

METHODOLOGICAL NOTES

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On incorrect formulations of the equivalence principle

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Abstract. **Early formulations of the equivalence principle (EP) [1 – 3, 6] are discussed and a critical analysis of them is given. It is shown that the main conclusions of papers [3, 5] are erroneous. The formulation of EP is presented as given by A Einstein in 1933 [9]. It is this formulation of EP that underlies GTR.**

1. In accordance with Einstein's works published in 1907 and 1911 [1], the equivalence principle (EP) is typically formulated in the following way [2]: "A reference frame that is absolutely static or undergoes inertial motion in a uniform gravitational field of the strength **g** is physically equivalent to a reference system moving translationally with acceleration **j** in the absence of a gravitational field, provided $\mathbf{g} = -\mathbf{j}$". A variant of this formulation has been suggested in Ref. [3]: "According to this principle (i.e. the equivalence principle), all physical phenomena proceed similarly in the inertial reference frame K_g having homogeneous gravitational field with acceleration of gravity **g** and in the uniformly accelerated system K_a which moves with acceleration $-\mathbf{g}$ relative to an inertial system without gravitational field". Based on this statement of EP, the author of Ref. [3] arrives at the following conclusion: "In the presence of a homogeneous gravitational field, a free charge is uniformly accelerated with respect to the inertial reference frame and, in agreement with the aforesaid, emits radiation. However, it is unlikely to radiate in the accelerated system K_a since it is not accelerated with respect to the inertial reference frame. Therefore, systems K_g and K_a appear to be nonequivalent, and EP seems violated. Actually, a charge in the system K_a radiates exactly as it does in the system K_g, that is the equivalence principle is absolutely fulfilled".

It is worthy of note that the above EP formulations [1 – 3] do not even mention locality (small space-time domains). Radiation in K_g is known to be defined by the exact formula

$$J = \frac{2}{3}\,\frac{e^2 a^2}{c^3}\,, \tag{1}$$

where a is the strength of the gravitational field.

Formula (1) has been obtained for a uniform gravitational field in the entire space. Actually, a uniform gravitational field never extends to infinity, and such an extreme situation should be regarded as an abstraction. However, it is not electrodynamically forbidden and, besides, may be considered in the framework of the above EP formulations.

According to these formulations, a charge in system K_a radiates precisely as it does in system K_g. However, this does not mean that radiation in system K_a is defined by the same formula (1). The problem of definition of radiation in system K_a is directly related to the problem of charge propagation in the framework of special rather than general relativity. In an earlier paper [4], we have demonstrated that in the above formulation EP is violated, and V L Ginzburg's conclusion concerning charge radiation in system K_a is incorrect. A charge at rest in an inertial system can not radiate, and this assertion does not depend on the system of coordinates in which the charge is considered. It is for this reason that the charge emits no radiation in the system K_a either. This problem is discussed at greater length in Ref. [4]. V L Ginzburg and Yu N Eroshenko [5] seem to be of the same opinion when they state that transition from the inertial "reference frame K to the reference frame K_a can not give rise to new particles: electrons, adrons, photons, etc".

This apparently settles the question, and V L Ginzburg agrees that a charge at rest in the inertial system K will not radiate in system K_a. In other words, EP as formulated in Refs [1 – 3] is not fulfilled. However, the authors of Ref. [5] go on as follows: "It is now time to prove that a charge which does not radiate in the inertial reference frame K where it is at rest does radiate in the reference frame K_a".

We have to treat this problem in greater detail in order to demonstrate that the main conclusions in Ref. [5] are erroneous. Unlike the authors of Ref. [5], we have made accurate calculations. It is generally known that the equation of motion for the charge in an arbitrary system of coordinates (including an accelerated one) of the Minkowski space has the form

$$m\,\frac{\mathrm{D}u^\nu}{\mathrm{d}\sigma} = F^\nu\,, \tag{2}$$

where $u^\nu = \mathrm{d}x^\nu/\mathrm{d}\sigma$ is the 4-velocity and F^ν is the force 4-vector. In the absence of force $F^\nu = 0$, the motion is along the geodesic line of the Minkowski space:

$$\frac{\mathrm{D}u^\nu}{\mathrm{d}\sigma} = \frac{\mathrm{d}u^\nu}{\mathrm{d}\sigma} + \Gamma^\nu_{\alpha\beta}\,u^\alpha u^\beta = 0\,, \tag{3}$$

$\Gamma^\nu_{\alpha\beta}$, the Kristoffel symbols of the Minkowski space, are

$$\Gamma^\nu_{\alpha\beta} = \frac{1}{2}\,\gamma^{\nu\sigma}\left(\partial_\alpha\gamma_{\beta\sigma} + \partial_\beta\gamma_{\alpha\sigma} - \partial_\sigma\gamma_{\alpha\beta}\right),$$

where $\gamma_{\alpha\beta}$ is the metric tensor of the Minkowski space. The motion along the geodesic line of the Minkowski space is free. But in the case of free motion, there is no radiation in any coordinate system, and the charge does not dissipate energy to maintain radiation. This is easy to see if one passes to an arbitrary (accelerated) system of coordinates or reads

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through paragraph 73 in "The Field Theory" by Landau and Livshitz. Radiation is not fiction but a real physical phenomenon.

It is equally well-known that field invariants $F_{\mu\nu} \cdot F^{\mu\nu}$, $F^{*}_{\mu\nu} \cdot F^{\mu\nu}$ for electromagnetic radiation are equal to zero. In the case under examination, when the charge is at rest in the inertial reference frame, invariant $F_{\mu\nu} \cdot F^{\mu\nu}$ differs from zero. Hence, it can not be turned into zero by any transformation of coordinates. It appears appropriate to dwell at greater length on the notion of radiation in order to elucidate the question. In classical electrodynamics, it is feasible to speak of radiation only if components of the stress-energy tensor, which determine the energy flow and are expressed through retarded potentials, fall at infinity as $1/r^2$. Only in this case, the Poynting vector is indicative of the loss of energy by the moving charge due to radiation. The radiation is physical reality in the form of a weak electromagnetic field, and this provides the basis for the statement of the Sommerfeld radiation condition.

If the field behaviour at the infinity does not satisfy this requirement, the surface integral of energy flow components has nothing to do with the presence of radiation.

2. Let us now consider the problem of a charge which is accelerated with respect to system K_a but remains at rest in the inertial reference frame K in the point with coordinates $x = y = z = 0$. We have solved this problem in a previous paper [4], but V L Ginzburg and Yu N Eroshenko [5] make calculations in the Möller reference frame and insist (without any good reason) that it is this system that is necessary to confirm EP as formulated in Refs [1–3]. However, validity of a physical principle does not depend on the choice of reference frame. Were it otherwise, we should have to do with mysticism rather than physics. Now, we shall demonstrate that even in the Möller system, which is referred to as system K_a in Ref. [5], a charge at rest in the inertial system K does not radiate.

In the inertial system K, the field of a static charge is a purely Coulomb field. For this reason, the stress-energy tensor of the field of the charge at rest in a point with coordinates

$$x = y = z = 0\,, \tag{4}$$

has the form

$$\begin{aligned}
&T'^{00} = \frac{1}{8\pi}\frac{e^2}{r_0^4}\,,\\
&T'^{01} = T'^{02} = T'^{03} = T'^{10} = T'^{20} = T'^{30} = 0\,,\\
&T'^{12} = T'^{21} = -\frac{e^2}{4\pi r_0^6}\,xy\,, \qquad T'^{13} = T'^{31} = -\frac{e^2}{4\pi r_0^6}\,xz\,,\\
&T'^{23} = T'^{32} = -\frac{e^2}{4\pi r_0^6}\,yz\,, \qquad T'^{11} = -\frac{e^2}{4\pi r_0^4}\left(\frac{x^2}{r_0^2} - \frac{1}{2}\right),\\
&T'^{22} = -\frac{e^2}{4\pi r_0^4}\left(\frac{y^2}{r_0^2} - \frac{1}{2}\right), \qquad T'^{33} = -\frac{e^2}{4\pi r_0^4}\left(\frac{z^2}{r_0^2} - \frac{1}{2}\right).
\end{aligned} \tag{5}$$

Now, let us introduce the Möller coordinates

$$t = \frac{1 + a\rho}{a}\sinh(a\eta)\,, \qquad x = \xi\,, \qquad y = \chi\,,$$

$$z = \frac{1 + a\rho}{a}\cosh(a\eta) - \frac{1}{a}\,. \tag{6}$$

The charge at rest in the point with coordinates (4) of the inertial system is accelerated in the noninertial system with coordinates η, ξ, χ, ρ along the trajectory

$$\xi = \chi = 0\,, \qquad \rho = \frac{1}{a\cosh(a\eta)} - \frac{1}{a}\,. \tag{7}$$

With a change η in region $-\infty \leqslant \eta \leqslant \infty$, the charge in the noninertial reference frame travels from point $\rho = -1/a$ towards point $\rho = 0$ and backwards.

The interval in variables η, ξ, χ, ρ has the form:

$$\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2\\
&= (1 + a\rho)^2\,\mathrm{d}\eta^2 - \mathrm{d}\xi^2 - \mathrm{d}\chi^2 - \mathrm{d}\rho^2\,.
\end{aligned} \tag{8}$$

For the metric in the noninertial system of coordinates, only Kristoffel's symbols

$$\Gamma^0_{03} = \frac{a}{1 + a\rho}\,, \qquad \Gamma^3_{00} = a(1 + a\rho) \tag{9}$$

will be different from zero. From transformation formulas (6), it is easy to find that

$$\begin{aligned}
&\frac{\partial\eta}{\partial t} = \frac{\cosh(a\eta)}{1 + a\rho}\,, \qquad \frac{\partial\eta}{\partial z} = -\frac{\sinh(a\eta)}{1 + a\rho}\,,\\
&\frac{\partial\rho}{\partial t} = -\sinh(a\eta)\,, \qquad \frac{\partial\rho}{\partial z} = \cosh(a\eta)\,.
\end{aligned} \tag{10}$$

Hence, the Jacobian of transformation D:

$$D = (1 + a\rho)^{-1}\,. \tag{11}$$

Therefore, transformation (6) has no sense at

$$\rho = -\frac{1}{a} \tag{12}$$

and produces an event horizon.

Let us now find components of the stress-energy tensor in coordinates η, ξ, χ, ρ using transformation

$$T^{\mu\nu} = \frac{\partial y^\mu}{\partial x^\alpha}\frac{\partial y^\nu}{\partial x^\beta}\,T'^{\alpha\beta}\,. \tag{13}$$

Here

$$x^\mu = (t, x, y, z)\,, \qquad y^\mu = (\eta, \xi, \chi, \rho)\,.$$

Hence:

$$\begin{aligned}
T^{00} = \frac{e^2}{8\pi r_0^6}\frac{1}{(1 + a\rho)^2}\bigg\{&(\xi^2 + \chi^2)\left[\cosh^2(a\eta) + \sinh^2(a\eta)\right]\\
&+ \frac{1}{a^2}\left[au\cosh(a\eta) + \sinh^2(a\eta)\right]^2\bigg\}\,,
\end{aligned} \tag{14}$$

$$T^{01} = \frac{e^2\xi}{4\pi r_0^6}\frac{1}{1 + a\rho}\left[au\cosh(a\eta) + \sinh^2(a\eta)\right]\frac{\sinh(a\eta)}{a}\,, \tag{15}$$

$$T^{02} = \frac{e^2\chi}{4\pi r_0^6}\frac{1}{1 + a\rho}\left[au\cosh(a\eta) + \sinh^2(a\eta)\right]\frac{\sinh(a\eta)}{a}\,, \tag{16}$$

$$T^{03} = -\frac{e^2}{4\pi r_0^6}\frac{1}{1 + a\rho}(\xi^2 + \chi^2)\sinh(a\eta)\cosh(a\eta)\,. \tag{17}$$

Here

$$r_0^2 = \xi^2 + \chi^2 + \frac{1}{a^2}\left[(1+a\rho)\cosh(a\eta) - 1\right]^2 ,$$
$$au = 1 + a\rho - \cosh(a\eta) . \tag{18}$$

Kristoffel's symbols in the Möller coordinates (6) being nonzero, the general form of the law of conservation of the stress-energy tensor is:

$$\nabla_\mu T^{\mu\nu} = \frac{1}{\sqrt{-\gamma}}\,\partial_\mu\left(\sqrt{-\gamma}\,T^{\mu\nu}\right) + \Gamma^\nu_{\alpha\beta}T^{\alpha\beta} = 0 . \tag{19}$$

Hence, we have for the case in question:

$$\partial_\mu\left(\sqrt{-\gamma}\,T^{\mu 0}\right) + 2aT^{03} = 0 , \tag{20}$$

or

$$\partial_\mu T^{\mu 0} + \frac{3a}{1+a\rho}\,T^{03} = 0 . \tag{20a}$$

Let us now find the surface of the constant phase of electromagnetic field using the geodesic line equation:

$$\frac{d^2 y^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}\,\frac{dy^\alpha}{d\sigma}\,\frac{dy^\beta}{d\sigma} = 0 . \tag{21}$$

It follows from expression (9) that

$$\frac{d^2\eta}{d\sigma^2} + \frac{2a}{1+a\rho}\,\frac{d\eta}{d\sigma}\,\frac{d\rho}{d\sigma} = 0 , \tag{22}$$

$$\frac{d^2\xi}{d\sigma^2} = 0 , \qquad \frac{d^2\chi}{d\sigma^2} = 0 , \tag{23}$$

$$\frac{d^2\rho}{d\sigma^2} + a(1+a\rho)\left(\frac{d\eta}{d\sigma}\right)^2 = 0 . \tag{24}$$

Eqns (23) yield

$$\xi = A_1\sigma + B_1 , \qquad \chi = A_2\sigma + B_2 , \tag{25}$$

where A_1, A_2, B_1, B_2 are integration constants.

Eqn (22) may be rewritten as

$$\frac{d}{d\sigma}\left(\ln\frac{d\eta}{d\sigma}\right) = \frac{d}{d\sigma}\left[\ln(1+a\rho)^{-2}\right] . \tag{26}$$

Hence:

$$\frac{d\eta}{d\sigma} = A_0(1+a\rho)^{-2} , \qquad A_0 > 0 . \tag{27}$$

Substitution of (27) into Eqn (24) gives

$$\frac{d^2\rho}{d\sigma^2} + aA_0^2(1+a\rho)^{-3} = 0 . \tag{28}$$

The solution of Eqn (28) has the form:

$$\rho = \frac{1}{a}\left[\frac{a^2A_0^2}{A_3^2} - (A_3\sigma + B_3)^2\right]^{1/2} - \frac{1}{a} , \tag{29}$$

where A_3 and B_3 are integration constants.

The function under the root sign in expression (29) must be positive. Therefore:

$$\frac{a^2A_0^2}{A_3^2} - (A_3\sigma + B_3)^2 = A_3^2(\sigma_+ - \sigma)(\sigma - \sigma_-) > 0 , \tag{30}$$

where

$$\sigma_+ = \frac{aA_0}{A_3^2} - \frac{B_3}{A_3} , \qquad \sigma_- = -\left(\frac{aA_0}{A_3^2} + \frac{B_3}{A_3}\right) . \tag{31}$$

It follows from (31) that $\sigma_- < \sigma < \sigma_+$. Thus:

$$\rho = |A_3|\,\frac{\sqrt{(\sigma_+ - \sigma)(\sigma - \sigma_-)}}{a} - \frac{1}{a} . \tag{32}$$

Let us now find function $\eta(\sigma)$ by substituting Eqn (32) into Eqn (27):

$$\frac{d\eta}{d\sigma} = \frac{A_0}{A_3^2}\,\frac{1}{(\sigma_+ - \sigma)(\sigma - \sigma_-)} = \frac{1}{2a}\left(\frac{1}{\sigma - \sigma_-} + \frac{1}{\sigma_+ - \sigma}\right) . \tag{33}$$

Integration of Eqn (33) yields

$$2a(\eta - B_0) = \ln\frac{\sigma - \sigma_-}{\sigma_+ - \sigma} , \tag{34}$$

where B_0 is an integration constant.

Taking into account Eqns (31) results in

$$\sigma = \frac{aA_0}{A_3^2}\tanh\left[a(\eta - B_0)\right] - \frac{B_3}{A_3} , \tag{35}$$

where variable η may acquire any value from $-\infty$ to $+\infty$, with quantities σ_+ and σ_- having limiting values:

$$\sigma_+ = \sigma(\infty) , \qquad \sigma_- = \sigma(-\infty) . \tag{36}$$

Let us write equations for the geodesic line (25) and (32) , taking into consideration expressions (35), in the following form:

$$\xi = \alpha_1 + a\beta_1\tanh\left[a(\eta - B_0)\right] , \tag{37a}$$

$$\chi = \alpha_2 + a\beta_2\tanh\left[a(\eta - B_0)\right] , \tag{37b}$$

$$\rho = \frac{\beta_3}{\cosh\left[a(\eta - B_0)\right]} - \frac{1}{a} , \tag{37c}$$

where

$$\alpha_1 = B_1 - \frac{A_1B_3}{A_3} , \qquad \alpha_2 = B_2 - \frac{A_2B_3}{A_3} ,$$
$$\beta_1 = \frac{A_0A_1}{A_3^2} , \qquad \beta_2 = \frac{A_0A_2}{A_3^2} , \qquad \beta_3 = \frac{A_0}{|A_3|} . \tag{38}$$

Since we are interested in isotropic lines, curves (37) must satisfy the equation

$$ds^2 = \left[(1+a\rho)^2 - \left(\frac{d\xi}{d\eta}\right)^2 - \left(\frac{d\chi}{d\eta}\right)^2 - \left(\frac{d\rho}{d\eta}\right)^2\right] d\eta^2 = 0 . \tag{39}$$

Substitution of expressions

$$\frac{d\xi}{d\eta} = \frac{\beta_1 a^2}{\cosh^2\left[a(\eta - B_0)\right]} , \qquad \frac{d\chi}{d\eta} = \frac{\beta_2 a^2}{\cosh^2\left[a(\eta - B_0)\right]} ,$$
$$\frac{d\rho}{d\eta} = -\frac{\beta_3 a\sinh\left[a(\eta - B_0)\right]}{\cosh^2\left[a(\eta - B_0)\right]} \tag{40}$$

into Eqn (39) yields the relation between integration constants

$$\beta_3^2 = a^2(\beta_1^2 + \beta_2^2) \,. \tag{41}$$

Let a charge produce the field at time $\eta_0 = 0$. Then, in accordance with (7), it must be in the point with coordinates

$$\xi = \chi = \rho = 0 \,. \tag{42}$$

Let us now require that isotropic geodesics (37) should pass through the point with coordinates (42). This can be achieved by the choice of integration constants

$$\alpha_1 = \beta_1 a \tanh(aB_0) \,, \qquad \alpha_2 = \beta_2 a \tanh(aB_0) \,,$$
$$\beta_3 = \frac{1}{a}\cosh(aB_0) \,. \tag{43}$$

Thus, for the isotropic lines passing through the point with coordinates (42), we obtain the following expressions:

$$\xi = \beta_1 a \frac{\sinh(a\eta)}{\cosh(aB_0)\cosh\bigl[a(\eta - B_0)\bigr]} \,,$$

$$\chi = \beta_2 a \frac{\sinh(a\eta)}{\cosh(aB_0)\cosh\bigl[a(\eta - B_0)\bigr]} \,,$$

$$1 + a\rho = \frac{\cosh(aB_0)}{\cosh\bigl[a(\eta - B_0)\bigr]} \tag{44}$$

with constants β_1, β_2, β_3 satisfying condition (41). It is worthwhile to note that constant B_0 is expressed in terms of β_3, in accordance with formula (43). Now, let us find the front of the constant phase. To this effect, let us express constants β_1, β_2, β_3 through variables ξ, χ, ρ, η. Taking into account expression for β_3 from formula (43), one can obtain from the last equality (44)

$$\beta_3 = \frac{1}{a}\frac{(1 + a\rho)\sinh(a\eta)}{\sqrt{2(1 + a\rho)\cosh(a\eta) - (1 + a\rho)^2 - 1}} \,, \tag{45}$$

β_1 and β_2 can be found in a similar way:

$$\beta_1 = \frac{\xi(1 + a\rho)\sinh(a\eta)}{a\bigl[2(1 + a\rho)\cosh(a\eta) - (1 + a\rho)^2 - 1\bigr]} \,,$$

$$\beta_2 = \frac{\chi(1 + a\rho)\sinh(a\eta)}{a\bigl[2(1 + a\rho)\cosh(a\eta) - (1 + a\rho)^2 - 1\bigr]} \,. \tag{46}$$

Substitution of these expressions into relation (41) allows the front of the constant phase to be found

$$\xi^2 + \chi^2 + \bigl[1 + a\rho - \cosh(a\eta)\bigr]^2 \frac{1}{a^2} = \frac{1}{a^2}\sinh^2(a\eta) \,. \tag{47}$$

It is quite clear that this expression satisfies the Hamilton–Jacobi equation

$$\frac{1}{(1 + a\rho)^2}(\partial_\eta S)^2 - (\partial_\xi S)^2 - (\partial_\chi S)^2 - (\partial_\rho S)^2 = 0 \,. \tag{48}$$

We may now turn to calculating the Poynting vector for the constant phase surface. For this, we may use variable

$$u = \frac{1}{a}\bigl[1 + a\rho - \cosh(a\eta)\bigr] \,. \tag{49}$$

The surface (47) takes the form

$$\xi^2 + \chi^2 + u^2 = \frac{1}{a^2}\sinh^2(a\eta) \,. \tag{50}$$

Here and hereafter, $\eta > 0$. Let us denote the volume bounded by the surface of the constant phase (47) as V. Then, in conformity with (20), integration over volume V gives

$$\int_V \partial_\eta\bigl[(1 + a\rho)T^{00}\bigr]\,d\xi\,d\chi\,du + 2a\int_V d\xi\,d\chi\,du\,T^{03} = -I \,, \tag{51}$$

where

$$I = \int_V \partial_i\bigl(\sqrt{-\gamma}\,T^{0i}\bigr)\,d\xi\,d\chi\,du \,.$$

Let us introduce spherical coordinates

$$\xi = r\sin\theta\cos\varphi \,, \qquad 0 \leqslant r \leqslant \frac{1}{a}\sinh(a\eta) \,,$$
$$\chi = r\sin\theta\sin\varphi \,, \qquad 0 \leqslant \theta \leqslant \pi \,,$$
$$u = r\cos\theta \,, \qquad 0 \leqslant \varphi \leqslant 2\pi \,. \tag{52}$$

Then, the use of the Gauss – Ostrogradskiĭ formula leads to

$$I = \int_V \partial_i\bigl(\sqrt{-\gamma}\,T^{0i}\bigr)\,d\xi\,d\chi\,du = \int_{\partial V}\sqrt{-\gamma}\,T^{0i}l_i\,r^2\,d\Omega \,, \tag{53}$$

where vector $\mathbf{l}$ has components

$$l_i = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \,. \tag{54}$$

At the surface (50), we obtain

$$\int_{\partial V}\sqrt{-\gamma}\,T^{0i}l_i\,r^2\,d\Omega$$
$$= \frac{1}{a^2}\sinh^2(a\eta)\int_0^{2\pi}d\varphi\int_{-1}^{1}\sqrt{-\gamma}\,T^{0i}l_1\,d\cos\theta \,. \tag{55}$$

Here, we assume that $r = (1/a)\sinh(a\eta)$, with $\eta > 0$. Simple calculations yield

$$T^{0i}l_i = \frac{e^2a^4(1 - \cos^2\theta)}{4\pi\sinh^9(a\eta)\bigl[\cos\theta + \coth(a\eta)\bigr]^7} \,, \qquad \eta > 0 \,,$$

$$r_0^2 = \frac{1}{a^2}\sinh^4(a\eta)\bigl[\cos\theta + \coth(a\eta)\bigr]^2 \,. \tag{56}$$

In the meantime, it should be noted that the approximation in Ref. [5] is not quite correct. Instead of the exact expression (56), the authors of Ref. [5] obtained formula

$$T^{0i}l_i = \frac{e^2\eta}{4\pi\rho a^2 r_0^6}\sinh(a\eta)\bigl[\cosh(a\eta) - 1\bigr](1 - \cos^2\theta) \,. \tag{56a}$$

Comparison of formulas (56) and (56a) in the range of small η shows that our expression (56) leads to

$$T^{0i}l_i = \frac{e^2a^2}{4\pi\eta^2}(1 - \cos^2\theta) \,, \tag{56b}$$

whereas expression (56a) gives

$$T^{0i}l_i = \frac{e^2a^2}{8\pi\eta^2}(1 - \cos^2\theta) \,, \tag{56c}$$

which is two times smaller than (56b). If the authors of Ref. [5] had not made a mistake, they would have obtained coefficient 1/3 instead of 2/3 in their formula (23).

Substitution of expression (56) into (55) and integration over angles θ and φ yields

$$\int_0^{2\pi} \mathrm{d}\varphi \int_{-1}^{1} r^2 \sqrt{-\gamma}\, T^{0i} l_i \,\mathrm{d}\cos\theta = \frac{2}{3}\frac{e^2 a^2}{c^3}\left[1 + \frac{6}{5}\sinh^2\frac{a\eta}{c}\right]. \tag{57}$$

Here and below, we restore dependence on the velocity of light c in the resultant expressions for the flow. Taking into account expression (57), the relation (51) may be written in the form

$$\int_V \partial_\eta\left[\sqrt{-\gamma}\, T^{00}\right]\mathrm{d}\xi\,\mathrm{d}\chi\,\mathrm{d}u + 2a\int_V \mathrm{d}\xi\,\mathrm{d}\chi\,\mathrm{d}u\, T^{03} = -I,$$
$$I = \frac{2}{3}\frac{e^2a^2}{c^3}\left[1+\frac{6}{5}\sinh^2\frac{a\eta}{c}\right], \qquad \eta > 0\,. \tag{58}$$

The authors of Ref. [5] integrate expression (20) over the volume. This leads to the relation

$$\int_V \partial_\eta T^{00}\,\mathrm{d}\xi\,\mathrm{d}\chi\,\mathrm{d}u + 3a\int_V \frac{\mathrm{d}\xi\,\mathrm{d}\chi\,\mathrm{d}u}{au+\cosh(a\eta)}\, T^{03} = -P\,. \tag{51a}$$

They perform an approximate calculation of the surface integral on the right-hand side of relation (51a) but do not report the procedure because it is 'cumbersome'. Meanwhile, integral P can easily be calculated exactly and is as follows:

$$P = \int_{\partial V} r^2 T^{0i} l_i\,\mathrm{d}\Omega$$
$$= \frac{2}{3}\frac{e^2a^2}{c^3}\cosh\frac{a\eta}{c}\left[1+\frac{8}{5}\sinh^2\frac{a\eta}{c}\right], \qquad \eta>0\,. \tag{57a}$$

In the first place, it should be emphasised that formula (1) obtained in system K_g does not coincide with the exact expression (57a) obtained in system K_a. Therefore, V L Ginzburg's statement [3] that "a charge in system K_a radiates exactly as it does in system K_g" [3] is utterly wrong even from the formal standpoint. It follows from expression (57a) or (57) that the surface integral infinitely grows with increasing distance which suggests the growth of volume integrals in (51a) and (58). Therefore, there can be no wave zone in this problem, hence no radiation. It is clear that the surface integral of the Poynting vector P (formula (57a)) for the given problem does not satisfy radiation condition which requires that with increasing distance, that is with the growth of η, P should tend to a constant non-zero value independent of η. It is for this reason that a charge which is not accelerated in the inertial system K does not radiate in system K_a either. Therefore, the inference of the authors of Refs [3, 5] that "a charge which does not radiate in the inertial reference frame K where it is at rest does radiate in the reference frame K_a" is simply erroneous.

Relations (51a) and (58) express energy balance and nothing else. Expressions (51a) and (58) are valid only if $\eta > 0$ because components T^{0i} have singularity in point $\eta = 0$. The authors of Ref. [5] write: "It will be shown below by means of direct calculation that a charge in the reference frame K_a emits energy provided transition from the reference frame K to the frame K_a was correct; moreover, the energy emitted is exactly (2)". What they really mean is formula (1) from our work. Setting aside the essence of the radiation problem, even formal comparison of formulas (1) and (57a) shows that the above inference is also wrong since these formulas have actually nothing in common, and the latter one is the result of exact calculation rather than approximation.

The presence of the surface integral is by no means indicative of radiation. V L Ginzburg and Yu N Eroshenko [5] seem to agree with this when they write that "identification of energy P and energy of free electromagnetic radiation (photon flow) is unsound". But they immediately go on to state: "It follows from EP that electromagnetic fields in reference frames K_g and K_a are similar which automatically implies equality of the quadratic-in-field variables P in these reference frames". But this is utterly wrong since the reference frame K_g exhibits radiation and hence contains free electromagnetic waves whereas a charge which rests in the inertial system does not emit in system K_a even though it is accelerated with respect to the uniformly accelerated system of coordinates. It is quite clear from the comparison of the field of a charge at rest in the inertial reference frame K and the description of this field in the uniformly accelerated reference frame K_a that the two fields differ only in that they are written in different systems of coordinates, taking into account the tensor transformation law. One must not compare this field with that arising when the charge propagates in system K_g. It is impossible not to see that formulas (1) and (57a) are absolutely different. This difference is largely due to the fact that a charge at rest in the inertial reference frame K travels along the geodesic line of the Minkowski space in the accelerated reference frame K_a; for this reason, its motion is free. A free charge in system K_g is forced to move which precludes its motion along the geodesic line of the Minkowski space. Therefore, physical equivalence of reference frames K_a and K_g is out of the question.

3. One may introduce a quantity the surface integral of which will be exactly equal to expression (1). But even this will not necessarily suggest radiation because it is associated with the behaviour of the stress-energy tensor on infinity.

Let us introduce a quantity

$$\mathcal{T}^{\mu\nu} = (-\gamma)^{3/2} T^{\mu\nu}\,. \tag{59}$$

Then, expression (19) leads to

$$\nabla_\mu \mathcal{T}^{\mu\nu} = (-\gamma)^{3/2}\nabla_\mu T^{\mu\nu} = \partial_\mu \mathcal{T}^{\mu\nu} - \frac{3}{2}\frac{1}{\gamma}\,\partial_\mu\gamma\cdot\mathcal{T}^{\mu\nu}$$
$$+ \Gamma^\mu_{\mu\lambda}\mathcal{T}^{\lambda\nu} + \Gamma^\nu_{\mu\lambda}\mathcal{T}^{\mu\lambda} = 0\,.$$

Since

$$\frac{1}{\gamma}\,\partial_\mu\gamma = 2\Gamma^\lambda_{\mu\lambda}\,,$$

the following expression holds:

$$\nabla_\mu \mathcal{T}^{\mu\nu} = \partial_\mu \mathcal{T}^{\mu\nu} - 2\Gamma^\lambda_{\mu\lambda}\mathcal{T}^{\mu\nu} + \Gamma^\nu_{\mu\lambda}\mathcal{T}^{\mu\lambda} = 0\,. \tag{60}$$

Owing to $\Gamma^\lambda_{\mu\lambda} = \delta^3_\mu\Gamma^0_{30}$, in the case $\nu = 0$ one has

$$\nabla_\mu \mathcal{T}^{\mu 0} = \partial_\mu \mathcal{T}^{\mu 0} = 0\,. \tag{61}$$

Integration of this expression over domain V gives

$$\int_V \partial_\eta \mathcal{T}^{00}\,\mathrm{d}\xi\,\mathrm{d}\chi\,\mathrm{d}u = -\int_V \left(\partial_\xi \mathcal{T}^{01} + \partial_\chi \mathcal{T}^{02} + \partial_u \mathcal{T}^{03}\right)\mathrm{d}\xi\,\mathrm{d}\chi\,\mathrm{d}u\,. \tag{62}$$

The use of the Gauss–Ostrogradskiĭ formula allows us to obtain

$$\int_V (\partial_\xi \mathcal{T}^{01} + \partial_\chi \mathcal{T}^{02} + \partial_u \mathcal{T}^{03})\, d\xi\, d\chi\, du = \int_{\partial V} \mathcal{T}^{0i} l_i r^2\, d\Omega\,. \tag{63}$$

On the surface (50),

$$\int_{\partial V} \mathcal{T}^{0i} l_i r^2\, d\Omega = \frac{1}{a^2}\sinh^2(a\eta) \int_0^{2\pi} d\varphi \int_{-1}^{1} \mathcal{T}^{0i} l_i\, d\cos\theta\,. \tag{64}$$

Simple calculations yield

$$\mathcal{T}^{0i} l_i = \frac{e^2 a^4}{4\pi}\, \frac{1-\cos^2\theta}{\sinh^6(a\eta)\big[\cos\theta + \coth(a\eta)\big]^4}\,, \qquad \eta > 0. \tag{65}$$

Substitution of this expression into (64) leads to

$$\int_{\partial V} r^2 \mathcal{T}^{0i} l_i\, d\Omega = \frac{2}{3}\, \frac{e^2 a^2}{c^3}\,, \qquad \eta > 0\,. \tag{66}$$

The resultant equation has the form

$$\int_{V(\eta)} \partial_\eta \mathcal{T}^{00}\, d\xi\, d\chi\, du = -\frac{2}{3}\, \frac{e^2 a^2}{c^3}\,, \qquad \eta > 0\,. \tag{67}$$

Thus, we have obtained the surface integral for quantity $\mathcal{T}^{0i}$ which is exactly equal to formula (1). However, this does not mean anything because whether radiation occurs or not depends only on the asymptotic behaviour of the components of the stress-energy tensor which describe the energy flow.

To sum up the results of our accurate calculations, the EP formulations in Refs [1 – 3] are at variance with electrodynamics. Therefore, our criticism of Ref. [3] as presented in Ref. [4] holds true and was not refuted by our opponents in Ref. [5].

4. Ref. [5] contains a few more incorrect inferences. It being published in the Uspekhi Fizicheskikh Nauk (Physics-Uspekhi), we have to treat them at greater length so that a thoughtful reader may compare the opinions and decide which is right and which is wrong. The authors of Ref. [5] allude to the formulation of EP as proposed in a book by V Pauli [6]. We cite here only one paragraph from this book: "*For an infinitely small four-dimensional world-region* (*i.e. a world-region which is so small that the space-time variation of gravity can be neglected in it*), *there always exists such a coordinate frame* $K_0(X_1, X_2, X_3, X_4)$ *in which gravitation has no influence either on the motion of a material point or any other physical process*". This formulation is altogether different from that in Refs [1 – 3]. Moreover, it is equally incorrect. Indeed, if one takes a particle with spin, the equation of motion for such a particle will inevitably involve the curvature tensor the action of which on the particle with spin is impossible to eliminate by any transformation of coordinates. A Eddington wrote about this in an even more general form as far back as 1924 "*There are more complicated events that obey equations containing components of the world curvature. Terms with these components are absent in equations which describe experiments performed in flat regions. But they must be reintroduced if the transition to a general case is needed. Evidently, there must exist events which allow for the discrimination between the flat world and the curved one. Otherwise, we should be ignorant about the curvature of the world. The equivalence principle is inapplicable to such events*". Some authors believe it possible to neglect the influence of second derivatives, i.e. the curvature. This is wrong because neglect of the curvature means disregard of the gravitational field. Moreover, the role of the space curvature is considered in detail in a book by J L Synge [8]. According to this author: "*If we accept the idea that space-time is a Riemann four-dimensional space* (*and if we are relativists, we must*), *then surely our first task is to get the feel of it just as early navigators had to get the feel of the spherical ocean. And the first thing we have to comprehend is the Riemann tensor, for it is precisely the gravitational field*: *if it vanishes, and only then, there is no field at all. Yet, strangely enough, this most important fact has been pushed into the background*". Synge went on as follows: "*In the Einstein theory, gravitational field is either absent or present depending on whether the Riemann tensor is zero or other than zero respectively. This is an absolute property, it is unrelated to any observer's world line*". It is quite clear that neglect of the curvature results in the loss of gravitational field. For this reason, when the authors of Ref. [5] write that they 'ignore possible tidal effects', this means that they get rid of gravitational field. In the absence of gravitational field, the space-time is exactly the Minkowski space. This statement is 'important, but it can hardly be called Principle' [8]. The authors of Ref. [5] also note: "*However, his views* (those of A Einstein) *were never in conflict with the formulation of EP as offered in Ref.* [6]". But this is not quite true because the formulation of EP given by A Einstein in 1933 [9] has the general form, and it is such EP that underlies the general theory of relativity (GTR). A Einstein wrote in Ref. [9]: "*Mathematically, it means that the physical* (*four-dimensional*) *space has the Riemann metric. Time-like extreme lines of this metric determine the motion of a material point on which other forces, excepting the force of gravity, have no effect. Coefficients* ($g_{\mu\nu}$) *of this metric concomitantly describe the gravitational field in relation to the selected coordinate frame. In this way, the natural formulation of the equivalence principle by us was found, and its application to arbitrary gravitational fields seemed very natural*". This EP formulation of A Einstein is different from that in Refs [1 – 3]. It also differs from the formulation offered by Pauli in Ref. [6]. Is it possible that V L Ginzburg and Yu N Eroshenko could take no notice of this? Strangely enough, they make the following statement: "*In fact, Logunov et al question the validity of EP or, to be more precise, Einstein's EP*". But this is also untrue. We do not object to the EP formulation proposed by A Einstein in 1933 [9]. By the way, we specially wrote about this formulation in our paper [4]. True, the formulation of EP in Einstein's early works sometimes looks very much like that of Pauli. Our criticism is not levelled at EP per se. What we really criticise are its early formulations which are incorrect even though they occur in the literature [1 – 3, 6].

Points 3 and 4 in Ref. [5] contain a reference to a paper of A Einstein [10] to which the authors apply in search of arguments to get rid of the undesirable systems of coordinates. Actually, there is limitation on the choice of coordinate frame in the Minkowski space. In accordance with this limitation, only those transformations of coordinates are possible which map, in case of transition to an accelerated coordinate frame, all space points in the inertial system with the Jacobian of transformation other than zero. Only in this case, it is possible to retain all objective information about physical phenomena. Apropos of this, V L Ginzburg and Yu N Eroshenko [5] write as follows: "We consider this statement to be totally incorrect because GTR does not invariably consider reference frames of the global (so to say) character". But the subject of the discussion has nothing to do with GTR because it concerns the Minkowski space, i.e the special theory of relativity (STR), calculations in system K_a being performed only in the framework of special relativity. Transformation of coordinates which maps only a part of the space in the inertial system leads to the loss of physical information. Such a transformation with reference to STR is irrelevant and inconclusive because the Minkowski space is essentially different from the Riemann space. Transformations with limitations are possible, but caution is necessary as regards general physical conclusions. The special theory of relativity containing no other limitations besides those mentioned above, there is no doubt that the system of coordinates used in our paper [4] is permissible. It may not only 'be assumed uniformly accelerated' as V L Ginzburg and Yu N Eroshenko put it — it is really such, being associated with a uniformly accelerated charge travelling in the inertial system (Galilean coordinates). Any general physical principle including EP must not depend on the choice of a coordinate frame as far as general physical conclusions are concerned. Our transformation [4] maps the entire space of the inertial coordinate frame into the accelerated system with the non-zero Jacobian. Arguments of V L Ginzburg and Yu N Eroshenko concerning the Lorentz contraction of length are equally groundless for it may change any way, decreasing for one kind of events and increasing or remaining unaltered for others, depending on how it is measured (this is true even for inertial reference frames). Turning now to A Einstein's ideas dating back in1912 [10] and referred to by V L Ginzburg and Yu N Eroshenko in Ref. [5], it should be emphasised that they reflect the early period of A Einstein's work when he believed that an accelerated reference system can not be used in the framework of special relativity. He adhered to this opinion for a quite long time. It is worthy of note that even in 1934, L I Mandelstam [11] wrote: "*The special theory of relativity can not explain how the accelerated clock ticks and why its rate varies because it does not examine accelerated reference frames at all*". Some text-books and papers still approach the problem from this standpoint although it is utterly wrong and is tantamount to erroneously maintaining that Euclidean geometry may use only Cartesian coordinates. Point 7 of Ref. [5] discusses our formulas (29). The authors expand them in powers of at/c and argue that such a transformation with accuracy $(at/c)^2$ eliminates relativistic effects of the order $(at/c)^2$ and hence radiation. However, this observation does not apply to our paper [4] where calculations have been accurately performed, without resorting to approximation. Ref. [4] was intended to demonstrate that a charge at rest in an inertial system does not radiate in an accelerated reference frame obtained by means of coordinate transformations which relates all points of the inertial system to the accelerated one. The present study shows that radiation is equally lacking in the Möller system used by V L Ginzburg and Yu N Eroshenko [5] to prove that radiation does exist in the reference frame K_a. Thus, the absence of radiation does not depend on the choice of the reference frame, as expected. This emphasises once again that the equivalence principle as stated in [1 – 3] is in conflict with electrodynamics, and the arguments of V L Ginzburg to the contrary are invalid.

Also, these authors maintain [5] that "*Transition to EP accomplished by A Einstein implies that all physical laws including mechanical and electromagnetic ones hold true in a falling elevator exactly as they do in the absence of gravitation*". But this is absolutely wrong. Arguments to disprove this erroneous statement have been given earlier. They can also be found in the "Field Theory" by L D Landau and E M Livshitz (7th revised edition, 1988, paragraph 91, Problem 2). Problem 2 in this book gives evidence that the curvature tensor explicitly enters equations of electrodynamics. This was noted in a book of A S Eddington published in 1924 and translated into Russian as far back as 1934 [7]. May be, this will make the authors of Ref. [5] understand that their statement is incorrect. Now, after the opinions have been expressed, and it is up to the thoughtful reader to decide which is right and which is wrong.

In conclusion, it is also noteworthy that V A Fock deduced from his analysis of A Einstein's GTR [12] that it is based on two principles: "The first one ... is the integration of space and time in a united four-dimensional manifold with an indefinite metric". "The second principle is the unity of metric and gravity which constitutes the essence of Einstein's theory of gravitation. It is these two principles that underlay the theory of gravitation, not the expansion of the notion of relativity allegedly possible due to local equivalence of gravity and acceleration". V A Fock is quite correct here, but the EP formulation later proposed by A Einstein [9] (in 1933) contains both principles of which V A Fock wrote. First and foremost, this confirms that the last definition of EP given by A Einstein is deeper than EP formulations in [1 – 3, 6] even though many qualitative arguments of A Einstein (e.g. Einstein's elevator) provided the basis for EP formulations which frequently occur in the literature. GTR is based on the EP as stated by A Einstein in Ref. [9] as opposed to EP cited in [1 – 3, 6] and favoured by the authors of Ref. [5]. Unfortunately, it takes certain physicists too long to grasp what A Einstein made clear a number of decades ago.

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