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Tachyons and the instability of physical systems

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Contents

Abstract. Not quite simple and rather obscure relations between the concepts of 'instability' and 'tachyons' are discussed.

1. Introduction

Hypothetical particles with imaginary mass were officially introduced into scientific usage in 1967 by Feinberg [1] under the term 'tachyons' (however, in the nameless form they were known to theoreticians of various countries long before [2]). Originally tachyons were considered as individual isolated particles similar to electrons, protons, etc. But in this understanding, tachyons, most likely, cannot be found naturally. Later, however, it was recognised [2] that they do widely occur in nature as elementary excitations (quasiparticles) in complex systems which lose stability and undergo a phase transition into a more stable state. Illustrations of this phenomenon from various fields of physics are given below. Perhaps, one of the most significant examples concerns with modern unified theories of elementary particles (see, for example [3]). In such theories, tachyons are introduced on purpose to make a vacuum state unstable and cause it to transform thereby imparting masses to massless particles [4]. The discussion of not very simple and not entirely known relations between the notions of 'instability' and 'tachyons' is just the subject of this paper.

To start with, let us formulate some questions which will right away acquaint the reader with problems to be considered and whose discussion, in essence, makes up the contents of the paper.

(a) Most readers associate tachyons with particles moving faster than light. If this is the case, how should we understand the words about the real participation of tachyons in the process related to the instability of a physical system? For the idea of the impossibility of motions faster than light is deeply rooted in us.

(b) There is a common explanation for the reasons of the occurrence of superconductivity at arbitrary weak interaction between fermions. Near the Fermi surface where the particles couple, the situation becomes two-dimensional, but in the two-dimensional case the Schrödinger equation yields bound states at any attraction. Moreover, the exponential dependence of the two-dimensional coupling energy on the potential leads to the similar dependence between the band gap and the critical temperature. But on the other hand, the Cooper pair is known to be a correlated rather than a bound state, which differs significantly from the state of a two-atomic molecule (see, however, [5]). Suffice it to say that pairing particles have opposite in direction (and equal in value) momenta. Does not this fact shake our faith in the above explanation?

(c) The Jeans instability is of the fundamental importance in cosmogony, resulting in the condensation of bulk matter at one or several centres [6]. This condensation manifests itself only under the condition that all the initial dimensions of the body exceed a certain length (the Jeans length). Therefore, a body for which this condition is not fulfilled (thin film, filament, etc) is more stable than the one for which it is met. Will such a body be stable? If not, how much will the increment of its density increase diminish?

(d) The Jeans instability relates to longitudinal (in terms of electrodynamics) degrees of freedom of gravitational field, which are generated by static gravitational charges, i.e., masses. This instability is eventually caused by the intrinsic property of the field, i.e., attraction of like charges. Does this property induce the instability of transverse (caused by the motion of charges, i.e., currents) degrees of freedom of the field, which are described by the off-diagonal components $g_{0\alpha}$ ($\alpha = 1, 2, 3$) of the metric tensor ?

(e) Such components arise as a heavy body rotates. Can the body self-rotate in the case of positive answer to the above question (which would obviously come in conflict with reality)? Can the increase in the relevant field be ceased due to the law of momentum conservation?

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2. Definition and the main properties of the tachyon

The tachyon is, by definition, an object for which in the usual formula $E^2 = p^2c^2 + M^2c^4$ relating the energy E to the momentum $\mathbf{p}$, the term M is replaced by the negative quantity $-\Gamma^2$. Assuming the Planck constant to be equal to unity, we rewrite this formula in terms of waves as:

$$\omega^2 - C^2k^2 + \Gamma^2 = 0\,, \tag{1}$$

where ω is the frequency, $\mathbf{k}$ is the wave vector, C is the characteristic velocity coinciding here with the speed of light. Equation (1) is valid for a uniform isotropic system where the wave is plane. In the general case, denoting the wave function by ψ, we can express the wave equation as:

$$(\omega^2 + C^2\Delta + \Gamma^2)\,\psi = 0\,. \tag{1a}$$

Let us consider some examples of various instabilities (references can be found in review [2]). The Jeans instability corresponds to a wave in which C is the speed of sound and $\Gamma^2 = 4\pi G\varepsilon c^{-2} = c^2/a^2$, where a is the Jeans length, G is the gravitation constant, ε is the density of the substance energy (see Section 4). The instability of a normal superconducting state (without Bose-condensate of Cooper pairs) at temperature below the critical one occurs at C equal to the electron velocity at the Fermi surface $\Gamma^2 = \Delta^2$, where Δ is the band gap. The instability of a set of pendula elastically coupled and placed into a gravity field 'head foremost' takes place at C equal to the speed of sound in the system, $\Gamma^2 = g/L$, where g is the acceleration of gravity, L is the pendulum length. The instability of an electromagnetic wave in a medium with inverse level population corresponds to $C = c$ and $\Gamma^2 = 8\pi\xi|d_{12}|^2$, where $\xi = (E_1 - E_2)/(N_1 - N_2)$, E and N are the level energy and population. And the last example is the wave of the Higgs scalar field ϕ, which plays a great role in the unified field theories. For this field $C = c$ and $\Gamma^2 = M^2 - \lambda\phi^2$, where M is the Higgs particle mass, and λ is the coupling constant of the Higgs field interaction.

Turning back to Eqn (1), one can easily verify that the group velocity of the wave $\mathrm{d}\omega/\,\mathrm{d}\mathbf{k}$ really exceeds the speed of light at $C = c$. If we deal with the information transmission, we come up against the violation of causality: there are reference systems where the event-cause occurs later than the event-effect. On the other hand, at $kC \leqslant \Gamma$ (for rather huge dimensions of the system: more than $C\Gamma^{-1}$), the frequency becomes imaginary $\omega = \pm\mathrm{i}\Gamma$, which, in view of $\psi \sim \exp(\mathrm{i}\omega t)$ means the exponential growth of the wave with time t, i.e. the system instability. To understand how so different properties as noncausality and instability can combine, we should consider the tachyon Green function $G(t, \mathbf{x})$ describing the tachyon propagation with time. The denominator of this function in the frequency–momentum representation is just the left-hand side of Eqn (1). Accordingly,

$$G(t,\,\mathbf{k}) = (2\pi)^{-1}\int_K (\omega^2 - C^2k^2 + \Gamma^2)^{-1}\exp(-\mathrm{i}\omega t)\;\mathrm{d}\omega\,, \tag{2}$$

where the contour K in the complex plane ω represents the way of bypassing the singularities of the integrand in Eqn (2) [zeros in the left-hand side of (1)]. This contour can conveniently include a distant semicircle in the upper (lower) frequency semiplane at t below (above) zero (Fig. 1). While $k > \Gamma/C$ the singularities lie along the real frequency axis and are bypassed in a usual way. In the opposite case the singularities displace into the complex plane and reside at points $\omega = \pm\mathrm{i}\Omega$, where $\Omega = \sqrt{\Gamma^2 - C^2k^2}$. Whether the Green function describes unstable or non-causal situation depends on the way of bypassing the singularities, i.e. on the choice of the contour K in Eqn (2).

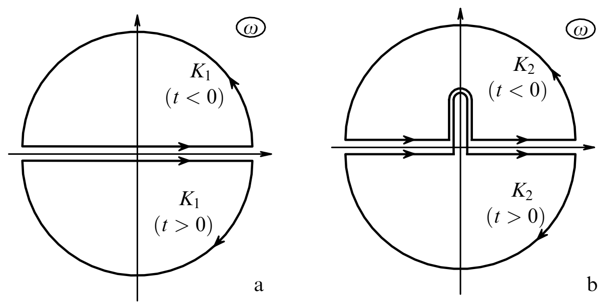

Figure 1.

In bypassing the contour K_1, one can readily see that the contribution of residuals at the singularities is proportional to $\exp(-\Omega|t|)$ which decreases with time (stability) but does not vanish at $t \leqslant 0$ (noncausality). The choice of the contour K in the form K_2 yields the expression which is proportional to $\sinh\Omega t$ at $t \geqslant 0$ and increases as $t \to \infty$ (instability), while is equal to zero at $t \leqslant 0$ (causality). The same results can be obtained from Eqn (2):

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \Omega^2\right) G(t, \mathbf{k}) = -\delta(t)\,.$$

One of the solutions to this equation, $\exp(-\Omega|t|)/2\Omega$, is stable but noncausal. The other one, $\theta(t)\sinh(\Omega t)/\Omega$, is causal [$\theta(x) = 1$ at $x > 0$, $\theta(x) = 0$ at $x < 0$] but unstable. The difference in these two solutions, $\exp(\Omega t)/2\Omega$, is the solution to the free equation for the Green function. The foregoing illustrates the general principle: the choice of rules for bypassing the singularities fixes a solution to the free equation.

Therefore, we can choose the rules for bypassing the singularities, which would meet the causality conditions but correspond to an increasing field and describe an unstable system. The Green function of a general type $G(t, \mathbf{x}_1 - \mathbf{x}_2)$ being considered as above, vanishes everywhere except for the upper part of the light cone, i.e. casuality conditions of the general type are fulfilled. It remains to clear up the question of the tachyon velocity exceeding the speed of light (see above). Referring the reader to the review [2] where the illustrations from electrodynamics (a wave in a medium with inverse level population, a wave in a dispersive absorbing medium) are discussed in detail, we only notice here that the group velocity of the signal fails to characterise the rate of the energy and information transfer as the wave packet transforms in the course of propagation. The transformation takes place in the case of absorbing or, on the contrary, unstable medium. However, in the case of tachyon the wave packet can be only constructed from harmonics with $k > \Gamma/C$, whose increment is equal to zero, though the group velocity exceeds C. Here, too, there is no information transmission faster than light and we deal with a sort of scrolling text in electrical advertising. Even at the initial moment the wave packet is not spatially localised (including only part of harmonics), and the occurrence of the wave packet maximum at a certain point is associated with the amplification of a signal already available at this point due to the system instability rather than with the energy (and information) transmission (see Ref. [2]). Notice that in the electric advertising there is also some instability; minor cause has a great consequence (an operation of a relay ignites a lamp).

Thus, we have answered the question (a) formulated in Section 1: in an unstable medium tachyon does not transmit information at velocity exceeding the speed of light, but it is this transfer that makes impossible motions faster than light. Therefore the participation of the tachyon in a real physical process of system transformations is not in conflict with any general principles.

3. The tachyon and the Schrödinger equation

To set the stage for answering the questions (b) and (c), it is worthwhile to establish a simple relation between the tachyon field equation and the general Schrödinger equation, which would enable us to use the well-known quantum mechanical regularities in the solution of tachyon problems. We begin with Eqn (1a) and consider Γ to depend, in the general case, on the spatial coordinate $\mathbf{x}$. Comparing this equation with the Schrödinger equation for the steady state of a particle with the mass equal to 1/2

$$\left[\Delta + E - V(\mathbf{x})\right]\psi = 0$$

one can readily see that

$$\frac{\omega^2}{C^2} \Leftrightarrow E, \qquad \frac{\Gamma^2}{C^2} \Leftrightarrow -V(\mathbf{x})\,. \tag{3}$$

Thus, the tachyon instability ($\omega^2 < 0$) corresponds to the state in the attraction field $-\Gamma^2/C^2$. Accordingly, a finite body where the tachyon moves corresponds to a potential well with dimensions depending on the body geometry. The bound state energy E_b in the well immediately determines the increment of the tachyon field increase:

$$\psi \sim \exp(\Omega t), \qquad \Omega = C\sqrt{-E_b}\,. \tag{4}$$

To start with, let us consider a three-dimensional body all dimensions of which are comparable and make up a quantity of the order of L. Since bound states appear in a three-dimensional potential well of depth V only at $V > L^{-2}$, the tachyon instability will arise only at $L > C/\Gamma$. This is just the condition for the body dimensions, which was already formulated in Section 1 in relation to the Jeans instability. Thus, the tachyon instability of a three-dimensional body takes place at rather huge dimensions of the body [otherwise, the second term in the left-hand side of Eqn (1) would be more in absolute value than the third one and the frequency would remain real].

Now let us consider a quasi-two-dimensional body (thin film), one dimension of which (thickness) is not subject to the above formulated condition, i.e. $d \ll C/\Gamma$. In essence, we deal here with the two-dimensional free motion in the direction parallel to the film surface and with the one-dimensional motion in a narrow potential well in the direction perpendicular to the surface. According to quantum mechanics, the bound states originate in a one-dimensional potential well of arbitrary small depth, the bound state energy E_b depending on the well depth quadratically. Finally, in the quasi-one-dimensional case (thin filament) the motion will be confined by two directions perpendicular to the filament axis, and, therefore, the potential well will be two-dimensional. In the relevant quantum-mechanical problem, the bound states also appear at arbitrarily small depth of the well and the energy E_b depends exponentially on this depth.

The foregoing consideration enables us to answer the questions (b) and (c) (see Section 1). Especially simple is the answer to the former one. Superconductivity arises when the ground state of a system becomes unstable with respect to the formation of Bose-condensate of Cooper pairs. This instability is of tachyon character (see Section 2) and therefore originates simultaneously with the appearance of the bound state in the relevant quantum-mechanical problem. The superconducting pairing occurs in the narrow region of the momentum space near the Fermi surface. Therefore, owing to the uncertainty principle, the motion in the coordinate space will resemble the motion in a quasi-one-dimensional cylinder, whose axis corresponds to the direction of the normal to the Fermi surface in the momentum space. Relatively free motion along the Fermi surface will correspond in the momentum space to a two-dimensional potential well in the coordinate space. Hence, we immediately infer the pairing at arbitrarily weak attraction, the exponential character of the dependence between the band gap and the well depth, etc. As for the nature of the Cooper pair, whether it is a bound state or something quite different, it is absolutely irrelevant to the above consideration. Therefore, the explanation formulated in the question (b) is, in essence, correct though lacking precision.

The answer to the question (c) is also straightforward. The value of increment is of the order of Γ for a three-dimensional body with rather huge dimensions. Bodies, one (film) or two (filament) dimensions of which are small, are still unstable (in one- or two-dimensional cases there is always a bound state) but the increment of their field increase is significantly less than in the three-dimensional case. Using the expression for the bound state energy, well-known from quantum mechanics: $d^{-2}\exp(-V^{-1}d^{-2})$ (filament of thickness d), V^2d^2 (film of thickness d), and formula (4), we can easily derive the following estimates for the relation between the increment Ω (for the filament and the film, respectively) and the increment Γ (for the three-dimensional body)

$$\frac{\exp(-\xi^{-2})}{\xi}\,,\ \xi\,. \tag{5}$$

Here $\xi = \mathrm{d}\Gamma/C \ll 1$ is a small parameter standing for the thinness of the filament and the film. Thus, 'thinned' unstable matter does live much longer than 'bulk' one.

4. Transverse instability in the theory of gravity

Bearing in mind question (b), we start with general considerations concerning the occurrence of transverse instability in the physics of gravity. It differs from electromagnetism by the attraction of like sign charges, i.e. masses (and the absence of unlike sign charges and their screening). This leads to the opposite signs of the coupling constants $-Gm^2$ and e^2 involved in the Newton and Coulomb laws, which manifests itself, in particular, as the Jeans instability. This instability increases the density oscillations and immediately results from the equation for the longitudinal (plasma) oscillations in electrodynamics:

$$\omega^2 = v^2k^2 + \omega_{\mathrm{p}}^2\,, \qquad \omega_{\mathrm{p}}^2 = 4\pi e^2 n m^{-1}\,,$$

where n is the concentration of particles, m is their mass, and v is characteristic velocity. Actually, replacing the electromagnetic coupling constant in the latter equation by the gravitation one, we can arrive at the Jeans equation, which is obviously of tachyon character

$$\omega^2 = c_{\mathrm{s}}^2k^2 - 4\pi G\varepsilon c^{-2} \tag{6}$$

(see below direct derivation of this equation).

For the same reason the transverse waves in heavy liquid might also be expected to be unstable as evidenced by the spectrum of transverse electromagnetic waves in the medium $\omega^2 = c^2k^2 + \omega_{\mathrm{p}}^2$ when in addition to the fields the current density oscillates rather than the charge density (as in plasma waves). Therefore, the arguments concerned with the well-known in electrodynamics Lenz's rule also hold true: a reactive current induced by a changing external field is directed opposite to this change and tends to decrease its effects. Reactive current is proportional to the coupling constant ω_{p}^2 and changes its sign in the case of gravitation where the reactive current brings the system away from the initial state rather than moves it close to it (see Appendix).

So far we have associated the notions of 'longitudinal' and 'transverse' with charge and, consequently, current degrees of freedom, appealing to physics of electromagnetism. Certainly, they can be interpreted independently by dividing the vector of current velocity $\mathbf{j}$ (or, which is, practically, the same, the velocity vector $\mathbf{v}$) into a potential (longitudinal) and a solenoid (transverse) components. The former is characterised by the fact that its rotor is equal to zero (vortex-free flow) and the latter by the fact that its rotor is nonzero, but the divergence is equal to zero. Having regard to the well-known continuity equation, one can readily see that oscillating in the transverse mode is really the rotor of current or velocity rather than that of density (concentration). Accordingly, oscillating in the longitudinal mode is genuinely the density or divergence of the current (velocity).

As applied to the gravitation, one can easily verify that in the Newton approximation only the longitudinal modes excite. This corresponds to the well-known conclusion about potentiality of small oscillations in liquid and is immediately seen from the Euler linearised equation

$$\dot{\mathbf{v}} = -\nabla(\delta p)(mn)^{-1} - \nabla\delta\phi\,,$$

which, together with the continuity equation, the state equation $\delta p = c_{\mathrm{s}}^2\delta(mn)$, and the Newton equation for the gravitation potential

$$\Delta\delta\phi = 4\pi G\delta(mn),$$

leads to the Jeans tachyon Eqn (6). As for transverse modes, their excitation can be only described in post-Newtonian approximation, when the off-diagonal components $g_{0\alpha} \equiv g_\alpha$ of the metric tensor generated, for example, by the rotation of a heavy body, come into play (see Section 5). Here distinction must be made between co- and contravariant components u_i and u^i of 4-velocity and their three-dimensional components (for relatively slow motions) v_α and v^α (see Ref. [7]). The relation between these quantities is as follows:

$$v_\alpha = g_{\alpha i}u^i = g_{0\alpha} + \tilde{v}_\alpha\,, \quad u^0 \approx 1\,, \quad \tilde{v}_\alpha = g_{\alpha\beta}v^\beta\,. \tag{7}$$

Particularly noteworthy is the case with superfluid liquids whose covariant velocity component is equal to the gradient of the phase of the order parameter [7]. Therefore, in terms of this component, the flow is potential and the relevant oscillations are longitudinal. However, v^α has nonzero rotor and it is its oscillations that are of transversal character. Especially simple is the case with rotating superfluid liquid (it is quite interesting for pulsar physics [7]). In spherical coordinates $x^{1,2,3} = r, \theta, \phi$ where an axis coincides with the rotation axis, the quantity v_3 (but not v^3!) is generally equal to zero due to the axial symmetry of the system. Therefore the first formula in Eqns (7) relates the contravariant velocity to the nondiagonal component of the metric tensor (Levi-Civita formula)

$$v^3 = -\tilde{g}^3\,, \quad \tilde{g}^\alpha = g^{\alpha\beta}g_\beta\,. \tag{8}$$

Now we can pass on from general observations to a particular problem where the transverse instability is obvious (Fig. 2). Let us consider a thin (of thickness d) film of 'bulk' superfluid liquid extending to infinity in the direction of the Cartesian axes x^2, x^3. Travelling on the film is the wave

$$\tilde{g}^3 = f(x^1)\,\exp\left[\mathrm{i}(kx^2 - \omega t)\right],$$

which is transverse due to Eqn (8) and zero divergence of the velocity. Restricting ourselves, for simplicity, to the lowest post-Newtonian approximation ($\kappa^2x^2 \ll 1$, $\kappa d \ll 1$, $\kappa^2 = 8\pi G\varepsilon/c^2$), we can easily arrive at tachyon equation (1) with $C = c$ and $\Gamma = \kappa$, which indicates the instability of the studied system (see, however, Section 5) and gives the positive answer to question (d) from Section 1.

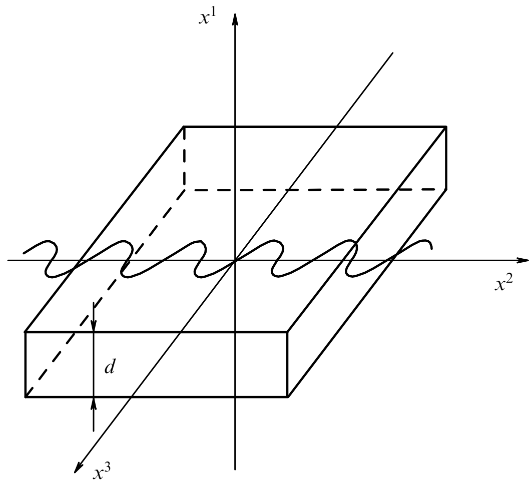

Figure 2.

5. Torsion oscillations of a heavy body

To conclude the paper, let us turn our attention to the last question (e) and show that the tachyon instability can be decreased by the action of the conservation laws. Particularly, we will deal with the conservation of momentum in the problem of torsion (obviously transverse) oscillations of a heavy spherically-symmetric body (superfluid liquid in a solid shell) at frequency ω (Fig. 3). The relevant component of the Einstein equation for $\tilde{g}^3$, which is the generalisation of the static equation [7], is

$$\left[(1-x^2)\left(\partial_x^2+\frac{4}{x}\,\partial_x\right)-\frac{3}{2}\,x\partial_x+\zeta^2\left(1-\frac{x^2}{2}\right)\right]\tilde{g}^3=0\,, \tag{9}$$

where $x=\kappa r$, $\zeta=\sqrt{6}\omega/\kappa$, $\kappa^2=16\pi G\varepsilon$. Equation (9) can be rewritten in the form

$$\left[\Delta+\frac{15-3x^2/4}{4(1-x^2)^2}+\zeta^2\,\frac{1-x^2/2}{1-x^2}\right]\mathbf{\Phi}=0\,,$$

where

$$\mathbf{\Phi}=(1-x^2)^{3/8}\tilde{g}^3[\mathbf{n},\,\mathbf{x}]\,,$$

$\mathbf{n}$ is the unit vector of the oscillation axis which is obviously of tachyon form (though the squares of the frequency and the mass depend on x). Therefore, the equation might be expected to describe a system unstable with respect to increasing $\tilde{g}^3$ (or, eventually, the angle velocity).

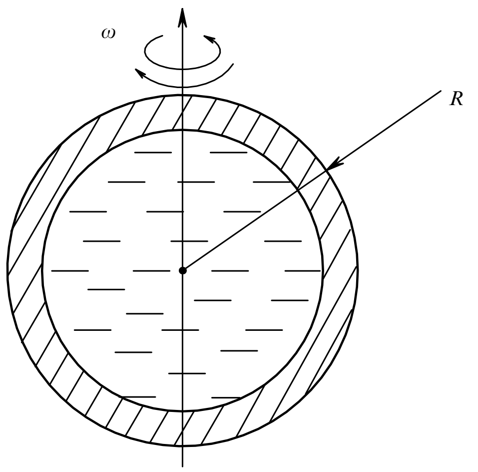

Figure 3.

It turns out, however, that the momentum conservation law prevents the 'self-rotation' of the body considered. This statement is, by no means, trivial since the momentum of the body is conserved together with the momentum of the gravitation field. In principle, these quantities might be of opposite sign and, increasing, they would completely compensate each other. This is just the case with the Jeans instability which takes place despite the law of energy conservation, since the increase in the kinetic energy is cancelled by the increase in the (negative) gravitation energy. The same is true of the transverse instability in a thin film (see Section 4) where the gravitation energy $-R^2/32\pi G$ ($R=\operatorname{rot}\tilde{\mathbf{g}}^\alpha$) is negative. Therefore the problem of the stabilising role of the law of momentum conservation deserves special consideration.

We will use the following definition of the total (including the gravitation term) momentum of the system M: in the static case ($\omega=0$) at large distances r from the body, the momentum behaves asymptotically as

$$\tilde{g}^3\to-2GMr^{-3}\,. \tag{10}$$

This case takes place at the stability boundary between the stable ($\omega^2>0$) and unstable ($\omega^2<0$) regions. This boundary corresponds to the value α_c of the dimensionless gravitational coupling constant $\alpha=r_g/R=\kappa^2R^2/6$, separating the stable ($\alpha<\alpha_c$) and unstable ($\alpha>\alpha_c$) regions; here $r_g=2Gm/c^2$ is the gravitation radius, R is the geometrical radius, and m is the body mass. In the state stable with respect to the gravitational collapse, $\alpha<1$ and the condition for the instability of transverse oscillations considered is $\alpha_c<1$.

To determine the quantity α_c let us turn to Eqn (9) at $\omega=0$. Its boundary conditions are set from the following reasoning. Initially, the momentum of the nonrotating body and, according to Eqn (10), the value of $\tilde{g}^3$ outside the body are equal to zero. The same holds true in the course of oscillations at $\omega\to0$ due to momentum conservation. Therefore at $r=R$ (or $x=x_0=\sqrt{6\alpha_c}$)

$$\tilde{g}^3=0,\qquad \partial_x\tilde{g}^3=0\,.$$

But a homogeneous differential equation of the second order with zero boundary conditions has only a trivial zero solution. Only at $x_0\to\infty$ there is a solution proportional to $x^{-9/2}$ and satisfying the boundary conditions. It corresponds to $\alpha_c\to\infty$ which means the absence of instability. Thus, the answer to the question (d) [more precisely, to the second question under label (d)] is also positive.

6. Conclusions

To conclude the paper, we would like to emphasise once again the main statement which makes up its pivot. Whether tachyons are found in nature as independent particles or not, today they constitute the all-important element of systems unstable with respect to the phase transition into the stable state. It is the tachyon mode that increasing with time performs a phase transition breaking down the pre-existing phase and forming a new one. Near the phase transition point, the 'soft mode' whose frequency tends to zero and its square passes on from positive values through zero to negative ones takes on the decisive role. This is just the tachyon degree of freedom which has be repeatedly spoken about in the foregoing. The tachyon parameters: velocity C and (imaginary) mass Γ determine the characteristics of the phase transition itself and the final properties of the system. It should be reiterated: despite unusual properties, the tachyon is not an idle idea of theoreticians, but a real component of the physical picture of universe.

7. Appendix. The Lenz rule in electrodynamics and the theory of gravity

We advance some arguments in support of the suggestion on the Lenz rule made in Section 4. The relation between reactive $\mathbf{j}_r$ and external $\mathbf{j}_e$ currents

$$\mathbf{j}_r(\omega,\mathbf{k})=C\,\frac{\mathbf{j}_e(\omega,\mathbf{k})}{\omega^2-c^2k^2+i\omega\delta} \tag{A.1}$$

can be expressed as

$$\mathbf{j}_{\mathrm{r}}(t, \mathbf{x}) = -C \int \frac{\mathbf{j}_{\mathrm{e}}(t - |\mathbf{x} - \mathbf{y}|\, c^{-1},\, \mathbf{y})}{4\pi|\mathbf{x} - \mathbf{y}|}\, \mathrm{d}\mathbf{y}\,.$$

It is seen that the sign of C indicates the Lenz ($+$) or anti-Lenz ($-$) behaviour of the system. From the Maxwell and post-Newtonian Einstein equations yield relation (A.1) with the parameter C equal to ω_{p}^2 and $-\kappa^2$, respectively. The signs of ω_{p}^2 and $-\kappa^2$ agree with the conclusions of Section 4.

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