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METHODOLOGICAL NOTES

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The Hawking effect in the sudden gravitational collapse model

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Contents

Abstract. The Hawking effect is studied on the basis of the sudden gravitational collapse model. Virtual particle – antiparticle pairs are shown to be pulled apart by the tidal forces of the black hole, in full accord with the current viewpoint.

1. Introduction

When studying the Hawking effect [1] for the quantum radiation of a black hole, and particularly when explaining it to students, one is prompted to find the simplest gravitational collapse model revealing all the important features of the phenomenon. Following Ref. [3] we have chosen the two-dimensional spacetime model, in which there are the time t and the radial coordinate r only. Unfortunately, the mathematical calculation is cumbersome and not straightforward even in this approximation. Therefore, it is only natural to try to find simpler models. One of them — the sudden gravitational collapse model — is considered in this paper. However, it turns out that such an approximation is too rough — the sudden gravitational collapse model leads to an interesting paradox. The resolution of the paradox, which is the subject of this paper, helps one to understand the physical nature of the Hawking effect.

A collapsed star of mass M emits a steady thermal radiation flux with temperature $T_0 = 1/(4\pi r_g)$, where $r_g = 2M$ is the gravitational radius (from here on $\hbar = c = G = 1$) [1]. In the literature there are two opinions on the nature of this fundamental effect (we shall call them Mechanisms 1 and 2). According to Mechanism 1 [3, 4], the crucial role belongs to the initial stage of collapse, during which the gravitational field is not static. Other authors [5–11], as well as Hawking himself, think that there is an analogy between the productions of particles in a black hole and a homogeneous electric field (Mechanism 2). In the static but inhomogeneous field of a black hole a virtual pair appears. The constituent particles fall in with different accelerations because of the tidal forces, the effect of which is significant since the characteristic size of the pair is r_g. As a result a real particle appears and goes to infinity while the other particle drops into the singularity. Clearly, the spectrum and intensity of the radiation do not depend on the peculiarities of the collapse in Mechanism 2. Mechanism 2 is more attractive because in the first case an infinite amount of energy must of necessity be concentrated in a finite spacetime region [8, 10]. Mechanism 2 is supported by calculations of the renormalised mean value of the energy – momentum tensor (see, for example, Ref. [2], Sec. 8.2), which show that the energy flux coincides at infinity with the result of Hawking in the steady state. These calculations also show that a negative energy of pure quantum origin falls upon the horizon of a black hole and exactly balances the positive energy going to infinity. The last fact conforms to the covariant conservation $\langle 0|T_{\mu\nu}|0\rangle_{\text{ren}}$.

In Section 2 the sudden collapse model is formulated and the intensity and spectrum of the Hawking effect are calculated by the Bogolyubov transformation method [2, 11, 13] in the two-dimensional space-time approximation. The radiation temperature turns out to be equal to $2T_0$ and it seems at first sight that this corroborates Mechanism 1 rather than Mechanism 2. To delve into the situation the energy – momentum tensor is studied in Section 3. It turns out that the 'odd' energy appears when the metric changes suddenly and is then emitted with constant intensity and superimposes upon the common Hawking radiation. Thus, Mechanism 2 is corroborated in the sudden collapse model too.

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2. Temperature of a black hole in the sudden collapse model

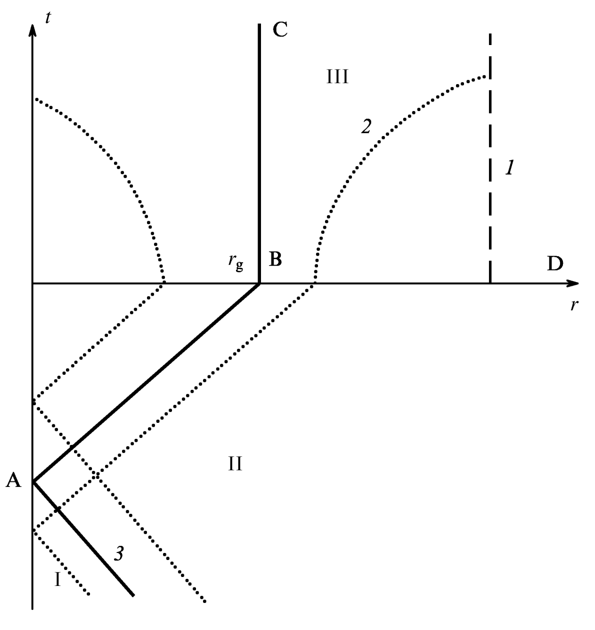
We shall calculate the number of particles in the massless scalar Hermitian field, produced in the two-dimensional spacetime with the metric

$$ds^2 = h(t,r)\,dt^2 - \frac{1}{h(t,r)}\,dr^2\,, \qquad (1)$$

where

$$r > 0\,, \qquad h(t,r) = \begin{cases} 1\,, & t < 0\,, \\ 1 - \dfrac{r_g}{r}\,, & t > 0 \end{cases}$$

This model describes an initially quiescent nonrelativistic star (of radius R: $r_g \ll R \to \infty$) collapsing with infinite velocity at $t = 0$. Although this geometry is physically unrealisable, the consideration of the sudden collapse model is prompted by the following fact. In Mechanism 2, quanta — which come to an observer at the point $r = \text{const} > r_g$ at $t \to +\infty$ (its world line is straight line *1* in the figure) — are produced near the event horizon (line ABC). The trajectory of such a quantum is the upper section of line *2*. Since in the sudden collapse model the gravitational field does not differ for $t > 0$ from the field of a black hole born in a real collapse, in accordance with Mechanism 2 the utter coincidence should be expected with the result of Hawking in the limit of $t \to +\infty$. However, as is shown in this section, these expectations are not realised.



We introduce the null coordinate in region I (see the figure)

$$u = t - r_*(r)\,, \qquad (2)$$

where

$$r_*(r) = \int \frac{dr}{h(t,r)} = r + r_g \ln\left(\frac{r}{r_g} - 1\right).$$

Then in regions I and II, $u = F(t - r)$, where the function F is determined from the sewing conditions for Eqn (2) on the line BD (i.e. $t = 0$),

$$u = t - r - r_g \ln \frac{-t + r - r_g}{r_g}\,. \qquad (3)$$

Another null coordinate (v) is chosen such that

$$v = t + r \qquad (4)$$

in regions I and II.

There is a critical incident ray $v = v_0 = -r_g$ (line *3*), which reaches the centre of the star and moves along the event horizon ABC upon reflection. This reflection of the ray in the two-dimensional model in question may be understood by taking into account the fact that such rays represent the motion of spherically symmetric wave packets in the real four-dimensional space.

The ray $v < v_0$ (curve *2*) goes upon reflection to infinity along the trajectory $u = f(v)$. The function $f(v)$ can readily be found from sewing the rays (3) and (4) on the line $r = 0$:

$$f(v) = v - r_g \ln \frac{v_0 - v}{r_g} \qquad (5)$$

where $v_0 = -r_g$.

Now we shall calculate the Bogolyubov coefficients (see, for example, Ref. [2]) for a massless scalar field. In the null coordinates, metric (1) takes the form

$$ds^2 = C(u,v)\,du\,dv\,, \qquad (6)$$

and the wave equation $\partial^2\Psi/\partial u\,\partial v = 0$. Its general solution is $\Psi = F_1(v) + F_2(u)$. The wave function of the in-basis, the boundary condition for which is imposed at $t \to -\infty$, is specified by the expression

$$\Psi_k = \frac{1}{2i\sqrt{k}}\Big\{\exp\big[-ikp(u)\big] - \exp\big[-ikv\big]\Big\}\,, \qquad (7)$$

where $p(u) = f^{-1}(u)$, $k > 0$. The physical meaning of Ψ_k can be understood by considering the wave packet

$$\Phi = \int_0^\infty dk\, C_k \Psi_k\,. \qquad (8)$$

When $t \to -\infty$, it is concentrated near the point $r + t = 0$ and consists of undistorted incident waves $\exp(-ikv) = \exp[-ik(r+t)]$. The contribution of the first term in Eqn (7), which describes reflected distorted waves, vanishes as $t \to -\infty$.

On the line $r = 0$ (which is represented by the curve $u = f(v)$ in terms of the null coordinates), the function Ψ_k satisfies the boundary condition $\Psi_k = 0$, which naturally arises when one goes from the four-dimensional world to the two-dimensional model. Similarly, if we consider the propagation of s-waves described by the equation $(\partial^2/\partial t^2 - \nabla^2)\psi = 0$ and substitute $\psi = \chi/r$, then we arrive at the 'two-dimensional' equation $(\partial^2/\partial t^2 - \partial^2/\partial r^2)\chi = 0$ and the boundary condition $\chi(t, r = 0) = 0$.

In contrast to the in-basis Ψ the complete set of quantum numbers for wave functions of the out-basis $\bar{\Psi}$ is the union $q = (k, \lambda)$, where k is the momentum and $\lambda = \pm 1$ is a new quantum number — 'observability' [2]. The reason why the out-solutions of the wave equation are decomposed into two orthogonal classes can readily be understood from the figure. The boundary conditions for the wave functions of the out-basis $\bar{\Psi}$ are imposed at $t \to +\infty$. If a wave packet of the type given by Eqn (8) is composed of $\bar{\Psi}$ (in essence these are the rays considered above), then it will move away from the star, fall into the singularity for a finite lapse of the affine parameter, or be a coherent admixture of these two states. In the first case the packet is registered by inertial detectors at infinity (let them be numbered by $\lambda = +1$). In the second case ($\lambda = -1$)

detectors are located under the event horizon and fall into the singularity. Since the detectors $\lambda = +1$ and $\lambda = -1$ are not causally related, the third case is physically meaningless. The figure shows the paths which these packets follow. Packets with $\lambda = +1$ fall onto the star along trajectories $v < v_0$ and packets with $\lambda = -1$ do so along trajectories $v > v_0$. It follows then that the incident wave is nonzero for $v < v_0$ only when $\lambda = +1$. It may be seen in the figure that the reflected wave $F_2(u)$ is nonzero outside the event horizon, i.e. on the right of the line ABC (for $\lambda = -1$ the case is just the reverse). If the wave function of the out-basis with $\lambda = +1$ satisfies the zero boundary condition at $r = 0$, i.e. on the line $u = f(v)$, then it takes the form

$$\bar{\Psi}_{k1} = \frac{1}{2\mathrm{i}\sqrt{k}}\left\{\exp\left(-\mathrm{i}ku\right) - \exp\left[-\mathrm{i}kf(v)\right]\theta(v_0 - v)\right\}. \quad (9)$$

A wave packet of the form given by Eqn (8), composed of $\bar{\Psi}_{k1}$, goes from the black hole as $t \to +\infty$ and consists of undistorted diverging waves $\exp\left(-\mathrm{i}ku\right) = \exp\left[-\mathrm{i}kt + \mathrm{i}kr_*(r)\right]$; while the second term in Eqn (9) makes a negligible contribution to Φ. The logarithmic dependence of the phase in $\exp\left(-\mathrm{i}ku\right)$ on r is explained by the long-range character of the gravitational field of the star and is fully analogous to the Coulomb phase in quantum mechanics.

The number of new particles with momentum k is specified by the expression (the detailed calculation is summarised in Ref. [2])

$$N_k = \sum_{k'} |\beta|^2, \quad \beta = (\bar{\Psi}_{k1}, \Psi^*_{k'})\,. \quad (10)$$

The particles are observed when $r \to +\infty$ (i.e. with the quantum numbers $\lambda = +1$). The inner product β (the Bogolyubov transformation coefficient) is the integral over a space-like surface Σ and does not depend on the choice of Σ. We shall use this fact to calculate β on the hyperplane $t = \mathrm{const} \to -\infty$, where the space is Euclidean and the expression for β has a simple form:

$$\beta = \mathrm{i}\int_0^\infty \mathrm{d}r\left(\bar{\Psi}^*_{k1}\frac{\partial\Psi^*_{k'}}{\partial t} - \frac{\partial\bar{\Psi}^*_{k1}}{\partial t}\,\Psi^*_{k'}\right). \quad (11)$$

In this case $t - r \to -\infty$ and, therefore, the summand with the logarithm in Eqn (3) may be omitted. The wave functions are then written in the form

$$\Psi_k = \frac{1}{2\mathrm{i}\sqrt{k}}\left\{\exp\left[-\mathrm{i}k(t-r)\right] - \exp\left[-\mathrm{i}k(t+r)\right]\right\} = \frac{1}{\sqrt{k}}\sin(kr)\exp\left(-\mathrm{i}kt\right), \quad (12)$$

$$\bar{\Psi}_{k1} = \Psi_k + G(v), \quad (13)$$

where

$$G(v) = \frac{1}{2\mathrm{i}\sqrt{k}}\left\{\exp\left(-\mathrm{i}kv\right) - \exp\left[-\mathrm{i}kf(u)\right]\theta(v_0 - v)\right\}. \quad (14)$$

The first term in Eqn (13) makes zero contribution to Eqn (11) because it is orthogonal to $\Psi^*_{k'}$; therefore, $\bar{\Psi}_{k1}$ can be replaced with $G(v)$. The function $G(v)$ tends to zero as one moves away from the line $v = v_0$; therefore, the lower limit in Eqn (11) can be substituted with $-\infty$. Considering $\partial G(v)/\partial t = \partial G(v)/\partial r$ we have upon integration that

$$\beta = \mathrm{i}\int_{-\infty}^\infty \mathrm{d}r\, G^*\left(\frac{\partial\Psi^*_{k'}}{\partial t} + \frac{\partial\Psi^*_{k'}}{\partial r}\right) = \sqrt{k'}\int_{-\infty}^\infty \mathrm{d}v\, G^*(v)\exp(\mathrm{i}k'v)\,. \quad (15)$$

From here on we omit unessential constant phase coefficients. The contribution of the first term in Eqn (14) to β is proportional to $\delta(k + k')$ and is zero since $k > 0$, $k' > 0$. Ultimately we get the expression for β in the two equivalent forms:

$$\beta = \frac{1}{2}\sqrt{\frac{k'}{k}}\int_{-\infty}^{v_0} \mathrm{d}v\exp\left[\mathrm{i}k'v + \mathrm{i}kf(v)\right]; \quad (16)$$

$$\beta = -\frac{1}{2}\sqrt{\frac{k}{k'}}\int_{-\infty}^{\infty} \mathrm{d}u\exp\left[\mathrm{i}ku + \mathrm{i}k'p(u)\right]. \quad (17)$$

To transform Eqn (16) into Eqn (17) we used the change of the variable of integration $u = f(v)$ and performed integration by parts. Expressions (16) and (17) are analogous to those obtained in Ref. [2], where the radiation of an accelerating mirror was studied. Note that expression (16) obtained in Ref. [2] as a result of a series of assumptions is actually exact.

Using the substitution $x = (v_0 - v)/r_{\mathrm{g}}$ we obtain from Eqns (5) and (16)

$$\beta = \frac{1}{2}\,r_{\mathrm{g}}\sqrt{\frac{k'}{k}}\int_0^\infty \mathrm{d}x\exp\left[\mathrm{i}(k'+k)r_{\mathrm{g}}x\right]x^{-\mathrm{i}kr_{\mathrm{g}}}.$$

It is necessary for this integral to be convergent as $x \to +\infty$ to make the change $k + k' \to k + k' - \mathrm{i}\delta$ and to consider it as the limit when $\delta \to +0$. Then,

$$\beta = \frac{1}{2}\,r_{\mathrm{g}}\sqrt{\frac{k'}{k}}\left[\delta + \mathrm{i}(k+k')r_{\mathrm{g}}\right]^{ikr_{\mathrm{g}}-1}\Gamma(1 - \mathrm{i}kr_{\mathrm{g}})\,,$$
$$|\beta|^2 = \frac{\pi k' r_{\mathrm{g}}}{2(k+k')^2}\,n_{\mathrm{B}}(k)\,, \quad (18)$$

where

$$n_{\mathrm{B}}(k) = \left[\exp\left(\frac{k}{T}\right) - 1\right]^{-1}, \quad (19)$$

$$T = \frac{1}{2\pi r_{\mathrm{g}}} = 2T_0\,. \quad (20)$$

It follows from Eqns (9) and (18) that

$$N_k = \sum_{k'}|\beta|^2 = \frac{1}{2}\,r_{\mathrm{g}}n_{\mathrm{B}}(k)J\,, \quad (21)$$

where

$$J = \int_0^\infty \frac{k'\,\mathrm{d}k'}{(k+k')^2}$$

and the rule of replacement of a sum with an integral, $\sum_{k'} \to \int \mathrm{d}k'/\pi$, has been applied. The integral J is divergent at the upper limit since radiation of constant intensity is established in the collapse; therefore, an infinite number of particles will be emitted for a finite time.

To understand the nature of the integral J we shall follow Ref. [2] in employing a simple, though not strict, method (there is another calculation in Ref. [2] — more strict but physically less meaningful). The Bogolyubov coefficient β (10) can be interpreted as the amplitude of transition of a particle with a momentum k' into a particle with a momentum k, which comes to the detector along the straight line *1* (see the figure) (for a nonhermitian field, β is the amplitude of transition of an antiparticle with an impulse k' into a particle with an impulse k). Let two photons be sent from the detector hanging at the point $r = r_0$ towards the black hole at t_0 and $t_0 + \Delta t_0$ along the trajectories $v = \text{const}$ and $v + \Delta v = \text{const}$, where $v = t_0 + r_0 < v_0$ and $v + \Delta v = t_0 + \Delta t_0 + r_0 < v_0$. Upon reflection from the centre of the star these momenta are directed away from it along the trajectories $u = u_1 = f(v) = \text{const}$, $u = u_1 + \Delta u = f(v + \Delta v) = \text{const}$. They arrive at the point $r = r_0$ at t_1 and $t_1 + \Delta t_1$, where $\Delta t_1 = \Delta u \approx f'(v)\Delta v = f'(v)\Delta t_0$. Consequently,

$$\frac{\Delta t_1}{\Delta t_0} = \frac{\Delta u}{\Delta v} = f'(v) = \frac{r_g}{v_0 - v} \propto \exp\left(\frac{u}{r_g}\right) \propto \exp\left(\frac{t_1}{r_g}\right).$$

Thus, reflected waves lag far behind as $t_1 \to \infty$; therefore, the frequency experiences a heavy red shift:

$$\frac{k'}{k} = \frac{\Delta t_1}{\Delta t_0} = \text{const} \times \exp\left(\frac{t_1}{r_g}\right).$$

Hence, it follows that $\mathrm{d}k'/k' = \mathrm{d}t'/r_g$. Therefore, $J = t_1/r_g$, where t_1 is the total time of detection of particles. Then, it follows from Eqn (21) that

$$\frac{\mathrm{d}N_k}{\mathrm{d}t_1} = \frac{1}{2}\, n_{\mathrm{B}}(k)\,.$$

The total number of particles entering the detector per unit time is

$$\frac{\mathrm{d}N}{\mathrm{d}t_1} = \sum_k \frac{\mathrm{d}N_k}{\mathrm{d}t_1} = \int_0^\infty \frac{\mathrm{d}k}{2\pi}\, n_{\mathrm{B}}(k)\,.$$

The last expression shows that the radiation is thermal and its temperature is T from Eqn (20).

3. The initial energy of the field

The calculation of the number of new particles performed in the previous section was based on the Heisenberg representation of states of a field when a state is considered to be fixed and the time dependence is 'transferred to' the field operators, through which the observed variables are expressed in their turn.

In particular, if the field was in the vacuum state $|\psi\rangle = |0,\text{in}\rangle \equiv |\hat{0}\rangle$ before the collapse, then the energy–momentum tensor of the new particles is the state-averaged operator of the energy–impulse tensor. Different approaches to regularisation of the average yield the same result. The situation is essentially more simple in the two-dimensional model since any two-dimensional metric $g_{\mu\nu}$ is conformally flat: $g_{\mu\nu} = C\eta_{\mu\nu}$. This fact enables the expression for the renormaliseded average energy–momentum tensor to be obtained without difficulties (see Refs [6, 2]). In terms of null coordinates (u, v) metric (1) has the form of Eqn (6). In this metric we have

$$\left\langle T_\mu^\nu(g_{\lambda\rho})\right\rangle_{\text{ren}} = (-g)^{-1/2}\left\langle T_\mu^\nu(\eta_{\lambda\rho})\right\rangle_{\text{ren}} + \theta_\mu^\nu - \frac{1}{48\pi} R\delta_\mu^\nu\,, \quad (22)$$

$$\theta_{uu} = -\frac{1}{12\pi}\, C^{1/2}\, \partial_u^2 C^{-1/2}\,,$$
$$\theta_{vv} = -\frac{1}{12\pi}\, C^{1/2}\, \partial_v^2 C^{-1/2}\,, \quad (23)$$
$$\theta_{uv} = \theta_{vu} = 0\,.$$

The first term in Eqn (22) vanishes for a specific choice of coordinates $(\hat{u}, \hat{v})$, which coincide with the coordinates of the plane spacetime $\hat{u} = t - r$, $\hat{v} = t + r$ in regions I and II and are specified by the expressions

$$\hat{u} = p(u) = -r_*^{-1}\left[-t + r_*(r)\right]\,,$$
$$\hat{v} = v = r_*^{-1}\left[t + r_*(r)\right] \quad (24)$$

in region III. These coordinates correspond to the metric (6) with

$$C(\hat{u}, \hat{v}) = \frac{1 - r_g/r}{(1 - r_g/\hat{v})(1 + r_g/\hat{u})}\,, \quad (25)$$

and the positive-frequency wave function (7) takes the simplest form.

Upon simple mathematical manipulations with the use of formulas (22) and (23) we obtain

$$\langle T_{uu}\rangle_{\text{ren}} = \frac{1}{24\pi}\left[\partial_u^2 \ln C - \frac{1}{2}(\partial_u \ln C)^2\right]\,,$$
$$\langle T_{vv}\rangle_{\text{ren}} = \frac{1}{24\pi}\left[\partial_v^2 \ln C - \frac{1}{2}(\partial_v \ln C)^2\right]\,, \quad (26)$$
$$\langle T_{vu}\rangle_{\text{ren}} = \langle T_{uv}\rangle_{\text{ren}} = -\frac{RC}{96\pi}\,,$$

where $R = -2r_g/r^3$ is the curvature of the space (1).

It may seem that, because of the singularity of the metric at $t = 0$, it is impossible to examine the energy–momentum tensor near this surface. However, $T_{\mu\nu}$ is a local object and the renormalisation procedure does not violate this locality. Therefore, expression (26) is true everywhere except the points on the surface $t = 0$.

In the metrics (24) and (25) calculations yield the following result:

$$T_{\hat{u}\hat{u}} \equiv \langle\hat{0}|T_{\hat{u}\hat{u}}|\hat{0}\rangle_{\text{ren}} = \frac{1}{24\pi}\left[\frac{3}{8}\frac{r_g^2}{r^4(1 + r_g/\hat{u})^2} - \frac{r_g}{2r^3(1 + r_g/\hat{u})^2} - \frac{3}{2}\frac{r_g^2}{\hat{u}^4(1 + r_g/\hat{u})^2} - \frac{2r_g^2}{\hat{u}^3(1 + r_g/\hat{u})^2}\right],$$

$$T_{\hat{v}\hat{v}} \equiv \langle\hat{0}|T_{\hat{v}\hat{v}}|\hat{0}\rangle_{\text{ren}} = \frac{1}{24\pi}\left[\frac{3}{8}\frac{r_g^2}{r^4(1 + r_g/\hat{v})^2} - \frac{r_g}{2r^3(1 + r_g/\hat{v})^2} - \frac{3}{2}\frac{r_g^2}{\hat{v}^4(1 + r_g/\hat{v})^2} + \frac{2r_g^2}{\hat{v}^3(1 + r_g/\hat{v})^2}\right], \quad (27)$$

$$T_{\hat{u}\hat{v}} \equiv \langle\hat{0}|T_{\hat{u}\hat{v}}|\hat{0}\rangle_{\text{ren}} = T_{\hat{v}\hat{u}} \equiv \langle\hat{0}|T_{\hat{v}\hat{u}}|\hat{0}\rangle_{\text{ren}} = -\frac{r_g}{48\pi r^3}\frac{(1 - r_g/r)}{(1 - r_g/\hat{v})(1 + r_g/\hat{u})}\,.$$

Now we shall determine the radiation intensity of the black hole and compare it with the results of calculations performed in the second section.

In the limiting case of $r = \text{const} \gg r_g$, $t \to +\infty$ the following expressions are true:

$$\hat{u} \approx -r_g - r\exp\left(-\frac{t}{r_g}\right),$$

$$t \approx r_g \ln\left(\frac{r}{-\hat{u}-r_g}\right), \qquad (28)$$

$$T_{\hat{u}\hat{u}} \approx \frac{1}{48\pi r_g^2 (1 + r_g/\hat{u})^2}.$$

Next, we pass to the null coordinate in the Schwarzschild metric,

$$u = t - r_*(r) \approx r_g \ln\left(\frac{1}{-\hat{u}-r_g}\right) + \text{const},$$

for which the energy flux is written in the form

$$T_{uu} = \left(\frac{d\hat{u}}{du}\right)^2 T_{\hat{u}\hat{u}} = \left(1 + \frac{r_g}{\hat{u}}\right)^2 T_{\hat{u}\hat{u}} = \frac{1}{48\pi r_g^2} \equiv I_t. \qquad (29)$$

Here I_t is the overall radiation intensity. This energy flux corresponds to thermal radiation with the temperature $T = 2T_0$:

$$\int_0^\infty \frac{k\,dk}{2\pi} n_B(k) = \frac{\pi T^2}{12}.$$

Now we shall calculate the radiation which is produced when the metric is 'shaken up'.

At $t = +0$ the metric has the form

$$ds^2 = h\,dt^2 - \frac{1}{h}\,dr^2 = d\tau^2 - dx^2,$$

where $d\tau = \sqrt{h}\,dt$, $dx = dr/\sqrt{h}$. In terms of the coordinates (τ, x) of a quiescent observer at a point $r > r_g$, the energy of the cloud of quanta produced at $t = 0$ within the interval $(x, x + dx)$ is

$$dE_p = T_{\tau\tau}\,dx.$$

To calculate this expression we shall employ the formulas:

$$T_{\tau\tau} = \left(\frac{\partial t}{\partial \tau}\right)^2 T_{tt} = \frac{1}{h} T_{tt}.$$

The component T_{tt} is expressed through the components of the energy–momentum tensor in terms of coordinates $(\hat{u}, \hat{v})$ [see Eqn (27)] by means of the common transformation formulas for a tensor. The desired expression on the surface $t = +0$, where the relationships $\partial\hat{u}/\partial t = \partial\hat{v}/\partial t = h$ are valid, has the form

$$T_{tt} = h^2 (T_{\hat{u}\hat{u}} + T_{\hat{v}\hat{v}} + 2T_{\hat{u}\hat{v}}).$$

One half of the energy dE_p will fall to the hole and the other half will go to infinity. (Note that the energy flux, i.e. $T_{\tau x}$, should be calculated from the start if we wish to be more consistent.) Since the quantity $\sqrt{h}\,dE_p$ is preserved when the cloud moves in the gravitational field of the hole, the energy that has gone to infinity is equal to

$$dE = \frac{1}{2} h (T_{\hat{u}\hat{u}} + T_{\hat{v}\hat{v}} + 2T_{\hat{u}\hat{v}})\,dr.$$

A quantum moves from a point r to the detector at point $r = r_0$ in the time

$$t(r) = \int_r^{r_0} \frac{dr}{h(r)},$$

i.e. the energy dE goes through the detector in the time $dt = dr/h$. Hence, the intensity of radiation produced in 'shaking up' is

$$I_s = \frac{1}{2} h^2 (T_{\hat{u}\hat{u}} + T_{\hat{v}\hat{v}} + 2T_{\hat{u}\hat{v}}). \qquad (30)$$

Here all the quantities are taken at $t = +0$. It follows from Eqn (27) that

$$T_{\hat{u}\hat{u}} = T_{\hat{v}\hat{v}} = \frac{r_g}{16\pi r^3 h}\left(1 + \frac{r_g}{4rh}\right), \qquad T_{\hat{u}\hat{v}} = \frac{r_g}{48\pi r^3 h}. \qquad (31)$$

When $t \to +\infty$ the radiation comes from $r \approx r_g$. Using this fact we can conclude from Eqns (30) and (31) that quanta produced in 'shaking up' make a constant contribution to the intensity at $t \to +\infty$, which is

$$I_s = \frac{1}{64\pi r_g^2}. \qquad (32)$$

As one would expect, the difference between I_t and I_s coincides with the intensity of the true Hawking radiation:

$$I_t - I_s = \frac{1}{192\pi r_g^2} = \frac{\pi T_0^2}{12}.$$

The results may be summarised as follows.

1. An infinite amount of energy is used to 'shake up' the metrics (this fact alone shows that the sudden collapse model is physically unrealisable).

2. The 'wrong' energy released in the shaking of the metric is emitted and superimposes on the regular Hawking spectrum. Thus, we have verified that the details of a collapse are not important in the sense that the energy flux is the sum of two parts: pure Hawking and nonphysical. The latter is due to the sudden change in the metric. The only important feature for the Hawking component is the presence of the horizon while the radiation itself is formed by Mechanism 2.

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Appendix

In this section we give a strict proof of the above results. The expression for the energy flux registered by a detector following an arbitrary trajectory with a 4-velocity U^{μ} has the form

$$I = \langle \hat{0}|T_{\mu\nu}|\hat{0}\rangle_{\mathrm{ren}} U^{\mu} n^{\nu} , \qquad (33)$$

where $n^{\mu}n_{\mu} = -1$, $U^{\mu}n_{\mu} = 0$. In terms of coordinates (24) we have for the detector with a trajectory $r = \mathrm{const}$ that

$$U^{\hat{\mu}} = \frac{1}{\sqrt{h}} \begin{pmatrix} 1 + \dfrac{r_{\mathrm{g}}}{\hat{u}} \\ 1 - \dfrac{r_{\mathrm{g}}}{\hat{v}} \end{pmatrix}, \qquad n^{\hat{\mu}} = \frac{1}{\sqrt{h}} \begin{pmatrix} 1 + \dfrac{r_{\mathrm{g}}}{\hat{u}} \\ -1 + \dfrac{r_{\mathrm{g}}}{\hat{v}} \end{pmatrix}.$$

Then the energy flux is

$$I_{\mathrm{t}} = \frac{1}{h}\left[\langle \hat{0}|T_{\hat{u}\hat{u}}|\hat{0}\rangle_{\mathrm{ren}}\left(1 + \frac{r_{\mathrm{g}}}{\hat{u}}\right)^2 - \langle 0|T_{\hat{v}\hat{v}}|0\rangle_{\mathrm{ren}}\left(1 - \frac{r_{\mathrm{g}}}{\hat{v}}\right)^2\right].$$

Note that the energy flux (33) that the detector measures consists of two terms of distinct physical nature: a term due to the polarisation of vacuum (it is zero for the trajectory $r = \mathrm{const}$) and a term related to the radiation going to infinity. If we consider only the second term, then the components $\langle \hat{0}|T_{\mu\nu}|\hat{0}\rangle_{\mathrm{ren}}$ in Eqn (33) should be replaced with $\langle \hat{0}|T_{\alpha\beta}|\hat{0}\rangle_{\mathrm{ren}} - \langle 0|T_{\alpha\beta}|0\rangle_{\mathrm{ren}}$, where the second term is calculated similarly to the first one (see Section 3) but with the use of other coordinates corresponding to the out-vacuum $|0\rangle$:

$$u = t - r_*(r) = -r_*(-\hat{u}) ,$$

$$v = r_*^{-1}[t + r_*(r)] = \hat{v} .$$

Simple manipulations yield the result of Eqn (29).

The energy which is emitted when the metric is shaken up can be calculated by means of the formula

$$\mathrm{d}E = \langle \hat{0}|T_{\mu\nu}|\hat{0}\rangle_{\mathrm{ren}} \xi^{\mu}\, \mathrm{d}\Sigma^{\nu} ,$$

where ξ is a time-like Killing vector present in the region $t > 0$, $r > r_{\mathrm{g}}$: $\xi^{\hat{u}} = 1 + r_{\mathrm{g}}/\hat{u}$, $\xi^{\hat{v}} = 1 - r_{\mathrm{g}}/\hat{v}$,

$$\mathrm{d}\Sigma_{\mu} = \sqrt{-g}\, \varepsilon_{\mu\nu} \frac{\partial x^{\nu}}{\partial r}\, \mathrm{d}r$$

is an elemental area of the surface $t = \mathrm{const}$ [11], and $\varepsilon_{\mu\nu}$ is the skew-symmetric tensor in the two-dimensional space. Simple calculations (see Section 3) yield the results of Eqn (32).