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BOOK REVIEWS

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Topology of gauge fields and condensed matter
by M Monastyrsky

Topology of gauge fields and condensed matter by M Monastyrsky (New York, London: Plenum Press and Moscow: Mir Publishers, 1993) 372 pp.

One of remarkable events in Theoretical Physics in the 1960s – 1990s may be considered to be the raising of gauge invariance and spontaneous symmetry breaking concepts to the rank of fundamental principles. There exists a justified persuasion that on the basis of these principles one would be able to obtain a unified description of fundamental interactions, and this might be considered as an eccentric revival of the dream to build up the Unified Field Theory. Furthermore, more and more proof of the validity of these principles is becoming available in the course of modelling localised structures (vortices, defects, textures, etc.), and also in modelling particles as extended objects in Condensed Matter Physics, in Astrophysics and Cosmology, in Particle and Nuclear Physics. Another way of putting it is that the aforementioned principles play a dominant role in describing extended objects and substantially nonlinear phenomena. This indicates that they are background principles in modern Nonlinear Physics. Mathematical language, which proved to be adequate to the physical nature both of nonlinear phenomena and nonlinear objects, has become the language of Algebraic Topology, to which physicists are still unaccustomed whence the title of the book — *Topology of Gauge Fields and Condensed Matter*.

The book consists of the following chapters (paragraphs):

I. Preliminaries. mathematical settings. Basics.
1.1 Manifolds; 1.2 Lie groups; 1.3 Group action; 1.4 Fibre bundles; 1.5 Connection in bundle

II. Elements of topology. How can two given manifolds be differentiated?
2.1 Homotopy theory; 2.2 Homology theory; 2.3 Cohomology theory; 2.4 Interrelations between homotopy and homology groups; 2.5 Degree of mappings and indices of vector fields; 2.6 Hopf invariant; 2.7 Characteristic classes

III. Physical principles and structures.
3.1 Gauge invariance; 3.2 Systems with spontaneously broken symmetry

IV. Topology of gauge fields.
4.1 Monopoles in gauge field theories; 4.2 Instantons

V. Topology of condensed matter.
5.1 Liquid crystals; 5.2 Topology of superfluid ^{3}He

VI. Conclusions.
6.1 Topology of gauge fields; 6.2 Topology of condensed matter; 6.3 Historical remarks

The monograph is completed with a List of References and a detailed Subject Index.

The author Mikhail Monastyrsky works at the Institute of Theoretical and Experimental Physics, and is widely known for his pioneering contributions to applications of Algebraic Topology Methods in the Gauge Theory and in Condensed Matter Theory. The appearance of the book was preceded by active lecturing at several universities and research centres around the world. The starting point for this activity was in 1977, when I S Shapiro made a proposal to the author to deliver a series of lectures on topology for theoreticians at the Institute of Theoretical and Experimental Physics. The preliminary intention to write the book occurred at that time, and the manuscript was accepted by the Publishing House "Nauka" but for well-known reasons, it was issued by foreign publisher.

As it can be traced from the Foreword, in the course of writing a twofold purpose was pursued. First of all, the author has tried to present in as clear form as possible the basic aspects and working instruments of those parts of Algebraic Topology, Differential and Algebraic Geometry, Theory of Lie Groups and Algebras, which are of frequent use in physics research, but are still not included into curriculums of basic mathematical education for physicists. These topics are explored in the two first chapters, and it should be emphasised, that the author, in a rather compact form, essentially succeeds not only in introducing basic definitions and to enumerating the most significant theorems, but also in giving visual demonstrations of a number of useful technical 'tricks and tips', which are known to practitioners, but are rarely found in textbooks. The material is presented rather pleasantly, without unnecessary general sentences and scrupulous proofs which usually frighten off theoreticians from reading mathematical monographs. It can certainly be used as a base for independent study or for delivering special lectures. Note that in spite of its compactness the given exposition is quite capacious and covers a substantial part of algebro-geometric technicalities used in modern physical literature.

The other evident purpose and distinguishing feature of the book is that it provides a nearly exhaustive outline for numerous applications of the Algebraic Topology and Differential Geometry methods† in Field Theory and in Condensed Matter Theory, known at the present time.

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Translated by I Sanyuk

† For good reasons most attention has been concentrated on applications of comparatively simple topological tools, whereas the modern achievements in the theory of monopoles, instantons and quantum strings, which involve rather advanced mathematical methods and results such as the Atiyah – Singer index theorem, the Chern – Simons characteristic classes, moduli spaces, twistors, and so on, are only mentioned.

In Chapter III the author first sets out in a popular manner the basic principles of modern Field Theory and Statistical Physics, the very principles which have made it possible to develop a unified approach to the description of various nonlinear phenomena. The same issues are exposed, for example, first in field-theoretical language, and then in terms of the fibre bundle theory. This expedient provides physicists with a rapid understanding of the so-called "physical meaning" of geometrical concepts, and, in turn, it provides mathematicians with an accessible formalisation of physical problems.

Chapter IV gives an exhaustive presentation of the topological structure of monopole-like and instanton-like† solutions. Since the book is mainly geared to theoreticians, preference is given not to general schemes and proofs of solution existence theorems, but instead to constructive derivations of solutions in an explicit form. Such, indeed, is the case for pure Yang – Mills equations, where instead of general but still formal Atiyah – Hitchin – Drinfeld – Manin scheme for getting *n*-instanton solutions, a detailed exposition is given of less general but derived in explicit formulas solutions, suggested by Belavin – Polyakov – Tyupkin – Schwarz, 't Hooft, and Witten. Nevertheless, the reader will find here results of more general character, namely: the topological criterion for monopole-like solution existence, the theory of harmonic mappings, topological aspects of existence of multi-dimensional solutions and much more.

The contents of Chapter V, dedicated to topology of ordered condensed media, is of special interest and there are practically no counter parts in the present literature of monographic or academic character. Defects in Liquid Crystals, singularities in *A*- and *B*-phases of superfluid ^{3}He, the phase structure in ^{3}He and in a neutron star, all these and other objects are subject to topological analyses in ordered media (this is in media admitting description in terms of order parameters). In order to classify and describe these objects there is used a rather various topological apparatus, including Milnor coefficients and Hopf invariant, Klein surfaces and Whitehead products, the general theory of links, knots theory and so on.

In his Conclusions the author acquaints the reader with modern trends in research activity, with currently predictable perspectives of the approach, and also formulates several unsolved problems.

In fact, we are still at an early stage in the application of topological methods to the studies of nonlinear dynamical systems, while conceptually the totality of such systems forms the very essence of the surrounding world. There is no question that the further enhancement of these methods by more and more theoreticians will provide not only an advance in solving current problems, but also a new vision of a number of problems. Undoubtedly, M I Monastyrsky's book *Topology of Gauge Fields and Condensed Matter* would be useful, first of all, for a wide range of specialists from various fields, willing to familiarise themselves with modern mathematical apparatus, for postgraduate and senior undergraduate students. Mathematicians will find here a rather clear formulation of physical problems, and to make use of their solutions one might need to further develop already known methods.

Needless to say, it would be inadvisable to recommend scientists in Russia (and especially to students) to purchase the English edition of this book. But due to the financial support of the Russian Fundamental Research Foundation it has become available a Russian edition of this book printed by PAIMS Publisher at the Peoples' Friendship University of Russia.

V I Sanyuk

† Remark on Terminology: the author uses an extended treatment of the term "instanton", applying it to kink solutions in ϕ^4-model and in sin-Gordon model, and as well to solutions in two-dimensional chiral models, determined on spaces with Minkowski's metric. It is more common to use the term "topological solitons" for these solutions, leaving the term "instanton" to solutions from Euclidean field theories.