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Graphic visualisation of fractal structures

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Contents

Abstract. A graphic method demonstrating formation and behaviour of fractal structures produced by interaction of metallic powder with the magnetic field of a coil is presented. The method is useful for investigating fractal structures by students.

1. Introduction

To simplify the studies of disordered systems and nonequilibrium phenomena, use is now being made of a nonconventional symmetry transformation, the scale invariancy, typical of fractal structures [1 – 3]. Fractals are exemplified by many random structures such as diffusion-limited aggregates, gels, colloid particles, etc. Fractals have been the subject of a number of recent reviews in *Physics – Uspekhi* [4 – 6]. We have developed a simple technique for visualising the chaotic motion of particles and many other features of fractal structure dynamics. In the device proposed by us the chaotic motion of particles is created by the interaction between an alternating magnetic field, induced by a rectangular coil, and iron dust situated within the coil.

The main component of the device is a rectangular coil with two windings. One of the windings, consisting of 900 turns of PEŌL-04 wire, draws an alternating current (ac) from the power supply through a voltage regulator; the other winding, consisting of 600 turns of the same wire, is fed from an adjustable direct current (dc) source and serves to magnetise the iron dust. The temporally alternating and spatially inhomogeneous magnetic field inside the coil induces chaotic motion of the iron dust, closely resembling Brownian motion. Magnetised particles of the dust (hereafter called 'particles') interact with each other. The spatial distribution function of the particles varies depending on the ratio between the energy of interaction of the particles with the external field, $B_i H_0 \cos[\omega t + \varphi_n(t)]$, and the energy of interaction of the particles with each other $B_i \langle H_i \rangle$. In these expressions, B_i is the magnetic induction in a particle, H_0 is the magnetic field strength induced by the coil, $\langle H_i \rangle$ is the mean value of the magnetic field strength induced by neighbouring particles, ω is the frequency of the external magnetic field, and the function $\varphi_n(t)$ characterises the random angle between the vectors $\boldsymbol{B}$ and $\boldsymbol{H}_0$.

Let us consider some illustrative experiments.

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2. Cluster formation

We mount the coil horizontally on the table of an overhead projector and insert a Petri dish with a transparent one-millimetre grid stuck to the outside of its bottom into the opening in the coil. We then pour the iron or iron oxide dust into the dish and try to make the density of the dust layer, as projected on the screen, the same over the bottom of the dish. Then we apply ac to the first coil, the current being such as to set the particles in motion. After that we supply the second coil with magnetising dc. By adjusting the currents in the windings we make particle aggregations (clusters) appear on the screen.

Cluster formation will be even more clearly seen if dust obtained by milling a ceramic magnet is used. By pouring water into the dish and stirring it with the dust we get a homogeneous suspension. By adjusting the currents in the windings we can make the suspension gradually clarify revealing the growth of clusters.

3. Demonstration of fractal formation

In the centre of the dish we place an iron washer with a flexible copper wire soldered to it and surround the washer with a copper ring which also has a flexible copper wire soldered to it. We then pour iron dust between the washer and the ring. By adjusting the alternating current through the coil we establish conditions under which particles drift towards the magnetised washer with a progressive build up of branches (Cayley tree).

To characterise growing fractal structures quantitatively we use relations from Refs [1, 9]. Let $\sum n_s$ be the number of cells at the bottom of the dish and $\sum n_s s$ be the total number of cells occupied by particles. $w_s = sn_s/\sum n_s$ is the probability that a randomly chosen occupied cell will belong to a cluster of size $S(R)$. The mean size of the cluster is then $S(R) = \sum sw_s = \sum s^2 n_s/\sum n_s s$. The number of particles forming a fractal is given by the relation

$$N(R) = \int^R \rho(R) r^{d-1}\,\mathrm{d}r = R^D \,, \tag{1}$$

where r is the radius of the circle inscribed in the grid cell (in our case it is 0.5 mm), $\rho(R)$ is the number of particles in one cell ($s = 1$ mm^2), R is the geometric size of the structure, d is the dimension (for plane figures $d = 2$), and D is the fractal dimension. From relation (1) we obtain the similarity law for the density

$$\rho(R) \sim \frac{M}{R^d} \sim R^{D-d} \,. \tag{2}$$

Since $D < d$, the density of the structure will decrease with increasing R. The number of branches intersecting the cylinder can be written as

$$n(R) = \frac{\mathrm{d}N(R)}{\mathrm{d}R} \approx R^{D-1} \,. \tag{3}$$

The area of each branch is

$$\langle \sigma(R) \rangle = \frac{R^{d-1}}{n(R)} \approx R^{d-D} \,. \tag{4}$$

This cluster model can be used to demonstrate the flow of an electric charge. To do this we assemble the circuit shown in Fig. 1. When the growing branches reach the copper cylinder the circuit closes and the lamp lights up, which indicates that the charge is flowing. The cluster resistance is equal to

$$R_\Omega = \frac{\rho_\Omega R}{\langle \sigma \rangle} \sim \rho_c R^{-(d-D)} \frac{R}{R_c} R^{d-D} = \rho_c R^{D-d+1} \,. \tag{5}$$

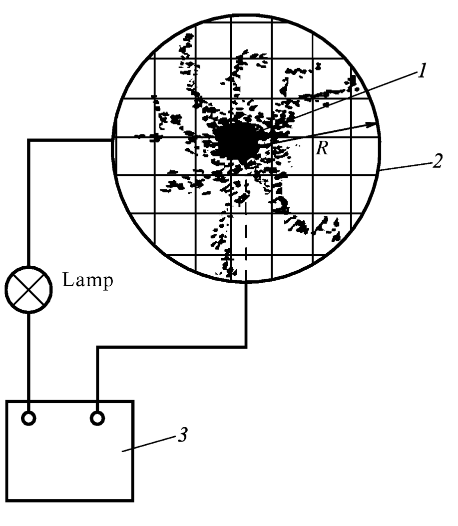

Fig. 1. The circuit for demonstrating the flow of electric charge through the growing branches of the cluster: *1* — iron coil, *2* — copper ring, *3* — audio frequency oscillator.

Here R_c and ρ_c are the critical radius and the density of the cluster, respectively.

4. The influence of fixed external fields on the shape of fractal structures

In this experiment we mount a plate of organic glass on the coil, with a copper wire about 1 mm in diameter passing through the centre of the plate and perpendicular to its surface. The ends of the wire are connected to a controlled dc supply. Iron dust is evenly spread over the plate. If ac is now fed to the coil, we first demonstrate chaotic motion of the dust particles. Then, when dc is passed through the straight wire, concentric rings of iron dust appear around the wire. Kinetics of the particles can be described by the Fokker – Planck equation [2, 7, 8]

$$\frac{\partial f(r)}{\partial t} = \frac{\partial}{\partial r}\left[v(t) f(r) + \frac{D}{2}\frac{\partial}{\partial r} f(r) \right] , \tag{6}$$

where $f(r)$ is the distribution function of the particles on the plane, D is the diffusion coefficient, which can be written down as $D = 2B_i H_0 \cos(\omega t + \varphi_n)/(m\gamma)$, v is the drift velocity of the particles, $\mathbf{r}$ is a vector connecting the point of intersection of the straight wire with the plate surface with the position of the particle at any given instant, γ is the friction coefficient, and m is the particle mass. The stationary solution to the Fokker – Planck equation for randomly walking ferromagnetic particles in an inhomogeneous magnetic field can be written as

$$f(r) = N \exp\left[-\frac{E_k + B_i\langle H_i \rangle - B_i\, i/(2\pi r)}{B_i H_0 \cos(\omega t + \varphi_n)} \right] , \tag{7}$$

where $E_k = \frac{1}{2} m(v^2 + \Omega^2 d^2)$ is the kinetic energy of a particle, $\Omega = [\mu_0 \mu H_0 \cos\omega t/(md^2)]^{1/2}$ is the frequency of particle oscillation, d is the particle size†, B_i is the induction of the magnetic field at the particle location.

The expression $E_k + B_i\langle H_i \rangle - B_i\, i/(2\pi r)$ can be considered as a functional whose minimum corresponds to the stationary distribution along induction lines with radii

$$r_i = \frac{1}{2\pi}\frac{B_i\, i}{E_k + B_i\langle H_i \rangle} \,. \tag{8}$$

Here i is the direct current in the straight conductor, $\langle H_i \rangle$ is the mean magnetic field at the particle location.

To demonstrate thermodynamic stability conditions for fractal structures we used ferromagnetic dust with the Curie point $T_c = 70\,^\circ$C. This dust can be obtained by milling a ceramic magnet from an IP-10 detector. The temperature is maintained close to T_c by pouring hot water into the Petri dish. Repeating the above experiments we can show that above T_c structures do not form, whereas below T_c particles are attracted to each other and form structures.

5. Simulation of the appearance of complex molecules with encoded information

For these experiments we prepare 16 – 20 balls of porous foam plastic, 6 – 8 mm in diameter and incorporate small pieces of ceramic magnet inside the balls. In an alternating magnetic field these balls execute chaotic motions closely resembling the Brownian motion. If we now put the balls in

†The dimensions and the mass of particles of ferromagnetic dust used to produce tapes and rods are usually provided in the specification.

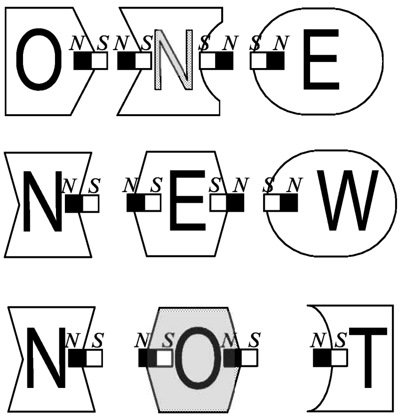

Fig. 2. The combinatory optimisation of 9 letters, which were originally randomly arranged in the opening in the coil.

the opening of the above coil and pass through it alternatingly ac and dc, we can obtain spatial structures similar to molecules of organic compounds. Thermodynamic conditions under which ordered structures arise in these experiments are simulated here by magnetic fields.

The appearance of encoded information can be illustrated as follows. We cut some figures with letters drawn on them out of a sheet of thin transparent celluloid or another transparent material (see Fig. 2). Then we stick pieces of ceramic magnet at the points where the letters are joined to each other. The poles of the magnets are directed so that the letters arrange themselves in sequences forming words with certain meanings (information carriers). For example, random joining of the letters 'O', 'N', 'E' can yield the combination 'ONE' possessing several meanings. For decoding, one of the letters 'N' out of several randomly arranged letters 'N' should be of a different colour (say, red). We can agree that the structure 'ONE' with red 'N' will mean 'first'.

The demonstrations described here are suitable for acquainting students with fractal structures and it is recommended that they be used in the laboratory when teaching physics.

References

1. Pietronero L, Tosatti E *Fractals in Physics* (New-York: North-Holland, 1986)
2. Feder E *Fractals* (New-York: Plenum Press, 1988)
3. Smirnov B M *Fizika Fraktal'nykh Klasterov* (Physics of Fractal Clusters) (Moscow: Nauka, 1991)
4. Smirnov B M *Usp. Fiz. Nauk* **163** (7) 1 (1993) [*Sov. Phys. Usp.* **36** 592 (1993)]
5. Olemskii A M, Flat A Ya *Usp. Fiz. Nauk* **163** (12) 1 (1993) [*Sov. Phys. Usp.* **36** 1087 (1993)]
6. Alkhimov V I *Usp. Fiz. Nauk* **164** 561 (1994) [*Sov. Phys. Usp.* **37** 527 (1994)]
7. Klimontovich Yu L *Usp. Fiz. Nauk* **164** 811 (1994) [*Sov. Phys. Usp.* **37** 737 (1994)]
8. Van Kampen N G *Stochastic Processes in Physics and Chemistry* (Amsterdam: North-Holland, 1981)
9. Gould H, Tobochnik J *Introduction to Computer Simulation. Applications to Physical Systems* (Reading, MA: Addison-Wesley, 1988)