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Spin light

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Contents

Abstract. The spin magnetic moment of an electron moving in a homogeneous magnetic field is a source of electromagnetic radiation. This radiation (spin light) shows up at high energies and it is available for measurements in contemporary electron accelerators. The spin light identification problem is considered when the spin radiation proceeds against the background of powerful synchrotron radiation, recoil effects, and other relativistic phenomena. A relativistic neutron is considered to be a source of pure spin radiation. The correspondence principle is formulated for spin radiation with and without the spin flip.

1. Experimental observation of the spin dependence of synchrotron radiation

As is known [1, 2], along with the electron-charge radiation W_e, the synchrotron radiation (SI) an electron emits when it moves in a magnetic field includes the spin magnetic moment radiation W_μ of the particle. In the classical formulation, the electron-charge radiation power is specified by the following expression [1]:

$$W_e = W^{\,\mathrm{cl}} = \frac{2}{3}\frac{e_0^2 c}{R^2}\gamma^4\,, \qquad \gamma = \frac{E}{m_0 c^2}\,, \tag{1}$$

where E is the electron energy, R is the orbital radius in the homogeneous magnetic field, m_0 is the rest mass, and $e = -e_0$ is the electron charge; the radiation power of an electron-spin magnetic dipole precessing in the outer magnetic field at a frequency $\omega_R = e_0 H_R/(m_0 c)$ may be evaluated by means of the familiar formula from the classical theory [3],

$$W_\mu = \frac{2}{3}\frac{1}{c^3}(\ddot{\boldsymbol{\mu}})^2 = \frac{2}{3}\frac{\mu_0^2}{c^3}\omega_R^4\zeta_\perp^2\,, \tag{2}$$

in the frame of reference in which the electron is at rest (the 'classical spin model').

In this expression, $\mu_0 = e_0\hbar/(2m_0 c)$ is the Bohr magneton, $\zeta_\perp$ is the component of the 'classical' spin vector $(\boldsymbol{\mu} = -\mu_0\boldsymbol{\zeta})$ perpendicular to the magnetic field.

On conversion to the laboratory frame of reference and bearing in mind that $H_R = \gamma H$ in this case, it follows from the preceding formula for W_μ that

$$W_\mu = \frac{2}{3}\frac{\mu_0^2}{c^3}\omega_R^4\gamma^8\zeta_\perp^2 = W^{\,\mathrm{cl}}\frac{1}{9}\xi^2\zeta_\perp^2 \tag{3}$$

in the classical interpretation. Here the quantum parameter ξ has the form

$$\xi = \frac{3}{2}\frac{\hbar}{m_0 c R}\gamma^2 = \frac{3}{2}\frac{H}{H^*}\gamma\,, \tag{4}$$

where $H^* = m_0^2 c^3/(e_0\hbar) = 4.4\times10^{13}$ G is the Schwinger magnetic field.

The magnetic dipole radiation power W_μ is proportional to $\hbar^2$ and depends on quantum transitions accompanied by the spin flip (see Refs [1, 2]). However, quantum corrections to the classical expression for the synchrotron radiation power (in the weak-field approximation $\xi \ll 1$) depend also on the linear expression in ξ [1, 2, 4]

$$W = W^{\,\mathrm{cl}}\left[1 - \left(\frac{55\sqrt{3}}{24} + \zeta\right)\xi + \ldots\right], \tag{5}$$

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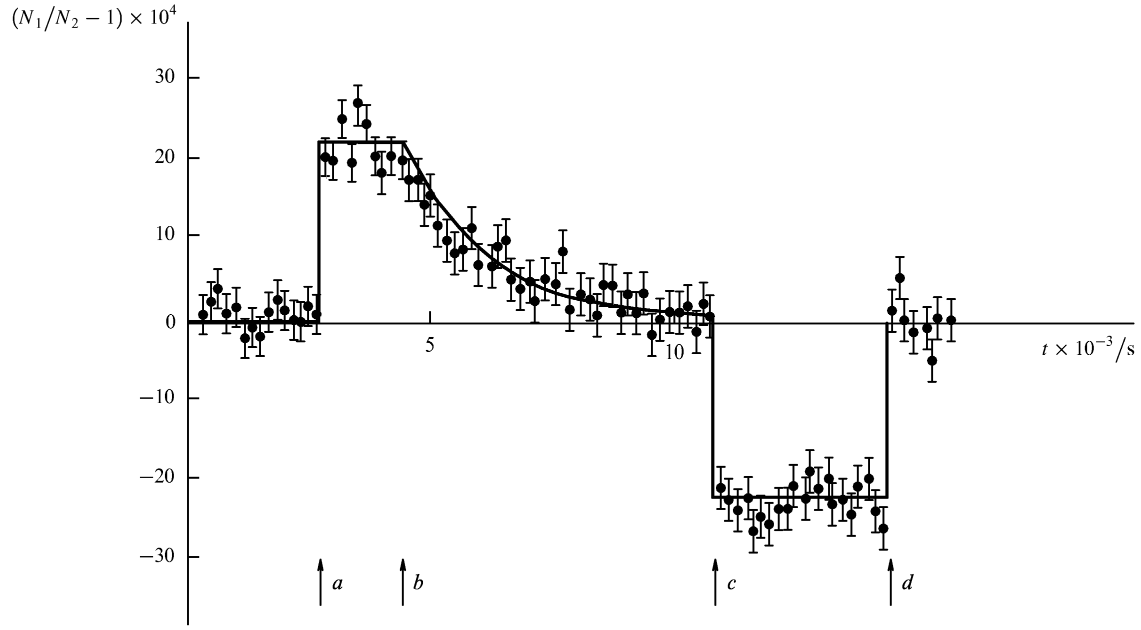

Figure 1. Results of observation of spin dependence of synchrotron radiation in the accumulator VEPP-4 (the experiment was performed in Novosibirsk).

where $\zeta = \pm 1$ is the spin projection onto the magnetic field vector (here only the linear term is retained in the expansion in terms of the invariant ξ). As shown by one of the authors [5], this value corresponds to the complex radiation power, which may be described by the expression:

$$W_{e\mu} = -W^{\mathrm{cl}} \xi \zeta_{\parallel} \tag{6}$$

(see also Ref. [4]).

The cited classical interpretation of an essentially quantum phenomenon of spin magnetic moment radiation can be considered only as a qualitative one. It is significant that, although the complex radiation makes a small contribution to the overall synchrotron radiation power, it is available for measurements in experiments. Let us now proceed to the experiment.

In 1983 the first quantum spin-orientation-dependent correction to the synchrotron radiation power was experimentally identified at the Institute of Nuclear Physics, Siberian Branch of the Academy of Sciences (Novosibirsk) [6]. In this experiment, synchrotron radiation was for the first time observed to be dependent on the spin orientation of a free electron moving in a macroscopic magnetic field.

The procedure for experimental observation of spin dependence of synchrotron radiation power was proposed in 1977 [7], and the experiment† itself was described in detail in Ref. [8]. The source of synchrotron radiation was a compensated tripolar wiggler‡ mounted on the rectilinear space in an accumulator. The field was about 20 kG in the central part of the wiggler, and its direction could be changed without disturbing the beam in motion. A lead filter 4 to 6 mm thick was employed to extract the hard part of the spectrum, where the complex radiation dominates the synchrotron radiation. The mean photon energy recorded was 200 – 250 keV for the field of 16 to 18 kG in the wiggler while the synchrotron radiation energy was 35 keV in the accumulator. About 2×10^3 photons hit the detector per pass of the cluster at the current of 0.5 mA in the beam.

In measurements the intensities of two electron clusters rotating simultaneously in the beam, with equal currents but with different degrees of polarisation, were compared. A special selective depolariser was designed to attain this goal. It enables an experimenter to depolarise only one of the two clusters circulating in the accumulator. At first the beams were adjusted by the rate of counting of the photons $\Delta_0 = N_1/N_2 - 1$. The quantity Δ_0 was $\sim 10^{-3}$ when the currents were equal at a level of 10^{-5}. Then, as the self-polarisation process occurred in the beams, control measurements were taken for stability of $\Delta_0 = \mathrm{const}$ over a period of one hour. At t_a (Fig. 1) the depolariser was switched on so that one of the clusters was depolarised. In this case, an abrupt change was registered in the ratio of counting rates. The difference in counts remained unchanged up to t_b when the depolariser was switched off. Then the quantity $\Delta = \Delta(t)$ smoothly regained its equilibrium value subject to the Sokolov – Ternov effect [9]. The degree of equilibrium polarisation could be determined by the time for which the polarisation came to the equilibrium state over the period $t_b - t_c$. In experiments, this value was measured to be 71%, i.e. it corresponded to the theoretically predicted value for such an accumulator. At t_c another cluster was depolarised. As a result, the quantity Δ changed its sign and remained constant up to t_d when both beams were depolarised.

In addition, measurements were carried out when the direction of the magnetic field in the wiggler was changed after each measurement. In this case, the quantity Δ changed its sign every time.

Note that the problem of electron spin observability has a long and intricate history. As Bohr showed, a direct

†The authors are very grateful to A E Bondarenko and V N Zhilich, scientists from the Novosibirsk Institute of Nuclear Physics, for the exhaustive information on the experiment.

‡A wiggler is an analogue of the magnetic undulator with a high magnetic field [4].

measurement of free electron spin is impossible in experiments like those of Stern and Gerlach, as the spin observability condition comes into conflict with the uncertainty relation and the wave nature of an electron when the Lorentz force comes into play. A comprehensive analysis of this problem may be found in the paper written by Pauli in 1930 [10].

Since the direct method of measuring the spin (by Stern and Gerlach) proved to be inconsistent, other methods were proposed to produce and observe free polarised electrons. Some of them were successfully realised [11, 12]. However, it was still unclear how the spin could be observed in a macroscopic magnetic field [13]. It is possible that the Bohr–Pauli 'taboo' syndrome is responsible for the fact that the cited experiment and its fundamental significance were hardly noticed.

2. Synchrotron radiation and spin light

In the magnetic field of a synchrotron, the electrons are naturally polarised because of the Sokolov–Ternov radiative self-polarisation effect [9]. The well-known explanation is as follows. The probabilities of quantum transitions are different for the two possible orientations of the spin in the magnetic field. As a result, electrons tend to occupy a more stable state when the spin is oriented opposite to the magnetic field. According to Ref. [9] (see also Refs [14, 15]), the degree of polarisation of an electron beam is 92.4% in ideal conditions. The spin relaxation time depends on the electron energy and magnetic field intensity. In particular, the relaxation time is about 1 hour for the typical parameters $E = 1$ GeV and $H = 10^4$ G in electron accelerators. Therefore, the effect is best observed in electron storage rings (for details, see Section 6 and also Ref. [16]).

The magnetic dipole radiation, which accompanies spin transitions, is negligible against the background of the electron-charge synchrotron radiation. In its pure form, the spin magnetic moment radiation (spin light) contributes to the synchrotron radiation power as a small correction proportional to $\hbar^2$ [17]. In the experiment performed in Novosibirsk the radiation observed was proportional to the first power of $\hbar$. It is shown below that this radiation is due to the interference of charge radiation and the spin magnetic moment radiation of the electron, and this gives rise to the complex radiation proportional to $\hbar$.

The progress made in the quantum theory of synchrotron radiation [1] based on the exact solutions to the relativistic Dirac equation and the rigorous methods of quantum electronics made it possible to describe the spectral-angular distribution of synchrotron radiation power with consideration for all quantum features, including spin evolution [1, 14, 18]. However, the physical interpretation of the quantum corrections to the classical theory remained unclear since there was no simple way of transition to the classical theory. It was established only relatively recently that synchrotron radiation has a complex structure depending on a number of fundamental phenomena: the charge radiation itself (conventional synchrotron radiation); recoil effects accompanying the radiation; interference of fields of charge radiation; the spin magnetic moment radiation (complex radiation); the spin moment radiation itself; and the radiation associated with the anomalous magnetic moment of the electron [2, 5].

The interpretation of spin corrections to the synchrotron radiation power is not simple since spin corrections have an equal competitor, namely, the recoil effects accompanying the radiation. They also make contributions to the synchrotron radiation power, proportional to $\hbar$ as well as to $\hbar^2$, and, moreover, the recoil effects are essentially different for an electron and a charged spinless particle (boson) in the second order in $\hbar$.

In this situation, the relativistic quasiclassical theory† proposed by Schwinger [19] and developed by Baier [20–22] proved to be a handy instrument for research. The basic conclusions of the strict quantum theory [1, 14] were replicated in the quasiclassical approach for high-energy particles following macroscopic paths. In particular, the quantum corrections to the classical formula for the synchrotron radiation power, radiative polarisation effects for electrons and positrons in storage rings, and quantum widening of orbits of particles were established anew. Certainly, this result was predictable, but the quasiclassical approach also proved to be appropriate for the physical interpretation of quantum effects [17, 25–27, 5].

3. Relativistic semiclassical radiation theory

The computational technique and scope of the relativistic semiclassical theory are presented in the excellent work of Jackson [24]. In this theory all features of motion of a relativistic electron are considered to be classical along the trajectory and the quantum processes associated with radiation are accounted for by the replacement of the electron velocity with the transition matrix element:

$$\beta \rightarrow \langle \mathrm{f}|\hat{\alpha}|\mathrm{i}\rangle \,, \tag{7}$$

where $\beta = \boldsymbol{u}/c$ is the dimensionless velocity of an electron, $\hat{\alpha} = \hat{\alpha}(t)$ are the well-known Dirac matrices in the Heisenberg representation, $|\mathrm{i}\rangle$ is the ket-vector of the initial state, and $\langle \mathrm{f}|$ is the bra-vector of the final state. The noninertiality of motion of an electron is not essential for quantum transitions in the range of temperatures typical of synchrotron radiation, and the matrix element in Eqn (7) is calculated for the wave functions of a free electron (quantum kinematics) with the ket-vector:

$$|\mathrm{i}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\dfrac{\gamma+1}{2}} \\ \sqrt{\dfrac{\gamma}{\gamma+1}}\,(\hat{\sigma}\cdot\beta) \end{pmatrix} |\zeta\rangle \,, \tag{8}$$

where $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic Lorentz factor, $\hat{\sigma}$ is the Pauli matrix, and the spin states in projection onto an arbitrary direction $\zeta = (\sin\nu\cos\lambda,\ \sin\nu\sin\lambda,\ \cos\nu)$ are given by the spin ket-vector

$$|\zeta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \zeta\sqrt{1+\zeta\cos\nu} \\ \sqrt{1-\zeta\cos\nu}\,\exp(\mathrm{i}\lambda) \end{pmatrix}, \quad \zeta = \pm 1 \,, \tag{9}$$

where λ and ν are the direction angles of the spin projection. The normalisation is chosen so that

$$\langle \mathrm{i}|\hat{\alpha}|\mathrm{i}\rangle = \beta \,, \quad \langle \zeta|\hat{\sigma}|\zeta\rangle = \zeta \,. \tag{10}$$

†The full mathematical justification of the quasiclassical approach is given in Ref. [23]. Here we use a simplified semiclassical version of this theory which we used earlier in Refs [17, 25–27].

Inhomogeneities in the leading magnetic field are assumed to have no effect on the electron polarisation process [28, 29].

The Fourier transform of the radiation field is specified in the same way as in classical electrodynamics:

$$\boldsymbol{E}^s_{\widetilde{\omega}} = -\mathrm{i}\,\frac{e\widetilde{\omega}}{Rc}\int\Big(\langle \mathrm{f}|\hat{\alpha}|\mathrm{i}\rangle \boldsymbol{n}^s\Big)\exp\left\{\mathrm{i}\widetilde{\omega}\left[t-\frac{\boldsymbol{n}\cdot\boldsymbol{r}}{c}\right]\right\}\mathrm{d}t\,, \tag{11}$$

where $\boldsymbol{n}^s$ are the basis vectors of the linear radiation polarisation ($s=\sigma,\pi$), $\boldsymbol{n}$ is the unit vector from the charge towards the point of observation, $\widetilde{\omega}$ is the radiation frequency, and all the other symbols are conventional notation.

In the ultrarelativistic approximation, all variables are expanded into series in terms of small parameters γ^{-1}, ct/ρ, and ψ, where ρ is the orbital radius of an electron in the homogeneous magnetic field, and ψ is the angle between the vector $\boldsymbol{n}$ and the orbital plane (the projection of $\boldsymbol{n}$ onto the orbital plane is parallel to $\boldsymbol{\beta}$). We have

$$\boldsymbol{\beta} = \left(1-\frac{1}{2\gamma^2}-\frac{\psi^2}{2}-\frac{1}{2}\frac{c^2t^2}{\rho^2}\right)\boldsymbol{n}+\frac{ct}{\rho}\,\boldsymbol{n}^{\sigma}-\psi\boldsymbol{n}^{\pi}\,, \tag{12}$$

$$\boldsymbol{n} = \left(1-\frac{\psi^2}{2},\,0,\,\psi\right),\qquad \boldsymbol{n}^{\sigma}=(0,\,1,\,0)\,,$$

$$\boldsymbol{n}^{\pi} = \left(-\psi,\,0,\,1-\frac{\psi^2}{2}\right).$$

correct to the second order of smallness ($e<0$, the magnetic field is parallel to the z axis).

The matrix element of Eqn (7) should be calculated with the laws of conservation of energy and momentum:

$$m_0c^2\gamma = m_0c^2\gamma' + \hbar\widetilde{\omega}\,,\qquad \gamma\boldsymbol{\beta}=\gamma'\boldsymbol{\beta}'+\hbar\widetilde{\omega}\boldsymbol{n}\,. \tag{13}$$

The spectral–angular distribution of the radiation is specified by the usual expression [1]:

$$\frac{\mathrm{d}^2W^s}{\mathrm{d}\widetilde{\omega}\,\mathrm{d}\Omega} = \frac{c^2R^2}{8\pi^3\rho}\left|\boldsymbol{E}^s_{\widetilde{\omega}}\right|^2. \tag{14}$$

4. Recoil effects and complex radiation

We introduce the recoil parameter $\varepsilon=\hbar\widetilde{\omega}/(m_0c^2\gamma')$. Then instead of Eqn (13) we have

$$\gamma'=\frac{\gamma}{1+\varepsilon}\,,\qquad \boldsymbol{\beta}'=(1+\varepsilon)\boldsymbol{\beta}-\varepsilon\boldsymbol{n}\,. \tag{15}$$

In the linear approximation in $\hbar$, the matrix element of Eqn (7) is calculated to be:

$$\langle \mathrm{f}|\hat{\alpha}|\mathrm{i}\rangle = \left(1+\frac{\varepsilon}{2}\right)\boldsymbol{\beta}\cdot\langle\zeta'|\zeta\rangle-\mathrm{i}\,\frac{\varepsilon}{2}\left[\langle\zeta'|\hat{\sigma}|\zeta\rangle\cdot\left(\boldsymbol{n}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}\right)\right]. \tag{16}$$

In this formula, the first term with $\boldsymbol{\beta}$ describes the radiation of a charge and the recoil effects associated with ε, whereas the second term with the Pauli matrices is responsible for the spin magnetic moment radiation [the Bohr magneton $\mu_0=e_0\hbar/(2m_0c)$].

By substituting the matrix element [Eqn (16)] into Eqn (11) we find

$$\widetilde{\boldsymbol{E}}^s_{\widetilde{\omega}} = \frac{e\widetilde{\omega}\rho}{Rc\gamma^2}\left\{\left(1+\frac{\varepsilon}{2}\right)\langle\zeta'|\zeta\rangle U_0^s+\frac{\varepsilon}{2}\langle\zeta'|\hat{\sigma}_z|\zeta\rangle U_1^s\right.$$
$$\left.+\,\varepsilon\Big[\langle\zeta'|\hat{\sigma}_+|\zeta\rangle U_2^s+\langle\zeta'|\hat{\sigma}_-|\zeta\rangle U_3^s\Big]\right\}, \tag{17}$$

where

$$U_0^{\sigma}=\frac{2\mathrm{i}}{\sqrt{3}}(1+\chi^2)K_{2/3}(x)\,,\quad U_0^{\pi}=-\frac{2}{3}\chi(1+\chi^2)^{1/2}K_{1/3}(x)\,,$$

$$U_1^{\sigma}=-\frac{2\mathrm{i}}{\sqrt{3}}(1+\chi^2)^{1/2}K_{1/3}(x)\,,\qquad U_1^{\pi}=0\,,$$

$$U_{2,3}^{\sigma}=\frac{\mathrm{i}}{\sqrt{3}}\chi(1+\chi^2)^{1/2}K_{1/3}(x)\,,$$

$$U_{2,3}^{\pi}=\pm\frac{1}{\sqrt{3}}(1+\chi^2)^{1/2}\left[K_{1/3}(x)\mp(1+\chi^2)^{1/2}K_{2/3}(x)\right], \tag{18}$$

and, in addition,

$$x=\frac{1}{3}\frac{\widetilde{\omega}}{\omega}\eta^3\,,\quad \eta=\frac{1}{\gamma}(1+\chi^2)^{1/2}\,,\quad \omega=\frac{e_0H}{m_0c\gamma}\,,\quad \chi=\gamma\psi\,.$$

The McDonald functions $K_{1/3}(x)$ and $K_{2/3}(x)$ appear as a result of expansion of the pre-exponential factor in Eqn (11) with retention of the linear terms in γ^{-1}, ct/ρ, and ψ. Generally, the time dependences of the $\hat{\sigma}(t)$ operators should be considered. These operators are solutions to the Heisenberg spin precession equation for a homogeneous magnetic field ($\mu<0$):

$$\hat{\sigma}_{\pm}(t)=\sigma_{\pm}\exp(\pm\mathrm{i}\omega t)\,,\qquad \hat{\sigma}_z(t)=\sigma_z\,, \tag{19}$$

where $\sigma_{\pm}=(\sigma_x\pm\mathrm{i}\sigma_y)/2$. However, it suffices to consider the principal term in the expansion $\exp(\pm\mathrm{i}\omega t)\approx 1\pm\mathrm{i}ct/\rho$ because all the other terms in $\langle \mathrm{f}|\boldsymbol{\beta}|\mathrm{i}\rangle\cdot\boldsymbol{n}^s$ are of the first order of smallness. This correlation will be considered in more detail for the 'true' magnetic moment of an electron (see Section 7).

If we then use the standard procedure of classical electrodynamics one obtains, according to Eqns (14) and (17), the spectral–angular radiation distribution in the form:

$$\frac{\mathrm{d}^2W^s}{\mathrm{d}\widetilde{\omega}\,\mathrm{d}\Omega}=\frac{e^2\widetilde{\omega}^2}{8\pi^3\gamma^4}\left\{\left|\left(1+\frac{\varepsilon}{2}\right)\langle\zeta'|\zeta\rangle U_0^s+\frac{\varepsilon}{2}\langle\zeta'|\sigma_z|\zeta\rangle U_1^s\right|^2\right.$$
$$\left.+\,\varepsilon^2\left|\langle\zeta'|\sigma_+|\zeta\rangle U_2^s+\langle\zeta'|\sigma_-|\zeta\rangle U_3^s\right|^2\right\}. \tag{20}$$

Here we account for the fact that the terms like the products $U_0^sU_{2,3}^{*s}$ and $U_1^sU_{2,3}^{*s}$ are odd functions of χ and vanish upon integration with respect to angles.

If we restrict ourselves to the linear approximation in ε, then only the charge radiation, spinless recoil effects, and complex radiation of the 'charge + electron' spin magnetic moment' system remain in expression (20):

$$\frac{\mathrm{d}^2W^s}{\mathrm{d}\widetilde{\omega}\,\mathrm{d}\Omega}=\frac{e^2\widetilde{\omega}^2}{8\pi^3\gamma^4}\,\frac{1+\zeta\zeta'}{2}\left[(1+\varepsilon)|U_0^s|^2+\zeta\cos\nu\,\varepsilon U_0^sU_1^{*s}\right]. \tag{21}$$

To deduce the last formula we used the relationships

$$\langle\zeta'|\zeta\rangle=\frac{1+\zeta\zeta'}{2}=\delta_{\zeta,\zeta'}\,, \tag{22}$$

$$\langle\zeta'|\sigma_z|\zeta\rangle=\frac{1}{2}\left[(1+\zeta\zeta')\zeta\cos\nu-(1-\zeta\zeta')\sin\nu\right].$$

Here ν is the spin orientation angle given by Eqn (9).

If we then integrate Eqn (21) with respect to the spectrum and angles in the usual way, we obtain the overall polarised radiation power:

$$W^s=W_{\mathrm{SR}}\,\frac{1+\zeta\zeta'}{2}\,f^s(\xi)\,, \tag{23}$$

where

$$W_{\rm SR} = \frac{2}{3}\frac{e^2 c\gamma^4}{\rho^2}$$

is the synchrotron radiation power and the function $f^s(\xi)$ specifies the polarisation of the linear radiation:

$$f^\sigma(\xi) = \frac{7}{8} - \frac{25\sqrt{3}}{12}\,\xi - \xi\zeta\cos\nu\,,$$

$$f^\pi(\xi) = \frac{1}{8} - \frac{5\sqrt{3}}{24}\,\xi\,,$$

where $\xi = 3\hbar\gamma^2/(2m_0 c\rho)$ is the well-known dimensionless invariant in the quantum theory of synchrotron radiation (sometimes another parameter, $\chi = 2\xi/3$, is used instead of ξ, see Ref. [19]). The presence of the coefficient $(1+\zeta\zeta')/2$ in Eqn (21) and others indicates that this radiation is not related to the spin flip since Eqn (21) equals zero for $\zeta' = -\zeta$.

Summation over $s = \sigma,\ \pi$ yields

$$W = W_{\rm SR}\,\frac{1+\zeta\zeta'}{2}\,f(\xi)\,, \tag{24}$$

where

$$f(\xi) = 1 + \frac{55\sqrt{3}}{24}\,\xi - \xi\zeta\cos\nu\,.$$

Eqns (23) and (24) are obtained on the assumption that $\xi \ll 1$. In the real situation, this condition is satisfied for contemporary accelerators. There is no principal difficulty in extending the result to $\xi \gg 1$ (see, for example, Ref. [16]).

The quantum corrections in Eqn (24) were calculated earlier by methods adopted in quantum electrodynamics. The first quantum correction $-(55\sqrt{3}/24)\xi$ was determined in Ref. [30] (later this result was replicated in Ref. [19]). Another quantum correction $-\zeta\xi$, which depends on spin orientation, was obtained in Ref. [18] (see also Ref. [31]). However, the physical meaning of these corrections remained unexplained for a long time. Today we know that the first describes the recoil effects and the second is the complex radiation [17], the interpretation of which is not, however, so simple as it may seem at first (see Section 5).

Let us consider the features of complex radiation in more detail. It follows from the common formulas for the radiation power [Eqns (23) and (24)] that this radiation is due to quantum transitions without the spin flip. The complex radiation is completely linearly polarised in the orbital plane (δ component). Electrons polarised along the magnetic field yield the largest contribution to the radiation when $\nu = 0$. The complex radiation power increases smoothly with the degree of polarisation of the electron beam [6]. The complex radiation disappears for nonpolarised electrons when $\nu = \pi/2$; it is also absent for electrons polarised along the velocity or orbital radius [32].

The angular complex radiation distribution is, in contrast to synchrotron radiation, more pointed (Fig. 2). The peak radiation frequency $\omega_{e\mu}^{\max}$ is perceptibly shifted towards high frequencies in comparison with $\omega_{e}^{\max}$ (Fig. 3).

Unfortunately, only a qualitative spin dependence of synchrotron radiation was recorded in the experiment performed in Novosibirsk. It is desirable to perform a more detailed study of the cited features of the complex radiation.

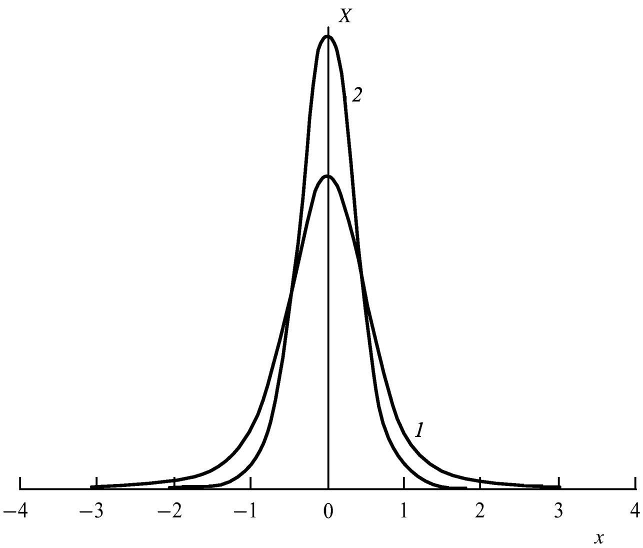

Figure 2. Angular distribution: synchrotron radiation $X_e^\sigma(x) = 3/4(1+\chi^2)^{5/2}$ (*1*) and complex radiation $X_{e\mu}^\sigma(x) = 35/32(1+\chi^2)^{9/2}$ (*2*).

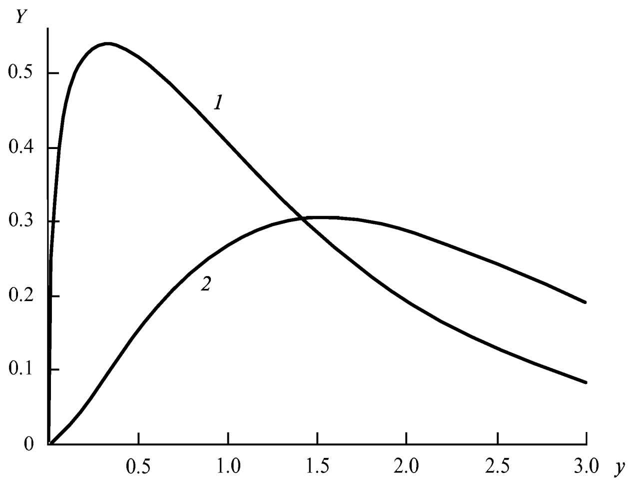

Figure 3. Spectral distribution: synchrotron radiation $Y_e^\sigma = (9\sqrt{3}/14\pi)y\left[\int_y^\infty K_{5/3}(x)\,dx + K_{2/3}(y)\right]$ (*1*) and complex radiation $Y_{e\mu}^\sigma = (9\sqrt{3}/8\pi)y^2 K_{1/3}(y)$ (*2*).

5. Interpretation of complex radiation

The recoil effects may be demonstrated clearly and convincingly by the example of the radiation of a spinless particle (boson). In this case, the radiation associated with the spin is absent and quantum corrections proportional to $\hbar$ and $\hbar^2$ are reduced to the recoil effects.

The matrix element Eqn (7) is calculated for a boson with the same semiclassical method for spin functions with the ket-vector

$$|{\rm i}\rangle = \frac{1}{\sqrt{\gamma}}\,.$$

The boson norming is chosen so that (see Ref. [1])

$$\langle {\rm i}|\gamma\beta|{\rm i}\rangle = \beta\,.$$

The formal difference from the first condition given by Eqn (10) is that $\hat\alpha$ is changed for $\gamma\beta$. It is easy to see that

$$\langle {\rm f}|\gamma\beta|{\rm i}\rangle = \sqrt{1+\varepsilon}\,\beta\,. \tag{25}$$

In the linear approximation in ε or in $\hbar$, this expression coincides with the spinless term in Eqn (16), which is not zero for $\zeta = \zeta'$ according to Eqn (22). The recoil effects for an electron and a boson are different only for the terms proportional to $\hbar^2$. This fact is also known in the exact quantum theory [32] (see also Section 6).

The fact that the term with the Pauli matrices in Eqn (16) is related to the spin magnetic moment radiation may be shown by considering the features of the spin precession for a Dirac particle in electromagnetic fields. We shall present the interaction between the electron's spin magnetic moment and the field in the form:

$$H_\mu^{\rm int} = -\frac{\mu_0}{2\gamma}\,\widetilde{H}^{\rm eff}_{\alpha\beta}\Pi^{\alpha\beta} = -\frac{\mu_0}{\gamma}\left(\zeta\cdot\widetilde{\boldsymbol{H}}_0^{\rm eff}\right) , \qquad (26)$$

where $\widetilde{H}^{\rm eff}_{\alpha\beta}$ is some effective radiation field, $\Pi^{\alpha\beta}$ is the dimensionless spin tensor ($\mu_0\Pi^{\alpha\beta} = \mu^{\alpha\beta}$ is Frenkel' tensor [33]), ζ and $\widetilde{\boldsymbol{H}}_0^{\rm eff}$ are, respectively, the unit spin vector and the effective magnetic field in the system of an electron at rest. The field $\widetilde{\boldsymbol{H}}_0^{\rm eff}$ may be determined from the Frenkel'–BMT (Bargmann–Michel–Telegdi) spin precession equation [34, 35]. Here we rewrite this equation in the form

$$\frac{{\rm d}\zeta}{{\rm d}t} = \frac{e}{m_0 c\gamma}\left(\zeta\times\boldsymbol{H}_0^{\rm eff}\right) , \qquad (27)$$

where

$$\begin{aligned}\boldsymbol{H}_0^{\rm eff} &= \gamma\left\{\left(a+\frac{1}{\gamma}\right)\boldsymbol{H} - \left(a+\frac{1}{\gamma+1}\right)(\beta\times\boldsymbol{E}) \right.\\ &\qquad\left. - a\,\frac{\gamma}{\gamma+1}\,\beta(\beta\cdot\boldsymbol{H})\right\}\\ &= \frac{g}{2}\,\boldsymbol{H}_0 + \frac{\gamma}{\gamma+1}(\beta\times\boldsymbol{E}_0) ,\\ \boldsymbol{H}_0 &= \gamma\left\{\boldsymbol{H} - (\beta\times\boldsymbol{E}) - \frac{\gamma}{\gamma+1}\,\beta(\beta\cdot\boldsymbol{H})\right\} ,\\ \boldsymbol{E}_0 &= \gamma\left\{\boldsymbol{E} + (\beta\times\boldsymbol{H}) - \frac{\gamma}{\gamma+1}\,\beta(\beta\cdot\boldsymbol{E})\right\} \end{aligned} \qquad (28)$$

are fields in the rest system of an electron, and the quantity $a = (g-2)/2$ characterises the anomalous magnetic moment of the electron $\mu_a = \mu_0 a$. The presence of the Lorentz factor in the denominator of Eqn (27) may be explained by the way in which the variation of ζ with time t is determined in the laboratory frame of reference.

If we neglect the anomalous magnetic moment of the electron in Eqn (27) and assume that $g = 2$, then

$$\boldsymbol{H}_0^{\rm eff} = \boldsymbol{H} - \frac{\gamma}{\gamma+1}(\beta\cdot\boldsymbol{E}) . \qquad (29)$$

In the nonrelativistic approximation,

$$\boldsymbol{H}_0^{\rm eff} \approx \boldsymbol{H} - \frac{1}{2}(\beta\times\boldsymbol{E}) , \qquad (30)$$

whereas

$$\boldsymbol{H}_0 \approx \boldsymbol{H} - (\beta\times\boldsymbol{E}) . \qquad (31)$$

In contrast to $\boldsymbol{H}_0$, the field $\boldsymbol{H}_0^{\rm eff}$ includes the coefficient 1/2 known as 'Thomas's half'. The reason that $\boldsymbol{H}_0^{\rm eff}$ is different from $\boldsymbol{H}_0$ is that the spin precession is the sum of the Larmor and Thomas precessions (see also Ref. [1]). Correspondingly, the field $\boldsymbol{H}_0^{\rm eff}$ can be presented in the form

$$\boldsymbol{H}_0^{\rm eff} = \widetilde{\boldsymbol{H}}_0^{\rm L} + \widetilde{\boldsymbol{H}}_0^{\rm Th} , \qquad (32)$$

where

$$\widetilde{\boldsymbol{H}}_0^{\rm L} = \frac{g}{2}\,\widetilde{\boldsymbol{H}}_0 , \quad \widetilde{\boldsymbol{H}}_0^{\rm Th} = \frac{\gamma}{\gamma+1}(\beta\times\widetilde{\boldsymbol{E}}_0) . \qquad (33)$$

Only in the ultimate nonrelativistic case when the Thomas precession can be neglected does Eqn (27) coincide with the equation for the magnetic moment precession in classical electrodynamics:

$$\frac{{\rm d}\zeta}{{\rm d}t} = \frac{ge}{2m_0 c}(\zeta\times\boldsymbol{H}_0) . \qquad (34)$$

It could be said that in this case the spin magnetic moment of an electron exhibits the properties of the 'true' magnetic moment (see the papers by Tamm [36] and Ginzburg [37]).

The 'true' character of the magnetic moment of an electron manifests itself also in the hypothetical case† $g \gg 1$ when the entire magnetic moment is anomalous and Eqn (27) takes the form

$$\frac{{\rm d}\zeta}{{\rm d}t} = \frac{ge}{2m_0 c\gamma}(\zeta\times\boldsymbol{H}_0) . \qquad (35)$$

In fact Eqn (35) describes the spin precession for a neutron, the magnetic moment of which is entirely anomalous. This equation may also be used in the case of the uniform rectilinear motion of an electron in Wien-filter-type fields when the Thomas precession disappears [38–40]. It is very interesting that the situation with the 'true' magnetic moment is the same in radiation theory (see Section 7).

However, it is time to return to the real electron. If $g = 2$, the effective interaction between the spin magnetic moment of an electron and the radiation field is specified, according to Eqns (26) and (29), by the expression

$$H_\mu^{\rm int} = -\frac{\mu_0}{\gamma}\left(\zeta\left\{\widetilde{\boldsymbol{H}} - \frac{\gamma}{\gamma+1}\left[\beta\times\widetilde{\boldsymbol{E}}\right]\right\}\right) , \qquad (36)$$

and the overall interaction of the charge and magnetic moment of the electron with the radiation field has the form

$$H^{\rm int} = H_e^{\rm int} + H_\mu^{\rm int} = -e(\beta\widetilde{\boldsymbol{A}}) - \frac{\mu_0}{\gamma}\left(\zeta\left\{\widetilde{\boldsymbol{H}} - \frac{\gamma}{\gamma+1}\left[\beta\times\widetilde{\boldsymbol{E}}\right]\right\}\right). \qquad (37)$$

With regard for

$$\widetilde{\boldsymbol{E}} = -{\rm i}\kappa\widetilde{\boldsymbol{A}} , \quad \widetilde{\boldsymbol{H}} = \boldsymbol{n}\times\widetilde{\boldsymbol{E}} , \qquad (38)$$

where $\kappa = \widetilde{\omega}/c$ is the wave number, Eqn (37) can be rewritten in the form:

$$H^{\rm int} = -e\left(\beta^{\rm eff}\cdot\widetilde{\boldsymbol{A}}\right) , \qquad (39)$$

where

$$\beta^{\rm eff} = \beta - {\rm i}\,\frac{\hbar\kappa}{2m_0 c\gamma}\left[\zeta\left(\boldsymbol{n} - \frac{\gamma}{\gamma+1}\,\beta\right)\right] . \qquad (40)$$

Clearly, Eqn (40) corresponds to the matrix element [Eqn (16)] in the relativistic semiclassical theory of radiation but without recoil effects which we have not considered in the above purely classical analysis. It is remarkable that

†We do not discuss here the issues of the complicated functional dependence of the anomalous magnetic moment of an electron on the outer magnetic field intensity and the energy level number [39, 1] since we assume that $\mu_a = {\rm const}$ and the Dirac–Pauli spin equation, from which the Frenkel'–BMT equation follows [2], remains valid for $a \gg 1$.

$\langle \zeta | \sigma | \zeta \rangle = \zeta$ for radiation without a spin flip and expressions (40) and (16) coincide one-to-one when the recoil effect is not taken into account; thus the origin of the complex radiation becomes clear.

6. Structure of quantum corrections to the synchrotron radiation power

Let us now study the synchrotron radiation with regard for the quantum corrections of the second order in $\hbar$. The recoil effects and complex radiation are also present here. However, there is a new specific type of radiation due to the natural magnetic moment of an electron (spin light). The $\hbar$-quadratic radiation is accompanied by transitions both with and without a spin flip and it has a very complex nature. In this approximation the radiation from the anomalous magnetic moment of an electron becomes perceptible and it is, as we shall see, principally different from the spin magnetic moment radiation (Bohr magneton).

In the quantum theory of radiation which is based on the Dirac–Pauli equation, the anomalous magnetic moment is taken into account through the following change [41]:

$$\hat{\alpha} \to \hat{\alpha}^{\text{eff}} = \hat{\alpha} - \mathrm{i}\, \frac{\mu_a \kappa}{e} \left(\rho_3 [\hat{\sigma} \times \boldsymbol{n}] + \rho_2 \hat{\sigma} \right) . \tag{41}$$

Conversion to the semiclassical theory of radiation yields

$$\beta \to \langle \mathrm{f} | \hat{\alpha}^{\text{eff}} | \mathrm{i} \rangle \tag{42}$$

instead of Eqn (7).

One must bear in mind when calculating the matrix element that $\mu_a \kappa / e = \varepsilon \gamma' a/2 \approx \varepsilon$ in the range of velocities typical of synchrotron radiation. Therefore, the matrix element may be calculated without taking into account the recoil effects ($\beta' = \beta$) in the linear approximation in ε. Otherwise we will follow the method described in Section 4 and obtain

$$\langle \mathrm{f} | \hat{\alpha}^{\text{eff}} | \mathrm{i} \rangle = \left(1 + \frac{\varepsilon}{2} \right) \beta \langle \zeta' | \zeta \rangle - \mathrm{i}\, \frac{\varepsilon}{2} \left[\langle \zeta' | \hat{\sigma} | \zeta \rangle \cdot \left(\boldsymbol{n} - \frac{\gamma}{\gamma + 1} \beta \right) \right]$$
$$- \mathrm{i}\, \frac{\varepsilon}{2}\, \gamma a \left\{ \left[\langle \zeta' | \hat{\sigma} | \zeta \rangle \cdot (\boldsymbol{n} - \beta) \right] - \frac{\gamma}{\gamma + 1} \left(\langle \zeta' | \hat{\sigma} | \zeta \rangle \beta \right) \cdot (\beta \times \boldsymbol{n}) \right\} . \tag{43}$$

In comparison with Eqn (16) there are additional terms with the anomalous part of the magnetic moment of the electron. In the classical interpretation (see Section 5), the energy of interaction of the magnetic moment and field is now presented in the form:

$$H_\mu^{\text{int}} = -\mu_0 \left[\left\{ \zeta \cdot \left[\widetilde{\boldsymbol{H}} - \frac{\gamma}{\gamma + 1} \left(\beta \times \widetilde{\boldsymbol{E}} \right) \right] \right\} \right.$$
$$\left. - \gamma a \left(\left\{ \zeta \cdot \left[\widetilde{\boldsymbol{H}} - \left(\beta \times \widetilde{\boldsymbol{E}} \right) \right] \right\} - \frac{\gamma}{\gamma + 1} (\zeta \cdot \beta) (\beta \cdot \widetilde{\boldsymbol{H}}) \right) \right] . \tag{44}$$

As in Section 5, it then follows that

$$\beta^{\text{eff}} = \left(1 + \frac{\varepsilon}{2} \right) \beta - \mathrm{i}\, \frac{\varepsilon}{2} \left(\left[\zeta \times \left(\boldsymbol{n} - \frac{\gamma}{\gamma + 1}\, \beta \right) \right] \right.$$
$$\left. + \gamma a \left\{ \left[\zeta \times (\boldsymbol{n} - \beta) \right] - \frac{\gamma}{\gamma + 1} (\zeta \cdot \beta) (\beta \times \boldsymbol{n}) \right\} \right) . \tag{45}$$

This formula is an extension of Eqn (40) and an analogue of Eqn (16).

If the terms in the Fourier transform of the field are grouped by the same criterion as in Eqn (17), then spectral-angular radiation distribution does not formally differ from that given by Eqn (20). However, the functions U_i^s ($i = 1, 2, 3$) have more complex forms:

$$U_1^\sigma = -\frac{2\mathrm{i}}{\sqrt{3}} (1 + \chi^2)^{1/2} (1 - a\chi^2) K_{1/3}(x) ,$$
$$U_1^\pi = \frac{2}{\sqrt{3}}\, \chi (1 + \chi^2) a K_{2/3}(x) ,$$
$$U_{2,3}^\sigma = \frac{\mathrm{i}}{\sqrt{3}}\, \chi (1 + \chi^2)^{1/2}$$
$$\times \left\{ (1 + a) K_{1/3}(x) \mp 2a (1 + \chi^2)^{1/2} K_{2/3}(x) \right\} ,$$
$$U_{2,3}^\pi = \pm \frac{1}{\sqrt{3}} \left\{ \left[1 + 2a(1 + \chi^2) \right] K_{1/3}(x) \right.$$
$$\left. \mp (1 + 2a)(1 + \chi^2)^{1/2} K_{2/3}(x) \right\} . \tag{46}$$

The functions U_0^s remain unchanged. In addition to Eqn (22) there is the relationship:

$$\langle \zeta' | \sigma_\pm | \zeta \rangle = \frac{1}{2} \begin{pmatrix} \zeta' \\ \zeta \end{pmatrix}$$
$$\times \left[1 \mp (\zeta - \zeta') \cos\nu - \zeta\zeta' \cos^2\nu \right]^{1/2} \exp(\pm \mathrm{i}\lambda) . \tag{47}$$

Further manipulations are performed in an ordinary fashion. Omitting insignificant details we present the final expressions for the linearly polarised radiation power in the linear approximation in a (in which the Dirac–Pauli equation is also true)

$$W^\sigma = W_{\text{SR}} \left\{ \frac{1 + \zeta\zeta'}{2} \left\langle \frac{7}{8} - \frac{25\sqrt{3}}{12}\, \xi + \frac{325}{18}\, \xi^2 \right. \right.$$
$$- \xi \cos\nu \left[\xi - \frac{245\sqrt{3}}{48}\, \xi^2 - \frac{1}{6}\, a \left(\xi - \frac{245\sqrt{3}}{72}\, \xi^2 \right) \right]$$
$$\left. + \cos^2\nu\, \frac{5 - a}{9}\, \xi^2 - \sin^2\nu \cos^2\lambda\, \frac{1 + 2a}{18}\, \xi^2 \right\rangle$$
$$+ \frac{1 - \zeta\zeta'}{2} \left\langle \frac{5 - a}{9} \sin^2\nu + (\cos^2\nu + \sin^2\nu \sin^2\lambda) \frac{1 + 2a}{18} \right.$$
$$\left. \left. + \zeta \cos\nu\, \frac{35\sqrt{3}}{216}\, a \right\rangle \xi^2 \right\} , \tag{48}$$

$$W^\pi = W_{\text{SR}} \left\{ \frac{1 + \zeta\zeta'}{2} \left\langle \frac{1}{8} - \frac{5\sqrt{3}}{24}\, \xi + \frac{25}{18}\, \xi^2 \right. \right.$$
$$- a\zeta \cos\nu \left(\frac{1}{6}\, \xi - \frac{245\sqrt{3}}{432}\, \xi^2 \right)$$
$$\left. + \left[\frac{5 + 22a}{9} \sin^2\lambda + \frac{13}{18} (1 + 4a) \cos^2\lambda \right] \sin^2\nu \xi^2 \right\rangle$$
$$+ \frac{1 - \zeta\zeta'}{2} \left\langle \frac{5 + 22a}{9} (\cos^2\nu + \sin^2\nu \cos^2\lambda) \right.$$
$$+ \frac{13}{18} (1 + 4a)(\cos^2\nu + \sin^2\nu \cos^2\lambda)$$
$$\left. \left. + \zeta \cos\nu\, \frac{35\sqrt{3}}{48} \left(1 + \frac{38}{9}\, a \right) \right\rangle \xi^2 \right\} . \tag{49}$$

The overall radiation power is

$$
\begin{aligned}
W = W_{\mathrm{SR}} \Bigg\{ & \frac{1+\zeta\zeta'}{2} \Big\langle 1 - \frac{55\sqrt{3}}{24}\,\xi + \frac{175}{9}\,\xi^2 \\
& -\zeta \cos\nu \left(\xi - \frac{245\sqrt{3}}{48}\,\xi^2 \right) + \cos^2\nu\,\frac{5-a}{9}\,\xi^2 \\
& + \sin^2\nu \left(\frac{5+22a}{9}\sin^2\lambda + \frac{7+27a}{9}\cos^2\lambda \right) \xi^2 \Big\rangle \\
& + \frac{1-\zeta\zeta'}{2} \Big\langle \frac{5-a}{9}\sin^2\nu + \frac{5+22a}{9}(\cos^2\nu + \sin^2\nu\cos^2\lambda) \\
& + \frac{7+27a}{9}(\cos^2\nu + \sin^2\nu\cos^2\lambda) \\
& + \zeta\cos\nu\,\frac{35\sqrt{3}}{48}\left(1 + \frac{40}{9}\,a \right) \Big\rangle \xi^2 \Bigg\} \, . \qquad (50)
\end{aligned}
$$

In these equations the spin orientation may be averaged over the azimuthal angle λ

$$
\begin{aligned}
& \overline{\sin^2\lambda} = \overline{\cos^2\lambda} = \frac{1}{2} \, , \\
& \cos^2\nu + \sin^2\nu\,\overline{\sin^2\lambda} = \cos^2\nu + \sin^2\nu\,\overline{\cos^2\lambda} \\
& \qquad = \frac{1}{2}(1+\cos^2\nu) \, , \qquad (51)
\end{aligned}
$$

and they then coincide with the results given in Ref. [26] (see also Ref. [42]). If instead of averaging over λ, we consider the spin to be oriented along the velocity of the electron ($\nu = \pi/2$, $\lambda = 0$) or along the orbital radius (perpendicularly to the magnetic field) ($\nu = \pi/2$, $\lambda = \pi/2$), then Eqns (48)–(50) go for $a = 0$ into the results true in quantum electrodynamics [43].

In the most interesting case of transversely polarised electrons ($\nu = 0$), Eqns (48)–(50) are simplified and upon averaging over λ take the form:

$$
W^s = W_{\mathrm{SR}} f(\xi) \, ,
$$

$$
\begin{array}{l|l|l|}
f^{\sigma}(\xi) = \frac{1+\zeta\zeta'}{2}\big[\frac{7}{8} & -\frac{25\sqrt{3}}{12}\,\xi + \frac{325}{18}\,\xi^2 & -\zeta\big(\xi - \frac{245\sqrt{3}}{48}\,\xi^2\big) \\
f^{\pi}(\xi) = \frac{1+\zeta\zeta'}{2}\big[\frac{1}{8} & -\frac{5\sqrt{3}}{24}\,\xi + \frac{25}{18}\,\xi^2 - & \\
\qquad\qquad (1) & \qquad (2) & \qquad (3)
\end{array}
$$

$$
\begin{array}{l|l|}
+\zeta\frac{a}{6}\big(\xi - \frac{245\sqrt{3}}{72}\,\xi^2\big) \;\; -\frac{a}{9}\,\xi^2 & +\frac{5}{9}\,\xi^2\big] \;\; + \frac{1-\zeta\zeta'}{2}\big[\frac{1}{18} \\
-\zeta\frac{a}{6}\big(\xi - \frac{245\sqrt{3}}{72}\,\xi^2\big)\big] & \qquad + \frac{1-\zeta\zeta'}{2}\big[\frac{23}{18} + \zeta\frac{35\sqrt{3}}{48} \\
\qquad (5) & \qquad\qquad (4)
\end{array}
$$

$$
\begin{array}{|l}
+\frac{a}{9}\big(1 + \zeta\frac{35\sqrt{3}}{24}\big)\big]\,\xi^2 \, , \\
+\frac{a}{9}\big(48 + 19\zeta\frac{35\sqrt{3}}{24}\big)\big]\,\xi^2 \, . \\
\qquad (5)
\end{array}
\qquad (52)
$$

The overall radiation power is specified by the expression:

$$
W = W^{\sigma} + W^{\pi} = W_{\mathrm{SR}} f(\xi) \, , \qquad (53)
$$

$$
\begin{array}{l|l|l|}
f(\xi) = \frac{1+\zeta\zeta'}{2}\big[1 & -\frac{55\sqrt{3}}{24}\,\xi + \frac{175}{9}\,\xi^2 & -\zeta\big(\xi - \frac{245\sqrt{3}}{48}\,\xi^2\big) - \\
\qquad (1) & \qquad (2) & \qquad (3)
\end{array}
$$

$$
\begin{array}{l|l|l|}
-\frac{a}{9}\,\xi^2 & +\frac{5}{9}\,\xi^2\big] + \frac{1-\zeta\zeta'}{2}\big[\frac{4}{3} + \frac{35\sqrt{3}}{48} & +\frac{a}{9}\big(49 - \zeta\frac{175\sqrt{3}}{6}\big)\big]\,\xi^2 \\
\quad (5) & \qquad (4) & \qquad (5)
\end{array} \; .
$$

The vertical lines separate the charge radiation (1) from the recoil effects (2), complex radiation (3), spin magnetic moment radiation (4), and the radiation associated with the anomalous magnetic moment (5).

If we do not separate the structural elements of the synchrotron radiation and neglect the anomalous magnetic moment of the electron, then we arrive at the familiar expression for the radiation power [see, for example, Eqn (14)], where

$$
f(\xi) = 1 - \zeta\xi - \frac{55\sqrt{3}}{24}\,\xi + \frac{64}{3}\,\xi^2 - \zeta\,\frac{35\sqrt{3}}{6}\,\xi^2 \, . \qquad (54)
$$

A similar calculation for a spinless particle on the base of the matrix element [Eqn (25)] yields

$$
f_0(\xi) = 1 - \frac{55\sqrt{3}}{24}\,\xi + \frac{56}{3}\,\xi^2 \qquad (55)
$$

instead of $f(\xi)$. This expression was first obtained in Ref. [44] (see also Refs [32, 45, 46]). Note that the 'role of spin' in synchrotron radiation was earlier limited to the quantum correction $(8/3)\xi^2$ [47]. Here it is obtained as the averaged difference

$$
\frac{1}{2}\sum_{\zeta=\pm1}\big[f(\xi) - f_0(\xi)\big] = \frac{8}{3}\,\xi^2 \, . \qquad (56)
$$

However, the structural role and physical meaning of this and other quantum corrections have been revealed only because of Eqn (50).

Certainly, the cited method enables the photon emission probability to be calculated as well. Although there is little sense in quoting these equations, we shall write down the expression for the probability of radiation with a spin flip, averaged over the azimuthal angle λ and summed over linear polarisations of photons [26]

$$
\begin{aligned}
w = \frac{1}{2T_0} & \left[1 + \frac{37}{9}\,a - \left(\frac{2}{9} - \frac{13}{9}\,a \right) \frac{\sin^2\nu}{2} \right. \\
& \left. + \zeta\,\frac{8}{5\sqrt{3}}\left(1 + \frac{14}{3}\,a \right) \cos\nu \right] , \qquad (57)
\end{aligned}
$$

where

$$
T_0 = \frac{8\sqrt{3}}{15}\,\frac{\hbar^2}{m_0 c e_0^2}\,\frac{1}{\gamma^2}\left(\frac{H^*_{\mathrm{cr}}}{H} \right)^3 \qquad (58)
$$

is the polarisation time in the Sokolov–Ternov effect [1].

This will be helpful in the analysis of the part of the anomalous magnetic moment of an electron in the radiative self-polarisation effect.

Since the magnetic field is about 10^4 G in a storage ring ($H \ll H^*_{\mathrm{cr}} = m_0^2 c^3/(e_0\hbar) = 4.4 \times 10^{13}$ G), the polarisation time takes a value, which is accessible for observation of the polarisation effect only in the high-energy range. In particular, this time is approximately equal to an hour for an electron with the energy about 1 GeV.

According to Eqn (57) the spin-flip transitions do not depend on the initial orientation of a spin for $\nu = \pi/2$ when the spin is oriented in the orbital plane, whereas the spin-flip transitions depend essentially on the orientation of the spin for $\nu = 0$ when the spin is oriented along the magnetic field and, thus, self-polarisation of the electron beam occurs. The spin relaxation time is specified by the expression

$$
\big[w(\zeta = 1) + w(\zeta = -1)\big]T = 1 \, . \qquad (59)
$$

It follows that (see also Ref. [28])

$$T = T_0\left(1 + \frac{37}{9}\,a\right)^{-1} \approx T_0\left(1 - \frac{37}{9}\,a\right) . \tag{60}$$

Thus the anomalous magnetic moment contributes only to ordering the spin orientation of the electron during radiation.

7. 'True' magnetic moment radiation

We have seen (see Section 5) that the spin magnetic moment of an electron exhibits the features of the 'true' magnetic moment only in the relativistic limit or during uniform rectilinear motion when the Thomas precession can be neglected. The case of uniform rectilinear motion when the electron charge radiation and complex radiation are absent may be obtained from Eqn (46) in the limit $\rho \to \infty$. Although the method of the relativistic semiclassical theory does not undergo any change in this case, the functions $U^s_{2,3}$ responsible for the 'true' magnetic moment radiation are essentially changed. In what follows we do not want to restrict ourselves by the assumption that the anomalous magnetic moment is small and proceed to consideration of an arbitrary a. Eqns (46) are no longer applicable in this case. In a more general notation they are written as [25]

$$U^{\sigma}_{2,3} = \mathrm{i}\pi q\chi\{(1+a)A(z_{\pm}) \pm qaA'(z_{\pm})\} ,$$
$$U^{\pi}_{2,3} = \pm\pi q\{[1 + (1 + \chi^2 \pm aq^3)a]A(z_{\pm}) \pm (1+a)qA'(z_{\pm})\} , \tag{61}$$

where

$$z_{\pm} = z_0 \pm aq , \quad z_0 = \frac{1+\chi^2}{q^2} , \quad q = \left(\frac{2c}{\widetilde{\omega}\rho}\right)^{1/3}\gamma .$$

The appearance of Airy functions A and their derivatives is explained by the fact that the argument depends on the anomalous magnetic moment and it is no longer possible to employ the McDonald functions in $U^s_{2,3}$ [24]. It may be shown that the previous equations for $U^s_{2,3}$ in Eqn (46) follow from Eqn (61) in the particular case of $a \ll 1$.

In the limit $\rho \to \infty$ the functions $U^s_{2,3}$ are simplified:

$$U^s_{2,3} = \mathrm{i}\pi q\chi a A(z_{\pm}) ,$$
$$U^{\pi}_{2,3} = \pm\pi q\left(1 + \chi^2 \pm \frac{1}{2}\,aq^3\right)A(z_{\pm}) . \tag{62}$$

Moreover, as $\rho \to \infty$, with regard for Eqn (19),

$$A(z_{\pm}) \to \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}x \exp(\mathrm{i}z \pm x)$$
$$= \left(\frac{\widetilde{\omega}_{\max}}{a\widetilde{\omega}}\right)^{2/3}\begin{cases} 0 , \\ \delta\left(1 + \chi^2 - \dfrac{\widetilde{\omega}_{\max}}{\widetilde{\omega}}\right) \end{cases} , \tag{63}$$

where $\widetilde{\omega}_{\max} = 2a\omega\gamma^3$, we have instead of Eqn (42) that

$$U^s_2 = 0 ,$$
$$U^{\sigma}_3 = \mathrm{i}\pi\,\frac{\widetilde{\omega}_{\max}}{\widetilde{\omega}}\,\chi\delta\left(1 + \chi^2 - \frac{\widetilde{\omega}_{\max}}{\widetilde{\omega}}\right) ,$$
$$U^{\pi}_3 = \pm\pi\,\frac{\widetilde{\omega}_{\max}}{\widetilde{\omega}}\,\frac{1}{2}(1+\chi^2)\delta\left(1 + \chi^2 - \frac{\widetilde{\omega}_{\max}}{\widetilde{\omega}}\right) . \tag{64}$$

If the magnetic moment is anomalous (neutron), it can be assumed that $g = 2a$, and then

$$\widetilde{\omega}_{\max} = \frac{4|\mu|H}{\hbar}\,\gamma^2 , \quad |\mu| = \frac{g\mu_0}{2} , \tag{65}$$

where μ_0 is the Bohr nuclear magneton. The frequency of spin precession depends generally on the direction of motion of the neutron in the magnetic field [48]:

$$\omega_{\mu} = \frac{2|\mu|H}{\hbar}[1 - \beta^2\cos^2\alpha]^{1/2} , \tag{66}$$

where α is the angle between $\boldsymbol{\beta}$ and $\boldsymbol{H}$. In the case of interest, the neutron (as the electron in Section 2) moves along the X axis, the magnetic field is parallel to the Z axis and therefore $\alpha = \pi/2$ and $\widetilde{\omega}_{\max} = 2\gamma^2\omega_{\mu}$. According to Eqn (64) the frequency $\widetilde{\omega}_{\max}$ plays the part of the maximal radiation frequency.

The full spectral-angular distribution of the 'true' magnetic moment radiation (of a neutron in our case) is calculated by means of Eqns (14), (20), and (64). As a result, we have

$$\frac{\mathrm{d}^2W}{\mathrm{d}\widetilde{\omega}\,\mathrm{d}\Omega} = \frac{64\mu^6H^4}{\pi c^3\hbar^4}\left|\langle\zeta'|\sigma_-|\zeta\rangle\right|^2\frac{4\chi^2 + (1+\chi^2)^2}{(1+\chi^2)^5}$$
$$\times\,\delta\left(\widetilde{\omega} - \frac{\widetilde{\omega}_{\max}}{1+\chi^2}\right) . \tag{67}$$

It then follows that the radiation proceeds at the frequency

$$\widetilde{\omega} = \frac{\widetilde{\omega}_{\max}}{1+\chi^2} . \tag{68}$$

Further integration with respect to angles and frequency yields, with respect to Eqn (47) for $\langle\zeta'|\sigma_-|\zeta\rangle$, the overall radiation power

$$W = \frac{128}{3}\,\frac{\mu^6H^4}{\hbar^4c^3}\,\gamma^4\,\frac{(1 + \zeta\cos\nu)(1 - \zeta'\cos\nu)}{4} . \tag{69}$$

In the quantum theory of radiation, spin transitions are considered only for time-invariant spin projections. In our case, adaption to such transitions corresponds to $\nu = 0$ when the spin of a neutron is oriented along the field. At $\nu = 0$ the spin transitions occur, according to Eqn (69), in only one direction — opposite to the magnetic field when $\zeta = 1$ and $\zeta' = -1$ ($\mu < 0$). The overall power,

$$W = \frac{128}{3}\,\frac{\mu^6H^4}{\hbar^4c^3}\,\gamma^4 , \tag{70}$$

coincides entirely with the spin-flip radiation of a neutron, calculated from the Dirac–Pauli equation [48]. At $\nu = \pi/2$ when the spin rotates in the plane perpendicular to the magnetic field vector, the overall radiation power of the neutron decreases by a factor of 4 but all the qualitative features of the radiation are the same. This case is adequate for the classical statement of the problem on the magnetic moment radiation and it is described fully within the scope of classical electrodynamics [47]. Note that the 'true' magnetic moment does not radiate at all in the classical theory at $\nu = 0$ when $\mu = \mathrm{const}$. This radiation is associated with spin-flip transitions and is purely quantum in nature.

In particular cases all the above results may be obtained by the Lorentz transformations and direct calculations as in Ref. [49] by quantum theory and by the classical theory in Ref. [50].

Let us consider in more detail the correspondence principle for the 'true' magnetic moment radiation. In classical field theory the magnetic moment radiation at a relativistic point is specified by the expression (see details in Ref. [49])

$$\widetilde{H}^{\mu\nu} = -\frac{\mu\widetilde{\omega}^2}{Rc}\,\frac{\Pi^{[\mu\rho}n_\rho n^{\nu]}}{n_\rho v^\rho}\,. \tag{71}$$

Here $\Pi^{\mu\rho} = (\boldsymbol{\Phi}, \boldsymbol{\Pi})$ is the dimensionless spin tensor; v^ρ is the four-dimensional velocity; $n^\rho = (1, \boldsymbol{n})$; and the square brackets in superscripts imply antisymmetry with respect to these superscripts.

The Fourier transform of the electromagnetic field tensor [Eqn (71)] has the form

$$\widetilde{H}^{\mu\nu}_{\widetilde{\omega}} = \frac{\mu\widetilde{\omega}^2}{Rc^2}\int_{-\infty}^{\infty}\pi^{[\mu\rho}n_\rho n^{\nu]}\exp\left\{\mathrm{i}\widetilde{\omega}\left[t - \frac{(\boldsymbol{n}\cdot\boldsymbol{r})}{c}\right]\right\}\mathrm{d}t\,, \tag{72}$$

where $\pi^{\mu\rho} = \Pi^{\mu\rho}/\gamma = (\varphi, \pi)$. It follows then that

$$\widetilde{\boldsymbol{E}}^{s}_{\widetilde{\omega}} = \frac{\mu\widetilde{\omega}^2}{Rc^2}\int_{-\infty}^{\infty}\Big(\big[(\boldsymbol{n} - \beta)\pi\big]\boldsymbol{n}^s\Big)\exp\left\{\mathrm{i}\widetilde{\omega}\left[t - \frac{(\boldsymbol{n}\cdot\boldsymbol{r})}{c}\right]\right\}\mathrm{d}t\,. \tag{73}$$

At the same time $\widetilde{\boldsymbol{E}}^{s}_{\widetilde{\omega}}$ is specified, according to the relativistic semiclassical theory of radiation, by means of Eqn (11), in which the substitution

$$\langle \mathrm{f}|\hat{\alpha}|\mathrm{i}\rangle \to \frac{\mathrm{i}\mu\widetilde{\omega}}{ec}\left\{\big[(\boldsymbol{n} - \beta)\langle\zeta'|\widetilde{\sigma}|\zeta\rangle\big] - \frac{\gamma}{\gamma+1}\big(\beta\langle\zeta'|\widetilde{\sigma}|\zeta\rangle\big)[\boldsymbol{n}\times\beta]\right\} \tag{74}$$

should be performed by taking Eqn (43) into account.

If we consider that the vector π is related to the unit classical spin vector ζ by expression [51],

$$\pi = \zeta - \frac{\gamma}{\gamma+1}\,\beta(\beta\zeta)\,, \tag{75}$$

then the semi-classical expression for $\widetilde{\boldsymbol{E}}^{s}_{\widetilde{\omega}}$ takes the form

$$\widetilde{\boldsymbol{E}}^{s}_{\widetilde{\omega}} = \frac{\mu\widetilde{\omega}^2}{Rc^2}\int_{-\infty}^{\infty}\Big(\big[(\boldsymbol{n} - \beta)\langle \mathrm{f}|\hat{\pi}|\mathrm{i}\rangle\big]\boldsymbol{n}^s\Big)\exp\left\{\mathrm{i}\widetilde{\omega}\left[t - \frac{(\boldsymbol{n}\cdot\boldsymbol{r})}{c}\right]\right\}\mathrm{d}t\,, \tag{76}$$

where $\hat{\pi}$ is different from Eqn (75) as a result of the substitution $\zeta \to \hat{\sigma}(t)$. Eqns (73) and (74) present the correspondence principle for the 'true' magnetic moment radiation.

We can now readily follow all the problems that arise in classical and semiclassical calculations. The classical expression for the radiation power, averaged over the period of spin precession, has the form ($\alpha = \pi/2$)

$$\overline{W} = \frac{2}{3}\frac{\mu^2}{c^3}\,\omega_\mu^4\gamma^4\,. \tag{77}$$

In a similar form, the corresponding semiclassical expression is, according to Eqn (69),

$$W = \frac{8}{3}\frac{\mu^2}{c^3}\,\omega_\mu^4\gamma^4\,\frac{(1 + \zeta\cos\nu)(1 - \zeta'\cos\nu)}{4}\,. \tag{78}$$

At $\nu = 0$ we have the result of quantum theory for radiation from a neutron [48] and at $\nu = \pi/2$ the classical Eqn (77).

We have considered the particular case when the 'true' magnetic moment (neutron) moves perpendicular to the magnetic field, but one may show that the correspondence principle works in a similar way for any other orientation of the velocity vector relative to the magnetic field vector. Moreover, the classical [49] and quantum [51–53] theories of radiation of a relativistic particle with the 'true' magnetic moment are in full accord for an arbitrary configuration of constant homogeneous electromagnetic fields.

Note in conclusion that these simple considerations may seem trivial from the point of view of contemporary theoretical physics but these issues will have to be resolved sooner or later since a number of topics of electrodynamics, associated with the spin magnetic moment, have not yet been considered. There are some single papers in this line ([33, 54–56]) and others; for a more complete list see Ref. [5], but note that the authors do not consider how their results correlate with quantum theory. This area of electrodynamics has been a terra incognita without the correspondence principle. Certainly, there are still some issues which need to be clarified. For example, it is unclear how the correspondence principle will work for inhomogeneous fields, what effect the radiation reaction will have on the magnetic moment (for a nonrelativistic discussion of the topic see Ref. [37]), etc. However, this does not exclude the possibility that the spin light will show itself in the near future. "Past experience, including very recent, shows that much new and interesting material can be found even in electrodynamics (including optics)..." (V L Ginzburg [57]).

This is an area open wide for experimentalists. Precise investigations of the features of spin light in contemporary accumulators would be an excellent application of experiment to the study of this new natural phenomenon.

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