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# Relative ordering criteria in open systems

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## Contents



**Abstract. Main concepts of a new interdisciplinary research area known as "Physics of Open Systems" are introduced with special reference to a criterion for the relative degree of order in nonequilibrium states of such systems. Based on this criterion, the notion of the 'norm of chaos' ('norm of order') is proposed and used to differentiate between degradation and self-organization processes. The possibility of applying methods of open system physics to investigations in economics, sociology and physiology is briefly discussed.**

## 1. An excursion into the history of open system physics

The rise of the theory of open systems has been prepared by the works of many eminent scientists in the 19th century, with physicist Ludwig Boltzmann, mathematicians Henri Poincare and Alexeĭ Lyapunov, and certainly Charles Darwin, a biologist, among them.

Ludwig Boltzmann called the 19th century the century of Darwin to emphasize that Darwin's theory of evolution based on the principle of natural selection was one of the greatest scientific achievements at his time. Such conclusion may seem rather unexpected. Indeed, the 19th century brought about a wealth of great discoveries in natural sciences, especially in physics. Suffice it to mention thermodynamics, the fundamentals of which were laid by Carnot, Clausius, and Thomson in the 19th century. It was also the century of Michael Faraday and James Maxwell, who created the theory of electromagnetism. The basis for the modern kinetic theory was also laid in the 19th century, and one of its initiators was Ludwig Boltzmann. He suggested the first kinetic equation for the description of irreversible processes in gases which even now remains one of the basic equations in the theory of nonequilibrium processes. Specifically, it describes processes leading to equilibrium in gases. Ludwig Boltzmann was the first to propose the statistical definition of entropy, one of the main thermodynamic characteristics, and proved the famous 'H-theorem'. According to this theorem, entropy of a closed system monotonically increases in the course of transition towards the equilibrium state and remains constant thereafter. Finally, it was Boltzmann who understood that entropy in closed systems may be a measure of the relative degree of chaos. Nonetheless, no less a person than Boltzmann defined the 19th century as the century of Darwin. Thereby, he gave priority to biological evolution [1].

Now, what underlies such a conclusion?

Boltzmann's choice of priorities was certainly prompted by his startling scientific intuition. There were no mathematical models of biological evolution at Boltzmann's time. But Boltzmann had little doubt that his own theory of time-dependent gas evolution in a closed system could be extended to open systems, including all biological objects. Therefore, he considered the Darwin theory to be the first step to the theory of evolution of open systems at large. Boltzmann was one of the few scientists at that time who came to understand the significance of this 'first step'. This induced him to speak about the Darwin theory as the greatest discovery of the 19th century.

Boltzmann could hardly rely on this view of his being shared by many of his contemporaries. To begin with, his own theory has met with objections by most scientists. In fact, an extremely heated controversy has arisen. He had among his

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most active opponents Henri Poincare, the great mathematician and one of the founders of the qualitative theory of differential equations and the theory of dynamical systems stemming from the equations of Newton's mechanics. Poincare simply denied Boltzmann's theory.

Here is a small fragment from a book by I Prigogine *From Being to Becoming* (Ref. [2] p 165): "Poincare went so far as to write that he could not recommend the study of Boltzmann's paper because the premises in Boltzmann's considerations clashed with his conclusions". Having analysed the reversible equations of mechanics, Poincare came to the conclusion that the theory of irreversible processes and mechanics are incompatible. This inference ensued in particular from the impossibility of constructing a function playing the role of entropy and implied that mechanics based on the reversible equations of motion was incompatible with the theory of irreversible processes.

There is another version of Poincare's saying cited in a paper by I Prigogine (see ref. [3]): "In connection with this, it is curious to recall a statement by Poincare that he could not recommend anyone reading the paper of Boltzmann since he cannot recommend studying scientific arguments in which the conclusions contradict the initial principles".

One cannot help noticing the striking difference between this comment of Poincare and the opinion of Erwin Schrödinger, an outstanding representative of the next generation of physicists and one of those who created quantum mechanics. A line in page 156 in the book by I Prigoine reads as follows: "His (Boltzmann's) direction of thought, I might call my first love in science. No other has ever thus enraptured me or will ever do so again".

Thus, it was clear as early as at the turn of the 20th century that the development of the theory of nonequilibrium processes in physical and biological systems is a most important problem in natural sciences. However, nearly a century has elapsed from the understanding of the significance of this problem till its partial solution.

The first serious contribution was made by Albert Einstein, Marian Smoluchowsky, and Paul Langevin with their theory of Brownian movement, i.e. chaotic motion of small macroscopic particles suspended in a fluid, first observed and described by R Brown, a Scottish botanist, in 1827 and thereafter referred to as the Brownian motion†. It is caused by the molecules of a liquid randomly colliding with one another. Therefore, a system of Brownian particles exemplifies an open system.

It follows from the Boltzmann equation that the average energy of gas particles is conserved during evolution. This condition is indispensable if entropy (hence, the degree of chaos) is to increase in the course of evolution towards the equilibrium state. On the contrary, the average energy of Brownian particles varies during this process. For this reason Boltzmann's H-theorem is no longer valid.

† The widespread opinion that R Brown was the first to observe 'Brownian particles' is wrong. Lens systems had been used to obtain a multiplied image of a subject as far back as the 16th century, and the first microscope suitable for laboratory observations had been invented almost 200 years before R Brown described his experiments. Therefore, investigators had an opportunity to see chaotic motion of small particles in liquids much earlier than Brown, e.g. Dutch scientists A Leeuwenhoek (1673 – 1677) who first described microorganisms or Jan Ingenhausz who observed movements of ground charcoal particles at the surface of alcohol. R Brown did not actually discover chaotic motion of suspended small particles, but he designed special experiments to study it and undertook to formulate physical laws underlying this phenomenon (*note by GR Ivanitsky*)

Note that according to the Boltzmann equation, the average energy value, rather than the real one, remains unaltered during evolution. This implies the possibility of energy fluctuations, which accounts for both the Boltzmann system and the system of Brownian particles being regarded as open systems.

The modern statistical theory of open systems has been created by the joint efforts of many scientists. We have already noted the key role of Ludwig Boltzmann and Henri Poincare in the development of the statistical and dynamical theories of open systems. Later, their mathematical aspects were elaborated by A M Lyapunov who greatly contributed to the theory of stability of motion, A N Kolmogorov with his works on dynamic systems (1957), physicists L I Mandelstam, A A Andronov, N S Krylov, Ya B Zel'dovich, and many others. Doubtless, V I Vernadsky, a father of the concept of the noosphere (anthroposphere), must be reckoned among those who greatly promoted the development of the theory of self-organization.

Following this extensive historical excursus, attention will now be devoted to the main topic. All other references shall hereafter be treated as digressions.

## 2. Physics of open systems. Dissipative structures. Synergetics

"Physics of open systems" — is an interdisciplinary field of science. Here is a brief list of key words and notions to characterize it: Chaos and Order; Open systems; Criteria for the relative degree of order in open systems; Norm of chaos; Degradation and self-organizing; Diagnostics of open systems; Constructive role of dynamic instability of atomic motion; Transition from reversible to irreversible equations; Kinetic and hydrodynamic description of nonequilibrium processes taking into account the structure of 'continuous medium'; Description of equilibrium and nonequilibrium phase transitions based on kinetic and hydrodynamic data; Unified kinetic description of laminar and turbulent motions; Quantum open systems. Certainly, many of these concepts are not at all 'new'. The purpose of "Physics of open systems" is to elaborate ideas and methods for the integrated description of this broad class of problems [4].

Open systems can exchange energy, matter, and (last but not least) information with the environment. We shall consider only open macroscopic systems. They are composed of many objects, constituent structural elements. These elements may be microscopic, e.g. atoms or molecules in physical and chemical systems. They may be small but macroscopic such as macromolecules in polymers or cells in biological structures.

Due to the complexity of open systems, they may host a variety of structures. Dissipation plays a constructive role in the formation of these structures. At first sight, this seems surprising because the dissipation concept, is associated with the attenuation of various forms of motion, energy scattering, and the loss of information. It is extremely essential, however, that dissipation is necessary for the formation of structures in open systems. To emphasize this, I Prigogine has coined the term 'dissipative structures'. This comprehensive and exact term covers all sorts of structures: temporal, spatial and time-space structures. The latter are exemplified by autowaves [5].

The complexity of open systems provides an ample opportunity for cooperative phenomena to occur. In order to emphasize the role of collective interactions in the formation of dissipative structures, H Haken has introduced the term 'synergetics', that means joint action [6, 7]. The objective of synergetics is to reveal common ideas, methods, and laws in totally different fields of natural science, sociology, and even linguistics.

Moreover, synergetics is an area where various special disciplines cooperate. The scope of synergetics is illustrated by a series of books under the common title of *Synergetics* published by Springer Verlag. The last issue in this series is volume 67 [8].

Synergetics stems from thermodynamics and statistical physics. This accounts for the word Physics being the first in the title of this Section. Thereby, it is emphasized that the theory of open systems is virtually based on fundamental physical laws.

## 3. Degradation and self-organization in evolution

Evolution is the process of changes and development in nature and society. Worded in this manner, it is a very general concept. In physically closed systems, evolution in time results in the equilibrium state to which maximum entropy and the maximum degree of chaos correspond.

In open systems, it is possible to distinguish two classes of evolutionary processes:

(1) Development in time towards the nonequilibrium stationary state

(2) Evolution via a series of nonequilibrium stationary states of an open system. The latter process is due to variation of the so-called control (governing) parameters.

Darwin's theory of evolution is based on the principle of natural selection. Thus, evolution can either lead to degradation or represent a self-organizing process during which more complex and sophisticated structures arise. Can self-organization be the unique outcome of evolution? The answer is negative because neither physical nor even biological systems display an 'intrinsic drive' towards self-organization. Therefore, evolution may also lead to degradation. A physical example is evolution to the equilibrium state, the most chaotic one, according to Boltzmann.

Thus, self-organization is only one of the possible routes of evolution. Criteria for self-organization are needed to answer the question along which route a process will develop, but such fundamental concepts as degradation and self-organization are not necessarily to be defined. This is very difficult to do, laying aside the ambiguity of such definitions. Of much greater importance is the comparative analysis of the relative degree of order (or chaos) in different states of the open system being examined. Only such analysis can answer the question whether the open system undergoes self-organization or degradation.

We have already emphasized the concepts of chaos and order. Now, what distinguishes order from chaos?

There are cases when the difference between them is rather clear-cut. However, it appears from the comparison of laminar and turbulent flows that a seemingly obvious inference may turn out to be totally incorrect. Quantitative criteria for the relative degree of order (or chaos) in different states of open systems allow a more reliable conclusion to be obtained.

The results of such analysis are objective and provide additional information which constitutes the basis for the establishment of the 'norm of chaos' and helps to reveal two-side deviations from the norm under the influence of various impacts. In biology, for instance, all kinds of stress may cause deviations in the degree of chaos from the norm. Deviations on either side suggest 'pathology', i.e. represent the process of degradation.

Therefore, a statement (based on a selected criterion) of the impaired degree of chaos does not necessarily mean that self-organization occurs and vice versa, an increase in the degree of chaos is not always identifiable with degradation. Such a conclusion is valid only for those physical systems where thermal equilibrium may be taken as the reference point for the degree of chaos. For example, in such, an open system as a generator of electrical oscillations, the equilibrium state is that of zero feedback when only thermal fluctuations exist in the electrical contour.

Since an organism normally functions only if a certain norm of chaos is available, corresponding to an essentially nonequilibrium state, the above reference point is nonexistent. This accounts for the lack of objective information about variation of the degree of chaos in biology as well as in economics and sociology which hampers distinguishing between self-organization and degradation in such systems.

However, another classification is equally relevant. The norm of chaos being determined, deviations on either side may be regarded as 'pathology', i.e. degradation. Therefore, it is possible to monitor the choice of 'therapy'. Here, a criterion for the relative degree of order is at stake again. If the 'treatment' normalizes the state of the open system, in terms of this criterion, self-organization occurs. Otherwise, 'therapy' leads to further degradation.

What are the criteria for the relative degree of order? What is the relative measure of order or disorder? These questions are very difficult to answer and have not until recently been clarified.

The difficulty in introducing the relative measure of order (or chaos, for that matter) in open systems is in the first place due to the absence of exact definitions for the initial concepts: chaos, order, degradation, and self-organization. It has already been mentioned that the definitions of these concepts are to a great extent arbitrary. We have just noted that the transition to a more chaotic state in sociology, economics, and especially biology should not necessarily be regarded as degradation. It is essential to consider deviations from the norm of chaos.

In this context, it appears useful to consider the principal concepts at greater length in order to formulate the criterion for the relative degree of order without which the notions of degradation and self-organization actually remain void of meaning.

## 4. Physical and dynamical chaos. Nonequilibrium phase transitions

Chaos and order are concepts given special emphasis as early as in ancient philosophic schools especially by Plato and his disciples. Disregarding minor details, it is worth mentioning two principles formulated by them which remain of interest even now.

According to Plato and his followers, chaos is a state of matter which persists as matter loses the ability to display its intrinsic properties. On the other hand, chaos gives rise to the whole contents of the Universe, i.e. chaos produces order (see [10]).

Although 'chaos' and 'chaotic motion' are fundamental physical concepts, their precise definitions are lacking.

Indeed, according to Boltzmann, motion in an equilibrium state is most chaotic. However, motions far from equilibrium are also called chaotic. Such is the 'motion' in noise generators intended for signals suppression.

Normally, different forms of turbulent motion in gases and fluids are also described as chaotic [11 – 13], e.g. turbulent motion in pipes which arises from laminar motion when the pressure difference at the ends of the pipe is sufficiently large. It seems natural to conceive turbulent motion as being more chaotic than laminar motion. However, such a view largely stems from the confusion of the concepts of complexity and chaos. Observation of turbulent motion primarily reveals its complexity, whereas additional analysis is needed to estimate the degree of chaos and appropriate criteria to quantify it [12].

Of late, the concept of 'dynamical chaos' has been extensively exploited to characterize complex motions in relatively simple dynamic systems [14, 15]. The word 'dynamic' implies the absence of fluctuation sources, that is sources of disorder.

For this reason, the 'dynamic system' concept is a somewhat idealized one. A more real chaotic motion, with random sources taken into account, might be called 'physical chaos'. An example is the chaotic motion of atoms and molecules in equilibrium.

The mathematical notion of 'dynamical chaos' can be traced back to the works of H Poincare and A N Kolmogorov. E Lorenz appears to be the first to have reported an example of dynamical chaos in 1963 [16]. He was attempting to solve equations which describe convective motion in gases and fluids. Convective motion results from the interaction between gravitational field and temperature gradient created by an external source of heat. Therefore, it occurs in an open system.

Imagine a layer of a liquid heated from below. Convective motion is manifested in that heated elements of the liquid move upwards and cold ones downwards which results in heat transfer from the bottom to the top. At rather small temperature gradients, heat transfer depends on thermal conductivity. It is a molecular (disorganized) process not accompanied by ordered hydrodynamic motion which could control heat transfer by analogy with the mode of traffic control.

The situation is altogether different when the temperature gradient exceeds a certain critical value. The change is in the appearance of a macroscopic motion in a fluid referred to as convective motion. It results in the self-adjustment of the thermal flow, that is in the upward movement of heated elements along one sort of channels and the downward motion of colder elements in different channels. This accounts for the strictly ordered distribution of thermal counterflows.

This situation is reminiscent of the control of traffic moving in opposite directions. There is however an essential difference. Indeed, traffic control is in conformity with driving regulations, whereas convective motion is subject to self-organization, and only a temperature gradient is given. The reorganization of motion is due solely to intrinsic properties of the system, and its outcome is apparent as the appearance of dissipative dimensional structures at the liquid surface, the Benar cells.

The liquid inside these cells lifts upward while it flows down at the edges. Therefore, reorganization results in enhanced transmission, in excess of that during a disordered molecular heat transfer. The appearance of the new structure may be regarded as a nonequilibrium phase transition.

Another example of nonequilibrium phase transition is the occurrence of coherent electromagnetic radiation in quantum optical laser generators.

## 5. Dynamical and statistical description of complex motion

In the historical Introduction, mention was made of the long-standing dramatic clash of opinions between adherents of the two theories, that is, statistical and dynamical descriptions of nonequilibrium processes. Nowadays, debates are not as heated as they used to be at the beginning of the century, but the two theories continue to develop as divergent trends. Evidently, their synthesis is in order especially in view of recent break-throughs made in the physics of open systems.

What is the reason for the opposition of these two basic trends for such a long period? Is their independent development justified?

The answer to the second question is self-apparent: a synthesis is indispensable. The first question is more challenging. It will be dealt with below.

We distinguish between two classes of systems, dynamic and stochastic (statistical) ones. Such categorization is arbitrary because it is actually difficult to discriminate between dynamical and physical chaos. However, differentiation is possible based on the results of a numerical experiment. It is justified because practically all mathematical models of interest have no analytical solutions.

The basis of the classification may be constituted by the property of reproducibility of motion under given initial conditions. Then, by definition, reproducible motions in nonlinear dissipative systems are reckoned as dynamic, whereas those non-reproducible in terms of initial characteristics as stochastic.

Naturally, all processes in a real experiment, where noise is inevitable, are more or less stochastic. A numerical experiment allows the initial conditions to be precisely reproduced (at a given word length of the computer). The reproducibility of a solution depends solely on the structure of the mathematical model. If the equations do not contain random sources, the process is easy to reproduce; hence, the motion is dynamic, albeit sometimes very complicated and practically unpredictable. Otherwise (in the presence of random sources), the motion is non-reproducible in terms of original characteristics and should be regarded as in nature stochastic.

It is essential for a study of stochastic processes in a numerical experiment that computer sources of random numbers be constructed in compliance with a certain algorithm and therefore actually determined. They may be considered to be random if their characteristic recurrence times are significantly longer than characteristic relaxation times of a dynamic system.

The main feature of dynamical chaos is dynamic instability of motion. It is expressed as strong (exponential) divergence of the originally close trajectories and leads to their mixing, which allows to proceed from the comprehensive description based on the equations of motion for all particles to simpler equations for the functions, smoothened over the mixing volume. Thereby, the manner of description is changed dramatically. The system of particles is replaced by a continuous medium [4].

A major contribution to the investigation into the relationship between the dynamical and statistical descriptions of complicated motion was made by the prematurely deceased N S Krylov. He was the first to raise the question of the role of dynamic instability and mixing as the basis of statistical physics in his posthumously published book *Works on the Foundations of Statistical Physics* (1950) [17].

## 6. Constructive role of dynamic instability of motion

Relatively simple dynamic systems can give rise to very complicated motions perceived as chaotic. This was the reason for the introduction of such new concepts, as the strange attractor and dynamical (or determined) chaos.

As a rule, the word 'chaos' has negative connotations in physics, biology and economics. However, the concept of chaos is a many-faceted one. For example, life can exist neither in complete chaos nor in perfect order. A normal organism needs a certain norm of the degree of chaos which can be estimated and maintained based on quantitation of the relative degree of chaos.

Given an opportunity to measure the relative degree of chaos, the word requires no additional attributes. It is therefore appropriate to ask whether the term 'dynamical chaos' is relevant. In fact, it was coined to characterize the complex states which arise from dynamic instability, i.e. exponential divergence of trajectories at a minor change of the initial conditions.

However, this term is somewhat in conflict with the fact that the trajectories computed from dynamic equations can be reproduced based on the initial data in a numerical experiment. Moreover, we shall demonstrate below that dynamic instability can play a constructive role in the physics of open systems [4,19,20].

Let us consider an illustrative example from sociology.

Imagine yourself participating in an international conference. Assume further that the conference is about to end. Take this situation as the initial one. The participants must choose between two possible courses of action when leaving the conference:

1. They withdraw together without moving too far from one another.

2. The participants depart each to his (her) residence or institution, that is 'diverge exponentially'. In other words, the motion of the participants is 'dynamically unstable'. Now, which way ('motion') is more efficacious in terms of using new information obtained at the conference?

Certainly, the former variant may be useful because it gives an opportunity to continue personal contacts and discussions. However, the latter type of 'motion', that is 'dynamic instability' and 'mixing' of the participants' trajectories is actually more conducive to further progress in science.

This example shows that dynamic instability of motion and mixing do not necessarily lead to 'chaos' and can play a positive constructive role.

Turning back to physical systems, it is appropriate to consider a rarefied gas. From the standpoint of mechanics, gas evolution may be described using a system of equations for all its atoms. Such a task is beyond the capabilities of even the most powerful computers. What is to be done? How can nonequilibrium processes be described in a gas, that is a system composed of a huge number of particles? The solution of the problem is possible by virtue of the constructive role of dynamic instability of atomic motion in a gas.

Dynamic instability of motion, i.e. exponential divergence of the trajectories, and effects of external factors are responsible for the mixing of trajectories in phase space. This accounts for the possibility of smoothening at physically infinitesimal scales and introducing the concept of 'continuous medium' to pass from reversible microscopic equations of motion for gas particles to irreversible kinetic and hydrodynamic equations for macroscopic functions of the continuous medium.

In this approach, the atomic structure of a system is taken in account to define 'a point of continuous medium'. This requires that physically infinitesimal time and length scales be defined as well as the corresponding physically infinitesimal volume which actually stands for the 'point' of a continuous medium [11, 18, 4].

Naturally, it is desirable that such definitions agree with the definition of the minimum mixing region and the minimum time for the development of dynamic instability.

## 7. Criterion for the relative degree of order in different states of open systems. S-theorem

Of all macroscopic functions, only entropy $S$ possesses a combination of properties that allow it to be used as a measure of uncertainty (chaos) in the statistical description of processes in macroscopic systems. Entropy was first introduced into thermodynamics as a state function whose change shows the amount of heat transferred to a system ($\mathrm{d}Q = T\mathrm{d}S$). This equality expresses the second law of thermodynamics for quasi-equilibrium, i.e. reversible processes. In a reversible adiabatic process at $\mathrm{d}Q = 0$, entropy is constant.

Boltzmann gave a statistical definition of entropy for both equilibrium and nonequilibrium (irreversible) processes and proved the famous H-theorem.

It states: The entropy of a system tending to an equilibrium state grows and remains unaltered after equilibrium is attained. According to Boltzmann, the degree of chaos monotonically increases during evolution and has the maximum value at equilibrium, entropy being a measure of uncertainty (chaos).

In this context, it is essential that during the evolution of a closed system to equilibrium in compliance with the Boltzmann equation, the average energy $\langle E \rangle$ remains constant. However, conservation of average energy in course of evolution is not a common property of all kinetic equations.

For the Brownian movement, it varies in the course of evolution to equilibrium. For this reason, Boltzmann's H-theorem cannot be straightforwardly applied to this case. The role of entropy in Brownian motion is played by a function which is an analog of thermodynamically free energy in nonequilibrium processes. However, free energy has no properties necessary to be a measure of uncertainty for the system's state. Only entropy possesses such a combination of properties.

It is only natural that the criterion for the relative degree of order needs to be universal. There is no reason it should be applicable only to a class of systems where average energy is conserved during evolution. Which route may lead to the solution of this problem?

Entropy being the sole function with the properties of a measure of chaos, there is but one option. It is necessary to redefine entropy so that the average energy remains constant in the course of evolution.

The previous paragraphs concerned evolution in time. It is equally possible to consider the evolution of stationary states in open systems at slowly changing governing parameters. It is for this type of evolution, that the criterion for the relative degree of order in various states of open systems will be introduced below. This criterion was for the first time formulated for specific cases [12] and called the 'S-theorem'. Later, its general formulation was suggested, to make possible the direct comparison between the relative degrees of order from experimental data [18, 4].

The letter S in the name of the 'S-theorem' is the first letter of the word 'Self-organization'. Curiously, the term 'H-theorem' was suggested by Berbery, an English physicist, only a few years after it had been proved. The letter H in the name of the H-theorem is derived from the English word 'Heat'.

Let us consider the evolution of stationary states of an open system at a varying control parameter $a$. Let us further assume the existence of two states with the control parameters $a = 0$ and $a = a_1$, e.g. the stationary states of the Van der Pol generator at different values of the feedback parameter [12]. Of course, the description of generation should take into account both current and charge fluctuations. Then, thermal fluctuations of the current and the charge in the electrical contour correspond to the former parameter value, when feedback is absent. The developed generation state, with the feedback parameter considerably exceeding the threshold, corresponds to the latter value.

In the general case, the degree of order of the distinguished states differs, which accounts for one of them being more chaotic than the other. Let us term it 'physical chaos'. As a rule, this state is nonequilibrium and more ordered than the equilibrium state. However, in the case of a generator with $a = a_0$, it coincides with the equilibrium state.

Let us denote the macroscopic characteristic of the stationary state as $X$. The role of $X$ for a generator can be played by the oscillation energy $E$. Let us further denote the distribution functions of two distinguished states as $f_0, f_1$ and the corresponding entropy values as $S_0$, $S_1$.

In the general case, there is no such notion as energy for an open system and only the effective energy can be introduced. It may just as well be termed the effective Hamilton function and denoted as $H_{\text{eff}}$. It is defined by the distribution function of the physical chaos state $H_{\text{eff}} = -\ln f_0$ and as a rule vanishes with a change of the control parameter. For this reason, functions substituted by the corresponding new values $\tilde{f}_0$, $\widetilde{S}_0$, if the entropy difference $S_0 - S_1$ needs to be used as a measure of the relative degree of order in the distinguished states. These new values are determined from the equality condition for the examined states of the average effective Hamilton function. When the physical chaos state coincides with the equilibrium state, as is the case with the Van der Pol generator, renormalization is carried out by substituting temperature $T$ by the new value $\widetilde{T}$. It is determined from the solution of the equation which describes the equality condition for the average effective Hamilton functions in the two states of interest. This equation has the form:

$$\int H_{\text{eff}} \tilde{f}_0(X,\, a = a_0)\,\mathrm{d}X = \int H_{\text{eff}} f_1(X,\, a = a_1)\,\mathrm{d}X\,. \qquad (1)$$

Given the correct choice of the 'physical chaos state', the solution of this equation has the form:

$$\widetilde{T}(a) \geqslant T\,. \qquad (2)$$

The sign of equality is relevant at $a = a_0$, i.e. for the state of physical chaos. Evidently, the state '0' should be 'heated' to equalize average energies. As the comparison is now made at identical values of the average effective energy, the entropy difference $\widetilde{S}_0$, $S_1$ can serve as a measure of the relative degree of order in the distinguished states.

The renormalized distribution function may be presented in the form of the canonical Gibbs distribution:

$$\tilde{f}_0(X) = \exp\frac{F_{\text{eff}}(\widetilde{T}) - H_{\text{eff}}(X)}{k\widetilde{T}}\,. \qquad (3)$$

The corresponding expression for the Boltzmann–Gibbs–Shannon entropy is derived from the equation

$$\widetilde{S}_0 = -\int \ln\bigl(\tilde{f}_0(X)\bigr)\tilde{f}_0(X)\,\mathrm{d}X\,. \qquad (4)$$

Now, let us turn back to the equation (1). If the '0' state is coincident with equilibrium, its solution has the form (2), with $T$ standing for the temperature. In the general case, the '0' state, i.e. the state of physical chaos, is a nonequilibrium one. The distribution (3) includes effective temperature which is equal to unity for the physical chaos state. Therefore, the solution (2) should be written in the form

$$\widetilde{T}(a) \geqslant 1\,. \qquad (5)$$

Here, the sign of equality also corresponds to the state of physical chaos. However, both effective temperature and free energy are now dimensionless. If the inequality (5) is valid at control parameter values $a > a_0$, the choice of the condition '0' in the form of physical chaos is correct, and the relative degree of order in the distinguished states is defined by the difference between the corresponding entropies.

Using the expression (3) for the distribution function $\tilde{f}_0$ and the constancy condition for the average effective Hamilton function, the expression for the entropy difference may be presented as the inequality:

$$\widetilde{S}_0 - S_1 = -\int\left(\ln\frac{f_1(X)}{\tilde{f}_0}\right) f_1(X)\,\mathrm{d}X \geqslant 0\,. \qquad (6)$$

on the condition that

$$\langle H_{\text{eff}} \rangle = \text{const}\,. \qquad (7)$$

The known inequality $\ln a \geqslant 1 - 1/a$ at $a = f_1/\tilde{f}_0$ is used to derive the formula (6).

To summarize, the result of computing the relative degree of order in the two distinguished nonequilibrium states is represented by two inequalities. One (5) confirms the correct choice of the '0' state as presenting physical chaos. Given the opposite inequality, physical chaos would be represented by the state '1'. The formula (6) provides a quantitative measure of the relative degree of order in the distinguished states.

The above calculations were made for the case of one $a$ parameter. When several control parameters are available, it is possible to optimize the search for the most ordered state.

Let us now apply the 'S-theorem' to estimate the relative degree of order upon the transition from laminar to turbulent flow [4, 13, 18, 21].

## 8. Is turbulent motion more chaotic than laminar motion?

The concept of 'turbulent motion' was first introduced more than one hundred years ago. However, until recently, the question of which of the two motions (laminar or turbulent) is more chaotic had not been answered convincingly. The majority of authors considered the answer to be perfectly clear: laminar motion must be more orderly. However, this opinion is due to the confusion of the concepts of 'complexity' and 'chaos'.

The conspicuous complexity of the turbulent flow is possible to see, so to say, with the naked eye. Nevertheless, the definition of the relative degree of its order requires that the relevant criterion be used. Calculations based on the S-theorem allowed the general results (5), (6) to be specified for the case of transition from the laminar flow in a pipe to the steady state turbulent flow.

It will be clear from the forthcoming discussion that laminar flow may be assumed to represent the state of physical chaos. The role of the effective Hamilton function is played by the average kinetic energy of the laminar flow. For the equality of this energy in both laminar and turbulent flows to be true, the laminar flow should be 'warmed up':

$$k_B T_{lam} = k_B T_{turb} + \frac{m}{3}\left\langle (\delta u)^2 \right\rangle \geqslant k_B T_{turb} . \qquad (8)$$

The temperature difference is defined by the sum of squared diagonal elements in the Reynolds stress tensor. Reynolds' stress representing collective degrees of freedom, the equality (8) may be interpreted as indicating that a part of thermal (chaotic) motion is replaced by the collective degrees of freedom during the transition from laminar to turbulent motion. This justifies the choice of the Reynolds stress tensor as the order parameter of the turbulent flow.

The above discussion accounts for the choice of laminar flow as the physical chaos state.

Therefore, the chaotic motion fraction decreases and that of a more ordered motion increases with the development of turbulence. We shall see that it is reflected in reduced entropy.

The result (6) which in the present case defines the relative degree of order in laminar and turbulent flows has the form:

$$T(S_{lam} - S_{turb}) = \frac{mn}{2}\left\langle (\delta u)^2 \right\rangle \geqslant 0 . \qquad (9)$$

Thus, the entropy of the turbulent flow is lower than that of the laminar one. This implies a higher degree of order in the turbulent flow. Here, the role of the control parameter is played by the pressure difference at the ends of the pipe. At its zero value, the fluid is in an equilibrium state characterized by the maximum degree of chaos. This is another important example of a physical system in which the equilibrium state is taken as the reference point for the degree of chaos. One more example is the Van der Pol generator.

When the pressure difference is other than zero, all states are better ordered. This adds weight to the argument, in accordance with what is said in Section 3, that the transition from laminar to turbulent flow is an example of the self-organization process. However, this does not mean that the degree of order grows monotonically with increasing Reynolds number.

A higher degree of organization of the turbulent motion compared with that of the laminar one is also apparent as demonstrated below.

The momentum transfer between layers in a laminar flow is mediated through a molecular mechanism which consists in independent changes of momenta of individual gas particles.

Conversely, in the case of a turbulent flow, the momentum transfer from one layer to another is a collective process. It other words, individual disorganized motion in a laminar flow changes, upon transition to the turbulent flow, into collective (hence, more organized) motion.

This results in the turbulent viscosity coefficient being much higher than the corresponding parameter for a laminar flow.

A higher degree of order in turbulent motion is also confirmed by calculations of entropy production.

## 9. Estimation of the relative degree of order from experimental data

Practical application of the S-theorem implies that the effective Hamilton function is known. It is easy to find provided a mathematical model of the process in question is available. In many cases, however, there are no adequate mathematical models for open physical systems. This problem is even more complicated as far as biological, social, and economic entities are concerned.

Therefore, it is sometimes necessary to be able to determine the relative degree of order in open systems directly from experimental data. This can be achieved in the following way:

1. By selecting control parameters for a given system, e.g. two states of the system with control parameters $a_0$ and $a_0 + \Delta a$.

2. By experimentally obtaining sufficiently long temporal realizations for the chosen values of the governing parameters.

$$X_0(t, a_0) , \qquad X(t, a_0 + \Delta a) . \qquad (10)$$

These data are loaded into a computer and used to construct the corresponding distribution functions:

$$f_0(X, a_0) , \qquad f(X, a_0 + \Delta a) . \qquad (11)$$

The two distributions are normalized to unity.

Further operations are routine.

3. By assuming one of the states, e.g. '0' state, to be the state of physical chaos and finding the effective Hamilton function:

$$H_{eff} = -\ln f_0(X, a_0) . \qquad (12)$$

Thus, it is derived directly from experimental data. Similar to what was said above, the term 'effective Hamilton function' is due to the fact that the distribution function renormalized to a given value of $\langle H_{eff} \rangle$ has the form of the canonical Gibbs distribution:

$$\tilde{f}_0(X) = \exp\frac{F_{eff}(\tilde{T}) - H_{eff}(X)}{k\tilde{T}} . \qquad (13)$$

Here, $\widetilde{T}$ is the effective temperature. For the state of physical chaos, $\widetilde{T} = 1$.

The effective free energy as a function of $\widetilde{T}$ is estimated from the normalization condition for the function $f_0$. The dependence of effective temperature on variation of the control parameter $\Delta a$ can be found (as above) from the condition that the average effective energy be constant

$$\int H_{\mathrm{eff}} \tilde{f}_0(X, a_0)\,\mathrm{d}X = \int H_{\mathrm{eff}} f(X, a_0 + \Delta a)\,\mathrm{d}X\,. \qquad (14)$$

The choice of the physical chaos condition is justified if the solution of this equation has the form (5). The relative degree of order is again calculated by the formula (6).

## 10. Diagnosis of medico-biological objects based on the S-theorem

Let us consider some applications of the S-theorem for the purpose of medico-biological diagnostics. Investigations into this problem were initiated in Kiev and Moscow in 1990, using both mathematical models and experimental data. In 1994, the first results of the analysis of cardiograms based on the S-theorem were obtained by the joint efforts of biologists and clinicians in the Laboratories of Nonlinear Dynamics at the Saratov and Potsdam Universities of [22–24].

Analysis of the relative degree of order for the purposes of medico-biological diagnosis was performed using data collected by examining both individual patients and selected groups of them.

Biological experiments reported by T G Anishchenko revealed significant differences in the responsiveness of male and female rats to the noise stress. Biochemical studies have also demonstrated opposite changes in the conditions of the two sexes. This finding provided the basis for a study of men and women's behavior in response to stress. The evaluation was also made using the S-theorem.

Two cardiograms were obtained from each subject included in the study, one before and the other after identical stress impact (a shrilly acoustic signal).

Two cardiograms being available from each subject, this allowed a change in the relative degree of order to be individually estimated using the S-theorem. The experiment has demonstrated opposite changes in the degree of order in men and women, the former showing a decreased degree of chaos, while in the latter it increased.

In both cases, there was a deviation from the 'norm of chaos' suggesting 'pathology'. It is for physicians to decide which 'disease' is more dangerous.

The return to the 'norm of chaos' may be spontaneous. Then, the 'recovery' occurs unaided, with time serving as the control parameter.

If the patient's conditions are normalized by drug therapy, its efficacy is possible to evaluate using the same criterion.

Naturally, each doctor has his (her) own criteria unrelated to the S-theorem. However, it may be equally useful to take advantage of the additional objective information derived from the analysis of the relative degree of order as described above.

The efficacy of medical diagnosis may be enhanced substantially if based not only on information about a given patient but also takes into consideration statistical data obtained by examining cohorts. However, the very first investigations in this sphere proceed from the assumption that both the cardiovascular system and the nervous system controlling it function in a similar way in all humans. This actually reduces the problem to the examination of different states of one and the same system.

The analysis was conducted using tachograms which depict time-dependence of cardiac rhythm patterns. The choice of the 'reference point' for the relative degree of order was based on the S-theorem using power spectra of tachograms obtained from a group of healthy subjects. The reference point for physical chaos was the state of a healthy subject producing a tachogram with the maximum degree of chaos. If entropy of this state is denoted as $\widetilde{S}_0$, then the main characteristic is the difference between entropies

$$\Delta S = S - \tilde{S}_0\,, \qquad (15)$$

which characterizes the degree of chaos relative to its 'norm' in the group of healthy subjects.

A large group of patients was then examined. It turned out that they could be subdivided into three groups based on selected criteria. One group was comprised by patients in which the disease was associated with a decrease in the degree of chaos, i.e. 'excessive order'. Two other groups included patients exhibiting an enhanced degree of chaos in the heart's work. In one of them, this rise was moderate, whereas in the other, the patients were characterized by an anomalously high degree of chaos in the cardiac performance.

It should also be recalled that these studies proceeded from the very strong assumption of the identity of cardiovascular function in all the patients. However, this restriction may prove rather weak because only relative characteristics were examined. At any rate, these investigations have demonstrated that the S-theorem may be used as a sensitive tool in diagnosing the state of medico-biological systems.

## 11. What is self-organization?

Two classes of systems were outlined in a previous Section.

One of them includes many physical systems exemplified in the foregoing discussion by two cases. To begin with, it is a Van der Pol generator in which losses (of electrical resistance) are first compensated as the feedback parameter grows while its further rise results in the transition to the developed generation region. According to the S-theorem, this is a case of self-organization. This process starts from equilibrium, that is, thermal fluctuations in an electrical contour in the absence of feedback. This leads to the conclusion that the process of self-organization may be defined as the transition from a most chaotic (equilibrium) state to a more ordered one (generation).

The situation is similar in the transition from laminar to turbulent flow in a pipe with increasing pressure difference (a higher Reynolds number).

Here, the reference point for the degree of chaos is also the equilibrium state of a fluid in the absence of pressure difference, that is at the zero control parameter. In this case, hydrodynamic motion is lacking and only chaotic motion of molecules occurs. Evidently, this state is most chaotic.

Again, the process of self-organization is the transition from a more chaotic to a less chaotic state. Is this the universal definition of self-organization? It can be inferred from the previous section that the process of self-organization is not necessarily associated with an increase in the degree of order.

Indeed, there is a broad class of systems (in the first place, biological systems) for which neither the state of complete chaos (thermodynamic equilibrium) nor that of ideal order can be realized. Biological systems would not function under such conditions.

A more fundamental notion for such systems is the 'norm of chaos' which has been used more than once in the previous discussion. This notion is compatible with that of 'health'. Then, self-organization is the process of reconvalescence.

Now, let us turn back to the aforementioned studies on the responsiveness of men and women to stress. Earlier, we have agreed to regard post-stress conditions as 'pathology'. This means that the transitions to the 'norm of chaos' in women is actually the 'recovery' referred to above as self-organization, i.e. the transition from a more chaotic to less chaotic state.

Conversely, the stress-induced state in males is 'illness' which corresponds to a more ordered state.

Hence, the 'recovery' (self-organization) in men is the transition from an ordered state to a more chaotic one.

Thus, the concepts of self-organization and degradation in biological systems cannot be unequivocally related to an enhanced (self-organization) or impaired (degradation) degree of order respectively.

A more fundamental notion for such systems is the 'norm of chaos' which can be estimated from empirical data using the 'S-theorem'.

To summarize, it appears from the above analysis that in certain cases self-organization is easy to observe, e.g. the generation developing in a Van der Pol system with an increasing feedback parameter. Other well-known examples are the appearance of a new structure (Benar cells) at the liquid surface heated from below and Taylor vortices between rotating coaxial cylinders.

Using the most fortunate term 'dissipative structures' coined by I Prigogine, the self-organization process may be described as the spontaneous occurrence of structures in nonlinear dissipative open systems, e.g. temporal dissipative structures in the Van der Pol generator and spatial dissipative structures exemplified by the Benar cells and Taylor vortices. The famous Belousov – Zhabotinsky reaction is an example of time-space structures. Elimination of the control parameter (feedback, temperature gradient, etc) in all these cases results in a 'system at rest', i.e. one in the state of thermodynamic equilibrium.

Such understanding of the term 'self-organization' underlies the theory of formation of dissipative structures. The first systematic exposition of this range of problems has been given in the well-known works of I Prigogine and G Nicolis [25]. The starting point was Prigogine's ideas on thermodynamics of irreversible nonequilibrium processes. H Haken's theory of self-organization is based on the appearance of structures due to collective interactions. In other words, cooperative processes are posited as being of primary importance. This prompted H Haken to use the term 'synergetics' for this new interdisciplinary field of research. The basic equations of synergetics are also nonlinear dissipative equations, e.g. reaction-diffusion equations or Ginzburg – Landau time equations.

In more complicated cases such as transition from one turbulent motion to another, in biological systems, it is possible to distinguish between the processes of degradation and self-organization based on the criterion for the relative degree of chaos (or order) in different states of open systems. In such cases, the understanding of self-organization as the appearance of new structures or the transition from less to more ordered states becomes insufficient.

This inference is valid for all systems in which the equilibrium state can not serve as the reference point for the relative degree of chaos (or order). Here, the 'norm of chaos' concept is of greater importance and, in the general case, certainly applies to the nonequilibrium state, with the transition from 'pathology' to 'health' corresponding to self-organization. Since deviation from the norm is possible in two directions (towards a greater or smaller degree of chaos), the self-organization process may in the general case also proceed in two directions.

Therefore, the traditional definition of self-organization as the spontaneous formation of structures in dynamic nonlinear dissipative open systems is too 'narrow'. A more comprehensive description of self-organization processes, even their mere identification, is feasible by the methods of the statistical theory of open systems [4].

The basic equations of this theory are kinetic equations for the distribution functions $f(X, R, t)$ of the values of the internal parameters X in space and time most essential for the problem being considered. These equations may be used to obtain a wide class of dynamic nonlinear dissipative equations for the moments of distribution function $f(X, R, t)$ including reaction-diffusion equations for the first moments $X(R, T)$, that is basic equations of the modern self-organization theory (synergetics).

It follows from the above that the theory of self-organization (synergetics) has numerous important implications despite the fact that it is a very young interdisciplinary field of science. However, the term 'self-organization' is actually rooted deep in ancient thought. This is a very interesting question worthy of illustration by the following facts.

In 1966, the book on 'Principles of Self-Organization' [26] was published in the Russian language. It is a collection of reports delivered to a Symposium at the Illinois State University, USA, in 1961. Here is a quotation from the Preface to the Russian edition by A Lerner, the editor:

"Despite the marked prevalence of self-organizing systems and persistent attempts of scientists to understand the phenomena occurring in such systems, self-organization has in a way remained for many centuries perhaps the most mysterious phenomenon, the most intimate of nature's secrets".

The Preface goes on to state: "... the reader will hardly find here a report which would not claim to disclose the mystery of self-organization".

Heinz von Foerster, the editor of the American publication, writes in the Introduction with reference to a story by Plato, a famous Greek philosopher: "The house of Agathon was the place where the first memorable symposium was held on the problems lying at the junction of different sciences, attended by philosophers, statesmen, dramatists, poets, sociologists, linguists, doctors and students learning various trades".

The report by Y Eshby, a known expert in the field, contains a statement to the effect that the word 'self-organization' can also mean 'transition from bad to good organization', even though the author does not explain how to distinguish between 'bad' and 'good'. An approach to this problem is illustrated by the above-mentioned analysis of cardiograms which allowed to differentiate between 'health' and 'pathology'. Such a distinction is also possible based on the above criterion for the relative degree of chaos in different states of open systems.

Naturally, there are more diagnostic criteria to evaluate the state of biological systems. However, the comparison of different diagnostic tools is a matter which requires special attention.

## 12. Physics of open systems for sociologists and economists

H Haken has reported one of the earliest applications of synergetics to sociology [6]. The expedience of applying synergetics for this purpose is due to the important role of collective effects in social processes. Specifically, they are to a great extent involved in shaping public opinion even though separate acts of choice are, by necessity, individual. A model survey of social systems was carried out by the group of W Weidlich [27] who suggested simple models for the description of the formation of public opinion, population migrations, and urbanization.

At present, methods of synergetics are extensively employed to simulate economic processes. Economics is an ancient science with deeply rooted traditions and advanced methods of qualitative and quantitative description of various processes. Nevertheless, there is a wealth of unresolved problems challenging the physics of open systems to apply its methodology in this field.

A substantial part of these problems is related to the optimization of the relationships between production, distribution, and consumption based on the criterion for the relative degree of order in open (social or economic) systems. Such an approach may hopefully provide additional information necessary to monitor the efficiency of the assumed control parameters, to estimate the 'norm of chaos', and to 'treat the disease', that is a deviation from the 'norm of chaos' on either side.

In case of spontaneous 'recovery', i.e. without interference from the outside ('medication'), 'reconvalescence' may also be regarded as a self-organization process. Certainly, similar to the situation with biological systems, the equilibrium state cannot serve as the reference point in estimating the relative degree of chaos in social and economic systems. Only the state corresponding to the 'norm of chaos' may be used for this purpose. Identification of such a state is the principal task which can be accomplished using criteria for the relative degree of order in physics of open systems.

## 13. Concluding remarks

A few years ago, G Cagliotti, an Italian investigator, published a popular-science book under the title of *Dynamics Ambiguity*. The two first Italian editions (1982, 1986) were later translated into German (1990) and English (1992) [28]. The English version had the foreword by H Haken. The Preface to the Russian edition due to appear soon was written by I Prigogine. The two experts welcome the publication. Now, why was the book received so favorably?

The book is first and foremost about the links and relationships between science and art, that is, the connection between 'two cultures' as the catchword goes. Suffice it to mention the author's interpretation of the transition from perception to idea. One of the first pages in the book reads as follows: "A study of perception may reveal integrating factors. That is, originally disordered sensory stimuli become correlated and organized in the brain into ordered coherent structures which are then converted to a thought". In other words, the transition from perception to idea is the transition from a less to more ordered state of the brain.

True, this is a very beautiful picture of the birth of an idea. The question is how close it is to reality. The book gives no answer since it does not consider the criteria for a relative degree of order in open systems which would allow for the distinction between 'order' and 'chaos'. Doubtless, some information about the modulation of orderliness accompanying the generation of an idea can be obtained from the analysis of brain activity using encephalograms and the above criteria from the physics of open systems, specifically the S-theorem.

Such an approach implies a series of experimental studies designed to elucidate 'the thought production rate', its difference in men and women, the influence on artistic performance, etc. Naturally, joint efforts of specialists representing different scientific disciplines are necessary to solve such a difficult problem.

Finally, all this brings to mind the famous book of Erwin Schrödinger *What is Life?* published in English in 1944 and in Russian in 1947. We shall refer to only two fragments directly related to the present discussion.

Chapter 6 entitled "Order, disorder, and entropy" queries: "What is a characteristic feature of life?" The answer is found on page 105 of the Russian edition:

"How would we express in terms of the statistical theory the marvellous faculty of a living organism by which it delays the decay into thermodynamical equilibrium (death)?"

We said before: "It feeds upon negative entropy", attracting, as it were, a stream of negative entropy upon itself to compensate the entropy increase it produces by living and thus to maintain itself on a stationary and fairly low entropy level.

If $D$ is a measure of disorder, its reciprocal, $1/D$, can be regarded as a direct measure of order.

Since the logarithm of $1/D$ is just minus the logarithm of $D$, we can write Boltzmann's equation thus:

$$-(\text{entropy}) = k_{\mathrm{B}} \log\left(\frac{1}{D}\right). \tag{16}$$

Hence, the awkward expression 'negative entropy' can be replaced by a better one: entropy, taken with the negative sign, is itself a measure of order. Thus, the device by which an organism maintains itself stationary at a fairly high level of orderliness (fairly low level of entropy) really consists in continually sucking orderliness from its environment."

Schrödinger's opinion as stated in this fragment is very interesting. It reflects a standpoint that was for many years shared not only by many biologists but also by physicists. We can compare Schrödinger's ideas with the results cited above.

In accordance with the S-theorem, entropy may be used as a measure of the relative degree of order in open systems only on the additional premise that the average effective energy is the same for the states being examined.

One of such states with entropy $S_0$ is assumed to represent physical chaos. The system in this state must be 'heated' if the equality condition for the average energy is to be met, which will result in $S_0 \to \tilde{S}_0$. Then, the difference between the renormalized entropy and entropy $S$ of a more ordered state

$$\widetilde{S}_0 - S \geqslant 0 \tag{17}$$

is the quantitative measure of chaotic motion in the case of physical chaos which becomes more ordered in the latter state.

This has been demonstrated earlier in the present paper using physical examples. One of them was a Van der Pol generator in which physical chaos was represented by thermal oscillations in the electrical contour in the absence of feedback. Another example was the state of developed generation. The former state was 'heated' to make average energies identical. As a result, the expression of the form (17) served as a measure of the quantity of energy associated with thermal fluctuations in the contour which was transformed to the energy of ordered oscillations. Such a transformation of the motion provides an example of the spontaneous (!) transition towards a more ordered state, i.e. self-organization.

One more example was the transition from laminar to turbulent flow. The former was taken to be the state of physical chaos and 'warmed up' to equalize average energies. This resulted in the transition of a part of chaotic motion in the laminar flow to the more ordered (collective) motion of the turbulent flow. Such a transition may also serve as an example of self-organization.

Understandably, the definition of self-organization in biological systems is not equally unambiguous. Here, the estimation of the norm of chaos (or order) is crucial, and the process of self-organization is regarded as the spontaneous (without 'therapy') return to the norm of chaos, i.e reconvalescence.

Therefore, an 'organism' as an open system maintains itself by virtue of its ability to transform the energy of chaotic motion to that of a more ordered motion. Evidently, this offers an opportunity to clarify Schrödinger's point of view.

Another very interesting statement which Schrödinger makes in the final chapter of his book is worth citing. A paragraph on page 108 of the Russian edition reads as follows: "An organism's astonishing gift of concentrating 'a stream of order' on itself, and thus escaping the decay into atomic chaos — of 'drinking orderliness' from a suitable environment — seems to be connected with the presence of the 'aperiodic solid', the chromosome molecules, which doubtless represent the highest degree of well-ordered atomic association we know of - much higher than the ordinary periodic crystal...".

This is truly a remarkable thought, but it cannot be considered here at greater length. Suffice it to answer the following question: "Is the degree of order in an aperiodic crystal higher than in an usual periodic one, in terms of the above theory?" There is every reason to argue that the answer must be in the affirmative!

Indeed, there is an analogy with the relative degree of order for laminar and turbulent flows. It seems natural to identify a laminar flow with a periodic crystal and a turbulent one with an aperiodic crystal. The thermal atomic motion in periodic crystals may be assumed to represent the state of physical chaos. Hence, collective degrees of freedom in aperiodic crystals are of greater importance than in periodic ones. This suggests, in conformity with the S-theorem, a higher degree of order in an aperiodic crystal than in a periodic one. This inference, however, remains to be quantitatively confirmed.

We have referred to only a few selected items in Schrödinger's book. But it actually contains many other valuable insights and will surely point the way to new developments in the statistical theory of open systems.

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