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Collective plasma processes in the solar interior and the problem of the solar neutrinos deficit

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Abstract. This review presents results of the recent calculations of collective plasma processes of radiation transport in the solar interior. The review introduces a remarkable number of previously neglected effects which are shown to reduce substantially the Rosseland opacity at the centre of the Sun (the decrease of opacity is approximately 10%, which is greater than the previously accepted possible errors in opacity). It is also shown that effects which were previously treated without taking into account the collective behaviour of plasma, change appreciably when the collective nature of the plasma is included. The analysis is based on modern concepts of plasma physics in which an essential role is played by photon scattering on ions and by the oscillations of ion electron shells in emission and bremsstrahlung absorption processes. The processes which contribute most to a decrease in opacity are: the broadening of the Raman resonance (due to both the Doppler effect and binary electron – ion collisions), frequency diffusion in radiation transfer processes, the processes of stimulated scattering and collective quantum corrections to the scattering. A list of collective plasma effects which influence photon transport in the dense central solar plasma is presented. The results of these new calculations could give a better agreement between the observed neutrino flux and theoretical predictions. New problems are discussed which can be of importance from the point of view of modern plasma physics for solar neutrinos production in different energy ranges.

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1. Introduction

There exists a widely held opinion that the measured flux of solar neutrinos is less than that predicted by the Standard Solar Model (SSM). A critical review of solar neutrino experiments and improvements in the SSMs was recently

given by Morrison [1] who indicated the need for detailed plasma physics calculations. Morrison [1] illustrated the tendency of a decrease with time of the discrepancy between the observations and the theoretical models, but the plasma aspect of the problem was only briefly mentioned and has not been dealt with in detail. The present article concentrates on the plasma aspect of the problem [2]. Concerning the discrepancy of the observations and the SSM it should be noted that in the first experiment (Homestake, Chlorine experiment, Davis et al.) this discrepancy was a factor of 8. At the present time there are four experiments going on and on average the discrepancy is a factor of 2 – 3 depending on the experiment. The four experiments are SAGE (Soviet-American Gallium Experiment), GALLEX (Gran Sasso, Italy), Kamiokande (Japan) and Homestake (USA). The different experiments have different thresholds and neutrino fluxes are measured within different ranges of energies. For comparison of theoretical predictions with observations in several cases a subtraction of the results of one experiment from the results of another experiment was performed. This is possible only if the absolute calibration of each experiment was performed which is questionable for some experiments. These problems will probably disappear soon but it is believed that the absolute calibration of the Homestake experiments will be difficult to perform in the nearest future. We shall leave these problems and concentrate on plasma collective effects in the SSM — the aspect of the problem which is rarely discussed in current literature dealing with the predictions of the neutrino flux from the Sun.

We shall concentrate on the question of whether or not in theoretical predictions of the neutrino flux the physics of the processes was treated in a correct manner. This question is more fundamental than the question which is often asked at present, namely, is the neutrino deficit due to an incorrect treatment of astrophysics of the solar interior or is it due to neutrino oscillations (MSW effect named after S P Mikheyev, A Yu Smirnov and L Wolfenstein or other effects of a similar kind)? In the literature the problem is stated as "Astrophysics or oscillations?". This was the name of a recent workshop held at Gran Sasso where the first results of detailed calculations of collective plasma effects in the solar interior were presented [2].

Why do we want to separate physics and astrophysics? The reason is that astrophysics usually uses the known and approved physical processes to construct the models of astrophysical phenomena. But the question is whether or not the physical processes for the conditions in the solar interior are known at a level necessary to predict the neutrino flux with the accuracy needed for a comparison with observations.

It is necessary to say a few words about how SSMs are calculated. In fact this can be considered as the standard astrophysical treatment. It is assumed that all the physical processes are well understood, the cross-sections of reactions are known and can be corrected if necessary in future laboratory experiments. Also known at the present time are the three main parameters of the Sun: the luminosity of the Sun, its radius and its mass and we know also of the abundance of elements on the surface of the Sun. It is assumed that the initial abundance at the stage of the formation of the Sun corresponds to the observed abundance of elements in the area of space adjacent to the Sun. One then follows the evolution of the initial plasma cloud which formed the Sun (in the literature one often finds the term 'gas cloud' which is certainly not correct). The evolution is followed up to the present time and determines the present composition of the elements in the solar interior in the way that it corresponds to the observed abundance of elements at the surface of the Sun. The relative abundance of different elements in the solar interior is an important parameter for predictions of the solar neutrinos flux in different energy channels. In the centre of the Sun hydrogen is burning and the abundance of helium is increasing. The abundance of such elements as C, N, O, Fe is important for predictions of solar luminosity and the neutrino flux in different energy channels. At the present time there exist many solar models which all go by the name SSM, they differ in the composition of different elements and in the dependences of temperature and the abundances as functions of the distance from the centre of the Sun.

The basic assumption in this 'astrophysical' approach is the assumption that all the physical processes are well known and the cross-sections for them can be determined from laboratory experiments or be at least improved in future laboratory experiments.

The question arises whether the last statement is correct.

Another question is, "What else is assumed in the construction of the SSM?" One of the assumptions is obvious: in the calculation of the radiation transport in the solar interior it is assumed that the local thermodynamic equilibrium is established with small deviations from it due to the radiation flux which is proportional to the gradient of the temperature. These deviations are described by the first Legendre polynomial with the angle related to the direction of the temperature gradient.

A commonly held opinion among the astrophysical community (which is the basis of the 'astrophysical' approach) is that the SSM is based on very well proven statements that the central regions of the Sun can be described with elementary mechanics and statistical physics and that the main processes necessary to determine the neutrino flux are the nuclear reactions and photon scattering on *free electrons* together with their absorption due to inverse bremsstrahlung (see Ref. [3]). However since the central region of the Sun is most definitely a plasma, photon scattering cross-sections can be determined by collective effects. The statement that the scattering is produced by *free electrons* can be completely wrong since the cross-section of collective scattering depends on the distribution of all other particles. This means that the cross-sections determined by individual particles have nothing in common with the cross-sections under real plasma conditions where the cross-sections depend on the surrounding plasma density, temperature, etc. In the process of scattering the statement that scattering is produced mainly by electrons is valid only for isolated electrons but not for electrons in plasma. Scattering on free electrons can be found in many astrophysical situations but it is valid only in the limit of very high frequencies when electrons behave as free particles (for free particles the Thomson cross-section is inversely proportional to the square of the mass and thus is negligible for ions as compared to electrons). We shall give the exact criteria when the electrons can be considered as free. The question then is whether or not in the solar interior these conditions are fulfilled and whether the electrons can be considered as free. In advance we may state that the answer is *negative*.

It is worthwhile mentioning that the Sun is a 'plasma sphere' but not a 'gas sphere' and should be treated as a plasma object with all the complications introduced by

collective plasma processes. Examination of the collective plasma processes in the centre of the Sun will be the main subject of the present review. Recently in some publications there has appeared the term 'plasma processes in the solar interior', which is very strange since the whole Sun consists only of plasma. Much experience, knowledge and data accumulated during the last decades in the investigation of laboratory plasma and the plasma of near space show very definitely that the collective effects determine the plasma properties. This knowledge should not be discarded in investigations of the solar interior.

Let us clarify why researchers not well acquainted with the physics of the solar interior but well acquainted with plasma physics can for a first attempt abandon the 'astrophysical' approach described above.

In the early days of controlled thermonuclear research (CTR) the belief was that plasma should follow the well established laws of statistical mechanics and should locally exhibit the tendency to form a thermodynamic equilibrium distribution. It is well known at the present time that plasma do not want to behave in the way prescribed by simple statistical physics and mechanics. After many years of research it is found that the main obstacles to controlled thermonuclear fusion are collective effects.

This term in plasma physics is used to describe two partially independent phenomena. The first phenomenon has already been discussed and is related to the radical change of cross-sections by collective effects. The second phenomenon is related to the development of different types of instabilities which make the state nonlinear, far from equilibrium and as a rule such a state could lead to self-organisation. Modern plasma physics mainly deals with the second phenomenon. But to ascertain whether or not collective effects lead to additional nonlinear transport phenomena which are very often observed in experiments, it is necessary to start by understanding the basic state when the instability is absent but taking collective effects in the cross-sections into account. Thus the question what could be the classical transport was the first one to be understood in laboratory plasma.

The analogy between the CTR research and the construction of SSMs is rather useful for understanding the general situation in plasma. In both cases in CTR devices and in the Sun the energy is transferred from the central part to the periphery, but in the CTR devices it is related to the thermal conductivity (in the case when instabilities are not developing) by plasma particles, while in the centre of the Sun the energy flux is formed by radiation via radiative conductivity. What is indeed known for certain in Tokamaks (which is a particular CTR device) is that the energy transport is never classical (more exactly in toroidal geometry it is called neoclassical). The classical theory of energy transfer in Tokamaks is constructed in a similar way to the energy transfer in SSM. Namely, it is supposed that locally a thermodynamic equilibrium is established with small deviations due to temperature gradients and proportional to the first Legendre polynomial. In the same way the theory of radiation transfer is constructed in SSM. It is assumed that the local thermal distribution is established both for photons and plasma particles and that deviations from the local distribution are due to the temperature gradient. Bearing this analogy in mind we can call the classical theory of energy transfer in Tokamaks as a Standard Model of Tokamaks (SMT). It is obvious that the experiments do not confirm the SMT. Why then should we rely on SSMs to confirm observations from the Sun?

The problem one should start with is: does there exist an SSM which is similar to the SMT in the sense that it takes into account all collective changes to the cross-sections? Thus the question is whether there exists the starting point from which we shall be able to discuss the possible role of instabilities.

We should answer also the question as to whether there exists at the present time a reasonable explanation of anomalously large energy transfer in Tokamaks. Unfortunately, the answer is no. More than three decades of investigation of the anomalous transfer in Tokamaks has not clarified the nature of anomalous transport although the real progress achieved in CTR is enormous (the maximal temperature reached is substantially larger than that in the centre of the Sun).

The deficit of solar neutrinos in some high energy channel is related to the Sun's luminosity. The plasma researcher on considering the neutrinos deficit will find it a natural phenomenon since he will consider it very probable in the presence of anomalous radiative transfer. But one can argue that the rate of binary collisions in the Sun is so high that processes should be considered as classical. But in Tokamaks the collision rate are also high, otherwise thermonuclear reactions do not occur. On the other hand, would even for a high rate of collisions there exists a set of dissipative instabilities. This brings us apparently to another question, "Why is the discrepancy between the measured neutrino flux and that calculated by SSM so small?"

In the present consideration we shall not discuss the problems of instabilities in the solar interior since our intention is only to discuss the *collective phenomena in energy transfer assuming that the instabilities are absent*. This is the problem one needs to start with before any further steps can be made in discussing the possibility of anomalous energy transfer.

Before going to the main subject we should make some comments on the possibility of anomalous energy transfer in the Sun due to the development of instabilities. The first question related to this problem is whether there constantly exists in the Sun a source of energy which can drive the instability. The answer will be *affirmative*. This source is the observed convection and continuous 'sunquakes' observed as oscillations of the Sun which can be considered as a continuous source of turbulence. The surface of the Sun is strongly turbulent and this is confirmed by observations. But it is unknown whether the interior of the Sun is also turbulent. But it is very probable that similar to Tokamaks the Sun is a self-organised system and what happens inside can not be separated from what happens on the surface. The nonlinear cascades can transfer energy to small scales important for energy transfer.

These comments are made here intentionally to have a picture of the Sun from a plasma point of view and also to demonstrate that there is no hope to find from the solar neutrinos deficit some definite conclusion about neutrino oscillations, since one can always include the effects of turbulence and instabilities.

Below we shall deal with more simple problems, namely, the classical (in the sense of the absence of anomalous transport phenomena) transport of radiation making a special note on the role of collective plasma phenomena. So we shall maintain the conservative position that instabilities are absent and discuss the problem whether the collective

plasma effects were taken into account with the necessary accuracy to predict the neutrino flux and whether all the collective effects taken into account are properly included in SSM. The answer to this question is *negative*. Many effects were missed, some were included improperly.

As concerns the collective properties, the traditional astrophysical approach is inadequate since it is impossible to use the cross-sections of nuclear and electromagnetic processes measured in laboratory experiments for the conditions in the solar interior. Even by laser compression of materials on very short time intervals it is rather difficult to obtain a conclusive answers. Under these conditions one should obviously use the plasma theoretical approach.

There exists a general modern plasma physics approach which should be used for this purpose. Only recently such an investigation has started to consider the properties at the very central part of the Sun [4]. It appears that the theoretical plasma physics approach discussed in this review can provide the predictions of solar neutrinos flux about 2 – 3 times lower than previous predictions. We consider this approach as an active approach as opposed to the passive astrophysical approach described above. We believe that such an active approach (including the collective plasma phenomena known from laboratory and near space measurements) is needed in many other astrophysical problems.

2. How sensitive is the neutrino flux to the plasma parameters in the centre of the Sun?

Is it possible to construct an SSM within the accuracy of a factor of 2 – 3? The question arose when the first SSM was discussed. But the fact is that the high energy neutrino flux (the only one measured in the first experiments) is very sensitive to small changes of temperatures in the solar interior. For example a change of the central temperature by only 2 – 3% changes the predictions of the high energy neutrinos flux by a factor of 2. It is worthwhile to give here an outline of neutrinos production from nuclear reactions in the centre of the Sun:

$$\mathrm{p} + \mathrm{p} \to \mathrm{d} + \mathrm{e}^+ + \nu\,, \tag{1}$$

$$\mathrm{p} + \mathrm{p} + \mathrm{e}^- \to \mathrm{d} + \nu\,, \tag{2}$$

$$\mathrm{p} + \mathrm{d} \to {}^3\mathrm{He} + \gamma\,, \tag{3}$$

$$\begin{array}{ccc} & 85\% \swarrow \quad \searrow 15\% & \\ {}^3\mathrm{He} + {}^3\mathrm{He} \to \alpha + 2\mathrm{p}\,; & {}^3\mathrm{He} + \alpha \to {}^7\mathrm{Be} + \nu\,, & \\ & \swarrow \quad \searrow & \end{array} \tag{4}$$

$${}^7\mathrm{Be} + \mathrm{e}^- \to {}^7\mathrm{Li} + \nu\,, \tag{5a}$$

$${}^7\mathrm{Be} + \mathrm{p} \to {}^8\mathrm{B} + \nu\,. \tag{5b}$$

The reaction (5b) shows the generation of ^{8}B neutrinos which give an 80% contribution in Chlorine experiments and about 100% in Kamiokande experiments. In Chlorine experiments 20% corresponds to the contribution of ^{7}Be neutrinos and in SAGE and GALLEX experiments the measured neutrinos correspond to the main processes of nuclear synthesis, which are described in the first two rows (1) and (2). The strongest dependence on temperature is for the most energetic ^{8}B neutrinos. For the neutrino flux Φ_ν the dependence is $\Phi_\nu^{\mathrm{B}} \propto T^{18}$. For ^{7}Be neutrinos, the dependence on the temperature is also rather strong ($\Phi_\nu^{\mathrm{Be}} \propto T^8$) and the weakest dependence on temperature exists for the proton–proton reactions ($\Phi_\nu^{\mathrm{pp}} \propto T^{-1.2}$).

The solar luminosity $L_\odot$ is determined by the relative value of the temperature gradient and is inversely proportional to the Rosseland opacity κ_{R} (solar opacity) and is proportional to T^4 (i.e., intensity of the blackbody radiation). Therefore a decrease of the Rosseland opacity of 12% for a given luminosity corresponds to a decrease of the temperature by only 3% which means a decrease of the ^{8}B neutrino flux by two or three times. Therefore it is recognised that the neutrino flux is very sensitive to small changes in the solar opacity.

At present there have been no physical reasons to change the opacity by as much as 12% or even 10%. The latter number corresponds to the change in temperature by 2.5% which is even more appropriate to the existing solar seismology data. However, in this paper we present new results of plasma collective processes which can change the opacity by as much as 10%.

The value of the solar opacity (more precisely the coefficient of the Rosseland opacity defined below) is determined mainly by scattering of photons, by bremsstrahlung absorption (not in 'free – free transitions', as was used previously; since collective effects are included the plasma particles can not be considered as free particles) and by line absorption. The value of the Rosseland opacity was corrected many times and together with the corrections of nuclear cross-sections the disagreement between predictions and observations was reduced from a factor of 8 to a factor of 2 – 3. The plasma collective effects in the coefficient of the Rosseland opacity have only been taken into account recently [5, 6], but many of them where omitted and the necessary change of 10% was not obtained.

The processes we are speaking about are well known in plasma physics, e.g. scattering, bremsstrahlung absorption and line absorption for elements which are not fully ionised, we shall discuss them in detail in this article. Although all the processes are known, for the Sun, contrary to laboratory experiments, what is also important are the integrated values over the frequencies, angles and thermal distributions of particles and such values are not given in plasma literature.

It should be noted that the plasma also effects nuclear reactions, since tunnelling is rather sensitive to small changes in the potential barrier which can be due to plasma shielding [7]. For this aspect of the problem there are still many unknown quantities and some effects are not completely clear, for example, we have in mind the capture of electrons by ^{7}Be nuclei and the generation of neutrinos in the reaction (5a). Laboratory experiments give in this case only the result for the case of a single electron bombardment of the nucleus and it is also needed to be extrapolated toward the lower energies [1]. But even if the experiments can be refined they will not give an answer for the electron capturing from a dense Debye shell which corresponds to the solar conditions and this capturing could be different than that of a single electron. Probably this reaction (5a) will be the most sensitive to the collective plasma effects. We restrict ourselves to this comment since the problem is still waiting to be analysed taking the plasma collective effects into account. This problem is real since some observations indicate that the deficit of beryllium neutrinos is larger than for other solar neutrinos (the above problem of absolute experimental calibration is also important for this problem).

It is necessary here to say a few words concerning the value of the coefficient of the Rosseland opacity. The possibility of introducing such a coefficient or to obtain its value without solving the transport equation is rarely dis-

cussed. The usual treatment in the SSM is to consider the solar opacity to be known and then with the known opacity to solve the transport equations integrated over frequency range. Such an approach can be used only if the structure of the radiation transport equation has a certain form, namely, the transport equation should not contain the derivatives of the intensity of radiation with respect to the frequency otherwise one should first solve the differential transport equation and then one can introduce the value called opacity. The solar opacity is defined as an integral with respect to the frequency characteristic of the radiation transport and in the case where the transport equations can not be first integrated with respect to the frequency the concept of opacity is useless. As we shall show in the presence of collective plasma effects the latter is always true (some arguments for the estimation of the effect is given below). The value of the Rosseland opacity κ_{R} is defined as a factor connecting the flux of radiation F (an integral with respect to all frequencies of the spectral flux of radiation) and the temperature gradient:

$$F = \int F_\omega \, \mathrm{d}\omega = -\frac{4\pi c}{3} \frac{1}{\rho \kappa_{\mathrm{R}}} \frac{\mathrm{d}B^T}{\mathrm{d}r} , \tag{6}$$

where B^T is the energy density of radiation of a Planck blackbody of temperature T, and ρ is the mass density of the matter. Naturally the right hand side of Eqn (6) can be written in terms of the temperature gradient:

$$\frac{\mathrm{d}B^T}{\mathrm{d}r} = \int \frac{\partial B_\omega^T}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}r} \, \mathrm{d}\omega , \tag{7}$$

where B_ω^T is the spectral density of the Planck distribution.

Let us illustrate the existence of the possibility to use the opacity κ_{R} by using the example of radiation scattering on electrons. Let the scattering cross-section $\sigma(\omega, \omega', x')$ be a function of the frequencies of the scattering and scattered waves, ω and ω', respectively as well as of the angle of scattering x'. The transport equation which takes into account direct and inverse scattering processes but ignores the processes of stimulated scattering can in its simplest form be written for the photon occupation number $N(\omega, x)$ (x is the cosine of the angle between the direction of propagation of photon with frequency ω and the direction of the temperature gradient; x'' is the same for the photon with frequency ω') as:

$$x \frac{\partial N(\omega, x)}{\partial r} = -N(\omega, x) n_{\mathrm{e}} \int \sigma_{\omega, \omega', x'} \, \mathrm{d}\omega' \, \mathrm{d}x' + \int N(\omega', x'') n_{\mathrm{e}} \sigma_{\omega, \omega', x'} \, \mathrm{d}\omega' \, \mathrm{d}x' . \tag{8}$$

The deviation of the photon distribution from the equilibrium Planck distribution is assumed to be negligible:

$$N(\omega, x) = N_\omega^T + x \delta N_\omega , \tag{9}$$

where N_ω^T corresponds to the Planck distribution (it is related to B_ω^T by the well known formulas), and δN_ω is related to the spectral density of radiation introduced above (the factor 4π in Eqn (6) corresponds to the total solid angle, and the coefficient $1/3$ corresponds to the average value of the square of the cosine of the angle. The left hand side of Eqn (8) is sufficient to take into account the dependence of the Planck distribution on r, while the right hand side of this equation will contain only the deviations from the Planck distribution related to the radiation flux. Due to axial symmetry of (8) we have $x'' = xx'$ and the equation which describes the transport of radiation is an integral equation of the form:

$$\frac{\partial B_\omega^T}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3 n_{\mathrm{e}}}{4\pi c} \int \sigma_{\omega, \omega', x'} \left(F_\omega - x' F_{\omega'} \frac{\omega^3}{(\omega')^3} \right) \mathrm{d}\omega' \, \mathrm{d}x' . \tag{10}$$

This equation (10) allows us to introduce such an integral characteristic as the opacity κ_{R} only in the case where one assumes that a good approximation could be $\delta N_\omega = \delta N_{\omega'}$ and then from Eqn (10) we obtain Eqn (6) where

$$\frac{1}{\rho \kappa_{\mathrm{R}}} = \frac{\int_0^\infty (\partial B_\omega^T / \partial T)(n_{\mathrm{e}} \sigma_\omega^{\mathrm{tr}})^{-1} \, \mathrm{d}\omega}{\int_0^\infty (\partial B_\omega^T / \partial T) \, \mathrm{d}\omega} , \tag{11}$$

where $\sigma_\omega^{\mathrm{tr}}$ is the transport scattering cross-section

$$\sigma_\omega^{\mathrm{tr}} = \int \sigma_{\omega, \omega', x'} (1 - x') \, \mathrm{d}x' \, \mathrm{d}\omega' . \tag{12}$$

This example was given not only to recall the definition of κ_{R}, but also to emphasise the conditions where the introduction of such a quantity is possible and useful. It is clear from a physical point of view that due to the Doppler effect the frequency of radiation is changed in each act of scattering and although such a value as κ_{R} can be introduced the expression for it can not in the general case be obtained from the equation of radiative transfer. To solve the general equation for radiative transfer and to find the intensity of radiation as a function of ω and $\mathbf{r}$ is rather difficult and no one has yet performed such calculations for the solar interior. The natural question then is whether such an integral characteristic of energy transfer as the opacity is a good approximation for describing the collective plasma effects in the solar interior. The answer to this is *negative*.

3. Physics of collective effects in scattering and bremsstrahlung

The physics of collective scattering and bremsstrahlung has already been illustrated in many textbooks and monographs on plasma physics. It has mainly been presented for the case of electrostatic plasma oscillations and not so much for electromagnetic waves, i.e. photons, although even in 1967 all necessary formulas for scattering of photons were given in Ref. [8] (see also Refs [9, 10]). And we shall use these results. It is worthwhile to recall the physics of collective scattering, which seems to be, at a first glance, rather simple, but indeed is not at all trivial. This can be the only excuse for the wrong statements appearing even at the present time in astrophysical literature such as "scattering occurs only on electrons and the ions can influence the scattering only through correlations in collective processes". At the present time much headway has been made in the physics of scattering in plasma and there can be no doubt that in an extreme collective regime, electrons and ions interchange their roles as compared to the case of isolated particles, i.e. the ions in the collective case are scattering almost as free electrons in vacuum and the electrons are scattering very weakly and in most cases as ions in vacuum. The main results were obtained in plasma physics for plasma oscillations for which the scattering is always collective. Usually, only plasma wave scattering on ions is taken into consideration in plasma physics. Most

attention in plasma physics focuses on stimulated scattering since it describes the nonlinear interaction of plasma waves. In the early construction of SSM the formulas of non-collective scattering were used and only spontaneous scattering was taken into account neglecting collective stimulated scattering.

Let us repeat the main principles of collective scattering and let us make it clear why in the collective regime it is impossible to speak of the influence of ions on the scattering on electrons but it is correct to say that the scattering is produced by the ions themselves.

The physical picture is at a first glance very simple. The charges in a plasma are screened at distances of the order of the Debye radius. In the case where the wavelength of the scattered wave becomes larger than the Debye radius the scattering becomes collective. Both electrons and ions have screening shells which consist of an excess of electrons and a suppression of ions in the vicinity of ions, and of an excess of ions and a suppression of electrons in the vicinity of electrons.

For high frequency waves it is the electrons which oscillate, both the screening and screened electrons, in the wave field. The electrons screen themselves by producing a deficit of electrons (electron hole) around the screened electron which experiences a positive charge. The scattering is equal and out of phase for the screened electron and screening electron with a net result of zero scattering. For the ions which do not oscillate in the high frequency field the screening electrons which have equal but opposite signs are responsible for producing the scattered radiation. For the wavelengths larger than the size of the screening cloud, the ions scatter like electrons in vacuum (for the case of singly charged ions).

For such a physical interpretation it is necessary to remove some doubts which the reader may have. Let us concentrate on the statement which may be considered to be unusual, namely, the presence of strong scattering on ions. The first point is that since in the case of ions only their electron shells scatter, it may be more correct to speak about the scattering on electrons correlated with ions as some physicists prefer to interpret this effect. It is easy to show that such a point of view is incorrect. A correct statement is that it is the ions which scatter the radiation, while the electrons play an intermediate role in the transfer of energy and momentum to the ions during the scattering process. This can be proved both mathematically and from physical point of view. To check this statement mathematically one can use the fluctuation theory to calculate the changes of the ion distribution during the scattering process. One can then easily see that the energy and momentum lost in the scattering process by waves is transferred to ions only. This calculation is based on the same fluctuation theory as the calculation for the scattering of waves. It should be noted that for a large system of particles there exist no other more exact approach than the fluctuation theory and all scattering processes have been previously calculated using it. The equations for the change of the ion distribution are obtained by the same procedure of averaging over the fluctuations as in the simpler approach when the distribution of particles which scatter the radiation is assumed (to a first approximation) fixed. Thus there is no doubt from the point of view of the mathematical procedure used in scattering theory that the scattering in the collective regime is due to the ions.

There is also no doubt from the physical point of view about this statement. Let us recall the process of the Cherenkov emission by a particle moving with a velocity greater than the light velocity in the medium. In this case there is no doubt that the polarisation cloud of particles in the medium plays an important role in the formation of radiation. But it is well known that the emitted energy and momentum of the wave is taken from the particle itself. The polarisation cloud in the case of a plasma is the Debye shielding cloud. In plasma physics the Cherenkov emission of plasma waves is a very common phenomenon and by using the quasilinear theory it was proved that in this case the sum of the energies of particles and waves is conserved (this statement has also been checked experimentally). The polarisation cloud in both cases, scattering and Cherenkov emission, plays only an intermediate role in transferring the energy and momentum. This last statement was known in the early stages of the investigation of the Vavilov – Cherenkov emission [11].

The other area of doubt concerns the statement that the electrons and ions of the plasma are screened also by electrons and ions. The question is how the plasma particles can be at the same time the scattering centres and be able to shield the other scattered particles? To resolve this doubt one should bear in mind that, by definition, in a plasma the number of plasma particles in the Debye sphere should be large (and this condition is fulfilled in the centre of the Sun). On the other hand, to treat scattering correctly, the only approach in plasma is the fluctuation approach. In the presence of fluctuations one should separate the average particles motion and the fluctuating part of their motion. For the average motion the particles appear as the centres of scattering and during the fluctuations they are able to screen the other particles. Since the number of particles in the Debye sphere is large there is no need for large fluctuations to produce the screening. The given picture is an adequate interpretation of the exact results of the fluctuation theory. In all processes such as particle collisions, scattering and bremsstrahlung the plasma looks more like a collection of 'neutral atoms' than free particles. But the screening is a dynamical screening and as soon as the particles move fast enough (their velocity is larger than the mean thermal velocity), they become 'undressed'. One should also keep in mind that the screening shell consists both of electrons and ions (the thermal velocities of electrons are much greater then the thermal velocities of ions) and by increasing their velocities the particles first 'take off their ion shell'. Such a plasma picture is the achievement of a long-term development of the plasma theory and the first steps toward it were made by Balescu [12] who proved that binary particle collisions are the collisions of dynamically screened particles. The screening during the collisions is produced by all other plasma particles. A similar picture also appears for bremsstrahlung processes. This statement was proved only recently [13].

It seems obvious that such a situation should indeed appear for all electromagnetic processes in a plasma since one can use the test particles approach. It is clear that any external charge inserted in a plasma is screened. But an 'external' charge can be any electron or ion from the plasma. The self-consistency of a plasma description does not allow one to distinguish an 'external' electron from the plasma electron.

The picture of plasma as a collection of dynamically screened neutral 'classical atoms' which seems to be more appropriate than the picture of a collection of free particles can be considered as a rather poor analogy since in the atoms

the screening is produced by the same bounded electrons while the screening shells of electrons and ions in a plasma are produced statistically by different electrons and ions of plasma. But it should be noted that the time needed, for example, for an electron to cross the Debye sphere is very short (of the order of the inverse plasma frequency) and the screening shell for the processes considered (including the scattering) behaves quasistatically in the case where the wavelength is much larger than the Debye length.

We shall write the criterion for collective effects in scattering to be dominant for electromagnetic waves and show that this criterion is usually fulfilled for frequencies much larger then the plasma frequency. The wavelength of electromagnetic waves for this case when their frequency is much larger than the plasma frequency is c/ω, while the size of the Debye screening shell is of the order of v_{T_e}/ω_{pe}, where ω_{pe} is the electron plasma frequency. By comparing these expressions we obtain the criteria when the collective effects for photons are strong which are

$$\omega_{pe} < \omega \ll \omega_{pe} \frac{c}{v_{T_e}} . \tag{13}$$

Since the factor c/v_{T_e} is rather large for a non-relativistic plasma the range given by expression (13) appears to be rather broad. Outside this range one can expect the usual picture of scattering when the scattering on free electrons dominate, while inside the range the scattering is described by the picture given in this section where the ions dominate in scattering and collective plasma effects are dominant. Even in some recent astrophysical publications one can find statements that the criterion for collective effects to dominate is that the frequency of photons should be close to the plasma frequency. The appearance of such statements is difficult to understand and they are obviously incorrect.

Before starting to construct an SSM one should answer a natural question which is whether or not the photons taking part in radiative energy transfer in the solar interior have frequencies in the collective range.

4. Plasma parameters in the central region of the Sun

An SSM which takes into account all the collective effects does not yet exist. The best thing we can do is to use the existing SSM to obtain the plasma parameters inside the Sun bearing in mind that future investigations should correct the SSM. We should also note that it is possible to change the solar opacity in certain limits (15% change in the solar opacity is probably the maximum allowed from solar seismology, but the latter statement is somewhat questionable since at the present time solar seismology does not give direct information on the central regions of the Sun). On the other hand it was also demonstrated that a large change of opacity is not needed.

In any case we shall take the plasma parameter data for the central solar region using the existing SSM [3, 14]. According to these models thermonuclear burning occurs only in the central part of the Sun up to distances from the centre of $0.1R_\odot$. It is assumed that the radiation flux is formed at these distances and this flux independently of transformations in the upper turbulent regions appears as emission in the visible domain and determines the solar luminosity (this conclusion is made from the conservation of flux, constancy of solar luminosity in time and domination of the optical radiation flux of the Sun as compared to other types of charged particle and electromagnetic emission from the Sun). This visible flux of radiation is what is measured from the Earth. The central part of the Sun is assumed not to be turbulent. Unfortunately solar seismology does not detect the central regions of the Sun and this statement or assumption is difficult to prove.

According to the present data the temperature in the central region of the Sun is 1.55 keV (which is less than temperatures obtained at the present time in CTR laboratory experiments).This corresponds to the electron mean thermal velocity $v_{T_e} = 1.53 \times 10^9$ cm s^{-1} and thus $c/v_{T_e} \approx 20$, the plasma density is 142 g cm^{-3}, which (for the abundance of hydrogen H, equal to 0.36, and abundance of He, equal to 0.62) corresponds to an electron density $n_e \approx 5.74 \times 10^{25}$ cm^{-3} and to an electron plasma frequency $\omega_{pe} = 4.27 \times 10^{17}$ s^{-1}. For an estimation of the frequency below which the collective effects dominate we should multiply the last value by 20 to obtain 8.54×10^{18} s^{-1}. This frequency should be compared with the frequency corresponding to the maximum of the blackbody radiation $3T/\hbar \approx 6 \times 10^{18}$ s^{-1}. We can also make a comparison with the frequency of the maximum of the weight factor $\partial B_\omega^T/\partial T$ in the Rosseland opacity κ_R, which corresponds to $3.7T/\hbar$ and corresponds to the frequency 7.4×10^{18} s^{-1}. Both comparisons definitely show that the whole frequency range responsible for the radiative energy transfer in the solar interior corresponds to the range of frequencies in which the collective effects dominate.

This is a very important conclusion which was not made in the early investigations of the Sun using SSMs (it was only taken into account in 1987 [5])

Another conclusion for the estimate given above is that the ratio of the maximum frequency to the frequency when collective effects start to dominate, is neither large, nor small, which means that in a theoretical description we can not use a small parameter and the collective effects should be treated strictly without using the asymptotic expressions. This also means that the contributions of electrons and ions to scattering in the solar interior are of the same order of magnitude. For the following it will be useful to define the collective electron parameter δ_e, which characterises the role of collective effects for scattering on electrons (later on we define also the collective ion parameter). The collective electron parameter is, by order of magnitude, equal to the square of the ratio of the wavelength of the scattered radiation to the electron Debye shielding length. The definition is

$$\delta_e = \frac{\omega_{pe}^2}{2\omega^2} \frac{c^2}{v_{T_e}^2} . \tag{14}$$

In the extreme collective regime $\delta_e \gg 1$, in the non-collective regime $\delta_e \ll 1$. For the centre of the Sun this parameter corresponds neither to the first inequality, nor to the second inequality but corresponds to $\delta_e \sim 1$.

We can also find another qualitative conclusion concerning the relation between the scattering and bremsstrahlung absorption (the process inverse to bremsstrahlung emission). Let us introduce an effective cross-section σ^{br} for inverse bremsstrahlung damping by using for the bremsstrahlung damping rate of the photon intensity $2\gamma^{br}$ the following formula: $2\gamma^{br} = n_e c \sigma^{br}$. Then one can easily show from the standard formula that for this absorption the ratio of

bremsstrahlung cross-section to the scattering cross-section is of the order $\delta_e^{3/2}$. This estimate definitely shows that the contributions to the solar opacity of the scattering and bremsstrahlung are of the same order of magnitude.

The third contribution to the opacity which is of the same order of magnitude is given by line absorption. The relative abundance of all elements neglecting hydrogen and helium in the centre of the Sun is only 2% and all atoms except iron atoms are completely ionised and do not absorb in lines. But iron ions have a line exactly in the range of frequencies important for energy transfer. Although the relative abundance of iron ions is small they have a large charge and the presence of the resonance line makes their contribution to the opacity almost of the same order of magnitude as scattering and bremsstrahlung. Therefore, for example, a change in scattering of 30% can change the opacity by only 10%.

In evaluation of the Rosseland opacity κ_R in different SSMs all three components were taken into account and the total cross-section (11) is equal to the sum

$$\sigma^{tot} = \sigma^{sc} + \sigma^{br} + \sigma^{L} . \tag{15}$$

The notations used for the three contributions are obvious. The bremsstrahlung cross-section is, as is known, proportional to the effective ion charge Z_{eff}

$$Z_{eff} = \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i} , \tag{16}$$

where n_i is the relative concentration of ions of type i, and Z_i is their charge. The effective charge does not differ much in different SSMs and is close to the value of 1.5. The ratio of the total κ_R to the value which takes into account only scattering and bremsstrahlung varies from one model to another but for each model this ratio is known. Therefore it is useful to relate the corrections to the Rosseland opacity to its value which takes into account only scattering and bremsstrahlung. The coefficient for transferring this value to the total opacity is known for each SSM but on the average it can be taken as a rough estimate to be equal to 2/3.

For calculations of the corrections to the Rosseland opacity $\kappa_R^{(0)}$ one can use a formula (11) with a total cross-section, assuming that both the cross-sections and the Planck distribution depend on the value

$$z = \frac{\hbar\omega}{T} , \tag{17}$$

then

$$\frac{\kappa_R - \kappa_R^{(0)}}{\kappa_R^{(0)}} = \frac{\int_0^\infty z^4 \exp z / \{\sigma_0(z)[\exp(z) - 1]^2\} \, dz}{\int_0^\infty z^4 \exp z / \{\sigma(z)[\exp(z) - 1]^2\} \, dz} - 1 . \tag{18}$$

The integral equation for radiative transfer can not be solved directly for all plasma collective corrections and the opacity can not be calculated directly from the transport equation. In this case the direct use of (18) is not possible. This difficulty can be overcome by perturbation theory in the case where the latter can be used in the transport equation and the explicit expressions for the change in the opacity can be obtained. The conditions where the perturbation approach can be used will be discussed below.

5. Zero approximation for opacity including bremsstrahlung and collective scattering

For $\delta_e \ll 1$ the collective effects are negligible and the cross-sections for scattering of photons on electrons and ions are well known and are given by equations:

$$\sigma^e \approx \sigma_T = \frac{8\pi e^4}{3 m_e^2 c^4} , \tag{19}$$

$$\sigma^i \approx 0 ,$$

where σ_T is the Thomson cross-section. These expressions are written for nonrelativistic particles (i.e. when the relativistic corrections are small), in the classical limit (i.e. where the quantum corrections are also small) at a first approximation in the parameter m_e/m_i:

$$\tau \equiv \frac{v_{T_e}^2}{c^2} = \frac{T}{m_e c^2} \ll 1 ,$$

$$\frac{\hbar\omega}{m_e c^2} = z \, \frac{v_{T_e}^2}{c^2} \ll 1 . \tag{20}$$

Due to the fact that the effective value of z in the Rosseland opacity is of the order of 3.7 the quantum corrections to Eqns (19) are somewhat larger than the classical relativistic corrections. One can write the expressions for the cross-sections taking the next order in the parameters (20) and write the first relation (19) in a form which differs from Eqn (19) by a factor $G(z, \tau)$, the explicit expression for which can be obtained by expansion of the general Klein–Nishina formula [15] with subsequent averaging on the thermal distribution. Such a factor was used in opacity calculations by taking into account the terms up to the second order in the parameters z and τ. Below we shall find the correct expressions for such a factor in the collective regime but only up to the first order in expansion in the parameters τ and $z\tau$ (even the collective corrections of this order of magnitude are very cumbersome calculations which have not been performed before). In the collective case a new parameter of expansion $z^2\tau$ occurs and in this parameter also the first order term in expansion will be taken into account. When considering linear corrections in τ the powers of z higher than 2 do not appear. For the problems of interest the previous use of the factor $G(z, \tau)$ is not correct since it is written for the non-collective regime and it can not be used in SSMs as was previously done in Refs [6, 16]. The real corrections, linear in τ, $z\tau$ and $z^2\tau$, in the collective case have nothing in common with that for the non-collective case and the use of the factor $G(z, \tau)$ for the solar opacity calculations is incorrect.

At this point we can give a definition of what will be meant by collective scattering in the zero approximation: it is the scattering where the collective effects are taken into account in zero approximation in the parameters (20) and in the new parameter $z^2\tau$. In this form the collective effects have already been taken into account in SSMs [5, 6]. In these papers an expression for the sum of the cross-sections of scattering on electrons and ions was used. We shall give this expression but then we shall consider separately the cross-sections for scattering on electrons and for scattering on ions to show a rapid decrease of scattering on electrons with an increase in the collective parameter and the growth of scattering on ions with an increase of this parameter and then we shall show how

the cross-sections change, if, for example, the temperatures of electrons and ions are not equal. The zero approximation for the scattering cross-sections can be obtained from the formulas given as early as 1967 [8]. On averaging over the thermal distributions one can use the so called fluctuation–dissipation theorem (see Refs [17, 18]) and find an analytical expression for the cross-sections for any value of the collective parameter δ_e used in Refs [5, 6]:

$$\sigma_0^{sc} \equiv \sigma_e^{sc(0)} + \sigma_i^{sc(0)} = \sigma_T \left\{ 1 - \frac{3}{8}\,\delta_e \left[\delta_i (2 + 2\delta_i + \delta_i^2) \times \ln \frac{\delta_i}{2+\delta_i} + 2\delta_i + 2\delta_i^2 + \frac{8}{3} \right] \right\}, \tag{21}$$

where

$$\delta_i = (1 + Z_{eff})\delta_e \tag{22}$$

is the collective ion parameter and the effective ion charge is given by expression (16). We have labelled the corresponding cross-sections with a subscript zero to emphasise that these expressions are given in the zero approximation. In the value of the Rosseland opacity given above for which the corrections will be calculated we also made the label zero. This is performed intentionally since below all the corrections will be counted from that zero approximation. To be complete we should also define the zero approximation for bremsstrahlung (see below). Relating all corrections to this zero approximation allows us to consider only the new collective effects not taken into account previously and to correct the other expressions. We shall give below the expression for $\kappa_R^{(0)}$; the corrections will be calculated relative to this value of the Rosseland opacity.

Here, however, we shall start with a more detailed consideration of the scattering in the zero approximation. We shall also discuss the corrections in the zero approximation in τ, $z\tau$ and $z^2\tau$ which are not taken into account in expression (21). Some of them are indeed small but we want to be precise in the analysis of all corrections in the same parameter τ. First of all, expression (21) is valid only if one neglects for photons the difference of the refractive index from 1 (the value in vacuum), i.e. it is valid for $\omega \gg \omega_{pe}$. Since the frequency range in the transfer of radiation in the solar interior consists of only one decade in frequency from ω_{pe} up to $10\omega_{pe}$ it is worthwhile to include this difference. We shall see that this correction contains the same relativistic factor $\tau = v_{T_e}^2/c^2$ as other relativistic corrections.

Secondly, it is rather easy to look at the process of scattering separately for electrons and ions in the case where their temperatures T_e and T_i are not equal. It is not quite certain that this case is of interest for the solar interior since in the dense plasma the characteristic time of equalising of the electron and ion temperatures is very short. An estimation shows that this time is still 5 times larger than that of heating of electrons by absorption of the radiation transferred. Although the question about the possibility of the existence of the difference between the electron and ion temperatures requires special investigation it is worthwhile to give general expressions for scattering on electrons and ions in zero approximation in the parameters τ, $z\tau$ and $z^2\tau$ not assuming that their temperatures are equal.

Third, contrary to the previously used formula (21) we shall write separately the expressions for the transport scattering cross-sections on electrons and on ions to show explicitly that the cross-section of scattering on electrons decreases rapidly with an increase of the collective parameter and the cross-section of scattering on ions increases with an increase of the collective parameter. We shall write down only the transport cross-sections of scattering which enter into the transport equation.

For the above conditions (arbitrary ratio ω/ω_{pe} but $\omega > \omega_{pe}$ and arbitrary ratio T_e/T_i) the collective parameters are determined by the relations:

$$\delta_i = \left(1 + Z_{eff}\,\frac{T_e}{T_i}\right)\delta_e\,, \tag{23}$$

$$\delta_e = \frac{c^2}{2v_{T_e}^2}\,\frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2}\,, \tag{24}$$

and the scattering cross-sections are described by the expressions:

$$\sigma_e^{sc} = \sigma_T \sqrt{1 - \frac{z_0^2}{z^2}} \left\{ 1 - \delta_e + \frac{3}{8}\,\delta_e^2 \left[(2 + 2\delta_e + \delta_e^2)\,\ln \frac{2+\delta_e}{\delta_e} - 2 - 2\delta_e \right] \right\}, \tag{25}$$

$$\sigma_i^{sc} = \sigma_T \sqrt{1 - \frac{z_0^2}{z^2}}\,\frac{3}{8}\,\frac{T_i}{T_e} \times \left\{ \delta_e^2 \left[-(2 + 2\delta_e + \delta_e^2)\ln \frac{2+\delta_e}{\delta_e} + 2 + 2\delta_e \right] + \delta_i\delta_e \left[(2 + 2\delta_i + \delta_i^2)\ln \frac{2+\delta_i}{\delta_i} - 2 - 2\delta_i \right] \right\}, \tag{26}$$

where $z_0 = \hbar\omega_{pe}/T$. The factor $(1 - z_0^2/z^2)^{1/2}$ is equal to $(1 - \omega_{pe}^2/\omega^2)^{1/2}$ and shows that the cross-sections of scattering tend to zero when the frequency becomes close to the plasma frequency. For the solar interior $z_0 \approx 0.21$. Shown in Fig. 1 are the dependencies of the transport cross-sections for scattering on electrons and ions separately and the total transport cross-section as a function of the frequency for $T_e = T_i$ but with exact values of the refractive index (not equal to 1). One observes a strong decrease of the cross-section for scattering on electrons and an increase of the cross-section for scattering on ions with decrease of the frequency. The curves were calculated for the parameters in the solar interior. An additional decrease of the cross-section close to the electron plasma frequency is related to the refractive index effect. Curve *3* is calculated without refractive index effect taking into account and corresponds to the cross-section which will be taken into account in $\kappa_R^{(0)}$. Finally curve *4* describes the weighting factor $z^4 \exp z/(\exp z - 1)^2$, which enters in κ_R.

From these curves it is clear that the most important frequency range corresponds to the collective range. The maximum frequency in the weighting factor is $\omega/\omega_{pe} \approx 18$, but even for $\omega/\omega_{pe} \approx 30$ a decrease of the total cross-section is $\approx 18\%$, and for $\omega/\omega_{pe} \approx 2$ it is as much as 40%.

Figure 2 shows the dependencies of the transport cross-sections and the usual cross-sections for scattering on electrons and ions as well as the total cross-section of scattering as a function of the collective parameter δ_e. These curves show the role of collective effects in a clear manner as well as the fact that the transport cross-sections do not differ

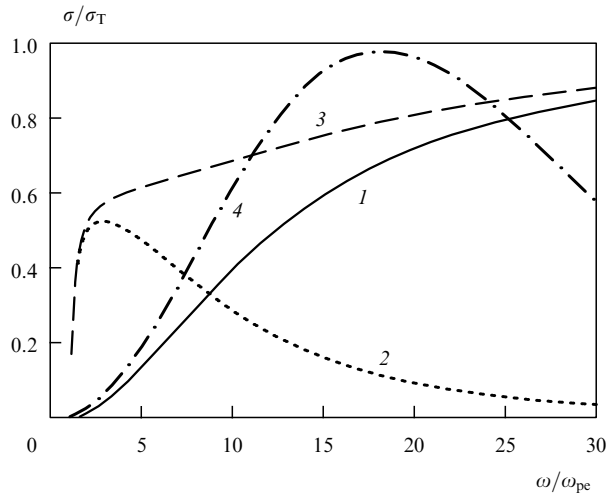

Figure 1. Dependences of the cross-sections of scattering of photons on electrons and ions in the solar interior on the photon frequency: (*1*) the cross-section of scattering on electrons; (*2*) the cross-section of the sum of scattering on ions (abundance of elements is taken from a standard solar model [3]); (*3*) the sum of scattering on electrons and ions; (*4*) the 1/5 of form factor $z^4 \exp z/\left[\exp(z)-1\right]^2$ which enters into the expression for opacity. Plasma electron density $n_e = 5.4 \times 10^{25}\,\mathrm{cm}^{-3}$, $T_e = T_i = 1.5\,\mathrm{keV}$, $v_{T_e} = 1.53 \times 10^9\,\mathrm{cm\,s}^{-1}$, $z_0 = \hbar\omega_{pe}/T = 0.21$, $Z_{eff} = 1.53$.

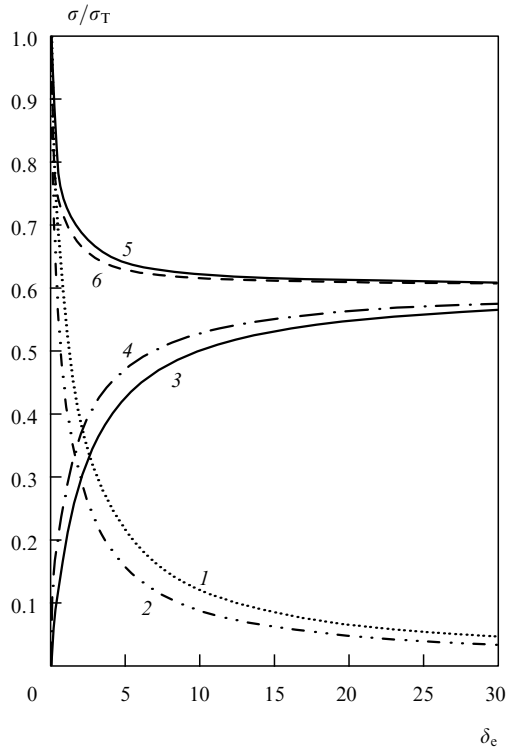

Figure 2. Dependences of cross-sections of scattering on the collective parameter δ_e for both the transport cross-sections (*2, 3, 6*) and the usual cross-sections (*1, 4, 5*): (*1, 2*) to scattering on electrons; (*3, 4*) to scattering on ions; (*5, 6*) to the sum of scattering on electrons and ions; $Z_{eff} = 1.53$; $T_e = T_i$.

substantially from the usual one. Thus the figures given before showing the change of the transport cross-sections of scattering at the centre of the Sun also give a good example of the dependencies of the usual cross-sections (which differs from the transport cross-sections not having the factor $1 - x$ in the angular integration). Figure 1 also shows that the influence of the effect of refractive index could not be large since the refractive index effects are the largest in the frequency range where the weighting factor is small.

Shown in Fig. 3 are the dependencies of the total transport cross-section on the parameter δ_e for different values of T_e/T_i from which it follows that the increase of the ratio T_e/T_i can decrease the total cross-section by up to 80%.

Let us consider bremsstrahlung absorption and define the zero approximation for it. Collective effects in bremsstrahlung were previously neglected in SSMs, therefore we shall not take into account the collective effect in zero approximation in the processes of bremsstrahlung absorption. The effect we discuss in exact terms arises as a balance between stimulated emission and stimulated absorption. In the low frequency limit, $\hbar\omega/T = z \ll 1$, it corresponds to classical absorption of electromagnetic radiation in a plasma due to binary electron – ion collisions. The classical limit is not quite appropriate for the energy transport problems, since the most important value of z is of the order or larger than 1. Since after an emission of a bremsstrahlung wave the particle energy decreases by $\hbar\omega$, the term describing stimulated absorption by thermal particles will contain the additional factor $\exp(-z)$ as compared to the term describing the stimulated bremsstrahlung emission. This leads for z of the order of 1 to a factor $\left[1 - \exp(-z)\right]/z$ in the expression for the wave damping (this factor is 1 in the classical limit) and thus one finds the general expression for damping for arbitrary z values. We can write the latter as $2\gamma^{br}(\omega) = n_e c \sigma^{br}(\omega)$ which serves as a definition of the already introduced effective cross-section of bremsstrahlung absorption $\sigma^{br}(\omega)$. It is useful to express this cross-section through the collective parameter (14) and the

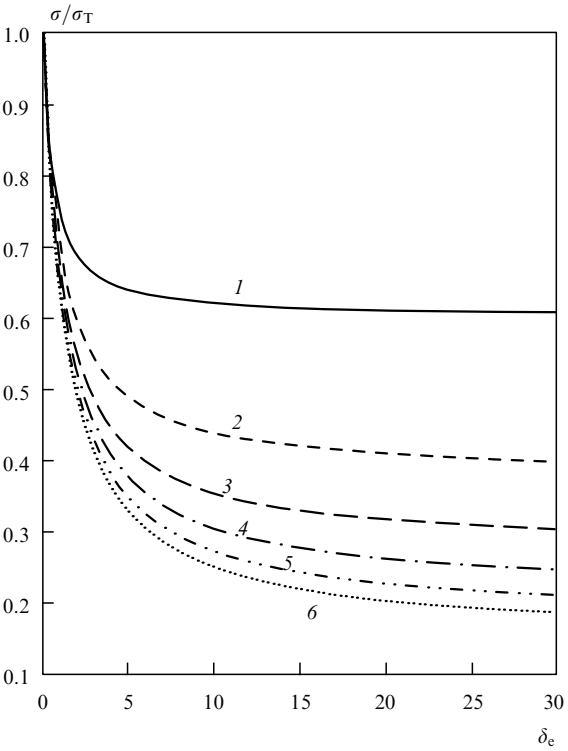

Figure 3. Dependences of the total transport cross-sections of scattering on the collective parameter for different ratios of electron to ion temperatures. The cross-sections decrease continuously with increase of $\tau = T_e/T_i$: (*1*) $\tau = 1$; (*2*) $\tau = 2$; (*3*) $\tau = 3$; (*4*) $\tau = 4$; (*5*) $\tau = 5$; (*6*) $\tau = 6$.

Thomson cross-section of scattering σ_T:

$$\sigma_0^{\rm br} = \sigma_T Z_{\rm eff} \frac{2\delta_e^{3/2}\left[\exp(z)-1\right]}{\sqrt{\pi}\, z_0} \mathcal{F}_0(\omega); \qquad z_0 = \frac{\hbar\omega_{\rm pe}}{T}\,, \quad (27)$$

where

$$\mathcal{F}_0(\omega) = \int_{\sqrt{2\hbar\omega/m_e}}^{\infty} \frac{\exp\left[-v^2/(2v_{T_e}^2)\right] v}{v_{T_e}^2} \times \ln \frac{v+\sqrt{v^2-2\hbar\omega/m_e}}{v-\sqrt{v^2-2\hbar\omega/m_e}}\, {\rm d}v = 2\int_{\sqrt{z}/2}^{\infty} \frac{{\rm d}x}{x} \exp\left[-\left(x+\frac{z}{4x}\right)^2\right]. \quad (28)$$

Expressions (21) and (27) for the zero approximations for cross-sections of scattering and bremsstrahlung will be the starting point for all the corrections we shall consider further and only they will be included in the zero approximation for the Rosseland opacity $\kappa_R^{(0)}$. The corrections to this value of the opacity will be calculated by using formula (18). Line absorption in the zero approximation will not be taken into account to find the universal result for the corrections (the coefficient for recalculating the ratio calculated in this manner to the ratio to the total opacity which includes line absorption is different for different SSMs and such a recalculation can be easily performed using a particular SSM). Thus $\kappa_R^{(0)}$ will be defined by $\sigma_0^{\rm br} + \sigma_0^{\rm sc}$. It is very important to take into account in the zero approximation both scattering and bremsstrahlung effects. For example, if we take into account only bremsstrahlung, then the cross-section will rapidly decrease with increasing z (proportional to $1/z^2$ or proportional to $1/z^3$) and then in the numerator of $\kappa_R^{(0)}$ a large factor z^6 (or a factor z^7) will appear and the effective value of z will be rather large, about 6–7. On the other hand, if we take into account only the scattering, the maximum effective z value will be quite different. The values of κ_R resulting from the contribution of scattering and from the contribution of bremsstrahlung are not additive since the sum of the cross-sections enters in the denominator of κ_R. It appears that for the solar interior both cross-sections are equal at z values corresponding to the exponential decrease of the weighting factor and therefore the results can be sensitive to small changes of the cross-sections.

6. Effects of refractive index in scattering and bremsstrahlung

We begin with consideration of the effect which is rather small but to take it into account will be necessary since firstly we may intend to search for any collective effect which contains the small parameter described by the relativistic factor $v_{T_e}^2/c^2$, secondly, it will allow us to describe and use further a simplified expression for the corrections of the Rosseland opacity, and thirdly, after giving this result of the estimation of the role of refractive index we shall be allowed in further considerations (for other collective effects) to neglect the refractive index effects.

We have already written the general expressions for scattering which take into account the refractive index effect. In calculations of the refractive index corrections in the opacity, we should also take into account an additional factor $(1-\omega_{\rm pe}^2/\omega^2)^{1/2}$ in the energy density of blackbody radiation which appears by changing the integration with respect to wave number to the integration with respect to frequency. We should also take into account the same factor due to the difference of the group velocity of photons from 1 in the expression for κ_R thus the factor $1-\omega_{\rm pe}^2/\omega^2$ will appear in the numerator. But the denominator in the cross-sections for scattering and bremsstrahlung will contain $(1-\omega_{\rm pe}^2/\omega^2)^{1/2}$, by dividing on the factor in the denominator we find that the first power of this factor in the denominator appears in the opacity. The change in the expression for the collective parameter and the change in the lower limit of integration on frequency are also essential. The change in the total value of κ_R due to all these changes is denoted as $\delta\kappa_R^{\rm refr}$ and numerical calculations give

$$\frac{\delta\kappa_R^{\rm refr}}{\kappa_R^{(0)}} = 0.135\%\,. \quad (29)$$

To illustrate the role of the low frequency part of the range of integration (for which the refraction index corrections are large) as compared to the high frequency range of integration in the expression of the opacity we can assume due to result (29) that the main part of the contribution is given by the high frequency range and thus expand the total cross-section κ_R, with refractive index effect taken into account, in the parameter $\omega_{\rm pe}^2/\omega^2 = 2\delta_e v_{T_e}^2/c^2$. Since in the centre of the Sun $\delta_e \sim 1$ we conclude that the refractive index corrections have the same factor $v_{T_e}^2/c^2$ as the other relativistic corrections and in principle collecting all the relativistic corrections we can not neglect this one. We use the following expression for the corrections to the refraction index entering in the expression for the opacity:

$$\sigma(z) = \sigma_0(z) + 2\,\frac{v_{T_e}^2}{c^2}\, H_{\rm refr}\,, \quad (30)$$

where

$$H_{\rm refr} = \delta_e(\sigma^{\rm br,0} + \delta\sigma^{\rm sc})\,, \quad (31)$$

$$\delta\sigma^{\rm sc} = \frac{1}{2}\,\sigma_T\left\{1 - \frac{3}{8}\,\delta_e\left[\delta_i(10+14\delta_i+9\delta_i^2) \times \ln\frac{\delta_i}{2+\delta_i} + 10\delta_i + 14\delta_i^2 + 8\right] - \frac{3\delta_e\delta_i}{2(2+\delta_i)}(2+2\delta_i+\delta_i^2)\right\}. \quad (32)$$

We shall give here also a simplified expression for the opacity corrections for the case where the corrections in the cross-sections are small and we can also expand the opacity in these corrections (this formula will be used also below in the case where we believe and finally find that the corrections are indeed small):

$$\sigma(z) = \sigma_0(z) + \delta\sigma(z)\,; \quad (33)$$
$$\delta\sigma(z) \ll \sigma_0(z)\,,$$

where $\delta\sigma(z)$ is the effective cross-section correction in the expression for the opacity, and for corrections to κ_R:

$$\frac{\kappa_R - \kappa_R^{(0)}}{\kappa_R^{(0)}} = \frac{\int_0^\infty \delta\sigma(z) z^4 \exp z/\left\{\sigma_0(z)^2[\exp(z)-1]^2\right\}{\rm d}z}{\int_0^\infty z^4 \exp z/\left\{\sigma_0(z)[\exp(z)-1]^2\right\}{\rm d}z}\,. \quad (34)$$

The change in κ_R for the case where the formulas (32) and (34) are used appears to be

$$\frac{\delta\kappa_R^{\rm refr}}{\kappa_R^{(0)}} = 0.134\% \,. \tag{35}$$

This result coincides closely with the exact result (29) which shows that the main contribution to the opacity is due to the high frequency range although the whole range of frequencies is not large (but $\omega_{\rm pe}^2/\omega^2$ changes in this range by two orders of magnitude). Further, in consideration of other collective corrections we shall neglect the refractive index effects.

Formula (34) will also be used below in the case where the corrections are small.

In the solar interior $\delta_e \sim 1$, and the corrections due to the difference of the refractive index from 1 are of the same order of magnitude as other relativistic corrections, and are much larger than the corrections considered now.

Some of the relativistic corrections to scattering have already been treated in Refs [6, 16] in the form of an additional factor $\sigma^{\rm sc} = \sigma_0^{\rm sc} G(z,\tau)$, where $\sigma_0^{\rm sc}$ corresponds to the zero approximation used above and the factor $G(z,\tau)$ was taken from the expression for scattering on free electrons. In the collective range of frequencies such an approach is incorrect and any use of such expressions in the calculation of the opacity can not be accepted for the solar interior. The real corrections as we show are much larger. Not only the relativistic corrections for scattering were treated previously incorrectly but the relativistic corrections and collective effects in bremsstrahlung as well as relativistic collective effects in the transport equation for radiation were ignored. It appears that the collective effects in bremsstrahlung have the same smallness as the other relativistic corrections. All these corrections will be considered below.

7. Collective effects in bremsstrahlung absorption

We start with the collective effects in bremsstrahlung. These effects are also small but contain the same factor as other relativistic corrections. They were previously neglected in the calculation of the opacity. In the existing literature the corrections due to Debye screening were considered [19, 41] with the conclusion that their contribution is less than 2%. We shall show that the correct expressions for collective effects in bremsstrahlung have nothing in common with Debye screening effects of ions during the process of bremsstrahlung and that they are indeed much smaller than that estimated in Ref. [19]. We shall also show that expanding the opacity in these corrections we again obtain (for $\delta_e \sim 1$) a factor $v_{T_e}^2/c^2$ in front of these corrections and thus these corrections should be included in the list of relativistic corrections to the opacity.

A correct calculation of collective effects in bremsstrahlung in the opacity appears to be not a simple problem. It was not considered previously in plasma physics in the form, which can be used in opacity calculations, this consideration has recently been treated in Ref. [20]. For a long time bremsstrahlung in plasma was calculated by using the theory of fluctuations and it was believed that the collective effects result in the Debye screening of the field of the ion in the process of emission of a photon in electron – ion collisions. Such expressions were given many years ago and one can find them in textbooks and monographs [21, 22]. The fact that those expressions are not correct both from a physical point of view and as mathematical expressions is explained in detail in the recent monographs [10, 23] and in the review [13].

In short, the physics of emission is the following: apart from the emission due to electron acceleration in the process of electron – ion collision a new type of emission appears due to the dipole moment produced by the displacement of the screening electron cloud; both effects interfere with each other and the resulting emission is not equal to the sum of the emission in the two processes. It was shown that one should add to the matrix element of the usual bremsstrahlung due to electron acceleration in the field of Debye screened ion $M^{\rm br,0}$ the matrix element $M^{\rm br,coll}$, describing the displacement of the screening shell of the ion. The total cross-section of bremsstrahlung is determined by the square of the absolute value of the sum of these two matrix elements:

$$\sigma^{\rm br} \propto \left|M^{\rm br,0} + M^{\rm br,coll}\right|^2 . \tag{36}$$

The additional matrix element (one can say the 'real collective matrix element') $M^{\rm br,coll}$ is of the same order of magnitude as $M^{\rm br,0}$ and moreover some terms in both of them are the same and opposite in sign which means that they partially compensate each other. This compensation is similar to that which leads to a decrease of the cross-section of scattering on electrons in the collective regime. This compensation in bremsstrahlung appears to be most important for low values of transferred momenta (from electron to ion) in the process of bremsstrahlung. For example, for fast electrons (on the tail of Maxwellian distribution) such a compensation leads to a cancellation of that part of the usual matrix element which corresponds to the difference between the field of the screened ion and the field of the unscreened ion with the matrix element, describing the oscillation of the polarisation shell. Thus, for fast electrons, the bremsstrahlung appears as if the ion is not screened at all ('stripping' shell effect described in details in Refs [13, 23]). This effect has a simple physical interpretation: the projectile electron collides both with the ions and its shielding electrons and for the fast electrons the shielding electrons can be considered as free electrons and it is known that the bremsstrahlung for particles with an equal charge to mass ratios is zero in the first approximation. Obviously there are not so many fast electrons in the thermal electron distribution in a plasma but the effect is pronounced even if the electron velocity is of the order of the thermal electron velocity. We shall show that the bremsstrahlung cross-section can be changed in certain domains of frequency by collective effects as much as 39%.

The result of the correct treatment of collective effects in bremsstrahlung is that the expression for the bremsstrahlung contains in the denominator not the square of the static dielectric constant (as it is written in many textbooks) but the square of the dielectric constant dependent on the frequency, which is determined by the velocity of the projectile electron (see Refs [13, 23]). This leads to the result that for velocities larger than the thermal velocity the screening totally disappears. On the other hand due to the fact that the electron shell of the ion has the charge equal in value and opposite in sign to the charge of the ion, in the correct expression for the collective effect in bremsstrahlung an additional factor appears which depends on the effective ion charge.

The final result can be written in the form (27) by changing the factor $\mathcal{F}(\omega)$ in the expression for the bremsstrahlung absorption (see Eqn (28) where this factor is given without the corrections described as ion field screening, which is not a correct expression, and without the collective corrections in a correct form). We denote as $\mathcal{F}_{\rm scr}(\omega)$ the expression for this factor which corresponds to pure Debye screening of the ion field and we denote by $\mathcal{F}_{\rm coll}(\omega)$ the expression for this factor for the correct treatment of collective effects in bremsstrahlung (taking into account the interference of the two processes of bremsstrahlung, i.e. taking into account the effect of ion 'stripping'). It is useful to express these factors through the integrals over the total normalised electron velocity $y = v/\sqrt{2}v_{T_{\rm e}}$ (note that below for the process of scattering we shall use the notation y for another value, the normalised component of the electron velocity along the difference of the wave vectors of scattering and scattered waves). The integration in the expression for bremsstrahlung will also be performed over the total transferred momentum q (naturally in units of $\hbar$):

$$\mathcal{F}(\omega) = \int_0^\infty \exp(-y^2) 2y\,{\rm d}y \int_{q_{\rm min}}^{q_{\rm max}} \frac{{\rm d}q}{q}\,\mathcal{H}(\omega, q) \tag{37}$$

and $\mathcal{F}_{\rm scr}(\omega)$ will contain $\mathcal{H}_{\rm scr}(\omega, q)$, while $\mathcal{F}_{\rm coll}(\omega)$ will contain $\mathcal{H}_{\rm coll}(\omega, q)$. The result is then given by the expressions

$$\mathcal{H}_{\rm scr}(\omega, q) = \left|1 + \frac{\omega_{\rm pe}^2}{q^2 v_{T_{\rm e}}^2}\right|^{-2} \tag{38}$$

and

$$\mathcal{H}_{\rm coll}(\omega, q) = \left(1 + \frac{\omega_{\rm pe}^2}{q^2 v_{T_{\rm e}}^2}\right) \times \left\{\left[1 + (1 + Z_{\rm eff})\frac{\omega_{\rm pe}^2}{q^2 v_{T_{\rm e}}^2}\right]\left|1 + \frac{\omega_{\rm pe}^2}{q^2 v_{T_{\rm e}}^2}\,W\!\left(\frac{\omega}{\sqrt{2}\,q v_{T_{\rm e}}}\right)\right|^2\right\}^{-1}, \tag{39}$$

where $W(x)$ is the well known plasma physics function describing the dispersion of plasma oscillations:

$$W(x) = 1 - 2x\exp(-x^2)\int_0^x \exp t^2\,{\rm d}t + {\rm i}\sqrt{\pi}\,x\exp(-x^2)\,. \tag{40}$$

For large values of its argument $W(x)$ is small but $\mathcal{H}$ is still not equal to unity ($\mathcal{H} = 1$ in the absence of both screening and the collective effects). The presence of $Z_{\rm eff}$ in $\mathcal{H}_{\rm coll}(\omega, q)$ reflects the role of ions in collective effects in bremsstrahlung. In the case of Debye screening the effect does not depend on ions, does not contain $Z_{\rm eff}$, and does not contain the plasma dispersion function $W(x)$ in the denominator. Thus the Debye screening effect is quite different from Eqn (39).

Before describing the possible influence of collective effects in bremsstrahlung on the solar opacity it is necessary to say a few words why the correct expression for the collective effects in bremsstrahlung was not obtained earlier. In fact the use of the full fluctuation approach gives a correct result, while previously the calculation was not performed in a full manner [13]. The complete calculation requires a nonlinear approach in a nonstationary and inhomogeneous media. One should take into account that in the absence of the wave (the damping of which is investigated) there exist sharp variations of the particle distribution functions both in time and in space. They are due, as usual, to the discreteness of the system and are described by standard methods of fluctuations in a plasma for a nonthermal particle distribution. In other words, in the initial plasma state unperturbed by the wave, there exist rapid and short length variations of refractive index. This leads to the processes of emission and absorption which influence wave propagation. The fluctuations of the particles distribution function lead also to the presence of fluctuating fields. The field of the propagating wave disturbs these fluctuations and to find the damping of the wave due to these disturbances one needs to use to a nonlinear theory of the field interactions in a strongly nonstationary and inhomogeneous plasma since the effects linear in the propagating field will be cubic in the total field which is the sum of the field of the propagating wave and the fluctuating fields. Thus the nonlinear response of the plasma being highly nonstationary and inhomogeneous should be used and, thus, taken in its entirety, the problem is not a simple one. All the above discussed problems were only recently treated [13].

It appears that independently a new effect called transition bremsstrahlung as an emission from the screening clouds of two colliding particles was investigated in Refs [10, 24, 25] and a simple recipe was given to calculate the additional matrix element $M^{\rm br,\,tr}$ which should be added to the usual matrix element for bremsstrahlung in the way we have already performed for the collective matrix element in expression (36). The latest research in this field shows that the expressions for $M^{\rm br,\,tr}$ and $M^{\rm br,\,coll}$ found from the fluctuation theory are identical. Therefore a possibility appears to use a simple recipe found for transition bremsstrahlung to calculate $M^{\rm br,\,coll}$ without dealing with cumbersome general expressions from the fluctuation approach which in turn allows us to find an expression for collective effects in bremsstrahlung in a rather general and compact form [13, 20]. The previous more simplified approach also uses the fluctuation theory [21, 22] but, as can be shown, neglects the effects of the same order of magnitude which appear from a nonlinear treatment of a highly nonstationary and inhomogeneous initial state. The nonlinear approach uses the nonlinear response coefficients and their approximate expressions which allows us to show the presence of the discussed compensation in a rather general form.

Thus, much headway has been made recently both in the understanding of the physics of collective effects in bremsstrahlung and its analytical description and these results should be used in applications to the solar opacity calculations.

Collective effects in bremsstrahlung are large for frequencies which do not differ significantly from the plasma frequency. For frequencies of this order, corresponding to the maximum of the weighting factor in the expression for the Rosseland opacity, both the screening approximation and the exact expression for collective effects in bremsstrahlung give small corrections. For frequencies close to the plasma frequency in the central part of the Sun for the case where all collective effects are taken into account the change in bremsstrahlung can be as large as 29%, while for the case of the screening approximation it is 24%.

Let us now calculate the role of collective effects in bremsstrahlung for the solar opacity. As in the case of refractive index corrections, one can expect that the most important frequencies are those much larger than the plasma

frequency. Therefore it is possible to expand the result in the parameter $\omega_{\rm pe}^2/\omega^2$:

$$\mathcal{F}^{\rm coll}(\omega) = \mathcal{F}_0(\omega) - 4\,\frac{v_{T_{\rm e}}^2}{c^2}\,\delta_{\rm e}\int_{\sqrt{z}/2}^{\infty} {\rm d}x\,x\exp\left[-\left(x+\frac{z}{4x}\right)^2\right]\left[Z_{\rm eff}+2\,{\rm Re}\,W(x)\right], \tag{41}$$

$$\mathcal{F}^{\rm scr}(\omega) = \mathcal{F}_0(\omega) - 4\,\frac{v_{T_{\rm e}}^2}{c^2}\,\delta_{\rm e}\int_{\sqrt{z}/2}^{\infty} {\rm d}x\,2x\exp\left[-\left(x+\frac{z}{4x}\right)^2\right]. \tag{42}$$

For calculation of the contribution of the collective effects in the Rosseland opacity we can use formula (34) to find

$$\frac{\delta\kappa_{\rm R}^{\rm br,\,coll}}{\kappa_{\rm R}^{(0)}} = -0.22\%\,, \tag{43}$$

$$\frac{\delta\kappa_{\rm R}^{\rm br,\,scr}}{\kappa_{\rm R}^{(0)}} = -0.28\%\,. \tag{44}$$

We should mention that the effect is small because the main contribution to the opacity $\kappa_{\rm R}$ is given by the high frequencies where the collective effects are small. Again it is necessary to mention that the collective effects in bremsstrahlung are also of the order of relativistic effects since for $\delta_{\rm e}\sim 1$ they are given as the Thomson cross-section times a factor $v_{T_{\rm e}}^2/c^2$. Therefore we are speaking only about the smallness of the numerical coefficient in front of this expression. We collect all the effects described by the cross-sections with such a dependence including those where the numerical coefficient is small. We shall not exclude any such expressions and do a complete search for them. Below we shall discuss other effects which have a much larger numerical coefficient although they are of the same smallness.

8. Bremsstrahlung absorption with relativistic effects

We shall discuss here the role of non-collective relativistic corrections in bremsstrahlung described by the well known classical Bethe – Heitler formula [27]. In the solar opacity the effective photon energies which make the major contributions are rather high since $\hbar\omega\approx 3.7T$ and therefore the energy conservation law in the process of bremsstrahlung allows only certain particles with an energy exceeding some energy threshold to take part in the absorption process. The threshold is found from the condition that the kinetic energy of the electron after emission is positive. Denoting by v the electron velocity before emission and denoting by v' the electron velocity after the emission we can write the threshold condition as $v'^2>0$. It is important that the relativistic corrections will lower the threshold. Indeed, by taking into account the first order relativistic corrections in the energy conservation law we get

$$v'^2 = v_0'^2 + \frac{3}{4c^2}\left(v^4 - v_0'^4\right), \qquad v_0'^2 = v^2 - \frac{2\hbar\omega}{m_{\rm e}}\,, \tag{45}$$

where v_0' is the final particle velocity in which the relativistic corrections are not taken into account.

According to Ref. [27] the relativistic correction on the other hand lowers the intensity of emission of a single particle. By taking into account only the first order relativistic corrections to the intensity of bremsstrahlung I_ω(intensity emitted per second in frequency interval ${\rm d}\omega$) we find from Ref. [27]:

$$I_\omega = \sum_i \frac{16Z_i^2e^6n_i}{3m_{\rm e}^2vc^3}\left\{\ln\frac{v+v'}{v-v'} - \frac{3vv'}{2c^2} - \frac{v^2-v'^2}{2c^2}\ln\frac{v+v'}{v-v'}\right\}. \tag{46}$$

In this equation v' is determined by Eqn (45) with the relativistic corrections taken into account, and thus the main term $\ln\left[(v+v')/(v-v')\right]$ also contains the relativistic corrections.

Due to a change in threshold by relativistic corrections a direct comparison of the curves with and without relativistic corrections is possible only by shifting the curves in energy or velocity in a way that the thresholds will coincide. Then it appears that the curve in which the relativistic corrections are taken into account is located always under the curve in which the relativistic corrections are not taken into account. For the values of frequencies $\hbar\omega\approx 3.7T$, mainly the particles in the tail of the thermal distribution are taking part in absorption and the increase of their number due to the lowering of threshold is larger than the decrease of absorption by each particle [26].

It is also necessary to take into account the relativistic effects in the electron distribution:

$$f^{\rm e}(v) \approx \frac{\exp(-v^2/2v_{T_{\rm e}}^2)}{(2\pi)^{3/2}v_{T_{\rm e}}^3}\left(1-\frac{3}{8}\,\frac{v^4}{v_{T_{\rm e}}^2c^2}+\frac{45}{8}\,\frac{v_{T_{\rm e}}^2}{c^2}\right).$$

We obtain by performing the integration with the threshold determined by Eqn (45) the following change in the expression $\mathcal{F}(\omega)$ which determines the cross-section of bremsstrahlung (we neglect here the collective effects):

$$\mathcal{F}(\omega) = \mathcal{F}_0(\omega) - \delta\mathcal{F}^{\rm br,\,rel}(\omega)\,,$$

$$\delta\mathcal{F}^{\rm br,\,rel}(\omega) = -\frac{v_{T_{\rm e}}^2}{c^2}\int_{\sqrt{z}/2}^{\infty}\frac{{\rm d}x}{x}\exp\left[-\left(x+\frac{z}{4x}\right)^2\right]f(x,z), \tag{47}$$

where

$$f(x,z) = \frac{3}{16}\,z^2\left(x+\frac{1}{x}\right)^4 - \frac{3}{4}\,z^2\left(x+\frac{1}{x}\right)^2 - \left[x^2+\frac{1}{x^2}+\left(\frac{1}{x^2}-x^2\right)\ln\frac{1}{x^2}\right]+\frac{15}{4}\,. \tag{48}$$

To calculate the change in the opacity we use the formula(18). The result is

$$\frac{\kappa_{\rm R}^{\rm br,\,rel}-\kappa_{\rm R}^{(0)}}{\kappa_{\rm R}^{(0)}} = +0.18\%\,. \tag{49}$$

Since the collective corrections to the bremsstrahlung are of the same order of magnitude and are opposite in sign to the non-collective corrections it is not possible to restrict the consideration to non-collective relativistic corrections only, as it was done in several of the latest papers. The reason for this is that the collective corrections apart from the same factor $v_{T_{\rm e}}^2/c^2$ contain a collective parameter $\delta_{\rm e}$ which in the solar interior is of the order of 1.

It is also known [38, 39] that non-collective bremsstrahlung in electron – electron collisions is of the relative order of $v_{T_{\rm e}}^2/c^2$. One can think that those corrections should be added

to the effect of non-collective relativistic corrections considered above. But this is not correct since the collective effects suppress to a large extent the bremsstrahlung in electron–electron collisions, making it of the order of $v_{T_e}^4/c^4$. Therefore the electron–electron bremsstrahlung in the solar interior should be neglected in calculations of opacity corrections which are of the order of $v_{T_e}^2/c^2$.

9. The Raman resonance in collective scattering

Before discussing relativistic effects in scattering we should consider a problem which can not be reconciled by a superficial look at the collective scattering but which is an important physical problem leading to an essential decrease in the scattering cross-section when necessary additional effects are taken into account. This is the problem of the contribution of the Raman scattering to the total cross-section of collective scattering on electrons. The Raman resonance corresponds to the case where the difference of the frequencies of the scattered wave and the scattering wave is equal to the electron plasma frequency. Plasma Langmuir waves can exist in the solar interior since the number of particles in the Debye sphere N_d for the temperatures and densities in the centre of the Sun which were already given above is $N_d = 4\pi v_{T_e}^3/\omega_{pe}^3 \approx 11.4\ \mathrm{cm}^{-3}$ and the relative damping of plasma waves (the damping rate divided by the plasma frequency) due to the binary collisions is approximately equal to 1/20. The necessary conditions for the Raman resonance are

$$\omega - \omega' \approx \pm\omega_{\mathbf{k}_p}\,, \qquad \mathbf{k} - \mathbf{k}' = \mathbf{k}_p\,, \tag{50}$$

where $\mathbf{k}_p$ is the wave vector of the plasma oscillation. For these conditions the longitudinal dielectric permittivity for the frequency which is the difference of the frequencies of the initial and scattered waves is close to zero:

$$\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'} \approx 0\,. \tag{51}$$

But the probability of scattering on electrons contains this dielectric permittivity in the denominator, which means that the scattering is of a resonant nature for the difference of two frequencies close to the plasma frequency. We write here the probability of scattering on electrons $W^{e}_{\mathbf{k},\mathbf{k}'}$ in the form it was defined in the book [8] (namely the probability of scattering of a single photon per unit time into the range $d^3k/(2\pi)^3$ from the range $d^3k'/(2\pi)^3$ related to these range intervals) assuming for simplicity that the electron velocity is much less than the average ion thermal velocity (which is a good approximation for averaging of the scattering on the thermal electron distribution):

$$W^{e}_{\mathbf{k},\mathbf{k}'} = \frac{(2\pi)^3 e^4}{2m_e^2\omega\omega'}(1+x^2)\left|\frac{1}{\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'}}\right|^2 \times \delta(\omega - \omega' - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v})\,. \tag{52}$$

From this expression we see indeed that the Raman resonance can be very pronounced. The question is what is the relative contribution of this resonance to the total cross-section of scattering averaged over the thermal electron distribution and integrated over all frequencies of the scattered wave? Another question arises, i.e. how does this relative contribution change with the collective parameter? It appears that for $\delta_e \gg 1$ the contribution of the Raman resonance is the largest in the total cross-section and this fact was not even mentioned or realised in the calculations of opacity in SSMs. The final answer is that the width of the Raman resonance for the approximation already used in SSMs can be extremely narrow and many effects not taken into account can broaden the resonance, thus decreasing its role in the total cross-section which then becomes much smaller and leads to a diminishing of the opacity.

To show this behaviour we should first say a few words about the Doppler effect in scattering. Since the velocity of the photons is close to the speed of light and $k \approx \omega/c$ the Doppler effect gives a correction of the order of the difference of the frequencies which is to a first approximation of the order of v_{T_e}/c. More exactly from the δ-function in the probability (52) (describing from a quantum point of view the conservation of energy and momentum in the scattering) we can find by expanding the terms describing the linear and the quadratic Doppler effect in the difference of the frequencies, namely,

$$\omega' \approx \omega\left[1 - 2\frac{v_{T_e}}{c}\,y\sqrt{1-x} + 2\,\frac{v_{T_e}^2}{c^2}(1-x)y^2\right], \tag{53}$$

where y is the normalised component of the electron velocity with respect to the vector along the difference of the wave vectors of the scattering and scattered waves respectively:

$$y = \frac{(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}}{\sqrt{2}\,v_{T_e}|\mathbf{k}-\mathbf{k}'|}\,. \tag{54}$$

In the zero approximation where the Doppler effect is neglected the frequencies of the scattered and the scattering waves coincide. Only in this approximation was the cross-section of scattering denoted as scattering in the zero approximation. Due to the symmetry of the electron distribution function in y, the Doppler corrections should be of the order of $v_{T_e}^2/c^2$. This means that we are again considering the terms of the order of $v_{T_e}^2/c^2$ which we are searching for. A new question nevertheless arises whether it is possible to expand them in the Doppler corrections close to the Raman resonance. In the case such an expansion is not possible the contribution of the Doppler effect could be larger than the rough estimation $v_{T_e}^2/c^2$ given above. To answer this question it is necessary to know the role of the resonance and its relative contribution to the total cross-section and to find its width (since in the case its width is less than v_{T_e}/c the answer to the question of the possibility of the expansion in v_{T_e}/c will be negative and one should treat the resonance with its Doppler broadening exactly). Of course the resonance can also be broadened by other means including binary collisions. At the present stage of consideration we wanted to consider the width of the Raman resonance without taking all those broadening effects, in the form it appears in the zero approximation used as a reference model. Then one should use the expression for the collisionless dielectric permittivity which takes into account only kinetic effects including as a damping effect only the Landau damping. From the radiation extinction coefficient using the probability (52) we find the following expression for the transport cross-section:

$$\sigma_e^{sc} = \frac{3}{8}\int_{-1}^{1}(1+x^2)(1-x)\,dx \times \int_{-\infty}^{+\infty}\frac{\exp(-y^2)}{\sqrt{\pi}}\,\frac{1}{\left|1+W(y)\delta_e/(1-x)\right|^2}\,dy\,, \tag{55}$$

where $W(y)$ is the plasma dispersion function given by expression (48). Under the integral with respect to y is the square of the absolute value of the dielectric permittivity. Thus the total cross-section includes integration over the Raman resonance. It appears that the integration over y can be performed in a general case using the dispersion relations which relate the real and imaginary parts of the dielectric permittivity [17, 18, 27] (or by using the fluctuation–dissipation theorem). Indeed the imaginary part of $W(y)$ in Eqn (48) contains an additional factor y as compared to the expression which enters in the numerator of Eqn (55). The expression under the integral (55) can be written as an imaginary part of $1/\omega\epsilon$ and the y integration can be considered as integration with respect to the frequency. Then by integrating in the upper complex plane of ω noting that the function $1/\epsilon$ has no poles, what is left is the integration due to circling the pole $1/\omega$ on the real axis. We then find

$$\int_{-\infty}^{+\infty} \frac{\exp(-y^2)}{\sqrt{\pi}\left|1+\delta_e/(1-x)\,W(y)\right|^2}\,\mathrm{d}y = \frac{1}{1+\delta_e/(1-x)}\,. \quad (56)$$

Using this expression in (55) leads immediately to the expression (25) for the cross-section of collective scattering used above.

In connection with relation (56) a new important question arises immediately concerning the asymptotic behaviour of the cross-section for large values of δ_e. In the case of large δ_e one can think it possible to neglect 1 as compared to the term containing δ_e in the denominator of the left hand side of Eqn (56). Then in the case where the integral converges one finds the asymptotic behaviour proportional to $1/\delta_e^2$. It appears that indeed the integral is converging and the result of the calculation will be

$$\int_{-\infty}^{+\infty} \frac{\exp(-y^2)}{\sqrt{\pi}\,|W(y)|^2}\,\mathrm{d}y = 3\,. \quad (57)$$

This means that the left hand side of expression (56) has an asymptote $1/\delta_e^2$, while the right hand side of it has the asymptote $1/\delta_e$, therefore there is a contradiction. Where was the mistake made in these calculations? It appears that the mistake is in the assumption that the main contribution in the integral is given by that part of the integration where the y values are of the order of 1 or less than 1. But how can it be different in the case where the function under the integral contains $\exp(-y^2)$? It appears that this is possible if there exists a very sharp, exponentially narrow resonance. This is the Raman resonance. The assumption that the main contribution comes from y of the order of 1 neglects the contribution of the Raman resonance which occurs for $\delta_e \gg 1$ in the range $y \gg 1$. But this will then mean that for large δ_e the value of the cross-section is almost completely determined by the contribution of the Raman resonance. Let us show that this is indeed the case.

The resonance corresponds to zero of the real part of the dielectric permittivity. This resonance should be reached at large values of y otherwise we shall be in contradiction with the previous considerations. For large values of y the function $W(y)$ has the following asymptote $W(y) \approx -1/2y^2$ which gives two possible values of y for which the resonance condition is satisfied:

$$y = y_r = \pm\sqrt{\frac{\delta_e}{2(1-x)}}\,. \quad (58)$$

The latter expression shows that indeed for large values of δ_e the resonance corresponds to large values of y_r. Then making an expansion close to the resonant points we get the left hand side of Eqn (56) in the form :

$$\int_{-\infty}^{+\infty} \frac{(1-x)^2\,\mathrm{Im}\,W(y_r)\,\mathrm{d}y}{\pi y_r \delta_e^2\left[(y-y_r)^2(\partial\,\mathrm{Re}\,W(y_r)/\partial y)^2 + (\mathrm{Im}\,W(y_r))^2\right]} = \frac{1-x}{\delta_e}\,, \quad (59)$$

which corresponds to the right hand side of Eqn (56).

The width of the Raman resonance is exponentially decreasing with increasing of δ_e:

$$\frac{\delta y_r}{y_r} \approx \frac{\mathrm{Im}\,W(y_r)}{y_r\,\partial\,\mathrm{Re}\,W(y_r)/\partial y_r} \approx \sqrt{\pi}\,y_r^3\exp(-y_r^2)\,. \quad (60)$$

The exponentially small thickness of the resonance makes it very dangerous to use the zero approximation for scattering but this was the only type of consideration of the role of collective scattering that was made at the present time in SSMs.

The broadening of the Raman resonance due both to the Doppler effect and to binary collisions can substantially decrease its contribution to the total cross-section, thus making it much smaller than previous accounts. Then the main contribution will be made by thermal particles ($y \sim 1$), the scattering cross-section will be proportional to $1/\delta_e^2$ and will be much smaller. But the problem is that this effect is very pronounced for large δ_e while the most important contribution to the opacity occurs for $\delta_e \sim 1$ when the weight function in the opacity is a maximum. Nevertheless we can see from Fig. 1 that the weight function in κ_R is rather broad and thus the part with high values of δ_e can be essential.

10. The Doppler and collisional broadening of the Raman resonance

This problem was recently considered in Ref. [28]. A simultaneous consideration of both the effects of broadening is important since it can be proved that in the absence of the Doppler effect the cross-section will be determined by the static dielectric permittivity which, as is known, does not depend on the collisions. This problem of the necessity of simultaneous consideration of Doppler effects and binary collisions is of general importance in nonlinear interactions [29]. Concerning the solar interior it will be very strange from a general point of view that for such a high collision rate of the order of 2×10^{16} s^{-1} one still uses the collisionless approximation for scattering and that the binary collisions have no way of influencing the scattering process but this was the way the scattering was previously considered. The collision frequency should be compared not with the frequency of radiation or even not with the plasma frequency but with the width of the Raman resonance. Then it becomes obvious that the broadening of the Raman resonance can be important for the reduction of the opacity. The contribution of the Doppler effect and binary collisions can be calculated by using the perturbation approach, but the expansion of these effects

close to the resonance is not possible. In using the perturbation calculation in the dielectric permittivity in the denominator we in fact use only the small parameters v_{T_e}/c and $\nu_{\rm coll}/\omega_{\rm pe}$, where $\nu_{\rm coll}$ is the effective frequency of the binary electron – ion collisions. We will use such an expansion in the dielectric constant but will not use it to expand in these parameters close to the point of the Raman resonance leaving the corresponding terms quadratic in v_{T_e}/c and linear in $\nu_{\rm coll}/\omega_{\rm pe}$ in the denominator. The quadratic terms in the Doppler effect should be taken into account since in the range outside the resonance where the expansion is possible only the quadratic Doppler terms will survive. As a result the cross-section will have the form (55) but with a denominator which takes into account the Doppler and collisional broadening of the Raman resonance and with an additional factor in the numerator which takes into account the first two terms in the expansion on the parameter v_{T_e}/c (the latter is necessary to take into account the Doppler corrections outside the resonance where the expansion is possible). We then find

$$\sigma_{\rm e}^{\rm sc} = \frac{3}{8}\int_{-1}^{1}(1+x^2)(1-x)\,{\rm d}x\int_{-\infty}^{+\infty}\frac{\exp(-y^2)}{\sqrt{\pi}}\,\frac{A(x,y)}{\left|F(x,y)\right|^2}\,{\rm d}y\,, \tag{61}$$

where previously for Eqn (55) we had

$$A(x,y) = A_0(x,y) = 1\,,$$

$$F(x,y) = F_0(x,y) = 1 + \frac{\delta_{\rm e}}{1-x}\,W(x)\,, \tag{62}$$

and now

$$A(x,y) = 1 - 3\,\frac{v_{T_e}}{c}\,y\sqrt{1-x} + 2\,\frac{v_{T_e}^2}{c^2}\,y^2(3-2x)\,, \tag{63}$$

$$\begin{aligned} F(x,y) = {} & 1 + \frac{\delta_{\rm e}}{1-x}\,W_{\rm R}(y) \\ & \times\left[1 + 2\,\frac{v_{T_e}}{c}\,y\sqrt{1-x} - 2\,\frac{v_{T_e}^2}{c^2}(2-x)\right] \\ & + {\rm i}\,\frac{Z_{\rm eff}\ln\Lambda\omega_{\rm pe}^3 y\delta_{\rm e}^{3/2}}{24\pi^{3/2}n_{\rm e}v_{T_e}^3(1-x)^{3/2}} \\ & \times\left\{\left[{\rm Ei}(y^2) - {\rm i}\pi\right]\exp(-y^2) - \frac{1}{y^2}\right\}, \end{aligned} \tag{64}$$

where $\ln\Lambda$ is the Coulomb logarithm, the function

$$\begin{aligned} W_{\rm R}(z) = {} & 1 - z\int_{-\infty}^{+\infty}\frac{\exp(-y^2)}{\sqrt{\pi}}\,{\rm R}(y)\,\frac{{\rm d}y}{z-y} \\ & + {\rm i}\sqrt{\pi}\,\exp(-y^2)\,{\rm R}(z) \end{aligned} \tag{65}$$

is a generalisation of the function $W(z)$ which takes into account the relativistic corrections in the distribution function of electrons:

$$\begin{aligned} {\rm R}(y) = {} & \exp\left(-\frac{3v_{T_e}^2 y^4}{2c^2}\right) \\ & \times\frac{\int_0^\infty x\exp\left\{-x^2\left[1 + 3v_{T_e}^2/(2c^2)(x^2+2y^2)\right]\right\}{\rm d}x}{\int_0^\infty 4/\sqrt{\pi}\,x^2\exp\left[-x^2 - 3v_{T_e}^2x^4/(2c^2)\right]{\rm d}x} \end{aligned} \tag{66}$$

and finally,

$${\rm Ei}(z^2) = \int_{-\infty}^{z^2}\frac{\exp t}{t}\,{\rm d}t\,. \tag{67}$$

Numerical calculations [28] of the corrections to the Rosseland opacity by using Eqn (18), for the parameters in the centre of the Sun result in

$$\frac{\kappa_{\rm R}^{\rm sc,\,broad,\,res} - \kappa_{\rm R}^{(0)}}{\kappa_{\rm R}^{(0)}} = -3.0\%\,. \tag{68}$$

The value given by Eqn (68) is a substantial contribution which was previously not taken into account in SSMs.

11. Relativistic corrections to collective scattering

The foregoing explanations and considerations suggest that the relativistic corrections to the scattering can not be found simply by multiplying the zero order approximation cross-sections by a factor taken from the expressions where the collective effects are neglected. Nevertheless it was the only consideration of relativistic effects in cross-sections previously performed for the solar opacity problem.

There are several reasons why this approach is faulty; the first being that the relativistic corrections are different for electrons and ions and one can not multiply the zero approximation by the same factor for electrons and ions; the second, that inside the Raman resonance the expansion in the parameter v_{T_e}/c is not possible and the third, that the relativistic corrections have another dependence on the collective parameter $\delta_{\rm e}$ from that of the zero approximation.

We can illustrate the latter statement by using the same argument as in the interference of the effects occurring in scattering on the charge itself and that of its shielding cloud. Let us denote the matrix element of scattering on an individual 'naked' electron by $M_{\rm T}$ and let us denote the relativistic corrections to it by $\delta M_{\rm T}^{\rm rel}$. Let us then denote the matrix element for scattering on the screening 'cloud' by $M_{\rm coll}$ and let us denote the relativistic corrections to it by $\delta M_{\rm coll}^{\rm rel}$. The total cross-section for scattering will be proportional to

$$\left|M_{\rm T} + M_{\rm coll} + \delta M_{\rm T}^{\rm rel} + \delta M_{\rm coll}^{\rm rel}\right|^2. \tag{69}$$

The corrections which were taken into account in SSMs correspond only to $2\delta M_{\rm T}^{\rm rel}/M_{\rm T}$ and they were multiplied by the square of the total zero order matrix element $|M_{\rm T} + M_{\rm coll}|^2$. But, even in the case one neglects the relativistic corrections to the collective matrix element, the procedure does not yield the correct result. After dropping the term $\delta M_{\rm coll}^{\rm rel}$ we can see from Eqn (69) that the corrections $\delta M_{\rm T}^{\rm rel}$, should be multiplied by $M_{\rm T} + M_{\rm coll}$, not $M_{\rm T}$. Apart from this one can not neglect $\delta M_{\rm coll}^{\rm rel}$ in which not only the effect due to the relativistic corrections in the particle motion should be taken into account, but also the relativistic corrections to the particle distributions which enter in the nonlinear plasma response on which the collective matrix element depends as well as the relativistic corrections in the electron distribution, while averaging the cross-sections should be taken into account. Previously in plasma physics such detailed calculations were not performed. They are very cumbersome and one should use general expressions for matrix elements given in Refs [9, 10] which take into account, in principle, relativistic effects exactly. Such calculations were performed

in Ref. [30] and the final result can be written using the already introduced function $F(x,y)$ (see Eqn (68)), which takes into account all the above effects of resonant broadening of the Raman resonance. We shall denote the relativistic corrections for scattering on electrons for an arbitrary value of the collective parameter δ_{e} by $\delta\sigma_{\mathrm{e}}^{\mathrm{rel}}$. We find

$$\delta\sigma_{\mathrm{e}}^{\mathrm{rel}} = -\frac{3v_{T_{\mathrm{e}}}^2}{4c^2}\,\sigma_{\mathrm{T}}\int_{-\infty}^{+\infty}\frac{\mathrm{d}y}{\sqrt{\pi}}\exp(-y^2)\int_{-1}^{1}\frac{\mathrm{d}x}{|F(x,y)|^2} \times\Big[(1-x)^{5/2}\,\mathrm{Re}\,F(x,y)\big(y^2 f_1(x)+2f_2(x)\big) - \delta_{\mathrm{e}}(1-x)^3 G(x,y)\Big]\,, \tag{70}$$

where

$$f_1(x) = 1+x+x^2-x^3\,,\qquad f_2(x)=\frac{1}{2}(3-x+x^2-x^3)\,, \tag{71}$$

$$G(x,y) = 1+x+x^2-x^3+\frac{1}{2}\,W(y)(9+4x+9x^2) - 2y^2 W(y)(1+x+x^2-x^3)\,. \tag{72}$$

Naturally when the collective effects are unimportant the expression for $\delta\sigma_{\mathrm{e}}^{\mathrm{rel}}$ converts to the known expression.

The relativistic corrections for scattering on ions are related only to the electron shell of ions (since the other relativistic corrections of the order of $v_{T_{\mathrm{i}}}^2/c^2$ are here naturally neglected completely) and are determined by $\delta M_{\mathrm{coll}}^{\mathrm{rel}}$ and therefore the ion velocity does not enter in these corrections. Therefore, for the averaging on the ion distribution the integrals are of the same form as in the zero approximation and the result can be obtained in an analytic form by using relation (56), which is a consequence of the fluctuation–dissipation theorem. The relativistic corrections for ions are denoted by $\delta\sigma_{\mathrm{i}}^{\mathrm{rel}}$ and they are given by the following expression:

$$\delta\sigma_{\mathrm{i}}^{\mathrm{rel}} = \frac{3v_{T_{\mathrm{e}}}^2}{4c^2}\,\delta_{\mathrm{e}}\big[g(\delta_{\mathrm{e}})-g(\delta_{\mathrm{i}})\big]\sigma_{\mathrm{T}}\,, \tag{73}$$

where

$$g(z) = \left(2-z+\frac{7}{2}\,z^2+\frac{3}{2}\,z^3+z^4\right)\ln\frac{2+z}{z} + \frac{28}{3}+\frac{35}{3}\,z+11z^2+2z^3\,. \tag{74}$$

A numerical calculation of the sum of corrections due to electrons and ions by using the formulas given in this section leads to the following change of the opacity in the centre of the Sun:

$$\frac{\kappa_{\mathrm{R}}^{\mathrm{e,\,screl}}-\kappa_{\mathrm{R}}^{(0)}}{\kappa_{\mathrm{R}}^{(0)}} = -0.2\%\,. \tag{75}$$

12. Effects of frequency change in the process of radiation transfer

In addition to the relativistic corrections to the transport cross-sections there appear also the corrections of the same order of magnitude in the equation for the radiation transfer. To clarify this point let us write here the equation for photons transport by taking into account the processes of bremsstrahlung and scattering. Since the starting point in the theory of photons transport is the assumption that the distribution of photons is locally an equilibrium distribution with small deviations from the equilibrium distribution due to the presence of temperature gradients, such an equation should include both the processes of spontaneous emission and scattering, and the processes of stimulated emission and scattering (in equilibrium the spontaneous and stimulated processes exactly balance each other giving the Planck distribution). In the theory of radiation transfer it is necessary to take into account both the small deviations from the equilibrium in the spontaneous processes and the small deviations from equilibrium in the stimulated processes. We will write down a general expression describing the propagation of photons for their occupation numbers $N_{\mathbf{k}}$ assuming that the photon distribution does not depend on time (see Ref [31]) and that the photon frequency is much larger than the electron plasma frequency:

$$\begin{aligned}\frac{\mathrm{d}N_{\mathbf{k}}}{\mathrm{d}t} &= \mathbf{v}_g\cdot\frac{\partial N_{\mathbf{k}}}{\partial\mathbf{r}}-\frac{\partial\omega}{\partial\mathbf{r}}\cdot\frac{\partial N_{\mathbf{k}}}{\partial\mathbf{k}}\\ &\approx\cos\theta\left(c\,\frac{\partial N_\omega^{\mathrm{T}}}{\partial r}-\frac{\omega_{\mathrm{pe}}^2}{2\omega^2}\,\frac{c}{n_{\mathrm{e}}}\,\frac{\partial n_{\mathrm{e}}}{\partial r}\,\omega\,\frac{\partial N_\omega^T}{\partial\omega}\right)\\ &= -\int\Big(W_{\mathbf{k},\mathbf{k}'}^{\mathrm{e}}f_{\mathbf{p}}^{\mathrm{e}}+\sum_i W_{\mathbf{k},\mathbf{k}'}^{\mathrm{i}}f_{\mathbf{p}}^{\mathrm{i}}\Big)(N_{\mathbf{k}}-N_{\mathbf{k}'})\,\frac{\mathrm{d}^3p\,\mathrm{d}^3k'}{(2\pi)^6}\\ &\quad+N_{\mathbf{k}}\int W_{\mathbf{k},\mathbf{k}'}^{\mathrm{e}}\hbar(\mathbf{k}-\mathbf{k}')\,\frac{\partial f_{\mathbf{p}}^{\mathrm{e}}}{\partial\mathbf{p}}\,N_{\mathbf{k}'}\,\frac{\mathrm{d}^3p\,\mathrm{d}^3k'}{(2\pi)^6}+2\gamma_{\mathbf{k}}^{\mathrm{br}}N_{\mathbf{k}}+q_{\mathbf{k}}^{\mathrm{br}}\,,\end{aligned} \tag{76}$$

where $W_{\mathbf{k},\mathbf{k}'}^{\mathrm{e,i}}$ are the probabilities of scattering on electrons (superscript 'e') and ions (superscript 'i') respectively, θ is the angle between $\mathbf{k}$ and $\mathbf{r}$; in the left hand side of the equation the thermal equilibrium distribution N_ω^{T} is substituted, while in the right hand side only the deviations from the thermal distribution contribute and the terms linear in these deviations are left. The last two terms describe the spontaneous and stimulated bremsstrahlung; since in equilibrium they balance each other and since the spontaneous emission has not been perturbed the only term left is the deviation of the stimulated bremsstrahlung related to the flux of radiation. This effect was already considered above and expressed through the effective bremsstrahlung absorption cross-section. In the first term of the right hand side of the transport equation, which describes the scattering, the frequencies of the photons before and after scattering do not coincide since due to the Doppler effect the frequency of photons changes in the scattering process. This term leads to a transport cross-section described above only if the deviations from the thermal distribution are proportional to the cosine of the angle between the direction of photon propagation and the direction of the temperature gradient and only if the Doppler effect is neglected in the expressions for the photons occupation numbers (the latter approximation is very important). Then we have $N_{\mathbf{k}} = N_\omega^{\mathrm{T}}+\cos\theta\delta N_\omega$, where δN_ω is proportional to the flux of radiation. In the case we want to calculate the quadratic corrections on the parameter $v_{T_{\mathrm{e}}}/c$, there appear new terms which are proportional to the derivatives of the flux of radiation with respect to the frequency since we need to use an expansion of the

occupation numbers on the frequency difference:

$$\delta N_{\omega'} \approx \delta N_\omega + (\omega' - \omega)\frac{\partial \delta N_\omega}{\partial \omega} + \frac{1}{2}(\omega' - \omega)^2 \frac{\partial^2 \delta N_\omega}{\partial \omega^2} \,. \quad (77)$$

The physical nature of these effects is obvious. In each act of scattering the frequency of the photons is changing and as a result of many scatterings the distribution of the photons diffuses in frequency.

The second term in the transport equation (76) describes stimulated scattering and since in the first approximation it is odd in velocities it will contain the first derivative of the flux of radiation with respect to the frequency and thus describes a systematic redshift of the photon frequencies in the process of scattering.

A systematic change of the photon frequency due to the density inhomogeneity is described by the second term in the left hand side of the transport equation (76). Its meaning is also obvious, it is to conserve the adiabatic invariant, the occupation number of photons. It is easy to see that the latter term is also the term containing, in front of it, the relativistic parameter $v_{T_e}^2/c^2$. Indeed, by introducing the collective parameter δ_e, one finds that the change to the left hand side of the transport equation due to the plasma density gradient is described by an additional factor in the left hand side having the following form:

$$1 + \delta_e \frac{v_{T_e}^2}{c^2} \frac{\partial \ln n_e}{\partial \ln T} \,. \quad (78)$$

Since for the solar interior $\delta_e \sim 1$ the density inhomogeneity effect is again of the order of the relativistic effects.

Thus the transport equation contains three new effects of the order of that already considered above and all of them are related to the change of frequency of photons in the process of radiative energy transfer.

13. Corrections due to density inhomogeneity

The existing SSM can be used to estimate the density gradients in the centre of the Sun and find the value of $\partial \ln n_e / \partial \ln T$. To calculate the change of Rosseland opacity due to the density inhomogeneity we shall use formula (34), which gives

$$\frac{\delta \kappa_R^{\text{inh}}}{\kappa_R^{(0)}} = -0.14\% \,. \quad (79)$$

Although this effect is rather small we do not exclude it from our consideration in order to obtain a complete result of the effects of the relative order of $v_{T_e}^2/c^2$.

14. Corrections due to photons frequency diffusion during the radiation transfer

This effect leads to a new type of transport equation for photons which raises several problems concerning possible solutions and the question whether the opacity approach can be at all applied to the transport of radiation inside the Sun. The corresponding contributions of effects of frequency diffusion in the transport equation are described by differential operators applied to the functions describing the radiation flux. The structure of the transport equation changes in some sense in a cardinal way since the transport equation became a differential equation in frequency containing the second order derivatives [31]. The terms with higher derivatives have a small parameter in front of them, which raises several mathematical problems not yet solved and the presence of the terms with derivatives do not allow us to integrate the transport equations over the frequencies to obtain the opacity directly from the equations. The mathematics reflects the physics and thus several physical problems arise in this context. We will first write the operators describing the effect of frequency diffusion in a form acting on the disturbance already introduced above of the photon occupation numbers δN_ω. It will be recalled that the flux of radiation $\mathcal{F}_\omega$ is proportional to this occupation numbers perturbation multiplied by ω^3 and therefore the operator $\hat{a}$, acting on δN_ω, corresponds in the transport equation to the following operator acting on the radiation flux $\mathcal{F}_\omega$:

$$\omega^3 \hat{a}\left(\frac{\mathcal{F}_\omega}{\omega^3}\right) . \quad (80)$$

In treating the frequency diffusion effects we shall take into account the broadening of the Raman resonance. In this case the first term on the right hand side of the transport equation (76) leads to the sum of the term already considered describing the transport cross-section (61) and an additional term which can be written in operator form and thus called an operator cross-section $\hat{\sigma}^{\text{fd}}$ describing the effect of frequency diffusion (remember that in a transport equation it will appear in accordance with relation (80)):

$$\hat{\sigma}^{\text{fd}} = \sigma_1^{\text{fd}}(\omega)\omega \frac{\partial}{\partial \omega} + \sigma_2^{\text{fd}}(\omega)\omega^2 \frac{\partial^2}{\partial \omega^2} \quad (81)$$

and

$$\sigma_1^{\text{fd}}(\omega) = \frac{3}{4}\sigma_T \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} \exp(-y^2) \int_{-1}^{1} \frac{(1+x^2)x\,dx}{|F(x,y)|^2} \times \left[\frac{v_{T_e}}{c} y\sqrt{1-x} - 4\frac{v_{T_e}^2}{c^2} y^2(1-x)\right] , \quad (82)$$

$$\sigma_2^{\text{fd}}(\omega) = -\frac{3v_{T_e}^2}{4c^2}\sigma_T \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} \exp(-y^2)\, y^2 \times \int_{-1}^{1} \frac{(1+x^2)x(1-x)\,dx}{|F(x,y)|^2} \,. \quad (83)$$

With these operator terms taken into account in the transport equation the latter becomes a differential equation in frequency with a small parameter in front of the highest derivative. This raises some questions already known from mathematical courses and textbooks. There are at least two questions in this connection. The first is whether it is possible to exclude the small parameter from the equation by changing the variables. The second is, "What kind of 'boundary' conditions in frequency should be taken and to what extent the final solution depends on these conditions?" Both questions are rather difficult to answer in general form. But these answers can alter the conclusions about the role of these effects in the solar interior. The question also arises why a problem does not arise in the energy transfer in a vacuum (more exactly for the case when the collective effects do not play an important role)? The answer is that the frequency diffusion terms were not previously obtained even for this simple case.

The seminal work in this field is the work of Sampson [32] who wrote the integral transport equation for radiative transfer using the Klein – Nishina quantum formula for the cross-section of scattering. The integral equation is of the type we wrote in the previous section (see Eqn (76)) and obviously it contains all the same or even a more complicated problem in the case one tries to solve it analytically. Incidentally, the transport equation we wrote in the previous section is exactly the same as in Ref. [32] except for the probability used and the equation is written in the classical limit. In Eqn (76) all collective effects are included while in Sampsons's paper the collective effects were ignored but the probability includes all the quantum effects. The collective quantum effects will be discussed in section 16. But here we discuss why similar problems with frequency diffusion do not appear in Sampson's approach. The answer is that in this paper they were simply neglected by two simplifying conditions which he needed to accept to solve the problem numerically (the assumptions I and II of Ref. [32]). Thus, the frequency diffusion terms are derived in Ref. [31] for the first time even for the simple case of 'undressed' particles. To have such an analytical equation with a diffusion term denotes substantial progress in the transport theory since it offers the chance to solve it analytically; today such an equation has been solved only numerically. In the subsequent papers [33, 34] the authors have excluded the two assumptions of Sampson but still only made numerical computations of the integral equation. Even by excluding the assumptions of Ref. [32] the authors of Ref. [34] were not able (due to numerical difficulties) to treat the temperature region corresponding to the solar interior (the calculations made in Ref. [34] were performed for temperatures greater than those in the solar interior). Therefore the consideration made in Ref. [31] is more consistent within a small parameter v_{T_e}/c and takes into account both collective effects and broadening of the Raman resonance. But there still remains a problem in all three papers which make another third assumption that the solution of the transport equation has a definite form, i.e. it is proportional to the cosine of the angle with the inhomogeneity direction (as usual in all transport equations) and the derivative of the black body radiation with respect to the coordinate. The latter assumption was not made in Ref. [31] and only the assumption that the solution contains the cosine of the angle was used. The form of the solution found in Ref. [31] is different from that used in Refs [32 – 34] and it contains the derivatives of the intensity with respect to the frequency.

Thus the equations with the frequency diffusion term can not be derived with the assumptions made in all three referred papers. The problem was in Ref. [31] even formulated in a more general form for the case of scattering in vacuum in the absence of collective effects. It can be seen that the terms in the form given in the papers [32 – 34] can be obtained from a more general equation written here if only in these terms one can use a perturbation approach, namely, first calculate the flux without frequency diffusion terms and then substitute this solution into the terms describing the frequency diffusion. But to use the perturbation approach for an equation with a small parameter in front of the highest derivatives is known from mathematics to be very dangerous. The only arguments of help not in resolving this problem but merely suggesting the way for a possible solution are the following: in the case of 'undressed' particles the frequency diffusion terms contain only the powers of the operator $\omega\partial/\partial\omega$, which does not allow the exclusion of the small parameter by changing the variables.

But we obtained above a more general result including in the frequency diffusion terms all collective effects which makes the coefficients in front of those operators depend on the collective parameter δ_e and thus they depend on frequency. In the collective case the question about the possibility of the use of the perturbation approach is even more serious.

At present we are unable to resolve it and the best we can do is to use the perturbation approach. But the important point is that in using such an approach we should bear in mind the existing uncertainty of the theoretical predictions, since, as we shall see, the total effect of the frequency diffusion on the solar opacity is significant (by finding some particular solutions without using the perturbation approach we can demonstrate that these effects can be even larger).

The problem recalls the 'boundary' conditions in the frequency diffusion terms. In the perturbation approach they do not appear and only in the perturbation approach we shall be able to find the corrections to the opacity explicitly. What do all these problems mean physically? Depending on the 'boundary' conditions in frequency we may or may not have the effect of photon accumulation in a certain frequency domain. This effect is well known in plasma physics for the Langmuir waves and has the name of the Langmuir condensation. Probably the condensation of photons will not appear since they have a large inverse bremsstrahlung absorption. Depending on 'boundary' conditions in frequencies, one can expect as another possible effect a 'runaway' of photons from absorption. All these problems mean that there could occur specific instabilities of photon distribution related directly to photon transport and generated by the transport phenomena. To investigate them is a new problem for the future research in this field.

In any case the possibility of writing down an analytic equation of transport including the frequency diffusion terms seems to have more advantages in future research in this field since up to the present time only the numerical solutions of integral equations was investigated. The differential transport equation we obtained has only the limitations that the temperature of particles should be nonrelativistic. A preliminary investigation of this equation for the vacuum case was performed in Ref. [35] and show that exact solutions are not at all trivial and the spectrum obtained has some peculiarities with rapid frequency variations the averaging of which lowers the total transferred flux. Having nothing more exact at the present moment we shall use the perturbation approach to at least estimate the order of magnitude of the possible effect of frequency diffusion on the opacity. One should bear in mind that this estimation can yield the lower limit of possible reductions of the opacity since the oscillations in the frequency distribution can only enhance the decrease of the opacity due to this effect.

The numerical results were performed together with effects of stimulated scattering since the latter also lead to derivative terms with respect to the frequency of the radiation flux. The results are illustrated in the next section.

15. Effect of stimulated scattering

This effect was also missed in the consideration of SSMs. The effect does not contain a term describing diffusion on frequency but only the term with a first derivative with

respect to the frequency which means it leads to a systematic change in the photon frequency during the transfer of radiation.

The corresponding operator we denote as $\hat{\sigma}^{\mathrm{st}}$:

$$\hat{\sigma}^{\mathrm{st}} = \sigma_0^{\mathrm{st}}(\omega) + \sigma_1^{\mathrm{st}}(\omega)\,\frac{\partial}{\partial\omega}\,, \tag{84}$$

where

$$\begin{aligned}\sigma_0^{\mathrm{st}}(\omega) = &-\frac{3v_{T_\mathrm{e}}}{4c}\,\sigma_\mathrm{T}\,\frac{z}{\exp(z)-1}\int_{-\infty}^{+\infty}\frac{y\,\mathrm{d}y}{\sqrt{\pi}}\exp(-y^2)\\ &\times\int_{-1}^{1}\mathrm{d}x\,\frac{(1+x^2)(1+x)\sqrt{1-x}}{\left|F(x,y)\right|^2}\\ &+\frac{3}{2}\,\sigma_\mathrm{T}\,\frac{v_{T_\mathrm{e}}^2}{c^2}\,\frac{z}{\exp(z)-1}\int_{-\infty}^{+\infty}\frac{y^2\exp(-y^2)\,\mathrm{d}y}{\sqrt{\pi}}\\ &\times\int_{-1}^{1}\mathrm{d}x\,\frac{2(1+x)-z\exp z/[\exp(z)-1]}{\left|F(x,y)\right|^2}\\ &\times(1+x^2)(1-x)\,,\end{aligned} \tag{85}$$

$$\begin{aligned}\sigma_1^{\mathrm{st}}(\omega) = &\frac{3}{2}\,\frac{v_{T_\mathrm{e}}^2}{c^2}\,\frac{z}{\exp(z)-1}\int_{-\infty}^{+\infty}\frac{y^2\exp(-y^2)\,\mathrm{d}y}{\sqrt{\pi}}\\ &\times\int_{-1}^{1}\mathrm{d}x\,\frac{x(1-x)(1+x^2)}{\left|F(x,y)\right|^2}\,.\end{aligned} \tag{86}$$

By using the perturbation theory in the transport equation one can consider the effect of the sum of frequency diffusion and stimulated scattering on the value of the solar opacity in the centre of the Sun (the opacity can be introduced phenomenologically as a factor between the radiation flux and temperature gradient). Then we find a modification of expression (34) due to the operator character of the corresponding contributions:

$$\begin{aligned}\frac{\kappa_\mathrm{R}-\kappa_\mathrm{R}^{(0)}}{\kappa_\mathrm{R}^{(0)}} = &\int_0^\infty\frac{z^3\,\mathrm{d}z}{\sigma_0(z)}\left\{\sigma_0^{\mathrm{st}}(z)+z\,\frac{\partial}{\partial z}\left[\sigma_1^{\mathrm{fd}}(z)+\sigma_1^{\mathrm{st}}(z)\right]\right.\\ &\left.\times z^2\,\frac{\partial^2}{\partial^2 z}\,\sigma_2^{\mathrm{fd}}(z)\right\}\frac{z\exp z\,\mathrm{d}z}{\left[\exp(z)-1\right]^2}\\ &\times\left(\int_0^\infty\frac{z^4\exp z\,\mathrm{d}z}{\sigma_0(z)\left[\exp(z)-1\right]^2}\right)^{-1}\end{aligned} \tag{87}$$

The result of numerical calculations taking into account the broadening of the Raman resonance is

$$\frac{(\delta\kappa_\mathrm{R}^{\mathrm{fd}}+\delta\kappa_\mathrm{R}^{\mathrm{st}})}{\kappa_\mathrm{R}^{(0)}} = -4.5\%\,. \tag{88}$$

This is a rather large effect which can not be neglected further in the construction of SSMs.

16. Quantum effects in scattering

Quantum effects in scattering are of relevance only for electrons. In vacuum for 'naked' electrons the quantum effects are described by the well known Klein – Nishina formula [15]. For the case of collective scattering those results can not be applied. Collective quantum scattering was considered only recently [36, 37] in connection with the problem of scattering in the solar interior and only the first quantum corrections in the parameter $\hbar\omega/m_\mathrm{e}c^2$ were obtained. But this is sufficient for our purpose.

We can start our estimation by excluding collective effects from the first stage. This estimation can be obtained from the first quantum corrections using the Klein – Nishina formula. Even from this estimate one can see that for problems of energy transfer the quantum corrections can be of the order of or larger than the relativistic corrections. Thus summing all the effects which have in front of them an additional factor $v_{T_\mathrm{e}}^2/c^2$ we shall need to include also the quantum corrections. Indeed, from the Klein – Nishina formula we obtain the first quantum corrections for scattering on nonrelativistic electrons in the form:

$$\sigma_\mathrm{e}^{\mathrm{sc,\,q,\,KL-N}} = \sigma_\mathrm{T}\left(1-\frac{2\hbar\omega}{m_\mathrm{e}c^2}\right). \tag{89}$$

The additional term $2\sigma_\mathrm{T}\hbar\omega/m_\mathrm{e}c^2$ can also be written in the form $2\sigma_\mathrm{T}zv_{T_\mathrm{e}}^2/c^2$. By taking into account that the weighting factor in the Rosseland opacity has a maximum at $z = 3.7$ we indeed see that the quantum corrections are of the same order of magnitude as the relativistic corrections.

But in the solar interior the scattering is collective and the use of the Klein – Nishina formula is not justified. In building a quantum collective theory of scattering it was necessary to reconsider many problems and solve cumbersome equations. One can use the general theory of fluctuations but to get from the general expressions even a first quantum correction is difficult since one needs first to separate those effects which are related with scattering from other effects. Even a classical theory of fluctuations is not that simple for such a separation. There exist three other approaches known in classical plasma physics to calculate the scattering probability from which the author of Ref. [36] used the quantum generalisation of the approach related to the momentum diffusion of the electron distribution on the electrostatic beat wave produced by the initial and the scattered waves. This approach allows us to find the expression of the collective part of the scattering matrix element including the first quantum corrections in the parameter $\hbar\omega/m_\mathrm{e}c^2$. Then it is possible to show that the matrix element of the Klein-Nishina scattering has no corrections to the matrix element of Thomson scattering to first order in this parameter [36]. Thus the Klein – Nishina corrections (89) appear only from the δ-function describing the quantum conservation law of momentum and energy in the scattering process (they describe only the recoil effect in scattering). Thus the matrix element of individual scattering can be taken as the matrix element of Thomson scattering in the case that we are interested only in the first order quantum corrections on the parameter $zv_{T_\mathrm{e}}^2/c^2$. A cumbersome calculation for the collective matrix element then show that it differs from the classical one only by containing the quantum expression for the dielectric permittivity instead of its classical limit in the classical expression. The quantum expression for the dielectric permittivity appears in the probability for the frequency equal to the difference of the frequencies of two waves (the scattered one and the scattering one) and the wave vector equal to the difference of the wave vectors of these waves. We shall write down here the expression for the quantum dielectric permittivity for the frequency ω and the wave vector $\mathbf{k}$:

$$\epsilon_{\omega,\mathbf{k}} = 1+\frac{4\pi e^2}{k^2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\,\frac{\Phi_\mathbf{p}-\Phi_{\mathbf{p}-\hbar\mathbf{k}}}{\hbar\omega+\epsilon_{\mathbf{p}-\hbar\mathbf{k}}-\epsilon_\mathbf{p}+\mathrm{i}0}\,. \tag{90}$$

With this dielectric permittivity the quantum expression for the scattering probability has the form similar with (52):

$$W_{\mathrm{p}}(\mathbf{k},\mathbf{k}') = \frac{(2\pi)^3 e^4}{2m_{\mathrm{e}}^2 \omega\omega'}(1+x^2)\left|\frac{1}{\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'}}\right|^2 \times \delta\left(\omega-\omega'-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}-\hbar\,\frac{(\mathbf{k}-\mathbf{k}')^2}{2m_{\mathrm{e}}}\right). \quad (91)$$

It is necessary at this point to stress that in Eqn (91) not only the corrections of the order of $zv_{T_{\mathrm{e}}}^2/c^2$ appear (as they appear in the Klein–Nishina formula) but also dielectric permittivity corrections of the order of $\hbar k/p \approx zv_{T_{\mathrm{e}}}/c$ and of the order of $z^2 v_{T_{\mathrm{e}}}^2/c^2$ can appear. The second small parameter is much larger than the parameter in the Klein-Nishina expansion. The second order term in this parameter has in front of it the factor $v_{T_{\mathrm{e}}}^2/c^2$, i.e. it is of the same order as the relativistic effects. These new terms of the order of $z^2 v_{T_{\mathrm{e}}}^2/c^2$ appear only from the expansion of the dielectric constant and thus are essential only in the collective regime of scattering. The expansion contains either terms of the order of $zv_{T_{\mathrm{e}}}^2/c^2$ or the terms of the order of $z^2 v_{T_{\mathrm{e}}}^2/c^2$. Taking into account the presence of z^2 in the last term we can then find when expanded in it the weighting factor in the opacity will have a maximum close to z equal to 6, but not 3.7 as it was before. This will enlarge all the quantum contributions. Thus although they all have the same factor $v_{T_{\mathrm{e}}}^2/c^2$ as the other relativistic contributions the presence of an additional factor z^2 can make their contribution large enough. There is another problem related to the first order corrections in this parameter $zv_{T_{\mathrm{e}}}/c$. Although in the expansion of all expressions the first order corrections vanish due to the symmetry of the distribution of electrons, the expansion is not valid inside the Raman resonance. When considering the Raman resonance broadening we shall keep all the linear terms in this parameter in the denominator of the Raman resonance as we kept above the linear terms for the Doppler effect.

Quantum dielectric permittivity can also be expressed through the plasma dispersion function $W(s)$, as its classical value. The parameter s will be given by

$$s = \frac{\omega-\omega'}{|\mathbf{k}-\mathbf{k}'|\sqrt{2}\,v_{T_{\mathrm{e}}}}\,. \quad (92)$$

We shall not need this general expression and give only its approximation up to the first order expansion in the parameter $z^2 v_{T_{\mathrm{e}}}^2/c^2$:

$$\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'} \approx 1 + \frac{\omega_{\mathrm{pe}}^2}{|\mathbf{k}-\mathbf{k}'|^2 v_{T_{\mathrm{e}}}^2}\left(W(s) + \frac{\varkappa^2}{6}\frac{\partial^2}{\partial s^2}W(s)\right), \quad (93)$$

where

$$\varkappa = \frac{\hbar|\mathbf{k}-\mathbf{k}'|}{2\sqrt{2}\,m_{\mathrm{e}} v_{T_{\mathrm{e}}}}\,. \quad (94)$$

It is sufficient to use the first approximation for $|\mathbf{k}-\mathbf{k}'| = \sqrt{2(1-x)}\,\omega/c$ and then in the correction term in Eqn (93) we have $\varkappa^2 = z^2(1-x)v_{T_{\mathrm{e}}}^2/(4c^2)$ which shows that indeed the corrections are of the order of $z^2 v_{T_{\mathrm{e}}}^2/c^2$, but not of the order of $zv_{T_{\mathrm{e}}}^2/c^2$ as in the case of non-collective scattering.

In the classical limit the conservation law of the momentum and energy in the scattering gives $s = y$, but not in the quantum case where we need to use an expansion up to terms of second order in the parameter $v_{T_{\mathrm{e}}}/c$; this is written below. We mention here another important point which shows that the expansion we wanted to use will be different in different terms of the transport equation. Indeed, the transport cross-section is obtained by balancing the processes of direct and inverse scattering. The inverse process is determined by the electrons, the initial momentum of which differs from the initial momentum of electrons in a direct process by the momentum transferred in the scattering process $\hbar(\mathbf{k}-\mathbf{k}')$. The particle momentum enters in the probability only under the sign of the δ-function describing the quantum conservation law of energy and momentum in the scattering process. By shifting the particle momentum by the amount $\hbar(\mathbf{k}-\mathbf{k}')$ we first convert the particle distribution in the final state to that of the initial one (which means that we should only average over the initial distribution in the transport equation) and secondly, the only change in the probability will be the sign of the quantum corrections in the term describing the process of inverse scattering.

Bearing in mind the possibility of performing this type of simplification in the transport equation, we shall write down the expression for s and other quantities such as the photon frequency ω' after scattering with two signs of z corresponding to different signs of the quantum corrections in the expressions for direct (+) and inverse scattering (−). We shall denote the corresponding expressions with a subscripts ± and shall take into account all terms of expansion up to second order in the parameter $v_{T_{\mathrm{e}}}/c$:

$$\omega'_{\pm} = \omega\left(1 - 2\,\frac{v_{T_{\mathrm{e}}}}{c}\,y\sqrt{1-x} + \frac{2\,v_{T_{\mathrm{e}}}^2}{c^2}\,y^2(1-x) \mp z\,\frac{v_{T_{\mathrm{e}}}^2}{c^2}(1-x)\right), \quad (95)$$

$$(\mathbf{k}-\mathbf{k}')_{\pm}^2 = \frac{2\omega^2(1-x)}{c^2}\left\{1 - 2\,\frac{v_{T_{\mathrm{e}}}}{c}\,y\sqrt{1-x} + \frac{v_{T_{\mathrm{e}}}^2}{c^2}\left[2(2-x)y^2 \mp z(1-x)\right]\right\}, \quad (96)$$

$$s_{\pm} = y \pm \frac{z}{2}\left[\frac{v_{T_{\mathrm{e}}}}{c}\sqrt{1-x} - \frac{v_{T_{\mathrm{e}}}^2}{c^2}\,y(1-x)\right], \quad (97)$$

$$A_{\pm}(x,y) = 1 - 3\,\frac{v_{T_{\mathrm{e}}}}{c}\,y\sqrt{1-x} + 2\,\frac{v_{T_{\mathrm{e}}}^2}{c^2}\,y^2(3-2x) \mp 2\,\frac{v_{T_{\mathrm{e}}}^2}{c^2}\,z(1-x), \quad (98)$$

$$F_{\pm}(x,y) = 1 + \frac{\delta_{\mathrm{e}}}{1-x}\,W(s_{\pm})\left[1 + 2\,\frac{v_{T_{\mathrm{e}}}}{c}\,y\sqrt{1-x} - 2\,\frac{v_{T_{\mathrm{e}}}^2}{c^2}(2-x) \pm \frac{v_{T_{\mathrm{e}}}^2}{c^2}\,z(1-x)\right] + \delta_{\mathrm{e}}\,\frac{z^2 v_{T_{\mathrm{e}}}^2}{24c^2}\,\frac{\partial^2}{\partial y^2}\,W(y)\,. \quad (99)$$

Then the transport cross-section which takes into account the collective quantum corrections can be written in the form:

$$\sigma_{\mathrm{e}}^{\mathrm{sc,\,q,\,coll}} = \frac{8}{3}\,\sigma_{\mathrm{T}}\int_{-\infty}^{+\infty}\frac{\exp(-y^2)\,\mathrm{d}y}{\sqrt{\pi}} \times \int_{-1}^{1}\mathrm{d}x\,(1+x^2)\left[\frac{A_+(x,y)}{|F_+(x,y)|^2} - x\,\frac{A_-(x,y)}{|F_-(x,y)|^2}\right]. \quad (100)$$

The result of numerical computations using the last formula without the collisional broadening of the Raman resonance is

$$\frac{\delta\kappa_{\mathrm{R}}^{\mathrm{sc,q}}}{\kappa_{\mathrm{R}}^{(0)}} = -0.7\% . \tag{101}$$

By taking into account the collisional broadening of the Raman resonance we get

$$\frac{\delta\kappa_{\mathrm{R}}^{\mathrm{sc,q,br}}}{\kappa_{\mathrm{R}}^{(0)}} = -1.0\% . \tag{102}$$

17. Quantum effects in frequency diffusion and stimulated scattering

The effects of frequency diffusion are related to the inverse process of scattering, which means that to take into account the quantum effects in them it is necessary to substitute $A_-(x,y)$ for $A(x,y)$ and substitute $F_-(x,y)$ for $F(x,y)$. For the stimulated scattering the result taking into account the quantum corrections can be written in a more compact form:

$$\sigma_0^{\mathrm{st,q}}(z) = -\frac{3}{8}\frac{z}{\exp(z)-1}\,\sigma_{\mathrm{T}}\int_{-\infty}^{+\infty}\frac{\exp(-y^2)\,\mathrm{d}y}{\sqrt{\pi}} \times \int_{-1}^{1}\mathrm{d}x\,(1+x^2)\left[\frac{A_+(x,y)}{|F_+(x,y)|^2} - \frac{A_-(x,y)}{|F_-(x,y)|^2}\right], \tag{103}$$

$$\sigma_1^{\mathrm{st,q}}(z) = -\frac{3}{8}\frac{z}{\exp(z)-1}\,\sigma_{\mathrm{T}}\int_{-\infty}^{+\infty}\frac{\exp(-y^2)\,\mathrm{d}y}{\sqrt{\pi}} \times \int_{-1}^{1}\mathrm{d}x\,(1+x^2)\,\frac{(\omega'_{\pm}-\omega)}{\omega} \times \left[\frac{A_+(x,y)}{|F_+(x,y)|^2} - \frac{A_-(x,y)}{|F_-(x,y)|^2}\right]. \tag{104}$$

The numerical calculations using formula (87) without the collisional broadening of the Raman resonance gives

$$\frac{\delta\kappa_{\mathrm{R}}^{\mathrm{fd,q}} + \delta\kappa_{\mathrm{R,q}}^{\mathrm{st}}}{\kappa_{\mathrm{R}}^{(0)}} = -1.0\% . \tag{105}$$

By taking into account the collisional broadening of the Raman resonance we get

$$\frac{\delta\kappa_{\mathrm{R}}^{\mathrm{fd,q,br}} + \delta\kappa_{\mathrm{R}}^{\mathrm{st,q,br}}}{\kappa_{\mathrm{R}}^{(0)}} = -5.5\% . \tag{106}$$

The last figure is preliminary. After subtracting the effect of frequency diffusion in which the quantum corrections are not accounted for, we find that the pure quantum correction both for scattering itself and for stimulated scattering and frequency diffusion is close to -2%.

18. Quantum corrections due to partial electron degeneracy

The change in solar opacity due to partial electron degeneracy was first considered in Ref. [16]. But together with degeneracy, relativistic corrections were taken into account by using the $G(\tau,z)$ factor derived from the Klein–Nishina formula which can not be used for the conditions in the solar interior. To separate the effect of degeneracy from the incorrect relativistic correction, we calculate using the results of Ref. [39] only the effect of degeneracy without the relativistic corrections which have already been discussed above in detail. The corrections due to electron degeneracy should be added to the results given above for relativistic corrections, including the collective effects. The result of our numerical calculation is

$$\frac{\delta\kappa_{\mathrm{R}}^{\mathrm{deg}}}{\kappa_{\mathrm{R}}^{(0)}} = -2.0\% . \tag{107}$$

19. The table of new collective effects in the Rosseland opacity at the centre of the Sun

We shall give the final Table for the ratio of the calculated new effects related to a zero order of the Rosseland opacity which does not take into account the line absorption (an approximate transition coefficient relating the results to the total Rosseland opacity accepted at the present time is 2/3).

Table

No.	The name of the effect	$\delta\kappa_{\mathrm{R}}/\kappa_{\mathrm{R}}^{(0)}$, %
1	Doppler and collisional broadening of Raman resonance	−3.0
2	Relativistic corrections for scattering on electrons and electron polarization cloud of ions	−0.2
3	Diffusion in frequencies and stimulated scattering	−4.5
4	Collective effects in bremsstrahlung	−0.2
5	Relativistic effects in bremsstrahlung	+0.2
6	Quantum effects in scattering	−2.0
7	Effects of electron degeneracy	−2.0
8	Refractive index effects	+0.1
9	Density inhomogeneity effects	−0.1
	Sum	−11.7
	2/3 of the sum	−7.8

Recently (see Refs [39, 40]) the corrections due to ion correlations were calculated in Ref. [39] and are of the order of -1.5%. In the case one added this value to the -7.8% given in the Table one gets -9.3%. The latest data indicate also the contribution of line absorption in the centre of the Sun could be less than that obtained previously and may be as low as 1/4 not 1/3, as assumed in the Table. If one takes this figure for iron absorption one should introduce not a factor of 2/3 but rather a factor of 3/4, which leads to -8.8% instead of -7.8% as given in the Table. After adding -1.5% for ion correlations we get $-10.3\,\%$. All these estimates are given to show the uncertainties which still exist in estimations of the value of solar opacity. Other uncertainties are discussed in the next section.

The total value of the change of the solar opacity due to the new collective effects is large and should be taken into account in future developments of any SSM. The Table gives the corrections in the central part of the Sun. For construction of a solar model it is necessary to have tables of opacities with collective effects taken into account which can be used in all regions of the Sun. Making such a table is complex and not a simple mathematical problem as our experience has shown.

But one important qualitative effect should be mentioned already. As soon as the distance from the centre of the Sun increases both temperatures and densities drop. The Debye radius containing the square root of the ratio of temperature

to density should change not as rapidly as the frequency of the maximum of the form-factor in the opacity which is $3.7T/\hbar$. So one may think that the collective effects should increase rapidly with distance from the centre of the Sun. They indeed increase but not so drastically as one can expect looking superficially at this problem. The point is that one should be interested only in regions of electromagnetic flux formation which corresponds to the region where the thermonuclear reactions take place. Burning ceases already at distances of about $0.25R_{\odot}$. In this region the temperatures decreases but not as fast as the density due to the nuclear burning. By using the existing SSM one finds the dependence of the collective parameter δ_{e} on the distance from the centre of the Sun. These data provide the proof that there exists a systematic growth of the collective parameter in the whole range of distances up to $0.25R_{\odot}$. At the edge of this region an increase of the collective parameter is 1.7 times. This shows that the collective effects are important in the whole region of formation of the electromagnetic flux and that they are growing towards the edge of this region. Thus the Table above gives only the lower limit of the collective corrections. These estimates can be improved in future after the collective effects are taken into account for all distances from the centre of the Sun. Therefore the complete coverage of the contribution of collective effects as a function of distance from the centre of the Sun will be an important problem in future research.

20. Conclusion. New problems

The results of our analysis can be summarised as follows. Previously it was accepted that the uncertainty in the value of the Rosseland opacity can not be larger than $\pm 5\%$. In this paper the total effect of collective corrections points to the conclusion that these corrections are of a definite sign and show a *decrease* in the Rosseland opacity of approximately 10%. Since solar luminosity is inversely proportional to the Rosseland opacity and proportional to T^4, this decrease of opacity leads to a decrease in the estimated temperature at the centre of the Sun by about 2.2%. A strong dependence of the high energy neutrino flux on temperature leads to a decrease in the predicted flux by a factor of 2 or 2.5. Although this result agrees better with the observations and theoretical predictions, it does not solve completely the problem of the neutrinos deficit, since, for example, the deficit of beryllium neutrinos decreases less than that of boron neutrinos, while the observations seem to indicate that the deficit of beryllium neutrinos is larger than that of boron neutrinos. But the beryllium neutrinos are not measured directly in any experiment. The difference of the deficits of neutrinos in different energy channels may also be due to the influence of collective plasma effects on the nuclear reactions and particularly on the process of capture of protons by beryllium ions in the dense solar plasma. Other problems also exist which have been underscored in the foregoing analysis. But independently of them, the 10% change in opacity is a rather large effect in the problem of the solar neutrinos deficit.

It bears repeating that we estimated *all* collective plasma effects which, for δ_{e} of the order of one, have a smallness of $v_{T_{\mathrm{e}}}^2/c^2$, a complete search of such effects was performed. We use new analytical results previously not calculated for most of these effects. The numerical results were double-checked by standard numerical programs and by a special numerical program developed for this problem at the Institute of Applied Mathematics (Naples). The numerical program developed in the Institute of Applied Mathematics is rather large and it checks the accuracy at each stage of the calculations. The standard numerical program takes more than one week on a PC to calculate one point of the corrections. In the Table the results of computations performed in the Institute of Applied Mathematics (Naples) are given.

We shall give the limits of the uncertainties of the results. These uncertainties are related to the new problems which arise from the considerations made in this paper. The following is a list of the problems and some comments to this list.

1. Collective effects in line absorption.

The most important is the Fe ion line the wavelength of which almost exactly coincides with the Debye length. Up to the present time the line absorption on Fe ions was considered as if it were in a vacuum. There exist methods to treat the problem when both the bounded electrons and the Debye shielding electrons contribute to the absorption. Bremsstrahlung absorption in the case where the frequency is close to the resonance has some specific features and the best term is line absorption [23]. The method is explained in detail in Chapter 6 of the monograph [23]. Since the relative contribution to the opacity of the absorption on iron ions is approximately $1/3$ even a relatively small correction to the absorption by iron ions can change substantially the opacity. This problem is not yet solved and should be the subject of future investigations. The collective effects in absorption on iron ions should be borne in mind when estimating the remaining uncertainties in opacity.

2. Solutions of the radiation transport equations with effects of frequency diffusion and stimulated scattering without using perturbation methods.

The problem is only formulated for the general collective case. For scattering in vacuum some preliminary results are obtained. The observed effect of rapid changes of intensity with frequency probably indicates that a more precise consideration of the frequency diffusion effects and the stimulated scattering effects will result in a further decrease of the value of the predicted opacity.

3. Collective effects in capturing protons and electrons by ^{7}Be ions.

As the problem is formulated above, in plasma the electrons (or protons) simultaneously play a role of the particle which can be captured and the role of the particle which contributes to the Debye screening. This has not yet been taken into account. The proper consideration can be made by developing the theory of fluctuations in a plasma which takes into account the possibility of electron capturing by the nuclei of ions.

4. The possibility of different electron and ion temperatures in the solar interior.

Although this possibility is interesting, making a definite statement that this possibility can be realised in the solar interior would require performing additional investigations. In many laboratory plasma the electron temperature is higher than the ion temperature. An example of this kind of plasma (which seems in some sense to model the solar conditions) is radio frequency discharges in plasma where the electrons receive the energy from the RF external field faster than they transfer the energy to ions. These experiments of course are opposite to the solar conditions in that they correspond to the case of optically thin plasma. Nevertheless in the solar interior radiative transfer occurs in a similar manner since the

radiation transferred is absorbed on electrons and the time of the conversion of this energy to ions via electron – ion binary collisions is five times larger than the characteristic time of receiving the energy by electrons from the radiation. This estimation does not mean that we insist on the existence of a difference between the electron and ion temperatures inside the solar interior. But this problem we think should be analysed in more detail bearing in mind the possibility of the development of instabilities which often make the electron temperature larger than the ion temperature.

Why is this effect, if it exists, of such importance? First of all, even from the formulas of the zero approximation given in this article for the case of an arbitrary difference between electron and ion temperatures it is easy to find that the additional decrease of the opacity taking into account the factor $2/3$ will be -2.7%. Secondly, solar seismology shows the presence of sound waves which, from a general point of view, can form the usual cascade toward smaller wavelengths. For the case of equal temperatures this cascade can not propagate in the collisionless range of sound frequencies but it may propagate in the presence of a temperature difference. If the latter is the case, the well known plasma physics effect predicts a formation of a tail in the proton distribution which can alter the predictions of the neutrino flux. All these possibilities are, however, at the present time purely speculative.

5. A possibility of the development of instabilities generated by the transport process.

This problem is only partially discussed in the text. Note that photons with different frequencies are absorbed over different lengths. The question is whether this effect can produce unstable nonthermal electron distributions.

6. Anomalous transfer of radiation due to the possibility of the presence of turbulence in the central regions of the Sun.

There exist some indications that the central region of the Sun may have a larger differential rotational velocity as well as other indications that the central region of the Sun may be convectionally unstable. If these statements are observationally valid it will be necessary to analyse the possibility of anomalous radiative transfer of radiation.

7. Energy transport by fast particles generated by turbulence.

This problem has yet to be calculated.

In the construction of future SSMs it will be necessary to take into account not only the new collective plasma effects discussed above, but also the problems stated in the conclusions.

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