Jubilee scientific session of the Division of General Physics and Astronomy and of the Division of Nuclear Physics of the Russian Academy of Sciences (14–15 June 1995), celebrating the centenary of the birth of I E Tamm

A jubilee scientific session of the Division of General Physics and Astronomy and of the Division of Nuclear Physics of the Russian Academy of Sciences, celebrating the centenary of the birth of I E Tamm, was held on 14-15 June 1995 at the P N Lebedev Institute of the Academy. The following papers were presented at the session:

(1*) **Feinberg E L** "Development of physics in our country and I E Tamm";

(2) Romanov Yu A "Reminiscences about a teacher";

(3) Tamm E I "Igor' Evgen'evich at home and at work";
(4) Ritus V I "Snapshots from I E Tamm's life. Twenty

years in close contact"; (5) **Bolotovskii B M** "I E Tamm's work on electro-

(5) **BOIOTOVSKII B M** "TE Tamm's work on electrodynamics";

(6*) Maksimov E G "Microscopic calculations of the photon spectra of crystals";

(7) **Kadomtsev B B** "I E Tamm and controlled thermonuclear fusion";

(8) **Kirzhnits D A** "Pulsars and rotation of a superfluid liquid";

(9*) **Gurevich A V** "Large-scale structure of matter in the Universe. Analytic theory";

(10) Chernavskii D S "I E Tamm and biological sciences";

(11*) Volkov B A "Spectra of defects and impurities, and their magnetic properties in narrow-gap IV-VI semiconductors";

(12) Kadyshevskii V G 'Field theory with curved momentum space: history, state of the art, future trends";

(13) **Vasil'ev M A** "Gauge theories of higher spins and rigorously solvable models in quantum mechanics".

The papers presented at the jubilee session and identified by an asterisk are represented in this issue of *Uspekhi Fizicheskikh Nauk* by monographic reviews. A brief summary of one of the papers is given below.

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Pulsars and rotation of a superfluid liquid

D A Kirzhnits

Igor' Evgen'evich Tamm was unable to judge the importance of the discovery of pulsars: a report of the

Uspekhi Fizicheskikh Nauk **165** (7) 829–830 (1995) Translated by A Tybulewicz discovery arrived in February 1968, when he was seriously ill. He did not work specifically on superconductivity and his contribution to this topic was limited to introduction of the term 'roton' for a short-wavelength excitation in helium. Nevertheless, the discovery of pulsars, which can be regarded as giant 'drops' of nuclear matter, happened several years earlier. I am sure it would have interested Igor' Evgen'evich. I would like to think that he would be interested also in the paradoxes of rotation of a superfluid liquid, described below and related to pulsar physics.

The conclusion of dissipation-free motion of matter in the core of a pulsar follows from the anomalously long relaxation time of its period after a disturbance. This implies an anomalously weak dynamic coupling between the core (containing the bulk of the mass of the star) and the crust of a pulsar (the rotation of which determines the period). This explanation can be accepted if other (apart from viscosity) mechanisms of coupling between the core and crust are sufficiently weak. These mechanisms include the action on the core of specific general-relativity magnetic-like forces generated by rotation of the crust and leading to the familiar Lense-Thirring effects. An estimate of this mechanism cannot be obtained without answering first a more general question: does a superfluid liquid rotate in the Lense-Thirring field of a slowly rotating vessel (like a superconducting liquid in a magnetic field) or does it remain at rest (like liquid helium)? This is a far from simple question, which I shall now discuss.

Application of simple qualitative arguments leads to answers to this question which are of mutually exclusive nature.

On the one hand, the answer depends on which gradient of the wave function of the condensate — 'lengthened' (in the gauge theory sense) or usual — enters the expression for the current vector. For example, for a superconductor and for helium (see above) the gradient has, respectively, the form $\nabla - ieA$ and ∇ , and the superfluid velocity v_s is $(\nabla \alpha - eA)/m$ and $\nabla \alpha/M$ (α is the phase of the wave function and $\hbar = c = 1$). In the axisymmetric case this gives, respectively, the London-London equation and the condition that the liquid is unentrainable:

$$v_{\rm s} = -\frac{e}{m}A , \qquad (1a)$$

$$v_{\rm s} = 0. \tag{1b}$$

However, in the general theory of relativity the 'lengthening' involves replacement of the usual derivative with the covariant one, which when acting on a scalar order parameter is identical with the usual derivative. Therefore, the Lense-Thirring forces are incapable of inducing rotation of a superfluid liquid [see Eqn (1b)].

On the other hand, however, there is an analogy between the equations of the general theory of relativity (in the case of weak fields) and electrodynamics, which is apparent in particular in the replacement $eA \rightleftharpoons mg$, where g_{ik} is the metric tensor and $g_{\alpha} = -g_{0\alpha}$. This leads to an analogy between the post-Newtonian physics of rotation of a superfluid liquid and the electrodynamics of superconductors. In particular, the London-London equation (1a) corresponds to the general relativity equation of B S De-Witt

$$v_{\rm s} = -g \,, \tag{2}$$

leading to the opposite conclusion that rotation of a superfluid liquid must take place in the field of the Lense-Thirring forces.

Above we did not distinguish the covariant and contravariant components of a vector. In the general theory of relativity, however, not only are these two components not identical, but more than that: one of them may vanish while the other remains finite. This is the root of the contradiction which we are discussing here.

Eqn (1b) relating to the phase gradient applies to the *covariant* component of the 4-velocity (the coordinate x^3 is the azimuthal angle)

$$v_{s3} = 0.$$
 (3)

The velocity occurring in the DeWitt equation (2), however, is the *contravariant* component of the 4-velocity $u^i = (1, 0, 0, v^3)$ (for a weak field) defined by the relationship $g_{3i}u^i = 0$:

$$v_s^3 = \frac{g_{03}}{g_{33}}.$$
 (4)

The disagreement between expressions (3) and (4) is the unavoidable consequence of the inequality $g_{03} \neq 0$. These features of the rotation of a superfluid liquid in the field of the Lense-Thirring forces manifest themselves physically as follows.

Relationship (3) leads to disappearance of the corresponding field source g_{03} (energy-momentum tensor T_{03}) in Einstein's equations. Therefore, rotation of a superfluid liquid, described by expression (4), does not create the Lense-Thirring field and the only source of this field is rotation of the normal component of matter. The contribution of the rotation of a superfluid liquid to the angular momentum of the system disappears at the same time. Consequently, there are no dynamic effects of such rotation which would be of the first order in the angular velocity of rotation of an external body. On the other hand, in the second order in terms of the angular velocity the rotation of a superfluid liquid gives rise to a meniscus on the free surface of this liquid: the square of the velocity given by expression (4) then occurs in the relevant Bernoulli equation.

The physical relationships formulated above can be understood if we bear in mind that in the general theory of relativity the rotation of a massive body drags with it an inertial reference system. This means that there is a system which rotates relative to the initial (Galilean at infinity) system in which the Lense-Thirring force compensates exactly the Coriolis force (inertia in the first order in terms of the angular velocity). The velocity of rotation of such an inertial system is exactly equal to the quantity given by expression (4). Therefore, a superfluid liquid is at rest relative to the initial reference system (from the point of view of the covariant velocity) and also relative to the inertial system (from the point of view of the contravariant velocity). This state of rest relative to the inertial system is responsible for the absence of dynamic manifestations of rotation.

It is important in that in the second order in respect of the angular velocity this inertial property is lost: the corresponding gravitational forces cannot compensate the centrifugal force. It is not surprising that in this (second) order there are dynamic manifestations of rotation (such as the meniscus mentioned above).

A detailed discussion of these topics can be found in the paper by Andreev A Yu, Kirzhnits D A, Yudin S N *Pis'ma Zh. Eksp. Teor. Fiz.* **61** 825 (1995) [*JETP Lett.* **61** 846 (1995)] and in a paper by Kirzhnits D A, Yudin S N 'Paradoxes of rotation of a superfluid liquid'' submitted to *Uspekhi Fizicheskikh Nauk*.