter (from estimations presented in Ref. [149]) to obtain a UCN density on the order of $(2-4) \times 10^5$ neutrons cm^{-3}. Deuterium is good at low temperatures because it is transparent with respect to UCN. But to make use of this advantage, its volume must be sufficiently large. In so doing, however, the difficulty arises that for maintaining a large volume of solid deuterium situated close to the reactor, powerful refrigerators are required at a low temperature. But, if the solid deuterium is used at some distance from the reactor, then the intensity of incident thermal neutrons is reduced and, given the high efficiency of deuterium, the absolute UCN intensity turns out to be insignificant.

The relatively high boundary energy of deuterium is also its disadvantage. Therefore, the UCN generated in it acquire additional energy upon leaving the converter and become useless for storage experiments in horizontal channels. When vertical or inclined channels are used, this disadvantage is overcome, but during transportation along such channels the losses of neutrons increase. A test series of experiments [152] with solid deuterium conducted in Gatchina confirmed the high efficiency of deuterium. The gain factor, i.e. the ratio of the yield from deuterium at a low temperature to the yield from gaseous deuterium at room temperature, was shown to amount to about 1000.

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† This technique of generating cold neutrons was first proposed by M Utsuro and K Okumura [151]. They calculated the spectrum of neutrons extracted from solid ortho- and para-deuterium.

‡ It is interesting to compare these results with the experimental data [115] gathered by the group of M Utsuro.

§ Some theoretical calculations (see Table 9.15 from Ref. [2]) yield less optimistic predictions.

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