

METHODOLOGICAL NOTES

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On superluminal light spots

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Contents

Abstract. The paper presents a model describing a moving emitter as a local polarization region which is produced by a light spot from a moving two-dimensional Gaussian beam and scans across a plane dielectric surface. The spectral composition, angular spectrum and the refraction, reflection, and polarization properties of such a beam are examined. The amplitude coefficients of refraction and reflection of the moving Gaussian beam are found and its total reflection behaviour and the total polarization angle are examined.

1. Introduction

The model of a 'reflected light spot' acting as a light emitter was originally proposed by I M Franck [1]. Suppose that two dielectrics with refraction indices n_0 and n are separated by a plane interface (Fig. 1). A narrow electromagnetic pulse whose wave vector $\mathbf{k}_0$ makes an angle μ with the normal to the interface, is incident on it. The pulse propagates at a rate $v_0 = c/n_0$. It is obvious that the light spot AB travels along the interface at a rate $v = c/n_0 \sin\alpha$. If this parameter satisfies the condition of Cherenkov-radiation occurrence in the material below the interface

$$v > \frac{c}{n}\,, \tag{1}$$

then travelling polarization generated by the light spot emits radiation at the Cherenkov angle θ (Fig. 1) determined by the formula

$$\cos\theta = \frac{c}{vn} = \frac{n_0 \sin\mu}{n}\,. \tag{2}$$

As was noted by Franck, condition (2) is identical to the Snell's law of light refraction. Really, the Cherenkov angle θ

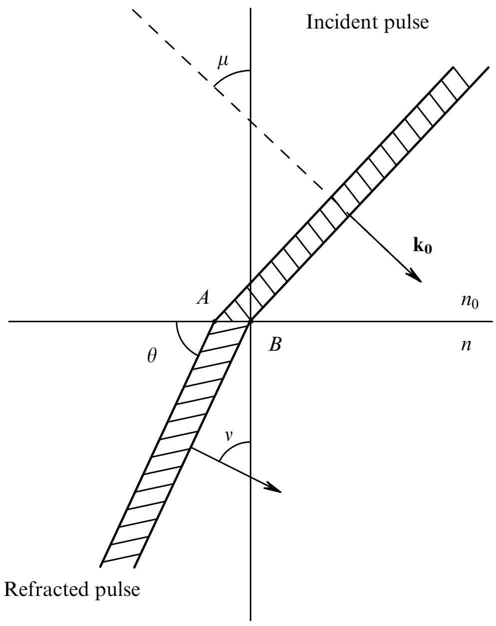

Figure 1. Incidence of an optical pulse on the interface between two dielectrics.

equals $\pi/2 - v$ (Fig. 1), and if this relation is substituted into Eqn (2), we obtain the well-known formula for the angle of refraction. Franck also noted that, if a diffraction grating is placed at the interface, the travelling polarization spot models a moving light emitter characterized by its inherent frequencies. Franck's idea of constructing the light emitters travelling faster than light was developed by several authors [2 – 5]. Beside an optical pulse, they considered an electron beam moving normally to the front of a flux and at an angle to a surface, a light spot produced by a rotating source, an electron beam generating a spot at the interface moving in a prescribed manner [4, 5], etc. (see the review [3] and Ref. [6]). One of the most feasible techniques for generating the travelling spots is to rotate or sway a sufficiently narrow beam. If the beam is shaped by spherical or parabolic mirrors,

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like those in laser cavities, it takes the form of a Gaussian beam with well-known properties [7]. Its characteristics, however, are different when it is moving. So, when a beam travels at right angles to its axis, its wave surfaces are no longer perpendicular to its axis, and its refraction demonstrates peculiar anisotropic properties. For simplicity sake we shall use a two-dimensional model of a Gaussian beam, which can be easily generalized to the case of a real three-dimensional beam.

2. Two-dimensional Gaussian beam

Let there be a reference frame x', y', z', t'. We shall describe the Gaussian beam as a superposition of plane waves:

$$\Psi_i'(x', z', t') = \int_{-\infty}^{\infty} A_i'(\eta) \exp(-\eta^2 r_0^2 + \mathrm{i}\eta x' - \mathrm{i}\varkappa z' - \mathrm{i}\omega' t') \, \mathrm{d}\eta \,, \tag{3}$$

where Ψ_i' is the electric vector component E_y in the case of TE-polarization or H_y in the case of TM-polarization, A_i are the amplitudes of the corresponding plane waves in the superposition determined by the emission properties of a beam, $\varkappa = (\omega'^2/c^2 - \eta^2)^{1/2}$, r_0 is the parameter characterizing the effective beam width. In what follows, the condition $\omega' r_0/c \gg 1$ is assumed to be satisfied.

In this case the integral on the right of Eqn (3) can be approximately calculated. Really, accurate up to a η^2 factor one gets $\varkappa = k' - \eta^2/2k'$, where $k' = \omega'/c$ and the integral in Eqn (3) is reduced to the well-known standard integral at small η:

$$\int_{-\infty}^{\infty} \exp(-\alpha x^2) \, \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}} .$$

Hence

$$\Psi_1'(x', z', t') = \sqrt{\frac{2\pi k'}{k' r_0^2 - \mathrm{i}z'}} A'(0) \times \exp\left\{ -\frac{x'^2}{2R^2} - \mathrm{i}\left[\omega' t' + k'\left(z' + \frac{x'^2}{2l}\right)\right]\right\} , \tag{4}$$

where

$$R^2 = r_0^2 + \frac{z'^2}{r_0^2 k'^2} \,, \qquad l = z' + \frac{r_0^4 k'}{z^2} \,.$$

The amplitude distribution in the beam is described by the function $\exp(-x'^2/2R^2)$. If we define the beam boundary as a geometric locus where its amplitude is lowered by a factor equal to e as compared to the beam axis value at $x' = 0$, then the boundary is described by the hyperbolic equation

$$\frac{x'^2}{2r_0^2} - \frac{z'^2}{k'^2 r_0^4} = 1 \,. \tag{5}$$

In the beam 'bottleneck' (for $z' = 0$) its half-width is r_0. Figure 2 shows a Gaussian beam in the region $z' > 0$.

The beam divergence due to diffraction is governed by the equation

$$\varphi' = \arctan \frac{1}{k_0' r_0} \approx \frac{\lambda'}{2\pi r_0} \,,$$

where $\lambda' = 2\pi c/\omega'$.

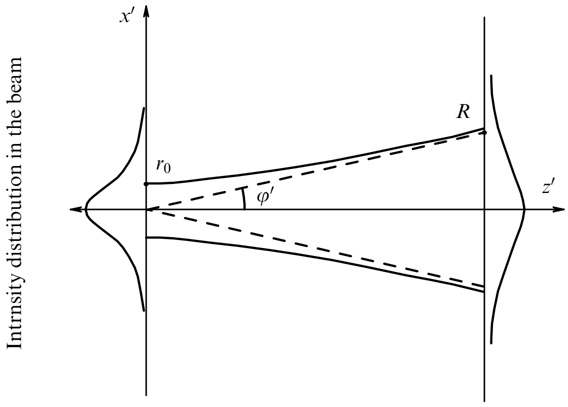

Figure 2. Gaussian beam.

The 'bottleneck' width of a tightly focused beam should accommodate a wealth of light wavelengths. In what follows we shall consider a beam moving in the direction perpendicular to its axis at a velocity v. Because of relativistic contraction, the beam is focused in the laboratory reference system more tightly and

$$\varphi' = \frac{\lambda'}{2\pi r_0 \gamma} \,,$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic factor.

The surface of constant phase in a two-dimensional Gaussian beam described by Eqn (4) is defined by the equation

$$z' + \frac{x'^2}{2l} = \text{const} \,. \tag{6}$$

If $z' \gg r_0^2 k$ (a Fraunhofer zone), then the surface is cylindrical.

3. Moving light beam

Let us now cause the beam defined by Eqn (1) to move normally to its axis and, at the same time, turn it through an angle $\pi/2 - \alpha$, where $0 < \alpha \leqslant \pi/2$, by going over to another reference frame with the coordinates x, y, z, t defined by the transformations

$$x' = \gamma(x \sin\alpha - z\cos\alpha - vt) \,,$$

$$z' = x\cos\alpha - z\sin\alpha \,,$$

$$t' = \gamma\left(t - \frac{1}{c}\beta x \sin\alpha + \frac{1}{c}\beta x\cos\alpha\right) ,$$

where v is the transverse velocity of the beam, $\beta = v/c$,

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}}$$

is the relativistic factor (Fig. 3).

The field in the beam is determined by the equation

$$\Psi_{1,2}(x, z, t) = \int_{-\infty}^{\infty} A_i(k) \exp(-k^2 r_0^2 + \mathrm{i}\Phi) \, \mathrm{d}k \,, \tag{7}$$

where $\Psi_1 = E_y$ (TE-polarization) or $\Psi_2 = H_y$ (TM-polarization), and

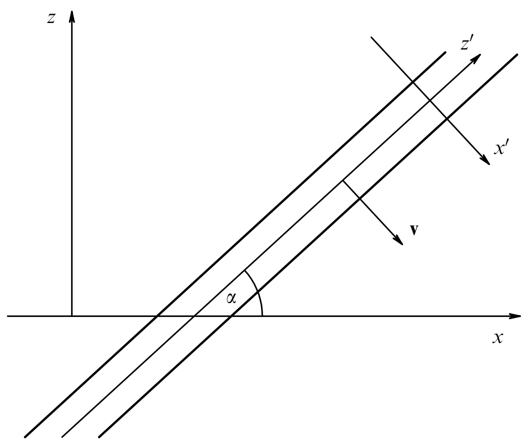

Figure 3. Configuration of a moving light beam.

$$\begin{aligned}\Phi &= k_x x - k_z z - \omega t\,,\\ k_x &= \gamma\left(k + \frac{\omega'}{c}\,\beta\right)\sin\alpha - \varkappa\cos\alpha\,,\\ k_z &= \gamma\left(k + \frac{\omega'}{c}\,\beta\right)\cos\alpha + \varkappa\sin\alpha\,,\\ \omega &= \gamma(kv + \omega')\,.\end{aligned} \tag{8}$$

Other components of the field can be easily calculated using the Maxwell's equations.

Let us introduce the wave vector $\mathbf{k} = (k_x, 0, -k_z)$ of plane waves whose superposition in Eqn (7) yields the field of a moving beam. The 'fundamental' wave in this superposition is the wave with $k = 0$. It is convenient for our further analysis to introduce the angle ψ_0 between the vector $\mathbf{k}_0$ of the fundamental wave corresponding to $k = 0$ and the x-axis. It directly follows from Eqn (8) at $k = 0$ that

$$\tan\alpha = \frac{\sin\psi_0 + \gamma\beta\cos\psi_0}{-\cos\psi_0 + \gamma\beta\sin\psi_0}\,. \tag{9}$$

Equation (9) allows us to rewrite Eqns (8) as

$$\begin{aligned}k_x &= \frac{\omega_0}{c}\cos\psi_0 + k(\sin\psi_0 + \gamma\beta\cos\psi_0)\\ &\quad + \frac{k^2 c}{2\omega_0}\,(\gamma\beta\sin\psi_0 - \cos\psi_0)\,,\\ k_z &= \frac{\omega_0}{c}\sin\psi_0 + k(\gamma\beta\sin\psi_0 - \cos\psi_0)\\ &\quad - \frac{k^2 c}{2\omega_0}(\sin\psi_0 + \gamma\beta\cos\psi_0)\,,\\ \omega &= \omega_0 + \gamma k v\,,\end{aligned} \tag{10}$$

where $\omega_0 = \gamma\omega'$. The summands in k_x, k_z of order k^3 and those of higher orders in k^3 are omitted in Eqn (10) since the main contribution to the integral in Eqn (3), as it follows from Eqs (3) and (7), comes from the interval of small k at small enough r_0.

Notice that, as seen from Eqn (10), different light frequencies correspond to different directions of $\mathbf{k}$, i.e. the beam spectrum is no longer monochromatic. If we again restrict our discussion to small k and consider the terms accurate within a k^2 factor, then the beam spectrum is described by the simple formula

$$\omega = \omega'\,\frac{1 + \beta(\psi - \psi_0)}{\sqrt{1 - \beta^2}}\,, \tag{11}$$

where ψ is the angle between the wave vector $\mathbf{k}$ and the x-axis. The domain of definition for this formula is given by the inequality

$$|\psi - \psi_0| < \frac{c}{r_0\omega'\gamma\cos^2\psi_0}\,.$$

At $\psi = \psi_0$ (fundamental wave) we have $\omega = \omega_0 = \omega'/(1 - \beta^2)^{1/2}$, and at relativistic velocities of the beam the light frequency is increased abruptly.

The shape of the moving Gaussian beam remains regular. Since it is moving perpendicular to its axis, all its transverse dimensions are reduced by a factor equal to $1/\gamma$. The beam axis by definition always makes the angle α with the x-axis, but the phase surfaces within the beam reshape. So, the normal to the wave front is directed at an angle ψ_0 related to α by Eqn (9) (Fig. 4).

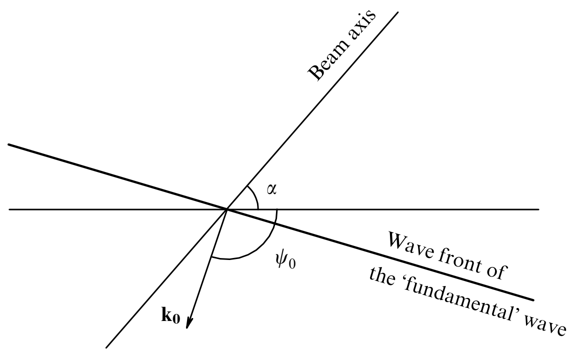

Figure 4. Turn of the wave front of the 'fundamental' wave in the beam motion.

4. Study on the 'geometry' of a refracted moving Gaussian beam

Let us consider the geometrical aspect of the refraction of the moving Gaussian beam at the plane dielectric boundary. In our discussion of its basic features, we assume that the dielectric is lossless and possesses a dielectric constant ε. The dielectric boundary is defined at $z = 0$. The beam in vacuum at $z > 0$ would be described by the equations

$$E_y^{(1)} = \int_{-\infty}^{\infty} A_1(k)\exp\left[-k^2 r_0^2 + \mathrm{i}k_x x - \mathrm{i}\omega(k)\,t\right] \times \left[\exp(-\mathrm{i}k_z z) + R_1(k)\exp(\mathrm{i}k_z z)\right]\,\mathrm{d}k \tag{12}$$

for the TE-polarization and

$$H_y^{(1)} = \int_{-\infty}^{\infty} A_2(k)\exp\left[-k^2 r_0^2 + \mathrm{i}k_x x - \mathrm{i}\omega(k)\,t\right] \times \left[\exp(-\mathrm{i}k_z z) + R_2(k)\exp(\mathrm{i}k_z z)\right]\,\mathrm{d}k \tag{13}$$

for the TM-polarization. The parameters k_x, k_z and ω have been defined by Eqn (8).

In the dielectric bulk, where $z < 0$, we have

$$E_y^{(2)} = \int_{-\infty}^{\infty} A_1(k)\,T_1(k) \times \exp\left[-k^2 r_0^2 + \mathrm{i}k_x x - \mathrm{i}k_{2z} z - \mathrm{i}\omega(k)\,t\right]\,\mathrm{d}k \qquad (14)$$

and

$$H_y^{(2)} = \int_{-\infty}^{\infty} A_2(k)\,T_2(k) \times \exp\left[-k^2 r_0^2 + \mathrm{i}k_x x - \mathrm{i}k_{2z} z - \mathrm{i}\omega(k)\,t\right]\,\mathrm{d}k\,. \qquad (15)$$

The functions $R(k)$ and $T(k)$ play the part of the amplitude reflection and refraction factors, respectively, for each wave in the superposition described by Eqn (7). The index 1 identifies the TE-polarization, and index 2 the TM-polarization. Notice that $R_1(k)$ is defined as the ratio between E-amplitudes, and $R_2(k)$ as the ratio between H-amplitudes in the reflected and incident waves. The transmission factors $T_1(k)$ and $T_2(k)$ are defined similarly. The wave vector component k_{2z} in the dielectric medium may be determined from the expression

$$k_{2z} = \sqrt{\frac{\omega^2}{c^2}\,\varepsilon - k_x^2}\,. \qquad (16)$$

If k_{2z} in Eqn (16) is expanded in a power series of the small parameter k and terms up to k^3 are retained, then the exponent in Eqns (14) and (15) can be approximated, as in Section 2, by the formula

$$-k^2 r_0^2 + \mathrm{i}k_x x - \mathrm{i}k_{2z} z - \mathrm{i}\omega t = -pk^2 + qk - r\,. \qquad (17)$$

Here we present only the expression for the factor q, which is necessary for our further discussion:

$$q = \mathrm{i}\left\{(\sin\varphi_0 + \gamma\beta\cos\varphi_0)x - \frac{1}{\sqrt{\varepsilon - \cos^2\psi_0}} \times \left[\gamma\beta(\varepsilon - \cos^2\psi_0) - \cos\psi_0\sin\psi_0\right] z - \gamma v t\right\}. \qquad (18)$$

Let us transform Eqn (17) to

$$-pk^2 + qk - r = -p\left(k - \frac{q}{2p}\right)^2 + \frac{q^2}{4p} - r\,.$$

The integral in Eqn (14), as in Section 2, can be calculated approximately:

$$E_y^{(2)} = \exp\left(\frac{q^2}{4p} - r\right)\int_{-\infty}^{\infty} A_1(k)\,T_1(k)\exp\left[-p\left(k - \frac{q}{2p}\right)^2\right]\mathrm{d}k = \sqrt{\frac{\pi}{p}}\,A_1(0)\,T_1(0)\exp\left(\frac{q^2}{4p} - r\right), \qquad (19)$$

using again the well-known integral

$$\int_{-\infty}^{\infty}\exp(-\alpha x^2)\,\mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}$$

and retaining only the first terms in the expansions of the functions $A_1(k)$ and $T_1(k)$ in terms of the small parameter k. The expression for $H_y^{(2)}$, determined by Eqn (15), is derived from Eqn (19) by replacing $A_1(0)$ with $A_2(0)$, and $T_1(0)$ with $T_2(0)$.

The quantities $|E_y^{(2)}|$ and $|H_y^{(2)}|$ achieve obviously their maximal values on the geometric locus defined by the formula $|q| = 0$ or under the condition

$$(\sin\psi_0 + \gamma\beta\cos\psi_0) - \frac{z}{\sqrt{\varepsilon - \cos^2\psi_0}} \times \left[\gamma\beta(\varepsilon - \cos^2\psi_0) - \cos\psi_0\sin\psi_0\right] - \gamma v t = 0\,, \quad (20)$$

which defines the axis direction for a moving Gaussian beam in the dielectric. It is easy to verify that this axis comprises a straight line directed with respect to the x-axis at an angle χ_0 such that

$$\tan\chi_0 = \frac{(\sin\psi_0 + \gamma\beta\cos\psi_0)\sqrt{\varepsilon - \cos^2\psi_0}}{\gamma\beta(\varepsilon - \cos^2\psi_0) - \cos\psi_0\sin\psi_0}\,, \qquad (21)$$

and it travels along the x-axis at a velocity

$$v_1 = \frac{\gamma v}{\sin\psi_0 - \gamma\beta\cos\psi_0}\,. \qquad (22)$$

Using Eqn (9), let us express the last two formulae in terms of an angle α:

$$\tan\chi_0 = \frac{\sin\alpha\sqrt{\varepsilon\gamma^2 - (\cos\alpha - \gamma\beta\sin\alpha)^2}}{\gamma\beta\varepsilon + \sin\alpha(\cos\alpha - \gamma\beta\sin\alpha)}\,, \qquad (23)$$

$$v_1 = \frac{v}{\sin\alpha}\,. \qquad (24)$$

Equation (24) determines the velocity of the light spot travel over the dielectric surface. If

$$\sin\alpha = \beta\,, \qquad (25)$$

then $v_1 = c$, and if $\sin\alpha < \beta$, the light spot velocity $v_1 > c$.

The refraction angle of the moving Gaussian beam is given by Eqn (23). It is obvious that at $\beta = 0$ one obtains

$$\tan\chi_0 = \frac{\sqrt{\varepsilon - \cos^2\alpha}}{\cos\alpha}\,,$$

thus we are led to the Snell's conventional formula.

5. Refraction of a moving Gaussian beam

It makes sense to undertake a detailed investigation of Eqn (23), which determines the refraction angle of a moving Gaussian beam. Figure 5 shows the angle χ_0 between the x-axis and the axis of a refracted beam plotted as a function of the angle α between the x-axis and that of an incident beam at $\varepsilon = 1.5$ and different values of β. It is clear that $\chi_0 \to 55°$ at $\alpha = \pi/2$, $\beta \neq 0$ and $\beta \to 1$. The axis of the refracted beam deviates from the normal, although the incident beam is normal to the surface, because the angle ψ_0 between the wave vector on the beam axis and the interface, determined by Eqn (9), is not $\pi/2$. At angles of incidence close to $\pi/2$, it follows from Eqn (23) that

$$\tan\chi_0 = \frac{\sqrt{\varepsilon - \beta^2}}{\beta(\varepsilon - 1)}\,. \qquad (26)$$

At small α, the axis of a refracted beam is close to the interface provided that $\beta \neq 0$ and $\beta \neq 1$:

$$\tan\chi_0 = \frac{\sin\alpha\sqrt{\varepsilon\gamma^2 - 1}}{\gamma\beta\varepsilon}\,. \qquad (27)$$

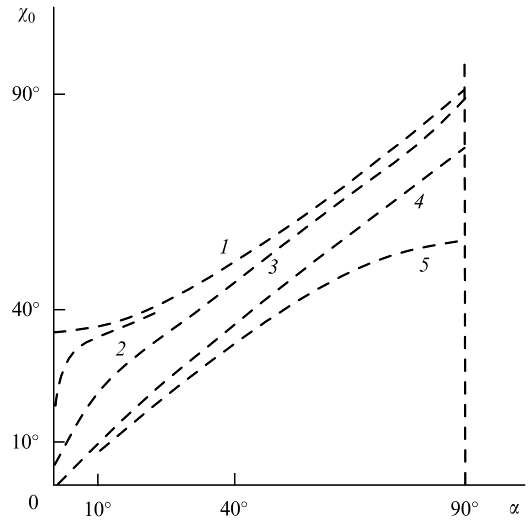

Figure 5. The angle χ_0 as a function of α at fixed $\beta > 0$ and $\varepsilon = 1.5$. Curve *1* corresponds to $\beta = 0$; *2* — $\beta = 0.01$; *3* — $\beta = 0.1$; *4* — $\beta = 0.5$; *5* — $\beta = 0.99$.

But if $\sin\alpha \approx \beta$ and $\beta_1 \approx 1$, i.e. the velocity of the light spot travel over the dielectric surface equals the light velocity, then

$$\tan\chi_0 = \frac{\sqrt{\varepsilon - 1}}{\varepsilon + 1}\left(1 - \frac{\beta^2}{\varepsilon}\right), \tag{28}$$

and in this case the refracted beam should deviate considerably from the interface at small α. For example, if $\beta \approx \sin\alpha = 0.1$, then $\alpha = 5°$ and $\chi_0 = 16°$. These properties of the refracted beam are illustrated by the curves in Fig. 5 at small α.

Figure 6 shows the plots of χ_0 versus α at various $\beta < 0$ ($\varepsilon = 1.5$). All these curves, except curve 5 corresponding to the ultrarelativistic case, have minima whose positions and depths are functions of the beam velocity. This means that at a given velocity of the light spot sliding, the refracted beams due to two beams incident at different angles α_1 and α_1 should leave the interface at the same angle χ_0. At small β, the minimum falls in the range of small α, and at larger β the minimum angle at which the refracted beam may be viewed is up to $\chi_0 = \pi/2$ or even larger, and the minimum shifts to the region of higher α. It is intriguing that at small velocities and angles of incidence $\sin\alpha \ll 1$, $\beta \ll 1$, $\sin\alpha \approx |\beta|$, and

$$\tan\chi_0 = \frac{1}{\sqrt{\varepsilon - 1}}, \tag{29}$$

for example, at $\varepsilon = 1.5$ we have $\sin\alpha = |\beta| = 0.1$ and $\chi_0 = 125°$.

Figure 7 shows the plots of the refraction angle versus β, which can be either positive or negative. The curves in the region $\beta < 0$ illustrate the situation when two different values of α correspond to one value of χ_0. At $\beta \ll 1$, Eqn (23) yields after simple calculations:

$$\tan\chi_0 = \frac{\sqrt{\varepsilon - \cos^2\alpha}}{\cos\alpha}, \tag{30}$$

which is a good approximation to the curves at $\alpha \neq 0$ and $\alpha \neq \pi/2$, whereas at ultrarelativistic velocities ($\beta \approx 1$)

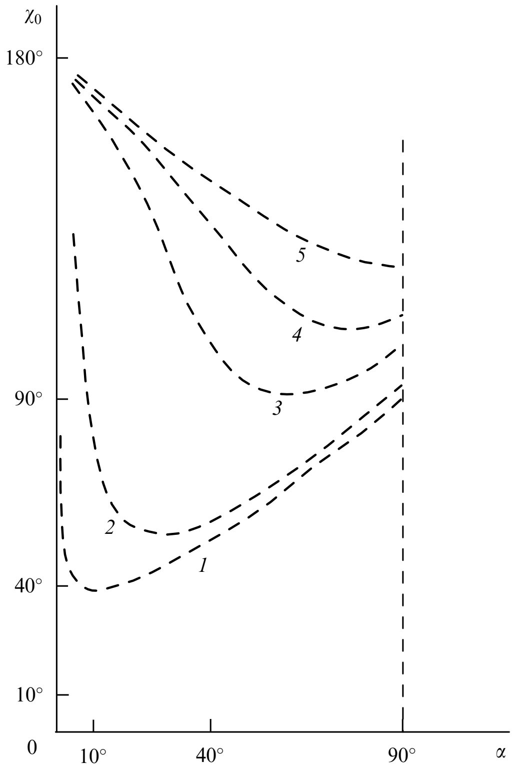

Figure 6. Plots of the angle χ_0 versus α at fixed $\beta < 0$ and $\varepsilon = 1.5$. Curve *1* corresponds to $|\beta| = 0.01$; *2* — $|\beta| = 0.1$; *3* — $|\beta| = 0.5$; *4* — $|\beta| = 0.75$; *5* — $|\beta| = 0.99$.

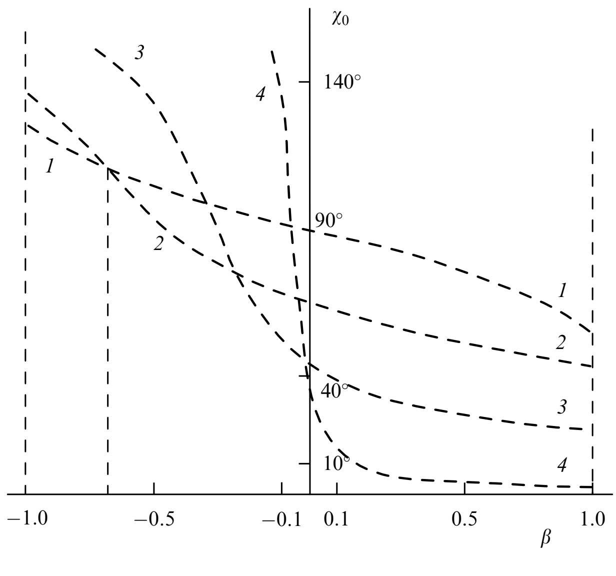

Figure 7. The angle χ_0 as a function of β at $\varepsilon = 1.5$ and the fixed angle α. Curve *1* corresponds to $\alpha = 90°$; *2* — $\alpha = 60°$; *3* — $\alpha = 30°$; *4* — $\alpha = 5°$.

$$\tan\chi_0 = \frac{\sin\alpha}{\beta\sqrt{\varepsilon - \sin^2\alpha}} \tag{31}$$

and $\chi_0(\beta) = \chi_0(-|\beta|) + \pi/2$.

Equation (23) also yields a simple formula for the total internal reflection angle:

$$\alpha_{\mathrm{TIR}} = \arccos\sqrt{\varepsilon} - \arcsin\beta, \tag{32}$$

whence it follows that this angle exists, as in the case of an immobile beam, only when $\varepsilon < 1$. For example, Fig. 8 shows the curves of $\chi_0(\alpha)$ at $\varepsilon = 0.5$ and various β. It is clear that, according to Eqn (32), the angle $\alpha_{\rm ITR}$ increases with β, and at $\beta = \sqrt{1-\varepsilon}$ the total internal reflection is lacking. At $\beta < 0$, the total internal reflection angle increases with $|\beta|$ up to $\pi/2$ at $\sqrt{\varepsilon} = |\beta|$. An interesting effect can be detected at velocities close to relativistic ones and at $\beta < 0$. At small α, the angle χ_0 is close to π. It decreases with α and has a minimum at $\alpha = 10°-40°$, $\beta = -(0.9-0.99)$. At larger α, the angle χ_0 increases up to π, and total internal reflection occurs at some critical value $\alpha_{\rm cr}$. It directly follows from Eqn (23) that the decrease in χ_0 is due to an increase in the denominator

$$\gamma\beta\varepsilon + \sin\alpha(\cos\alpha - \gamma\beta\sin\alpha)\,,$$

and the curve has a minimum at the point where the square root

$$\sqrt{\varepsilon\gamma^2 - (\cos\alpha - \gamma\beta\sin\alpha)^2}$$

turns to zero. The position of the sharp minimum shifts to the region of larger α with the increase in $|\beta|$, and when $|\beta| \to 1$ $\alpha_{\rm cr} \approx 44°$ at $\chi_0 = \pi/2$.

Figure 9 displays plots of χ_0 as a function of β at fixed α and $\varepsilon = 0.5$. Under normal incidence the curve is symmetrical and at smaller α the domain of definition of this function shifts to the region of positive β, and in the case of sliding angles of incidence a refracted beam exists only at ultrarelativistic values of β. When $\beta < 0$ and $|\beta| \to 1$, it is clear that in this configuration the curve of $\chi_0 = f(\alpha)$ has a minimum, i.e. two values of α correspond to a given χ_0 at the fixed β.

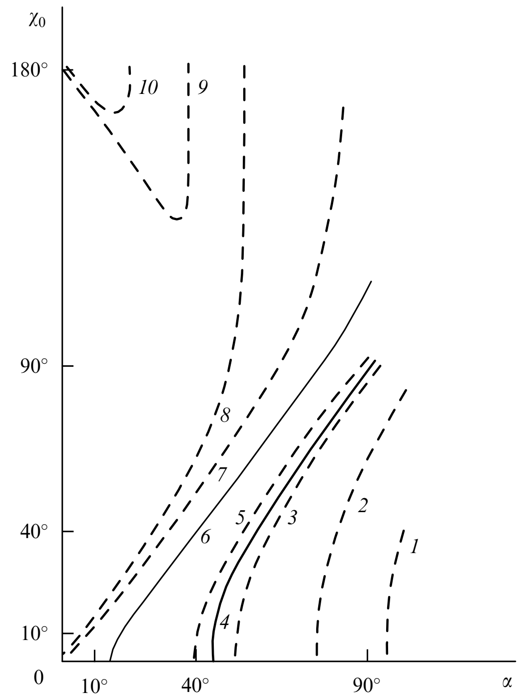

Figure 8. Curves of the angle χ_0 plotted against α at fixed β and $\varepsilon = 0.5$. Curve *1* corresponds to $\beta = -0.65$; *2* — $\beta = -0.5$; *3* — $\beta = -0.1$; *4* — $\beta = 0$; *5* — $\beta = 0.1$; *6* — $\beta = 0.5$; *7* — $\beta = 0.8$; *8* — $\beta = 0.99$; *9* — $\beta = -0.99$; *10* — $\beta = -0.3$.

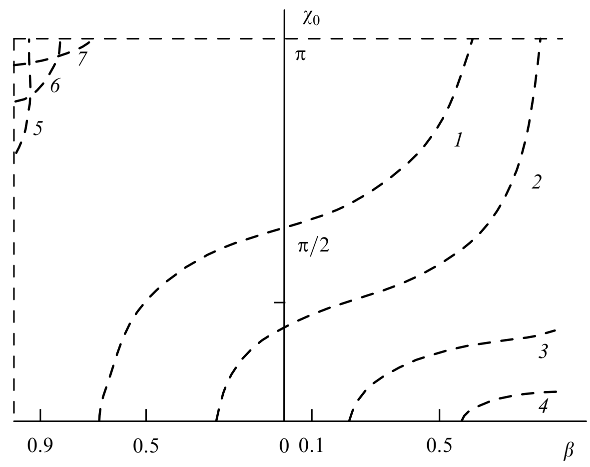

Figure 9. Plots of the angle χ_0 against β at the fixed angle α and $\varepsilon = 0.5$. Curve *1* corresponds to $\alpha = 90°$; *2* — $\alpha = 60°$; *3* — $\alpha = 30°$; *4* — $\alpha = 5°$; *5* — $\alpha = 30°$; *6* — $\alpha = 15°$; *7* — $\alpha = 5°$.

6. Angular spectrum of the field generated by a moving Gaussian beam in a dielectric

The angular spectrum of the field generated by a moving Gaussian beam in a dielectric can be derived as follows. Let us introduce the angle θ such that

$$k_x = \frac{\omega}{c}\sqrt{\varepsilon}\cos\theta\,, \tag{33}$$

whose physical meaning can be got through consideration of the angle between the wave vector of a plane wave in the dielectric and the x-axis. In the approximation linear in k, the first equation from (8) yields

$$k_x = \frac{\omega}{c}\sqrt{\varepsilon}\cos\theta = \frac{\omega'}{c}\cos\alpha + \gamma\left(k + \beta\,\frac{\omega'}{c}\right)\sin\alpha\,.$$

From the third line of Eqn (8) we derive

$$k = \frac{\omega - \gamma\omega'}{\gamma v}\,.$$

After eliminating k from the last two equations, we obtain

$$\omega = \omega'\,\frac{\sqrt{1-\beta^2} + \beta_1\cos\alpha}{1 - \beta_1\sqrt{\varepsilon}\cos\theta}\,, \tag{34}$$

where $\beta_1 = v/c\sin\alpha$. At $\alpha = \pi/2$, Eqn (34) goes over into the conventional Doppler formula for a source of light moving through a refractive medium. Notice that the pronounced intensity of spectral components described by Eqn (34) will be observed at $\theta \approx \pi - \chi_0$, where χ_0 is defined by Eqn (23), i.e. around the direction of the refracted light beam. A quantitative estimate of this effect can be submitted by calculating amplitudes of reflected and refracted plane-wave components of the Gaussian beam.

7. Reflection and refraction factors of a moving Gaussian beam in terms of field amplitudes

The reflection and refraction coefficients are derived from boundary conditions, which involve, as known, the requirement of continuity of tangential field components at the interface. Let us consider first the case of the TE-polarization. The expressions for the electric field in vacuum ($z < 0$) and in the dielectric medium ($z > 0$) are given by Eqns (12) and (14). The tangential component of the magnetic field may be expressed as follows:

$$H_y^{(1)} = \int_{-\infty}^{\infty} A_1(k)\,\frac{ck_z(k)}{\omega(k)}\exp\left[-k^2 r_0^2 + \mathrm{i}k_x x - \mathrm{i}\omega(k)\,t\right] \times \left[\exp(-\mathrm{i}k_z z) - R_1(k)\exp(\mathrm{i}k_z z)\right]\,\mathrm{d}k \qquad (35)$$

when $z > 0$, and

$$H_y^{(2)} = \int_{-\infty}^{\infty} A_1(k)\,T_1(k)\,\frac{ck_{2z}(k)}{\omega(k)} \times \exp\left[-k^2 r_0^2 + \mathrm{i}k_x x - \mathrm{i}k_{2z} z - \mathrm{i}\omega(k)\,t\right]\mathrm{d}k \qquad (36)$$

when $z < 0$. By substituting Eqns (12), (14), (35), and (36) into boundary conditions at $z = 0$, we obtain the following formulae for $R_1(k)$ and $T_1(k)$ coefficients:

$$R_1(k) = \frac{k_z - \sqrt{\omega^2\varepsilon/c^2 - k_x^2}}{k_z + \sqrt{\omega^2\varepsilon/c^2 - k_x^2}}\,, \qquad (37)$$

$$T_1(k) = \frac{2k_z}{k_z + \sqrt{\omega^2\varepsilon/c^2 - k_x^2}}\,, \qquad (38)$$

where k_x, k_z, ω were defined in Eqn (8).

Taking only the summands of the zeroth and first orders in k, we get

$$R_1(k) = \frac{\sin\psi_0 - \sqrt{\varepsilon - \cos^2\psi_0}}{\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}} + \frac{2kc}{\omega'\gamma} \times \frac{\cos\psi_0(\varepsilon - 1)}{\sqrt{\varepsilon - \cos^2\psi_0}\left(\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}\right)^2}\,, \qquad (39)$$

$$T_1(k) = \frac{2\sin\psi_0}{\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}} + \frac{2kc}{\omega'\gamma} \times \frac{\cos\psi_0}{\sqrt{\varepsilon - \cos^2\psi_0}}\,\frac{\varepsilon - 1}{\left(\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}\right)^2}\,. \qquad (40)$$

At $k = 0$, Eqns (39) and (40) go over into Fresnel's formulae for the reflection and refraction factors of a monochromatic plane wave.

The equations for the TM-polarization are derived similarly:

$$R_2(k) = \frac{\varepsilon k_z - \sqrt{\omega^2\varepsilon/c^2 - k_x^2}}{\varepsilon k_z + \sqrt{\omega^2\varepsilon/c^2 - k_x^2}}\,, \qquad (41)$$

$$T_2(k) = \frac{2\varepsilon k_z}{\varepsilon k_z + \sqrt{\omega^2\varepsilon/c^2 - k_x^2}}\,, \qquad (42)$$

where k_x, k_z and ω are defined by Eqn (8). Let us recall that $R_2(k)$ was defined as the ratio between H_y components of the reflected and incident waves. If $R_2(k)$ is defined in terms of the ratio between E_y components of the electric field vector, then Eqn (41) should have an opposite sign.

It seems interesting to analyze Eqn (41) in detail. The reflection coefficient of the plane wave entering the wave superposition in Eqn (35) comes to nought provided that

$$\varepsilon\left[\gamma\left(k + \frac{\omega'}{c}\,\beta\right)\cos\alpha + \sqrt{\frac{\omega'^2}{c^2} - k^2}\sin\alpha\right] = \left\{\frac{\gamma^2(kv + \omega')^2}{c^2}\,\varepsilon - \left[\gamma\left(k + \frac{\omega'}{c}\,\beta\right)\sin\alpha - \sqrt{\frac{\omega'^2}{c^2} - k^2}\cos\alpha\right]^2\right\}^{1/2}, \qquad (43)$$

from which it follows that

$$\sin\alpha = \gamma\,\frac{k\beta + \omega'/c}{\sqrt{\varepsilon + 1}}\,\frac{\sqrt{\omega'^2/c^2 - k^2} \pm \gamma\left[k + (\omega'/c)\beta\right]\sqrt{\varepsilon}}{\gamma^2\left[k + (\omega'/c)\beta\right]^2 + \omega'^2/c^2 - k^2}\,. \qquad (44)$$

For the fundamental wave in the beam, which corresponds to $k = 0$, we derive from Eqn (44) the formula

$$\tan\alpha = \frac{1 \pm \gamma\beta\sqrt{\varepsilon}}{\sqrt{\varepsilon} \pm \gamma\beta} \qquad (45)$$

or, after going over to ψ_0 in Eqn (9), we have

$$\tan\psi_0 = \pm\frac{1}{\sqrt{\varepsilon}}\,, \qquad (46)$$

which is identical to a well-known Brewster's formula.

It follows from Eqn (44) that $\alpha \to \pi/2$ at $\sqrt{\varepsilon} \pm \gamma\beta = 0$, hence

$$\beta = \pm\sqrt{\frac{\varepsilon}{\varepsilon + 1}}\,. \qquad (47)$$

Consequently, under the condition of normal incidence, when $\alpha = \pi/2$, and at the beam velocity determined by Eqn (47), the 'fundamental' wave of the beam ($k = 0$) is incident at the interface at a Brewster's angle given by Eqn (46) and entirely finds its way into a dielectric medium. If

$$\beta = +\sqrt{\frac{\varepsilon}{\varepsilon + 1}}\,,$$

i.e. the beam moves in the positive direction of the x-axis, the normal to the wave front in the beam makes an obtuse angle with the x-axis (minus sign in Eqn (46)). If the beam moves in the negative direction of the x-axis at the velocity defined by Eqn (47), the normal to the wave front makes an acute angle with the x-axis.

8. Conclusions

In conclusion we present expansions of $R_2(k)$ and $T_2(k)$ in a power series of k and restrict ourselves to linear terms:

$$R_2(k) = \frac{\varepsilon\sin\psi_0 - \sqrt{\varepsilon - \cos^2\psi_0}}{\varepsilon\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}} + \frac{2kc}{\omega'\gamma}\,\frac{\cos\psi_0}{\sqrt{\varepsilon - \cos^2\psi_0}}\,\frac{\varepsilon(\varepsilon - 1)}{\left(\varepsilon\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}\right)^2}\,, \qquad (48)$$

$$T_2(k) = \frac{2\varepsilon\sin\psi_0}{\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}} + \frac{2kc}{\omega'\gamma}\,\frac{\cos\psi_0}{\sqrt{\varepsilon - \cos^2\psi_0}}\,\frac{\varepsilon(\varepsilon - 1)}{\left(\varepsilon\sin\psi_0 + \sqrt{\varepsilon - \cos^2\psi_0}\right)^2}\,. \qquad (49)$$

If the angle of incidence of the beam α satisfies the condition (46), the first summand on the right of Eqn (48) vanishes and it is reduced to the equation

$$R_2(k) = \frac{kc}{2\omega'\gamma} \frac{\sqrt{\varepsilon}(\varepsilon^2 - 1)}{\varepsilon^2} .$$

Thus we cannot totally eliminate the reflected beam. The fraction of the reflected power, however, is considerably lowered in this case.

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