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The phenomenological theory of world population growth

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Contents

Abstract. **Of all global problems world population growth is the most significant. Demographic data describe this process in a concise and quantitative way in its past and present. Analyzing this development it is possible by applying the concepts of systems analysis and synergetics, to work out a mathematical model for a phenomenological description of the global demographic process and to project its trends into the future. Assuming self-similarity as the dynamic principle of development, growth can be described practically over the whole of human history, assuming the growth rate to be proportional to the square of the number of people. The large parameter of the theory and the effective size of a coherent population group is of the order of 10^5 and the microscopic parameter of the phenomenology is the human lifespan. The demographic transition — a transition to a stabilised world population of some 14 billion in the foreseeable future — is a systemic singularity and is determined by the inherent pattern of growth of an open system, rather than by the lack of resources. The development of a quantitative nonlinear theory of the world population is of interest for interdisciplinary research in anthropology and demography, history and sociology, for population genetics and epidemiology, for studies in evolution of humankind and the origin of man. The model also provides insight into the stability of growth and the present predicament of humankind, and provides a setting for discussing the main global problems.**

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1. Introduction

The growth of world population is the single most important global problem and is certainly the most complex of all these issues. An understanding of the process of growth can be sought by extending the scope of analysis and sensibly simplifying our approach. Growth is usually expressed by the data of demography as presented for countries or regions [1]. These statistics contain the clue to working out the quantitative laws that could in principle describe the dynamics of world population growth. Starting with demographic data, one can develop an approach to discern the laws that could describe development over many generations and encompassing the whole world. By applying the methods of systems analysis and synergetics for the phenomenological description of the human population system a mathematical model is suggested.

An alternative approach is pursued by global modellers, mainly known by the first reports of the Club of Rome. In these studies an attempt is made at developing a complex computer based model using comprehensive inputs from extensive data bases. Following a reductionist methodology most of the factors which influence human development are purportedly taken into account. At present the results of this detailed analysis have been extensively criticised and such an approach has not led to any recent advances.

On the other hand, one can apply the methods developed initially in physics to describe systems with many particles and degrees of freedom. These methods, based on first principles, were later successfully applied to complex physical and chemical systems, then to biology and the environment, and finally to economics and social phenomena. Extending the methods so successfully used in physics to the population of the world is not self-evident, if a meaningful result is expected. In the course of study this point will be reviewed in the light of the results of modelling, since growth and development of humankind over an immense timescale is to be described.

In pursuing interdisciplinary research with methods and data coming from very different fields of knowledge it is not easy to establish a common frame of reference. One of the main complications is that what may seem to be obvious, if not trivial, to one party is far from the usual concepts of others. In these cases some facts and ideas will necessarily demand a more detailed and in-depth presentation for the sake of interdisciplinary understanding. Bringing together facts and figures of anthropology and history, and of demography — a well established science with an old and thorough mathematical tradition, has proved that a dialogue, a constructive intercourse is not only possible, but most necessary in dealing with global problems.

Finally, the whole issue of world population growth as a complex global problem deals with data of a very different nature and involves issues well beyond science. Ideology and conceptions of some societies and people inevitably touch on the convictions and values of others.

At present, our common horizon is dominated by the population explosion. The annual addition to the population of the world is approaching 100 million with 250,000 people arriving daily. This ever increasing growth sets the pace for demands of food and energy. The growth rate is alarming and has, when simplistically extrapolated, led to apocalyptic forecasts and doomsday projections [2] (see Table 1). By using up more and more resources of our planet humankind is exerting a growing pressure on our environment and biosphere. Ultimately, this pressure will have an effect on our growth. In this case it is all the more important to develop lines of rationale in assessing factors which determine the growth rate and what limits there are to our numbers. Only if we obtain a broader and in-depth vision of human growth and evolution can we hope to further our understanding of these universal issues.

In spite of all the drama of the population explosion and the high emotions it generates, what really matters is that at present we are passing through a *population transition*. This phenomenon is experienced by all populations and involves a rapid rise in the growth rate followed by a similarly fast decrease in growth, finally leading to the stabilisation of the population (Fig. 1). This transition has been experienced by all developed countries, and now the same process is happening worldwide. The demographic transition is accompanied by a general increase in economic growth and the corresponding resettlement of people from rural areas to towns. At the closing stages of the population transition a marked change in the age profile is observed, with a predominance of the older, rather than younger generation.

In the modern interconnected and interdependent world this transition will practically come to an end within the next 50 years. It is happening much faster than in Europe, where a similar process began at the end of XVIII century. The demographic transition is certainly the most significant worldwide transformation and is intensively studied by modern demography [3].

Following the approach and methods of demography the changes in a population can be worked out for the next one or

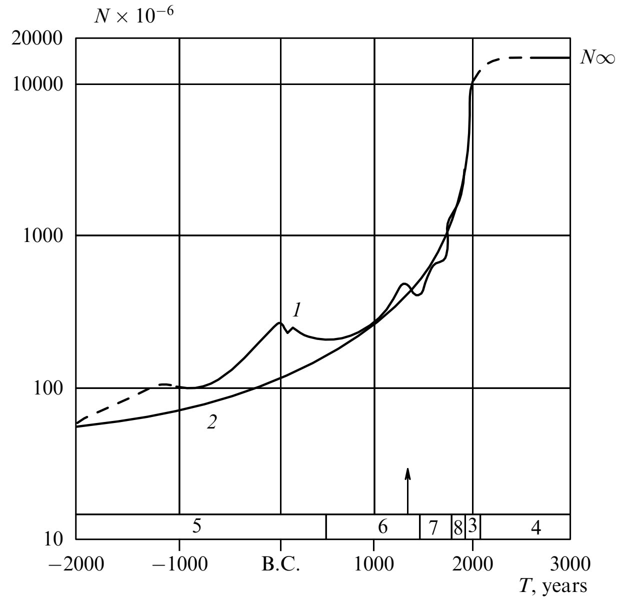

Figure 1. *1* — World population from 2000 B.C. to 3000 A.C. [1, 26], *2* — population blow-up, 3 — demographic transition, 4 — stabilized population, 5 — ancient world, 6 — middle ages, 7 — modern history, 8 — recent history, ↑ — the Plague.

two generations. For these projections the world is divided into a number of regions, where certain growth scenarios are assumed. Patterns of changes in fertility and mortality, considered to be more elementary processes, lead to a description of the demographic transition. Unfortunately, it is difficult, if at all possible, to provide more than an adequate description of the transition following this reductionist approach passing from a more fundamental level of concepts to the next level of complexity. But in no case can fertility or mortality be considered to be elementary processes. These concepts are also phenomenological, generalising in these indicators many factors. In a system of such complexity as humankind, most, if not all, connections and interactions are essentially nonlinear and cannot be summed up, assuming linear cause-and-effect relations, a direct connection between part and the whole.

The alternative is to pursue a systemic approach, whereby the entire population of the world is seen as an evolving and self-organising system. This is the main feature of the mathematical model for the development of all humankind. The feasibility of this approach is far from obvious. What should be considered in the first place is the extent to which the concept of a system may be meaningfully applied to the total population of the world and the question of whether the process of growth is statistically regular and historically predictable.

The acceptance of a synthetic rather than analytic, reductionist treatment is the key to the model, much as in modern cosmology the dynamic treatment of the Universe is

Table 1. UN Data (1992)

Area	Population, billions			Percentage of population (mid 1990)			
	1990	2000	2025	Children $(0 \div 4)$	Youth $(15 \div 24)$	Elderly > 65	Urban residence
World total	5.3	6.3	8.5	12	19	6	45
Developed regions	1.2	1.3	1.4	7	15	12	73
Developing regions	4.1	5	7.1	13	20	4	37
Europe	0.50	0.51	0.51	6	15	13	73

only possible when the whole of the world is considered to be the object of study.

2. The world population as a system

For many years the mere possibility of considering the population of the world as a single system was not taken seriously. Most demographers saw in world population data a mere sum of the populations of all countries, a number that had no objective dynamic meaning [4]. At present this extreme attitude has been abandoned, and in modern demography a systemic and historic approach is becoming prevalent [1].

Data on the population for the main regions of the world show both an overall growth trend and at the same time a diversity in the individual patterns of development. The sum growth of the world population follows a more regular pattern, culminating in a rapid escalation of the population explosion (see Figs 1 and 2).

One prerequisite for a *system* as applied to the world population is the interaction of all its regions. Interactions are instrumental for a system and the interconnections by transport and trade, migration and information bring all interdependent parts of the world together. Now these connections unequivocally permit one to treat the population of the modern world as a system. In the course of this study on a number of occasions we shall see proof of the systemic behaviour of the world population and phenomena that support this attitude.

To what extent is this true for the past? In the development of the model the criteria for systemic behaviour will be obtained. Even in the distant past, when the number of people was much less than today and the world was divided, the populations of different regions slowly but surely did interact. The time of this interaction may be estimated and it can be shown that in most cases the systemic approach to the world population is valid.

In the demographics of regions and countries the change of population by migration is a noticeable contribution to the balance of people. But on a global scale, migration should not be taken into account, as there is yet no possibility to leave the Earth [16]. On a planetary scale, migration is simply one of the internal interactions of the system. Because of migration and wars, on a historically significant time scale we get a mixing of people and cultures that is part of the systems dynamics. When these interactions and exchanges are for a long time cut off, this isolates a part of the population and leads to the subsequent stagnation of a subsystem.

It should also be noted that biologically all people belong to the same species *Homo Sapiens*. We all have the same number of chromosomes — 46, different from other primates. All races can intermix and socially interact [5] and, biologically, humankind is a comparatively uniform species [6]. But in terms of numbers we are by *five orders* of magnitude more numerous than any mammal of comparable size and position in the food chain. Only domestic animals, living with and around humans, are not naturally limited in their numbers, as other creatures in the wild, occupying their own area and ecological niche. Humans practically inhabit the whole world, all parts that are fit to live in. By industry humans have created an environment, a habitat much of our own making. There are good reasons for assuming that during the last million years mankind has hardly evolved biologically and human development and self-organisation is primarily social.

These processes of social and technological development are to be described by a quantitative phenomenological model. The data of demography and concepts of anthropology and history will be interpreted in terms of systems analysis and synergetics [7, 8]. This may help to introduce some new ideas and concepts into traditionally humanistic studies. The model is an attempt to develop a quantitative statistical approach to anthropology, a theory that may lead to new insights into a problem of concern for us all.

3. Modelling world population growth

Constructing a model entails the application of methods developed for the study of dynamic systems to data provided poorly by demography and anthropology. These facts and figures are poorly known to physicists and one of the purposes of this paper is of introduce them to a new set of problems. In a number of cases it will be possible to identify concepts and recognise well-known ideas in a new setting. On the other hand it should be kept in mind that both the data and the model itself are but crude images of the real world. In

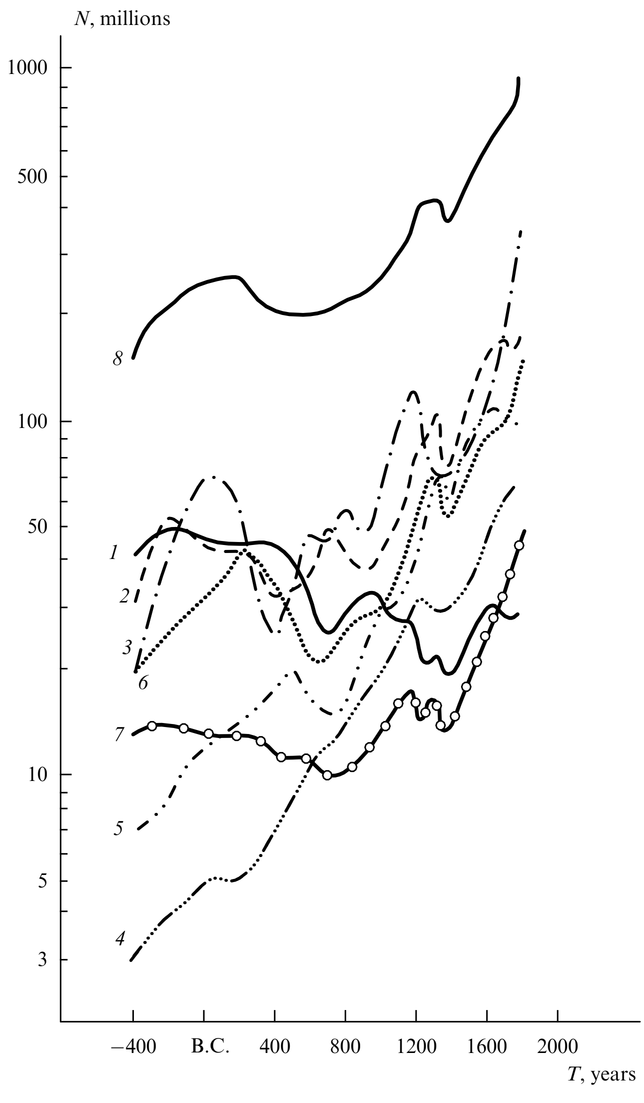

Figure 2. Regional population growth since 400 B.C. to 1800 [1]. *1* — South-East Asia, *2* — Indian subcontinent, *3* — China, *4* — rest of Asia, *5* — Africa (without North Africa), *6* — Europe (without USSR), *7* — USSR, *8* — the world.

interdisciplinary studies, one of the main difficulties is that of taking ideas and methods from one field and transfering them into another. This process is perhaps best developed in mathematics when models are suggested, and then in theoretical physics, although it is difficult to say when a model can reach the status of a theory, serve not as a description of events, but lead to greater insight into the nature of the phenomena treated.

For a system as complicated as the population of the world it is this complexity that provides an opportunity. When many factors are relevant and different interactions simultaneously occur, it can be expected that a statistical approach is feasible. In this case, most of the spatial and temporal variations will average out with the consequence that of all the different processes taking place, those finally determining the systemic behaviour are left. Then we can expect to explore the resulting pattern of changes that have an objective nature and can be expressed numerically. The following treatment of the world population is based on the development of such a model [9–12].

The population of the world N at the time T years is to be determined by the function $N(T)$. Changes in N will be considered over a large number of generations. In this case it can be assumed that the time span of a single generation, of the human life-time will not explicitly enter the formula, just as the distribution of people by age and sex. In systems, where a multifactored process is considered with many degrees of freedom and development can be treated as statistically uniform, one may expect that growth will follow a self-similar pattern and scale in time. This main assumption of scaling can be expressed as the invariance of the relative rate of change in the system

$$\lim_{\Delta N, \Delta T \to 0} \frac{\Delta N}{N - N_1} \frac{T - T_1}{\Delta T} = \frac{\mathrm{d} \ln |N - N_1|}{\mathrm{d} \ln |T - T_1|} = \alpha \,, \qquad (1)$$

where T_1 and N_1 are the points of reference for time and population. In most cases $N_1 = 0$.

In self-similar processes of growth and development, the ratio of relative changes in population and time is constant and as a necessary consequence of scaling (1) growth is described by a power law $N = C(T_1 - T)^{\alpha}$, where C and α are constants. The simplest case is linear growth, where $\alpha = 1$. Exponential growth or growth following a logistic law [13, 14] are not self-similar, as they have an internal scale of time — the time of doubling, so these growth laws are not scaleable.

Forster [15] was probably the first to suggest

$$N = \frac{179 \times 10^9}{(2027 - T)^{0.99}} \qquad (2)$$

as an empirical formula to describe world population growth, where the following values for the constants were obtained

$$C = (179 \pm 0.14) \times 10^9 \,, \qquad T_1 = 2027 \pm 5 \,,$$
$$\alpha = -0.99 \pm 0.009$$

by the least square fit of a large collection of population data from A.D. to 1960. The accuracy implied for α seems somewhat excessive and in the following treatment $\alpha = -1$ is assumed. Later Horner [16] suggested a similar expression

$$N = \frac{C}{T_1 - T} = \frac{200 \times 10^9}{2025 - T} \,, \qquad (3)$$

where $\alpha = -1$ and $C = 200 \times 10^9$. These formulae describe with good accuracy world population growth over thousands of years and more, now culminating in the population explosion.

Expressions for growth (2) and (3) should be seen as a physically meaningful law for self-similar growth, following a hyperbolic growth curve. These self-accelerated blow-up processes in great detail have been studied in recent research of nonlinear phenomena [17, 18]. In other words, the population explosion is seen as a global instability, well known in the behaviour of essentially nonlinear systems.

On the other hand (2) and (3) are limited in their validity both in the past and present for, as we approach year 2025 the population of the world will go off to infinity. This has led some to believe that the world is approaching Doomsday, indicating an impending crisis. In a very similar way these expressions are not valid in the distant past. Thus, 20 billion years ago we could still expect ten people to be around, presumably cosmologists, observing the Big Bang! In other words, the self-similar expression (2) is limited in the past and present due to the divergences of hyperbolic growth. This is exactly what should be expected for scaling power laws are only intermediately asymptotic [19].

Consider for a moment those ten elders who presumably would be present 20×10^9 years ago. If they ever existed, they should have lived a billion years, which is just as meaningless as their very existence. The blow-up of humankind, as we approach T_1 means that in 2024 the population would double in less than a year, which is equally nonsensical. What has not been taken into account is the duration of the human life, an interval of time that characterises our reproductive capacity and lifespan. It is this factor that has to be taken into account to set the limits of scaling.

It is well established that all countries pass through a maximum growth rate of their demographic transition, followed by a rapid decline. This has been observed for all developed countries and is now seen in countries of the developing world representing regions of Africa, Asia and South America (Fig. 3). The demographic transition for the world population is illustrated in Fig. 8. It is significant that the population growth rate passes through a pronounced maximum, and does not stabilise at its highest point.

At the very outset of human development at T_0 it can be assumed that the growth rate is also limited. It cannot in general be less than one person, presumably a hominid, in a

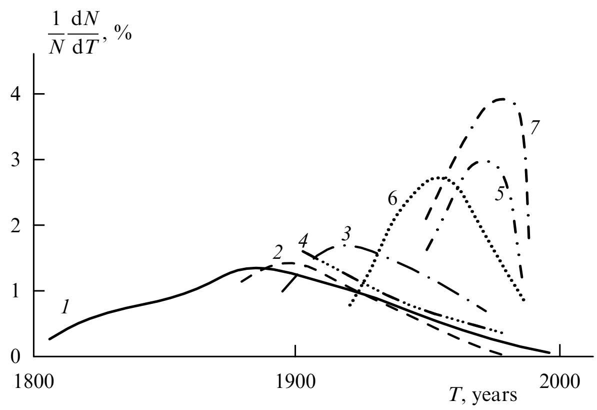

Figure 3. The population transition [3]. *1* — Sweden, *2* — Germany, *3* — USSR, *4* — USA, *5* — Mauritius, *6* — Sri Lanka, *7* — Costa Rica.

generation, a requirement for the continuity of growth. As a very approximate description of this early stage, it provides the necessary asymptotic cut-off for the divergence in the past

$$\left(\frac{\mathrm{d}N}{\mathrm{d}T}\right)_{\min}\Bigg|_{\substack{N\to 1\\ T\to T_0}} \geqslant \frac{1}{\tau} \geqslant \left(\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}T}\right)_{\max}\Bigg|_{T\to T_1} \tag{4}$$

and the cut-off of growth rate as the demographic transition at T_1 is approached.

By introducing a cut-off time constant τ years the divergences both in the past and present can be eliminated, regularising the growth rate

$$\text{A:}\quad \frac{\mathrm{d}N}{\mathrm{d}T} = \frac{N^2}{C} + \frac{1}{\tau}\,, \tag{5a}$$

$$\text{B:}\quad \frac{\mathrm{d}N}{\mathrm{d}T} = \frac{C}{(T_1 - T)^2} \tag{5b}$$

and for the last stage of growth and the demographic transition

$$\text{C:}\quad \frac{\mathrm{d}N}{\mathrm{d}T} = \frac{C}{(T_1 - T)^2 + \tau^2}\,. \tag{5c}$$

Bringing into the model a specific time τ years as the microscopic parameter of the phenomenology, the expressions for the growth rate are extended both into the past and the future, beyond the divergence at T_1. These expressions exclude singularities, have the required asymptotics and can be expected to describe human population growth over our entire history.

The numerical values for the constants appearing in this case can be determined by comparing modern demographic data with different models obtained by integrating 5(c)

$$N = \frac{C}{\tau}\,\mathrm{arccot}\left(\frac{T_1 - T}{\tau}\right), \tag{6}$$

that describes the population growth before and after the demographic transition.

The following values for the relevant constants

$$C = (186 \pm 1) \times 10^9\,, \qquad T_1 = 2007 \pm 1\,,$$

$$\tau = 42 \pm 1\,, \qquad K = \left(\frac{C}{\tau}\right)^{1/2} = 67000$$

provide for Model III the best fit to demographic data (see Table 2). It should be noted that even recent demographic data is accurate only from 3 to 5% [21], although in demography traditionally more digits are indicated than those having a meaning. This is partially due to the ethical difficulty in rounding off numbers that supposedly represent real people, officially counted during a census.

The results of modelling show that T_1 and C are not sensitive to the value of τ and on the other hand τ is primarily dependent only on recent data describing the demographic transition. For this purpose it is useful to compare demographic data for the growth rate (5c) and the relative growth rate

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}T} = \frac{\tau}{\left[(T_1 - T)^2 + \tau^2\right]\mathrm{arccot}\left[(T_1 - T)/\tau\right]}\,, \tag{7}$$

that passes through its maximum value

$$\left(\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}T}\right)_{\max} = \frac{0.725}{\tau}$$

before (5) and reaches its maximum value of 1.7% at $T_{\max} = T_1 - 0.43\tau$ in 1989 for Model III. The growth rate (5c) will pass through its high point in $T_1 = 2007$. With the introduction of a finite $\tau = 42$ years renormalisation shifts T_1 from 2027 to 2007 years (Fig. 4).

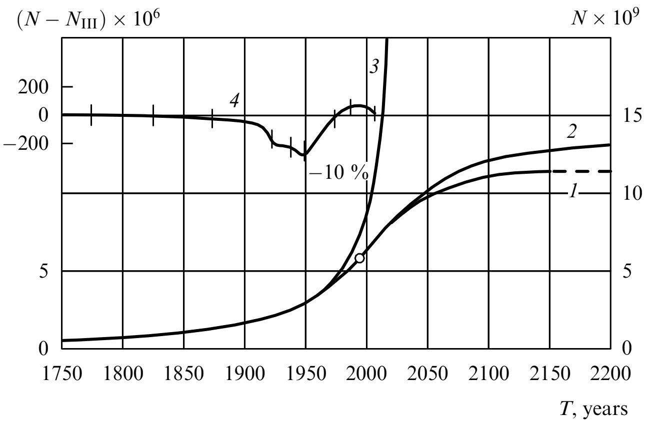

Figure 4. World population growth since 1750 to 2200. *1* — Projections of UN and IIASA, *2* — Model III, *3* — runaway scenario (2), *4* — difference between model and world population X5 times. ○ — present.

Two values of τ can be introduced — for the past and present, but the model shows that a plausible estimate for T_0 is obtained with the same τ as for the present. This value for $\tau = 42$ years is a reasonable time constant to describe the human life from our every-day experience, although the value

Table 2

Model	N_∞ 10^{-9}	C 10^{-9} year^{-1}	τ year	T_1 year	$\left(\frac{\mathrm{d}N}{\mathrm{d}T}\right)_{T_1}$ 10^{-6}, max year^{-1}	$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}T}$ max, %	$T_{\max}$	N_{1990} 10^{-6}	$T_{0.9N_\infty}$	K 10^{-4}	T_0 10^{-6} year	$P_{0,1}$ 10^{-9}
I	10	180	55	1998	60	1.31	1964	5260	2157	5.72	4.9	99
II	13	185	45	2005	92	1.60	1986	5135	2143	6.41	4.5	102
III	14	186	42	2007	105	1.73	1989	5253	2138	6.66	4.4	103
IV	15	190	40	2010	119	1.81	1993	5259	2133	6.89	4.3	106
V	18	195	33	2017	180	2.18	2003	5230	2119	7.69	4.0	110
VI	25	200	25	2022	320	2.88	2011	5306	2099	8.94	3.5	114
VII	∞	200	(20)	2025	—	—	—	5713	—	(10)	(3.1)	115

is obtained from the global demographic transition, as an average for many countries at very different stages of development.

Introduce dimensionless variables for time

$$t = \frac{T - T_1}{\tau} \tag{8}$$

and population

$$n = \frac{N}{K}, \tag{9}$$

where time is reckoned from T_1 in units of τ and as the unit of population the constant $K = 67000$ appears. K is really the single large parameter of the theory that enters into all formulae and describes all relevant proportions in the phenomenology. One could ascribe to K a certain dimension $[K]$ — number of people in the case when K serves as a unit of population (9), in other cases K is a real dimensionless number. The distinction arises from the two meanings of τ as in (4). The constant K can be interpreted as the natural size of a coherent population unit and numbers of this order turn up in human genetics, city planning etc.

Introducing t and n into (5) and on integrating the rate the growth the following expressions are obtained

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{n^2 + 1}{K}, \qquad n = -\cot \frac{t}{K}, \tag{10a}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{n^2}{K}, \qquad nt = -K, \tag{10b}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{K}{t^2 + 1}, \qquad n = -K \operatorname{arccot} t, \tag{10c}$$

In these expressions time and population appear in a symmetric way indicating the reciprocal connection of the variables. The singularities arise as $n \to 0$ and $t \to 0$. In the beginning of growth at T_0 the independent variable is t, as it should be. But as we approach T_1 the significant variable is n that really determines the change during the transition at T_1, although time certainly is physically independent. This is the consequence of the essentially nonlinear nature of the population system for which three distinct epochs can be identified. The first — epoch A is dominated by linear growth. Epoch B is described by a hyperbolic growth curve and epoch C is the transition to a stabilised world population.

The asymptotic transition of one solution into another is best seen in the series expansion

$$n = -\frac{K}{t}\left(1 - \frac{1}{3t^2} + \frac{1}{5t^4} - \cdots\right), \qquad t^2 > 1, \tag{11}$$

and

$$n = -\frac{K}{t}\left(1 - \frac{t^2}{3K^2} - \frac{t^4}{45K^4} - \cdots\right), \qquad t^2 < K^2\pi. \tag{12}$$

These functions intersect at $t_* = -\sqrt{K}$ at an angle $2/3K$ in an effectively smooth transition from one curve to another for any large K.

The beginning of growth at epoch A is determined by

$$T_0 = T_1 - \frac{\pi}{2} K\tau = T_1 - \frac{\pi}{2}\sqrt{C\tau} \tag{13}$$

or 4.4 million years ago for Model III. This expression shows that T_0 is rather insensitive to τ. Obviously, it is possible to start the solution at T_0 or $t_0 = -\pi K/2$ and, by excluding t from (10c) to have one autonomous equation for n, valid for all times

$$\frac{\mathrm{d}n}{\mathrm{d}t} = K \sin^2 \frac{n}{K} + \frac{1}{K}. \tag{14}$$

By integrating (14) a complete, although cumbersome solution for growth can be obtained

$$n = K \arctan\left(\frac{1}{\sqrt{K^2+1}} \tan \frac{\sqrt{K^2+1}}{K^2}\, t'\right), \tag{15}$$

where t' is reckoned from t_0 and $n_0 = 0$. These rather simple calculations provide a general description of the dynamics of the global demographic system. Developing this theory further will require the application of more advanced methods of nonlinear mechanics to the demographic problem, a problem this model has helped to bring into the realm of physics.

To gain insight into the growth mechanism it is best to consider the primary expression for the rate of growth during epoch B

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N^2}{K^2},$$

where K is the main parameter determining the rate of growth for a collective binary interaction in a generation. This growth rate is well known in chemical kinetics and has been extensively studied in systems analysis. In the case of the human system this growth rate should be seen as the net result of all partial processes, that contribute to growth. Thus the growth rate is the outcome of all inputs of an economic, social and biological origins. The human capacity to multiply, that taken alone could lead to exponential growth is only a part of all mechanisms contributing and limiting growth in this essentially nonlinear expression. To break up growth into the separate channels that sum up to (15) is beyond the agenda of phenomenology, expressed by the quadratic growth rate. In fact, this should be seen as the effective mechanism of growth described in phenomenological terms. The parameters τ and K are indeed constants, as no evolution is implied to adjust their values to changing circumstances (Fig. 5).

In a formal sense the N^2 term in (15) can be considered as an effective field and first in an expansion in powers of N, to describe higher order interactions. As K is a large number and both spatial distributions and temporal fluctuations have to be taken into account, it would, at any rate at the present stage, be premature to extend the expansion in terms of powers of N.

The main interaction (15) is a *cooperative* societal interaction valid over many generations and for the whole human system. The phenomenology is truncated by the microscopic time constant τ. The limit occurs in the distant past where the initial growth is postulated. Then the self-accelerating quadratic growth rate reaches its limit, culminating in the break down of scaling at the demographic transition. The hyperbolic runaway growth curve diverges in a finite time — more rapidly, than exponential growth, that diverges only in infinity. This growth rate is limited kinematically, by the internal nature of the growth process, not by the lack of external resources, as for example it is assumed in logistic growth. In other words, we are dealing

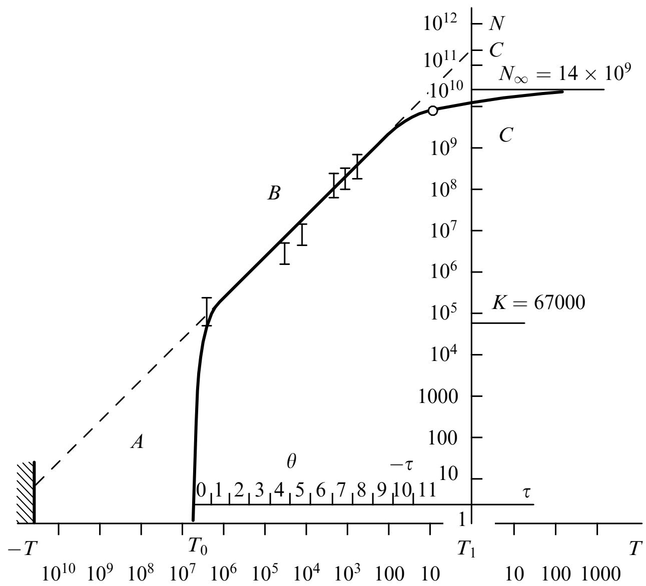

Figure 5. World population since the origin of humankind to the foreseeable future. Data [5, 26] and Coppens [23], $\theta = \ln t'$ (22), – – – —(2), ∘ — present.

with an open system, limited not by external, but by internal factors.

4. Limit of population and the number of people who ever lived

The expression for growth (6) indicates a limit for world population

$$N_\infty = \pi K^2 = 14 \times 10^9\,, \tag{16}$$

in the foreseeable future. Of this asymptotic limit 90% will be reached for Model III by year 2135, or in 3τ years after $T_1 = 2007$.

For human evolution and population studies, estimates of the number of people who ever lived have been made. In the framework of the model this may be done by integrating $N(T)$ from T_0 to T_1, or t_0 to t_1

$$P_{01} = K\int_{t_0}^{t_*} \cot\frac{t}{K}\,\mathrm{d}t + K\int_{t_*}^{0} K \operatorname{arccot} t\,\mathrm{d}t$$

$$= \frac{1}{2}K^2 \ln K + \frac{1}{2}K^2 \ln(1+K) \simeq K^2 \ln K \tag{17}$$

and for Model III $P_{0,1} = 2K^2 \ln K = 100 \times 10^9$. In these estimates $\ln K = 11.1$, and the average life time is assumed to be $\tau/2 = 21$ years, doubling the value of the integrals (17, 18). These estimates are made with the same assumptions for the average life span as in calculations by Weiss [20] and Keyfitz [21]. In these cases growth over the ages is described by a sequence of exponents and have led to P from 80 to 150 billions. For different models $P_{0,1} = (C/21)\ln K$ is practically independent of τ (Table 2).

By the time of the intersection point $t_* = -\sqrt{K}$, $T_* = T_1 - \tau\sqrt{K} \simeq 9000$ years B.C. half of all people have already lived. During the initial epoch A the corresponding number of people, or rather hominids was

$$P_A = 2K\int_0^K \tan\frac{t'}{K}\,\mathrm{d}t' = 2K^2 \ln\cos 1 \; = 5.5 \times 10^9\,. \tag{18}$$

It is best to present the growth process of the human system on a $\operatorname{Log} N - \operatorname{Log} T$ plot. This is not only a matter of convenience, when variables changing over 10 orders of magnitude are to be shown. On a double logarithmic plot all power laws, describing self-similar growth appear as straight lines. This is a graphic indication of the constancy of the relative growth rate, of scaling. Later we shall see that a logarithmic time chart leads to greater insight into the meaning of time for the development of the global human population system.

5. Population growth and the model

By comparing the model with data of paleoanthropology and paleodemography a description of human development over a vast period is possible. The initial epoch A began 4.4 million years ago and lasted $\Delta T_{\mathrm{A}} = K\tau = 2.8$ million years. Thus the model describes in general terms the initial period of development, an extensive interval that may be identified with the time when, according to modern data some 4.5 My ago humanids began to be separated from the humanoids [5]. By the end of period A *Homo Habilis* appeared and the number of these primeval hominids would have reached $N_{\mathrm{A,B}} = K \tan t/K = K \tan 1 = 1.04 \times 10^5$. This number corresponds well with the estimate $\simeq 10^5$ suggested by Coppens for this decisive moment in the development of humankind [23]. It was then in Africa at the beginning of Paleolithic that the toolmaking humanid first appeared. What matters is that a reasonable estimate can be provided for the time and numbers matching the data of anthropology, when some 1.6 million years ago a critical step in human development occurred. It was then that the cooperative social and technological pattern of self-accelerating development began, and since that time humankind started to spread throughout the world and grow in numbers, well beyond any other comparable creatures. This initial process of differentiation could be accompanied by fluctuations and the appearance of different parallel evolutionary lines. These events are beyond the scope of the model [5, 24, 25] (Fig. 6).

Epoch B encompasses the Paleolithic, Neolithic and history. During 1.6 million years the number of people once more increased K times. By the onset of the population

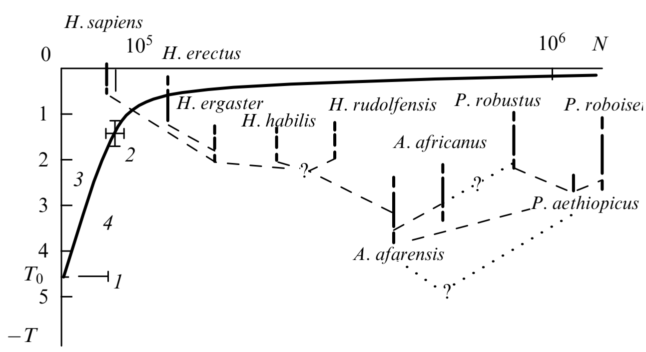

Figure 6. Separation of hominids from hominoids in Epoch A [22] and the growth of early hominids. *1* — [20], *2* — [23], *3* — (11a), *4* — (11b).

transition in $T_1 - \tau = 1965$ the global population had reached $(\pi/4)K^2 = 3.5$ billions. Growth and development mainly took place on the Euroasian continent. Across these vast spaces, tribes migrated, languages and cultures developed, civilisations grew and vanished developing in a systematic, although turbulent pattern of growth. During the last millennia the Silk Road, connecting the civilisations of East and West, of China and Europe with an input from India was active. Along this main trade route world religions spread, technical and social information diffused. As yet slowly, then faster and faster did the human system develop, incessantly growing in numbers.

Most of population data to the extent that it is known within reasonable limits, fits the model. Although the further we go into the past, the accuracy of demographic data rapidly decreases. In general, both in history and anthropology dates are known much better than population numbers. For example, for the time of A.D. paleodemography provides estimates from 100 to 250 million. The model indicates 100×10^6, that corresponds to the number mentioned incidentally in the joint statement of 50 Academies of Sciences on demography [15]. Since XVI century, after the great geographic discoveries the quality of world demographic data improves. It was then that the world population system rapidly became more and more interconnected, although in the past this interdependence was always sustained and the gross features of our past are reasonably well described by the model.

Of interest is to compare the results of modelling with projections of demography into the foreseeable future. The model indicates a rapid transition asymptotically limited at 14 billion and reaching the 90% point of 12.5 billion in 2135. These numbers may be compared with projections of UN [27] and IIASA [28] (Fig. 7).

The projections of UN are based on summing up scenarios for fertility and mortality for nine regions of the world and are extended to 2150. By the optimal UN scenario the world population will reach a constant level of 11 600 million, then extrapolated to 2200. Projections of IIASA cover a shorter time horizon — up to 2100 and are based on six regions with ten different patterns of growth. The optimal scenario is No. 7 — that of a slow decrease of fertility, when the projections of UN and IIASA practically coincide. Both modelling and demographic projections indicate a levelling off of the world population (Table 3).

It should be indicated that the calculations of demography are not only in a certain sense arbitrary by assuming a scenario taken to be operative, but are also computationally unstable. A shift of 2–3 years in assuming a change of fertility or mortality may lead to rapidly growing discrepancies. That is why such calculations are at best valid for short range projections. According to Sadic [29], most projections of demography have over the last decades been systematically revised upward.

6. The demographic transition and world population stabilisation

For the world population the demographic transition is a very special event, a phenomenon of great interest and complexity. Its beginning can be attributed to 1965 and its end to $T_1 + \tau = 2049$. It is during these 84 years that the world population will on one side increase its numbers three times and on the other side undergo a most profound transformation. To describe the population transition Chesnais [3] introduced the transition multiplier that for the model is

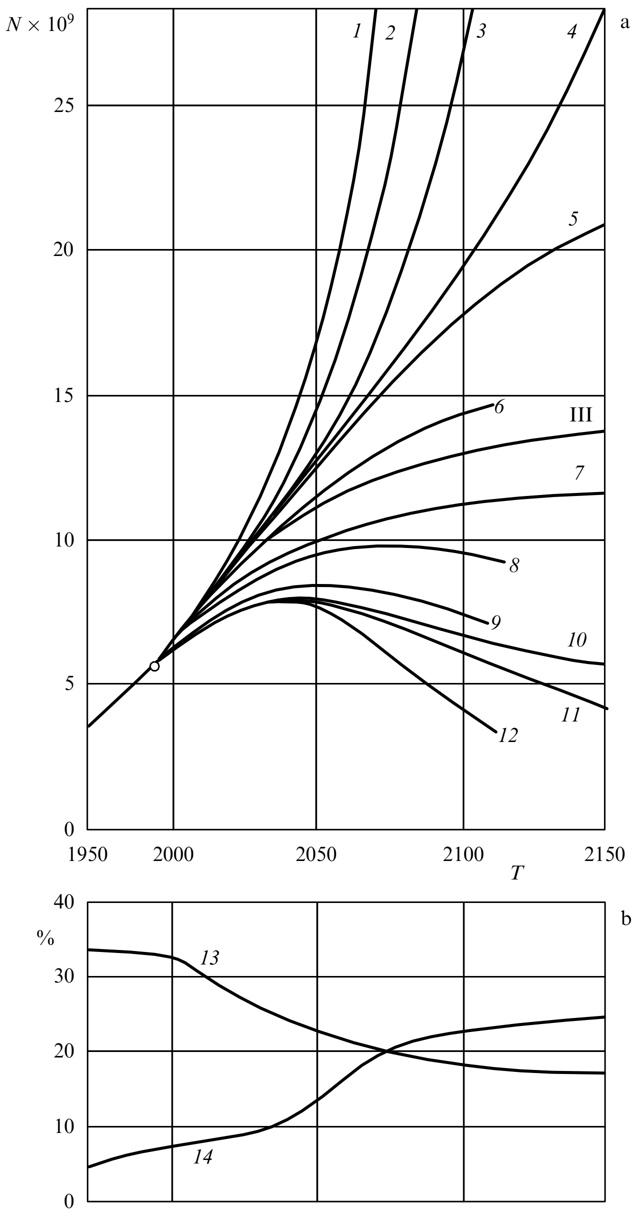

Figure 7. (a) World population projections by UN [27] and IIASA [28]. *1* — constant fertility, *2* — constant rate of growth, *3* — III World crisis, *4* — high UN, *5* — medium high UN, *6* — low decrease of fertility, *7* — medium decrease of fertility, *8* — slow decrease of mortality, *9* — constant mortality, *10* — medium low UN, *11* — low UN, *12* — rapid decrease of fertility, III — Model III, ∘ – present. (b) Changing age distribution during the demographic transition. *13* — less than 15 years, *14* — older that 65 years, according to medium UN projection [27].

$$M = \frac{N(T_1 + \tau)}{N(T_1 - \tau)} = \frac{\operatorname{arccot} 1}{\operatorname{arccot}(-1)} = 3\,. \tag{19}$$

In this case the beginning of the transition is at the point of most rapid increase and its end — at the point of most rapid decrease of the growth rate. Chesnais provides the following data for M: China $M = 2.46$, India $M = 3.67$ and for the world $M = 2.95$, numbers that compare well with (19), although in some cases as France $M = 1.67$ or Mexico

Table 3. World population growth

Year	$10^{-6}N$	$10^{-6}N_{\mathrm{III}}$
-4.4×10^6	(0)	0
-1.6×10^6	0.1	0.1
-35000	$1\div5$	5
-15000	$3\div10$	11
-7000	$10\div15$	21
-2000	47	46
0	$100\div230$	93
1000	275	185
1500	450	366
1650	550	519
1750	728	717
1800	907	887
1850	1170	1158
1900	1617	1656
1920	1811	1992
1930	2020	2211
1940	2295	2480
1950	2515	2812
1955	2752	3009
1960	3019	3230
1965	3336	3478
1970	3698	3758
1975	4080	4073
1980	4450	4426
1985	4854	4820
1990	5292	5253
1995	5765	5724
2000	6251	6265
2005	6729	6746
2010	7561	7572
2025	8504	8749
2050	10019	10427
2075	10841	11462
2100	11185	12034
2125	11390	12398
2150	11543	12648
2200	11600	12946
2500	UN ↑	13536

$M = 7$ the discrepancy may be large. What matters are not these special cases, that to a certain extent are dependent on when the beginning and end of the transitional period is determined, but the overall correspondence of modelling and the value of M for the largest contributors to the world population.

On the scale of history the transition is remarkably short — 1/50 000 of the time of growth, although 1/10 th of all people who ever lived are to experience this special period. This can be attributed to the strong interaction existing nowadays between countries and regions of the world. The world population definitely does behave as a system and the synchronisation of all the partial transitions is a measure of this interaction. Synchronisation and narrowing is a well-known phenomenon in nonlinear systems and points to the systemic nature of the global demographic transition.

The rate of the transition with a characteristic time of 42 years is in fact shorter than the life expectancy of 70 years in developed countries and practically equal to the world average of 40 years. This rapid non-equilibrium transition leads to the break-up and disruption of traditions and customs long established in human societies, factors that to a large extent stabilised the life-style in the past, setting up long-term correlations between generations. Today it is customary to say that the connections between generations are severed and many see in this one of the reasons for the strife and stress of modern life, a factor that should be attributed to the transition through which we are passing.

The concept of a future stabilised world population is of significance. This is an immediate result of the model that describes the transition from quadratic growth during epoch B, culminating in the population explosion and a basic change in our growth after the transition (Fig. 8). Recalling Adam Smith, the invisible hand of self-organisation is seen as the collective agent that changes the paradigm of our growth. The change is due to a systemic crisis and is described by the change in the asymptotic behaviour of the human population system. Its fundamental reason may be connected with the analytic properties of the function $N(T)$, its singularities and asymptotics. This point needs further consideration, as it could pave the way to greater insights into the foreseeable future of humankind. If we had only to extrapolate an established pattern of growth without this qualitative change, it would be much more simple to make this forecast, but its significance would be correspondingly less.

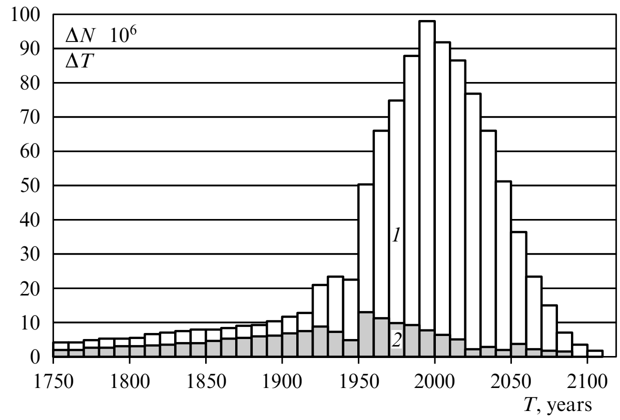

Figure 8. Demographic transition. World population increase since 1750 – 2100, averaged over a decade. *1* — developing world, *2* — developed countries.

The systemic transformation of a country passing through the demographic transition has been graphically depicted by Vishnevsky [30], who introduced a phase plane to describe this process when the system rapidly moves from one attractor to another, indicating the stability of motion at each of these states of equilibrium (Fig. 9).

Chesnais in his detailed description of the demographic transition finally comes to the conclusion that it is impossible to explain this complex phenomenon in terms of cause and effect, of linear modelling for only an inherently nonlinear model can describe this transformation.

For a physicist accustomed to concepts of statistical physics and systemic behaviour, the demographic transition displays many features of a phase transition. It is the age distribution of the population that changes during the transition and this is the most important thing happening. In the framework of the model, the age distributions are not taken into account. An extension of the model could treat the changes in the age distribution, that were fairly constant during epoch B and then rapidly change to a new pattern. These transformations can be described by the standard methods of demography, when this change is postulated, rather than as it appears in a natural way from a model (Fig. 10).

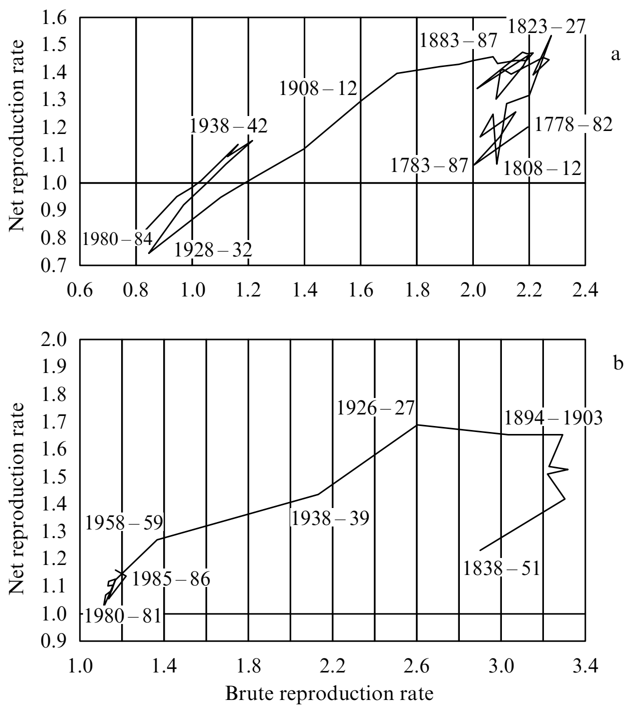

Figure 9. Phase diagrams for population reproduction rates [30]: (a) Sweden (1778 – 1984), (b) USSR (1838 – 1986).

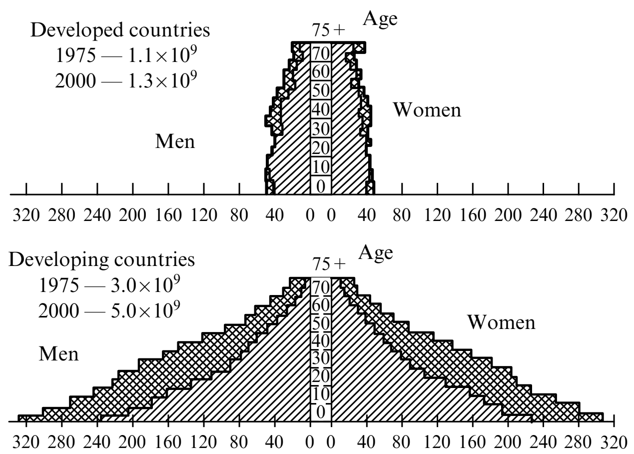

Figure 10. Distribution of population by age and sex (1975 and 2000).

7. Transformation of time

A significant result of modelling is a change in the scale of time as we recede into the past. The transformation and compression of the time scale is best seen on the logarithmic plot when the critical date T_1 is approached. Mathematically this change in the effective time scale is best described by considering the instantaneous exponential time

$$T_e = \left(\frac{1}{N}\frac{dN}{dT}\right)^{-1} = \frac{1}{\tau}\left[(T_1 - T)^2 + \tau^2\right] \operatorname{arccot} \frac{T_1 - T}{\tau} \tag{20}$$

of growth is introduced (see (6)). For the past of epoch B

$$T_e(T) \simeq T_1 - T\,. \tag{21}$$

At present we are very close to T_1 and T_e is simply equal to the time before present (B.P.). For example, 1.6 million years ago with the onset of epoch B in the lower Paleolithic a marked change could happen only in a million years. This very slow rate of change is well known in anthropology, although a plausible explanation has yet to be offered.

By the end of the Stone Age a marked change is associated with the Neolithic Revolution — the transition from a hunter-gatherer life-style to agriculture. This transition is close to $T_* = 11\,000$ B.P. — the time when on a logarithmic scale half of the human systems life span has passed and half of all people who ever lived have appeared. As the average pattern of growth is described by the model, there is no discontinuity in the world population. But by that time the growth rate is 10,000 times greater than at the beginning of the Paleolithic and the world population was approaching 15 million, a number that is close to most estimates [5].

The changing sense of time is a kinematic property of the model, a direct consequence of the kinetics of hyperbolic growth. As blow-up at T_1 is approached $T_e(T)$ is no longer a linear function of time and T_e now follows the exact expression (20). In 1989 T_e passes through its minimum value of 58 years, corresponding to the relative growth rate of 1.7% per year or a doubling time of $T_2 = 40$ years. This indicates how rapid the transition really is and that at no point an extended exponential pattern of growth is kept up. As the population stabilises, T_e rapidly grows as $T_e \simeq (T - T_1)^2/\tau$ for $T > T_1$ (Fig. 11).

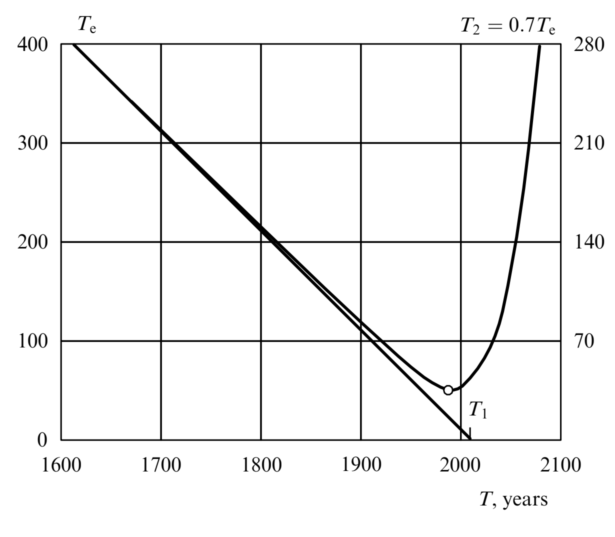

Figure 11. Instantaneous exponential time T_e vs T, ∘ — present.

It is instructive to apply this transformation of time to large-scale historical events (Table 4). For example, the history of ancient Egypt spans 3000 years and ended 2700 years B.P. According to Gibbon the decline and fall of the Roman Empire took 1500 years [33]; nowadays empires built in centuries collapse in decades, if not less.

This transformation of time is best seen if the main periods designated in history and anthropology are plotted on a logarithmic scale. The tradition of history and observations of anthropologists identify major periods that on a logarithmic scale are more or less equally spaced from $T_0 = 4.4$ million years B.P. to $T_1 = 2007$. Both the limits defining the historic periods and periods of the more distant past seen by archeologists cannot be established with any great accuracy

Table 4. The development of humankind on a logarithmic scale. Glacial ages: 1 — Wurm 10–54, 2— Riss 200–250, 3 — Mindel 400–500, 4 — Gunz 600–800, 5 — Danube 800–2000 thousand years ago. Recent studies have shown a more detailed pattern of glaciation. The dates of the periods indicated are known with better accuracy than the data for population, shown by order of magnitude only

Geology		Demographics and anthropology							History technology
Epoch	Glacial ages	Epoch	Age	θ	T dates	N	Period	ΔT years	
HOLOCENE		C	HISTORY		2175 2050	13×10^9 10.5×10^9	Stabilized population	125	Limit 14×10^9 Urbanization Age distribution change
					2007	7×10^9	World demographic transition	42	$\leftarrow T_1$
				11	1965	3.5×10^9		42	$\leftarrow$ now
		B		10	1840		Recent	125	computers, I, II world wars electric power industrial revolution
				9	1500	10^9	Modern	340	French revolution, great geographical discoveries, printing
				8	500 A.D.		Middle ages	1000	
				7	2000 B.C.	10^8	Ancient world	2500	Fall of Rome $\leftarrow$ A.D. Greece, Buddha, writing
			STONE AGE	6	9000 B.C.		Neolithic	7000	Egypt, China, India Domestication Agriculture
PLEISTOCENE	1			5	29000	10^7	Mesolithic	20000	Ceramics, bronze Microliths Populating America
				4	80000		Moustier	51000	
				3	220000	10^6	Acheulean	140000	*Homo Sapiens* speech, fire Populating Europe, Asia
	2 3			2	600000		Chelles	380000	
	4 5			1	1600000	10^5	Olduvai	1000000	Handaxe *Homo Habilis*
		A		0	4400000	(1)		2800000	Hominids separate from Hominoids T_0

as they are defined not by changes in population growth, but by the less objective, although meaningful criteria of technological and social changes. With the accuracy that one could expect each cycle is 2.5 to 3 times shorter than the previous one, with the corresponding growth in population. This cyclic pattern can be described by a sequence of intervals of a geometric progression

$$\Delta T(\theta) = K\tau e^{-\theta}, \tag{22}$$

where θ is the integer part of $\ln|t - t'|$ — the number for the cycle from $\theta = 0$ to $\theta = 11 = \ln K$, assuming that each cycle is shorter by $e = 2.72$ times. Then the whole duration of our development from T_0 to T_1 is

$$T_1 - T_0 = K\tau \sum_0^{\ln K} e^{-\theta} = \frac{e}{e-1} K\tau = 1.582\, K\tau\,. \tag{23}$$

This estimate is very close to $T_1 - T_0 = \pi K\tau/2 = 1.571\, K\tau$ (13). The slight discrepancy should be ascribed to a different

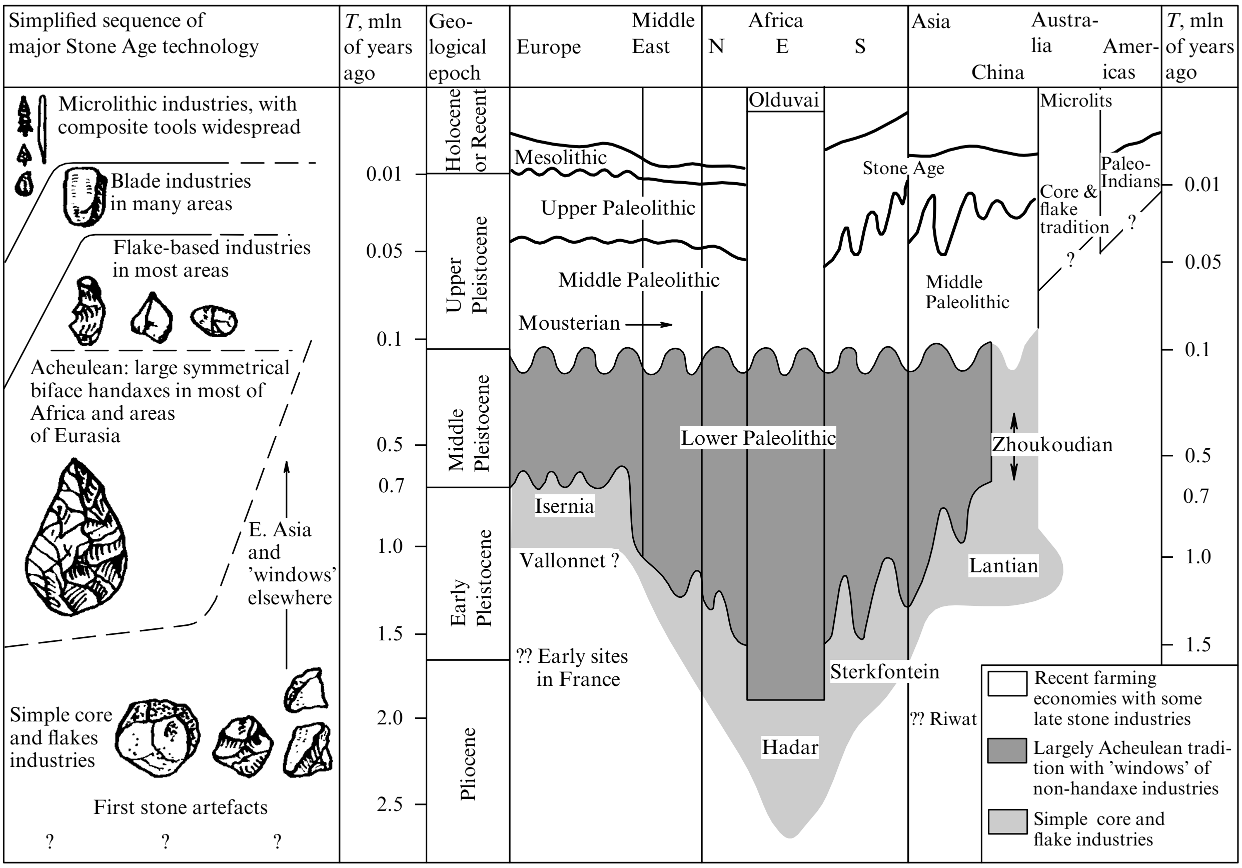

Figure 12. Stages of Paleolythic [5].

way of determining the initial epoch A $\Delta T_0 = 2.8 \times 10^6$ of the 0-th cycle.

During epoch B $\ln K = 11$ cycles took place and through each cycle $\Delta P = 2K^2 = 9 \times 10^9$ people lived, as the duration of each period changed from 1 million years to 42 years. The invariant population step $\Delta P = 9$ billion appears as a constant for this periodic sequence of major cycles. As a conjecture this periodicity could be ascribed to the inherent periodicity of the logarithmic function for a complex variable, but in that case a phase has to be ascribed to $P_{\mathrm{B}}(t') = K^2 \ln t'$ of unspecified origin; or these exponentially sequenced cycles could be bifurcations of a more complicated set of equations.

These global transitions happen more or less synchronously on a world-wide scale and the whole subject of the simultaneity of major periods in history has been extensively discussed by historians [34]. Recently D'yakonov in a review of human history [35] indicates that the sequence of gross features of our past explicitly follow a geometric progression, culminating in a singularity, but offers, apart from this insightful observation, no explanation for this phenomenon.

The simultaneity of transitions is due to the interactions in the world population system and this is indicative of its systemic behaviour over very extensive epochs. The time of transition is usually marked by the first appearance of a new technology or social changes, which then spreads throughout the world. Probably Kondratieff was the first to indicate such cycles [36]. The hyperbolic periodicity is seen only on a logarithmic scale, but it extends throughout our history.

From the systemic process of global population growth it follows that any long term breakup of the global community will lead to a slowing down of development in any lesser part of the greater system. Even today in isolated communities societies at a neolithic or even paleolithic stage of development can be found. Probably the most instructive case is that of the separation of the Americas from the Eurasian-continent. This led to a slower rate of development for the precolumbian civilisations and came to an abrupt and tragic end with the discovery of the New World.

8. Stability of growth

Of importance is the stability of growth. If the standard Lyapunoff criteria of systemic stability is applied to the human system, then a variation δn will grow as $\delta n = \delta n_0 \exp(\lambda t)$. For the Lyapunoff exponent λ the following expression

$$\lambda = \frac{\partial}{\partial n}\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right) = \sin 2\,\frac{n}{K} \tag{24}$$

is obtained by differentiating (14). The instability reaches its maximum value between T_0 and T_1 $n = \pi K/4$ at $t = -1$ in 1965 when $\lambda_{\max} = 1$, halfway to N_1. Only after T_1 does λ change sign and systemically stable development if possible. In spite of this instability world population growth is in general stable and the reasons for this have to be understood.

The trend towards stable development is demonstrated by the large scale behaviour of the human population system. The system was destabilised by at least two global events — the Plague in the XIV century and World Wars I and II in the

XX century. Demographic data show that after these major disasters, when in Europe from 30 to 40% of the population died and with a 10% loss of the global population during the World wars, the global system rapidly regained its losses and a generally stable pattern of development is sustained.

In "The Economic Consequences of the Peace" [36] Keynes cites as one of the reasons for WWI the rapid build-up of population in the warring countries. The collapse of global security then showed many signs of a systemic loss of stability. It may be simplistic to interpret history in such mechanistic terms, but a predisposition to a loss of stability, as indicated above, should be kept in mind. At present the developing world is now in a similar stage, with 2-digit growth figures for the economy of China and the 6 to 7% growth of India, indicating that these countries passing through the demographic transition may become a source of global insecurity.

Major instabilities cannot be predicted, but a critical predisposition should not pass unnoticed. In the developing world of today, changes are happening twice as fast as in the developed world at a similar stage of growth and the population involved is 15 times greater. Probably of all global problems that of security is of the greatest importance, while both an understanding of these threats and an effort to avert general instability should be uppermost on the global agenda, for no one can afford a world war of a scaled-up magnitude of the previous XX century conflicts. In considering military, economic and ecological factors of global security, the demographic factor is the first to be assessed. In this case not only quantitative factors, but qualitative, including ethnic ones, should be accounted for.

An estimate of fluctuations to be expected in the global population system

$$\delta N = \sqrt{KN} \simeq 20 \times 10^6 \tag{25}$$

for the present world population were conjectured in [9]. In relative terms the fluctuations were largest at the beginning of quadratic growth a million years ago.

For the stability of the world population system the spatial distribution should be taken into account. If diffusion is introduced into the kinetic equation one can expect a damping of systemic instabilities, as the eigenvalues of the Laplacian $\nabla^2 N$ are negative [17]. This will require a more detailed analysis of fluctuations and instabilities of the solutions of partial differential equations describing changes in space and time.

It is well known that the distribution of populations in space is far from uniform, whether on a regional scale or of nations and cities. The distribution of the population of France has recently been studied by Le Bras, using the multifractal method. [37]. The urban concentration of people in cities can be described by a hyperbolic distribution $U(R)$

$$U(R) = \frac{U_0 \ln U_0}{R + \ln U_0}\,, \tag{26}$$

where R is the rank of a city with U people, in spite of the difficulty in defining the population of a city in the present or past. The fractal distribution is well established for $R > \ln U_0$. This expression is valid from U_0 — the population of the largest city in the world downwards to $U_{\min} = 1$ as the lower limit (Fig. 13). By integrating (27) from $R = 0$ to $R_{\max} = U_0 \ln U_0$ at $U_{\min} = 1$ the population of the world is normalised $N = U_0 \ln^2 U_0$, so U_0 and $U(R)$ can be found from N.

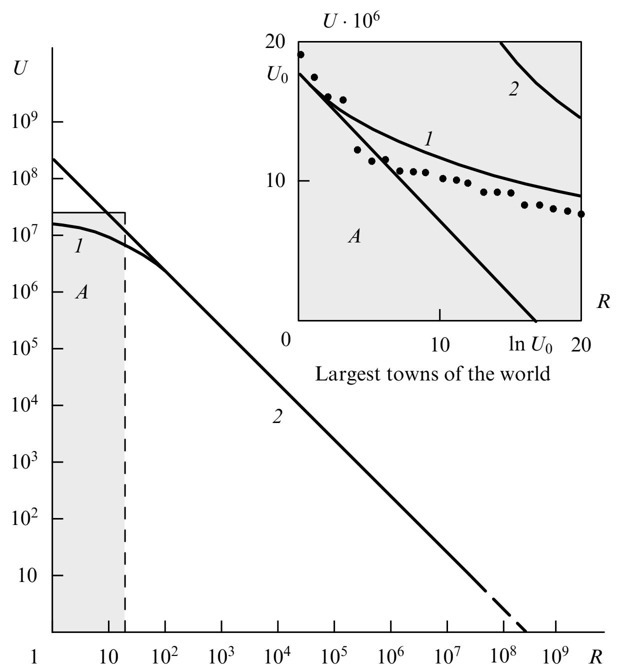

Figure 13. Distribution of population of towns for the world 1985. *1* — $U(R)$, *2* — $U(R) = U_0 \ln U_0 R^{-1} = 290 \times 10^6 R^{-1}$ for $R > \ln U_0 = 16.67$. A. $R = 0$ — Tokyo, 1 — Mexico City, 2 — San Paulo, 3 — New York, 4 — Shanghai, 5 — Calcutta, 6 — Buenos Aires, 7 — Rio de Janeiro, 8 — London, 9 — Seoul, 10 — Bombay, 11 — Los Angeles, 12 — Osaka, 13 — Beijing, 14 — Moscow, 15 — Paris, 16 — Djakarta, 17 — Tianin, 18 — Cairo, 19 — Teheran, 20 — Delhi.

The distribution of cities is described on a global scale without introducing any new parameters. For example, at A.D. $N \simeq 200 \times 10^6$ for ancient Rome, where, according to historic data the Coliseum could seat 50,000, we get $U_0 \sim 1$ million. It should be noted that $U_0/N = 1/\ln^2 U_0 \simeq \simeq 0.4\%$ does not on the average vary for the model. For obvious reasons (26) is not applicable to a separate country, where locally fractal distributions are valid only asymptotically, with large cities excluded [39]. The way U_0 is brought in may be conjectured, rather than proven by assuming $\ln U_0$ as the natural inherent unit of scaling for ranking cities and justifying the statistical approach to the global system [51].

Incomes in a society also follow a power law — Pareto's law — indicating the non-equilibrium pattern of the distribution of wealth. These fractal laws should be taken into account when future projections for humankind are considered. The underlying reason for these distributions may be seen in the correlations, both spatial and temporal that are established in a developing system, as indicated by Scarrot [39]. In the global population system a number of instabilities may develop and various stabilising factors, migration in the first place operate. This is best demonstrated by the large scale cycles observed in the human population system, although a whole hierarchy of instabilities of a lesser scale do develop.

9. Influence of the environment

From the cyclic pattern of our growth, even without going into the mathematical details of modelling, it follows that a major period in our growth and development has now come to an end. On the other hand, we see that all through the ages the human population system can be seen as a single and open entity. That the system is open means that the external resources do not directly affect growth, have not limited this growth in the past and should not in the foreseeable future.

The approaching population limit is determined by internal systemic factors, factors operating at all times for more than a million years, over many cycles of our development. At all stages of growth humankind always had enough resources for sustained development. If at any point, resources were lacking, then by migrating to other lands new resources were brought into play and growth continued unabated. During each cycle of development fewer and fewer people were engaged in feeding the population, so that today in advanced societies 2–3% can feed the whole country. According to estimates by FAO in principle the world at present could feed 20–25 million people, roughly twice the expected limit.

Conventional Malthusian wisdom says that demographic transitions are due to the depletion of resources. But a growing body of evidence points to the opposite and only by treating the human population as an evolving system can we obtain some insight into the reasons for the demographic transition [42]. Recently, Le Bras illustrated this point in a number of cases, showing that a naive Malthusian limit of resources cannot explain the transition [43]. Moreover, in the future it seems that such resources will be available and in principle allow for a crossover unhindered by resources of humankind through the global demographic transition. In this case the population from 5.7×10^9 of today will grow by less than 2.5 times to reach the limit of 14 billion, or only double in the next hundred years.

In other words the model shows that the world population growth in the first approximation is independent of resources. If a cut off of resources should come, the first reaction of the global population system would be a much more uniform worldwide distribution of people, signaled by a change in constants, that describe growth. The independence from global resources may be stated as the principle of *the demographic imperative*, as the expression of the fundamental independence of growth in an open model.

The non-uniformity of the use of resources is in a sense amazing. A good example is the comparison of India and Argentina. If the area of the subcontinent is only 30% larger, the population of India is 30 times greater than in Argentine. There conditions are such that, if modern agricultural methods were used, by exploiting the resources to the limit it could feed the whole world. The non-uniform distribution of population and resources existed also in the past, for at the height of the Roman Empire more than a million people lived in Rome.

The concepts of the systemic model contradict most of the assumptions of "Limits to Growth" [44]. In the last publication of Medows, special importance is ascribed to exponential growth. Data and modelling show that exponential growth hardly ever finds its expression in a pattern of development for any length of time. It should be noted that the Club of Rome in its last and very significant report "The First Global Revolution" by King and Schneider has definitely parted with the initial Malthusian reductionist pronouncements of the Club, and moved towards a more synthetic and holistic approach to describing the destiny of humankind [45, 46].

On the agenda of these studies for the foreseeable future an important issue will be the new criteria of growth, that are to appear in a stabilised world population. Will this state be stable and what will be the measure of growth? With a pronounced change in the age structure of society a profound change in values and connections between generations is to be expected. Greater social security expenditures and educational outlays can be anticipated. Of significance is the distribution of wealth and its global evolution. These issues are beyond the scope of the present model, but the general change in the pattern of growth will set the scene for this emerging new world of a stabilised global population, if and when it is to happen.

10. Conclusions

For the population of the world the model provides a description of the gross features of growth and evolution for humankind. The invariance of the self-similar pattern of growth, where inherent limits are set, is the main feature and result of the modelling, when a bare minimal number of constants are introduced.

The model provides for a phenomenological, macroscopic treatment but does not profess to explain in detail the processes leading to growth. The model is an open one and growth is not explicitly dependent on resources as it is the result of cooperative interactions of all relevant forms of human activities. This interaction, summing up all contributions to development that can be seen as the ultimate mechanism of growth.

The model is justified not only by the extent to which the results of modelling correspond to the facts of life, but also by the fundamental principles of systemic growth that it is based on. The concept of self-similar growth is an expression of systemic dynamics, initially developed by Haken and Prigogine and now applied to the description of global population growth. Describing the overall process of development by an essentially nonlinear model it should be kept in mind that it cannot be directly applied to local or regional growth. But the global process of development does definitely influence any of its parts by the connections and interactions implied in the world model.

The transformation of the effective time scale is a significant result of the theory, a kinematic consequence of self-accelerated growth. The moment from which time should be measured is set and the scale changes as we go into the past, corresponding to the intuitive insight of historians and anthropology on past cycles. The model indicates that humankind is now rapidly passing through a critical period. A fundamental change in the growth paradigm is occurring, a change never experienced before. Some historians have pronounced the end of history [48]. Today we are witnessing a much more profound transformation, a critical period compressed into a remarkably short time of drastic global changes [49].

In this study the demographic factor is taken to be decisive. Until recently the approach through demography to global problems was to a great extent blocked by some parties and excluded from most of the international debate. Now this has hopefully changed [50]. As these discussions excite high feelings on matters concerning our common future and the issues at stake are great, it is most necessary to develop and foster interdisciplinary research, following different intellectual traditions. In these studies mathematical modelling should be seen not only as a useful tool, but hopefully as a theoretical basis for analysis and projections of world population growth and the consequences it may have for sustainable development and global security.

In the foreseeable future we can hardly expect to significantly change and influence the overall growth pattern. The sheer size of the world population and given the pace of events it is difficult to imagine how the world community can have a major effect on global population growth. The fundamental understanding of growth is still rather limited and definitive advice for action is hard to provide, apart from very general recommendations. Probably the most important issue is by all means to ensure the stability and security for the world to be, as the prerequisite for resolving global problems.

If the model is to be supported by further research and the insight it provides is valid, then it may help to lead to greater understanding of the present state of affairs. It can offer a common frame of reference for anthropology and history, demography and sociology, for studies in human evolution and genetics. For doctors and politicians it may elicit an understanding of the sources of stress and tensions in this transient period, so unique in human development, both for an individual and at a broader societal level. In this case interdisciplinary studies and experiences are worth the effort in developing the model and the promise it can provide in facing the predicament of humankind.

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