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PACS numbers: 12.90.+b; 12.38.–t

Theoretical search for collective effects in multiparticle production

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Contents

Abstract. The properties of QCD vacuum and of the confinement of quarks and gluons certainly influence the multiparticle production processes. Some phenomenological attempts at the consideration of related collective effects and the possibilities of their experimental detection are briefly discussed. Particular consideration is given to the correlation characteristics of pion systems, statistical and hydrodynamic analogies, the problem of phase transition from a quark – gluon plasma to a multipion state, and the possible modifications of the evolution equations of the quark – gluon jets. The presentation is somewhat simplified and could be interesting for those just entering the field.

Foreword

This review is not a traditional one, as can be seen from its title. Usually a review contains a series of original papers providing a relatively exhaustive presentation. Here we are attempting to describe certain directions of the theoretical search for the collective effects in particle interactions at high energies. Using the modern terminology one could somewhat conventionally unify these directions and speak about some phenomenological attempts to find nonperturbative effects which cannot be described by the quantum chromodynamics (QCD) perturbation theory. They are related in particular to the QCD vacuum structure and the confinement of quarks and gluons. Although the applied methods and the level of the results vary drastically, we have decided to give their brief description in the hope that their comparison could be useful for the development of new ideas and will lead to new suggestions. This is the reason for the presentation not being detailed. We mention only the final results and for the detailed explanations and calculations the reader should turn to the original papers. Our goal is to give a general perspective of several parallel approaches.

Additional motivation for this is a recollection of how the lively Igor' Evgen'evich Tamm, to whose centenary this volume is dedicated, always faced new ideas and participated in their discussion. Moreover, the last paragraph of the review is directly related to the work of I E Tamm on the development of electron – photon showers in the medium and on Cherenkov radiation, because there we discuss the hypothetical analogous effects in quark – gluon jets.

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Received 14 March 1995
Uspekhi Fizicheskikh Nauk **165** (7) 759 – 772 (1995)
Submitted in English by the authors; edited by J R Briggs

1. Introduction

A collision of elementary particles or nuclei at very high energies is usually accompanied by the production of many new particles. Multiparticle production processes were discovered first in cosmic rays and then in accelerators. Their origin is a strong interaction of colliding particles which, according to modern understanding (see Refs [1 – 6]), is described as an interaction of quarks and gluons in the framework of the QCD.

Owing to the asymptotic freedom property of QCD, when the coupling constant becomes small at small distances, an approach based on the QCD perturbation theory turned out to be extremely fruitful in the study of hard processes with large momentum transfer. Moreover, although it seems amazing, it turns out to be possible to describe a number of properties of the soft processes by taking into consideration higher-order contributions in perturbation theory and conservation laws.

At the same time the problem of translating from the language of quarks and gluons into that describing the experimentally detectable hadrons is still solved either at the axiomatic or at the model level. In the first case one usually exploits the hypothesis of a local parton – hadron duality, when the momentum distributions of partons and hadrons are identified up to a multiplicative factor. This hypothesis is supported both by theoretical considerations [the formation of colourless parton clusters (preconfinement)] and by a number of experimental facts. The model approach is usually used in Monte Carlo modelling of a transition from partons to hadrons.

Despite the impressive successes of the perturbative approach, one should not forget that the problem of confinement of colour objects, quarks, and gluons, still remains unsolved. In particular the influence of confinement on the properties of hadrons in multiparticle production processes could turn out to be nontrivial. One cannot exclude the possibility that the collective aspects of the system's behaviour as a whole and the specific properties of the quark – gluon QCD vacuum could show up. For a theoretical description of such features, the string models taking into account confinement and methods of statistical physics (or, more generally, macroscopic approaches) are usually more relevant than the perturbative calculations. The study of the collective excitation modes of a hadron (quark – gluon) medium is a very complicated problem and is still at the infancy stage.

To give an example of the possible effects let us turn to the analogies with the electrodynamics of continuous media. An electric charge (e.g. an electron), crossing the boundary of two electrically neutral media with different refractive indices, radiates photons. The radiation properties are determined precisely by this difference, which describes the different collective properties of the two media. At sufficiently high electron velocities, Cherenkov radiation might also appear. Analogously, in the process of interaction of two hadrons or nuclei, the quarks that are hidden in one of them can in principle 'see' the target as a whole, i.e. can radiate as in the case when they pass through a colour neutral medium. This is of course possible only at relatively small momentum transfer.

At the same time one should not forget that this analogy is not complete. The radiation of a photon does not lead to a change of the charge of the radiating particle. On the contrary, the colour current changes in the process of gluon emission, because the gluons are themselves colour objects. The space – time analysis of the process is especially relevant in that situation [6].

Collective interactions can also be important in the case when, in the process of interaction, a quark – gluon plasma is formed. Such a possibility has been discussed for a long time. The main criterion is the appearance of a sufficiently high hadronic density in the collisions of heavy nuclei.

The above-listed examples can only serve as guidelines in attempting to apply the methods of statistical physics to the problem of multiparticle production. Here it is appropriate to stress that in the statistical physics as well as in the study of multiparticle production processes one often applies the same mathematical method. It is based on the calculation of correlation characteristics in the phase space. The similarity of the electro- and chromodynamics Lagrangians allows one to use certain analogies. At the same time their difference leads to important distinctions, one of them having been mentioned above. This shows up, in particular, in the similarity and distinctions of the corresponding equations.

We shall try to give a brief review of the theoretical attempts to analyse the problem of collective effects in particle physics. Unfortunately, it is difficult to choose a main stream of thought. The development of these ideas follows several directions, slowing down or completely stopping at certain obstacles. We shall begin with a brief review of the history of the subject and then give a more thorough discussion of recent ideas.

2. Early history of the problem. Statistical and hydrodynamic models

Processes in which a large number of particles is produced were discovered in cosmic ray showers more than 60 years ago. The first attempt to describe them with the ideas of statistical physics and hydrodynamics dates back to Heisenberg [7]. Somewhat later, analogous attempts were made by Wataghin [8]. An active discussion of this approach began, however, after the appearance of the paper by Fermi [9] in which he introduced a particular statistical model of the multiparticle production processes in nuclear collisions (a detailed description of the model and its further development is given in Refs [10, 11]). According to the main assumption of the model the process of multiparticle production occurs via the creation of a unique system, in which a thermodynamic equilibrium is established. The distributions of secondary particles are therefore described by the thermodynamic formulas for blackbody radiation. The legitimacy of such an assumption was discussed many times (see Ref. [12] and Refs [10, 11]). It was agreed that if models of this type are valid, this takes place only in the domain of relatively low energies. At higher energies (and higher multiplicities) the interaction of generated particles can lead to such an expansion of this unified system that can be described by the equations of hydrodynamics. This idea was put forward by Landau [13] and was later widely exploited in the papers by many authors (see Ref. [14]). Here it is worth mentioning that the possibility of applying the thermodynamic formulas for the description of an ensemble of strongly interacting particles at realistic energies and multiplicities is of course far from being evident. The critical analysis of the basics of this approach can be found, for example, in the interesting paper, Ref. [15].

According to the basic postulates of the statistical physics, the Fermi model used the assumption of a dominant role of the phase space in the probability of a final state with n particles, when a quantum-mechanical matrix element is just a normalisation factor. Let us write a general expression for this probability:

$$P_n \sim \int |A_n|^2 \delta^4(\Sigma p_{\mathrm{i}} - \Sigma p_{\mathrm{f}}) \prod_f \mathrm{d}^3 p_{\mathrm{f}} \,, \qquad (1)$$

where A_n are the transition amplitudes, and p_{i} and p_{f} are the four-momenta of initial and final particles. If we consider the transition amplitudes A_n as being independent of the final momenta p_{f}, the integral over the phase space will factorially vanish with growing n, because the mean particle momentum will be proportional to $1/n$, i.e.

$$P_n \sim n^{-(3n-4)} \qquad (2)$$

at $n \to \infty$ and fixed total energy E (the particle mass is neglected). Factorial behaviour of an analogous type is known in statistical physics as well. However, if the assumption of the weak amplitude dependence on the final momenta and particle number n can be justified, this can happen only at low energies. With growing energy and number of created particles the strong interaction in such a quasiclassical system forces one to describe its evolution rather as the hydrodynamic expansion of a blob of the nuclear matter. In the process of its expansion the temperature drops and the blob decays into final particles. This idea immediately explains an important experimentally known fact of the limited transverse momenta of the particles. Let us mention that quantum field theory is still unable to describe this phenomenon of the cut-off of large transverse momenta.

If one tries to take this into account phenomenologically in the general relation (1), it is necessary to note that a 'complete' theory should provide this transverse momenta cut-off in the integral (1) through the corresponding behaviour of the amplitudes A_n. If (according to experiment) this cut-off happens at finite values of the transverse momenta, then instead of the estimate (2) one gets a much slower decrease with growing n:

$$P_n \sim n^{-(n-2)} . \tag{3}$$

In this case the phase space does not already have the form of a $3n$-dimensional sphere and takes the form of an n-dimensional cylinder in this space† (if one disregards the restrictions imposed by the conservation laws that slightly deform this cylinder).

Another substantial factor, which is not accounted for in the statistical approach to multiparticle production, is the rapid growth of the number of field theory diagrams contributing to the amplitude A_n at large n. For example, for the process of transition from two gluons to n gluons this growth even exceeds factorial one (see Table 1 in Ref. [16], where the following numbers can be found: at $n = 2$ one has 4 diagrams, and at $n = 8$ their number already reaches 10525900). Of course, the relative phases of different contributions are highly significant and it is currently impossible to conclude how strongly the growth of a number of diagrams with increasing n affects the estimate (3). However, this factor cannot be neglected.

At high multiplicities this can lead, for example, to a change of an expansion parameter in quantum chromodynamics, describing the strong interactions, from the coupling constant α_s to its product at a factor of the order of n, and it will turn out that

$$|A_n|^2 \sim n!\alpha_s^n \sim (n\alpha_s)^n . \tag{4}$$

As a result, the law of decreasing of P_n at large n will change from the factorial (of Poisson type) distribution to the exponential one or to the one close to it. Precisely this type of behaviour is currently being discussed in quantum chromodynamics [17].

Here the appearance of a new expansion parameter is clearly seen when one analyses the corresponding distributions over n with the help of their moments [18, 19]. It is easy to see that the processes with high multiplicity n determine the moments of multiplicity distribution of a high order q. For example, only the processes having the multiplicity $n > q$ contribute to the factorial moments of order q. One finds that precisely the quantity $q\alpha_s^{1/2}$, corresponding to the above-mentioned factor $n\alpha_s$, provides [18] an expansion parameter for the solutions of equations on the generating functions of P_n distributions (see also Ref. [19]).

The appearance of this parameter actually supports the idea that at high multiplicities the collective interaction effects become significant‡ although possibly not describable within the simplest statistical approach. Nevertheless, the mathematical methods applied in the studies of many-body systems (be it a statistical physics problem or a multiparticle production process) are the same and are based on the analysis of the distributions of particles and their correlations in the system under investigation. Thus we begin with its description.

3. Correlation characteristics and methods of description of multiparticle systems

As was already stressed, the approaches used in the analysis of the properties of multiparticle systems are quite similar both in statistical physics (to mention one particular example, in laser physics) and in the physics of multiparticle production at high energies. At fixed given particle number n in some phase space volume Ω the system can be characterised by the probability density $W_n(1, 2, ..., n)$, where the arguments $1, 2, \ldots, n$ denote the corresponding (generally speaking, multidimensional) coordinates of these particles in the phase space. Such an approach is called exclusive.

However, it is often more convenient to use the so-called inclusive approach, where the total number of particles in the system is not fixed and one considers just its q-particle characteristics. This is the way followed by the majority of the experiments studying multiparticle production. In this case the inclusive densities ρ_q are related to the experimentally measured inclusive differential cross-sections as follows:

$$\rho_q(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_q) = \frac{1}{\sigma_{\text{in}}} \frac{d\sigma}{d^3p_1 \ldots d^3p_q} , \tag{5}$$

where σ_{in} is the total cross-section of inelastic processes, and the distribution over the 3-momenta of the final particles is considered. Of course, the less detailed distributions are also used, when one integrates over the certain momenta components. The integration of the inclusive densities over the total phase space volume Ω gives the nonnormalised factorial moments:

$$\begin{aligned} \tilde{F}_q &\equiv \int_\Omega d^3p_1 \ldots \int_\Omega d^3p_q \rho_q(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_q) \\ &= \langle n(n-1) \ldots (n-q+1) \rangle \\ &= \sum_0^\infty n(n-1) \ldots (n-q+1) P_n = \langle n \rangle^q F_q , \end{aligned} \tag{6}$$

where P_n is the probability of finding an n-particle state of a system (the so-called particle multiplicity distribution), and F_q are the normalised factorial moments. The inclusive densities of order q are given by a sum of the exclusive density of the same order and the integrals from the

†In this connection one often speaks of a 'cylindrical' phase space.

‡It is interesting that the resulting distributions are not infinitely div-

exclusive densities of higher order over all variables that are not taken into account, i.e. in formal notation

$$\rho_q(1,\ldots,q) = W_q(1,\ldots,q) + \sum_{m=1}^{\infty} \frac{1}{m!}\int_{\Omega} W_{q+m}(1,\ldots,q,q+1,\ldots,q+m)\prod_{j=1}^{m} \mathrm{d}(q+j) \ . \quad (7)$$

The inclusive densities ρ_q are nonvanishing even if the particles are statistically independent. Therefore (analogously to the cluster decomposition in statistical mechanics) it is convenient to introduce the so-called cumulant correlation functions C_q, which vanish in the case when the particles are completely statistically independent [20–22]. The general formulas relating them to the inclusive densities are quite cumbersome (see e.g. Refs [23, 24]). Therefore we shall reproduce only the formulas for the cases $q = 2$ and $q = 3$:

$$C_2(1,2) = \rho_2(1,2) - \rho_1(1)\rho_2(2) \ , \quad (8)$$

$$C_3(1,2,3) = \rho_3(1,2,3) - \sum_{(3)} \rho(1)\rho_2(2,3) + 2\rho(1)\rho(2)\rho(3) \ , \quad (9)$$

where it is clearly seen that the contributions from lower-order correlations are subtracted from the higher-order ones (the notation $\sum_{(3)}$ stands for the summation over three possible particle permutations).

All these results can be obtained in a unified form with the help of a generating functional,

$$G(z) = 1 + \sum_{q=1}^{\infty} \frac{1}{q!}\rho_q(1,\ldots,n)\, z(1)\ldots z(q)\prod_{j=1}^{q} \mathrm{d}(j) \ , \quad (10)$$

where $z(j)$ is a subsidiary function depending on $\boldsymbol{p}_j$. Then

$$\rho_q(1,\ldots,q) = \frac{\delta^q G(z)}{\delta z(1)\ldots\delta z(q)}\Big|_{z=0} \ , \quad (11)$$

$$C_q(1,\ldots,q) = \frac{\delta^q \ln G(z)}{\delta z(1)\ldots\delta z(q)}\Big|_{z=0} \ . \quad (12)$$

At $z = \mathrm{const}$ the generating functional becomes a generating function of the multiplicity distribution, and the variational derivatives in Eqns (11) and (12) become ordinary ones, which leads to the nonnormalised factorial and cumulant moments of this distribution, respectively. The normalised factorial moments F_q and the cumulants K_q are given by the formulas

$$F_q = \frac{1}{\langle n\rangle^q}\,\frac{\mathrm{d}^q G(z)}{\mathrm{d}z^q}\Big|_{z=0} \ , \quad (13)$$

$$K_q = \frac{1}{\langle n\rangle^q}\,\frac{\mathrm{d}^q \ln G(z)}{\mathrm{d}z^q}\Big|_{z=0} \ , \quad (14)$$

and the multiplicity distribution P_n is given by

$$P_n = \frac{1}{n!}\,\frac{\mathrm{d}^n G}{\mathrm{d}z^n}\Big|_{z=-1} \ , \quad (15)$$

i.e. it is related to the generating function by the formula

$$G(z) = \sum_{n=0}^{\infty}(1+z)^n P_n \ . \quad (16)$$

We see that, differentiating the generating function, one can compute both inclusive and exclusive characteristics of a system depending on the point z at which the derivatives are taken.

As is clear from all the above, the direct computation of Feynman diagrams within the perturbative approach in quantum field theory is not well suited to the description of the multiparticle production processes. On the one hand the number of diagrams grows catastrophically with the increasing particle number; on the other, in the perturbative approach one considers the matrix elements of the scattering operator for the transitions between the states having a fixed number of particles. Therefore even if a calculation at fixed multiplicity is made, one gets the exclusive quantities, whereas according to Eqn (7) for computing inclusive characteristics one has to perform the infinite summation of the exclusive probabilities. One usually tries to bypass these difficulties [1, 6] either by resumming an infinite number of specially chosen (leading logarithms, etc.) terms in the perturbative series, or using the equation for the generating functions (their validity is again proved by comparing the results with the same series calculated up to a definite order in the coupling constant). We should note that these approaches turned out to be quite successful and fruitful in predicting and describing many characteristics of multiparticle production processes in QCD (see Refs [1, 6]). However this approach leaves unsolved the basic question of whether the perturbation theory can reproduce all the effects corresponding to an interaction Lagrangian in principle (in particular, the possible collective effects). To shed light on the latter, it seems more promising to exploit the analogies with statistical physics which we shall try to discuss in the subsequent sections.

The inadequacy of the perturbative calculations of given subsets of Feynman diagrams in the problem of finding the inclusive characteristics of the multiparticle production processes is due to an overwhelming complexity of the scattering operator in the basis of the eigenfunctions of the particle number. Therefore, attempts are made to find a more adequate representation. A well-known example of this kind is laser radiation, where it is preferable to use the basis of coherent states. Below we shall consider analogous attempts in the physics of multiparticle production processes. Of course, in this case one often has to rely upon specific models rather than upon the initial QCD interaction Lagrangian.

In the study of correlations in the multipion systems it also proved fruitful to follow a direct analogy with hydrodynamics. The self-similarity of vortices in hydrodynamics corresponds to a growth of correlation functions at small scales. This led to the introduction of the notions of intermittency and fractality, giving rise to the powerlike growth of the factorial moments (13) with decreasing phase space volume, in particle physics. We shall not discuss these questions (a detailed review can be found in Ref. [24]).

4. Feynman – Wilson liquid. Statistical analogies

Each individual event in particle interactions at high energies can be fully characterised by specifying (apart from the masses and quantum numbers) the three-dimensional momenta of the secondary particles. The endpoints of these vectors define a set of points, lying as a rule in the above-mentioned cylindrical phase space. The correlations in the position of these points are defined by the interaction Lagrangian and conservation laws. A large enough set of such events can be considered as a statistical ensemble. In particular, Feynman [25] put forward the analogy with a usual liquid by assuming the presence of short-range correlations in the ensemble. This idea was further developed by Wilson [26], and so the ensemble is called a Feynman – Wilson liquid. Let us stress that in contrast to the Fermi model, one does not develop here a new statistical interaction model, but attempts to consider the statistical properties of the ensemble of particles created in the interaction process at high energies using the analogy with statistical mechanics. This topic is discussed in a large number of publications (from simple analogies to specific models, see Refs [11, 27 – 37, 110] and references therein). As the exclusive probabilities P_n characterise the volume of the phase space filled with the ensemble of n-particle events, they play the role of a partition function for the canonical ensemble. The generating function defined by the relation (16) (in the general case it is a functional) is analogous to a grand canonical partition function. The role of the volume is played by the maximal rapidity

$$Y = \ln \frac{s}{m^2}\,, \tag{17}$$

where s is the total energy squared in the centre-of-mass system (CMS), m is a particle mass. It characterises the size of the 'cylinder' in the direction of the longitudinal momentum. The variable z is related to activity [or the chemical potential $\mu = \ln(z+1)$] in statistical mechanics. Thus, one can calculate the 'pressure' in such a liquid in the 'thermodynamic limit' $Y \to \infty$ as

$$p(z) = \lim_{Y\to\infty} \frac{\partial \ln G(z,Y)}{\partial Y}\,, \tag{18}$$

and its density as

$$\rho(z) = \lim_{Y\to\infty} \frac{\partial}{\partial z} \frac{\ln G(z,Y)}{Y}\,, \tag{19}$$

and thus determine the equation of state (for details see Refs [32 – 34]).

The analysis of experimental data on e^+e^- annihilation and nondiffractive hadron – hadron processes at high energies within this approach was carried out in Ref. [34]. Using the formula (16) and the data on P_n one can calculate the generating function $G(z,Y)$ at

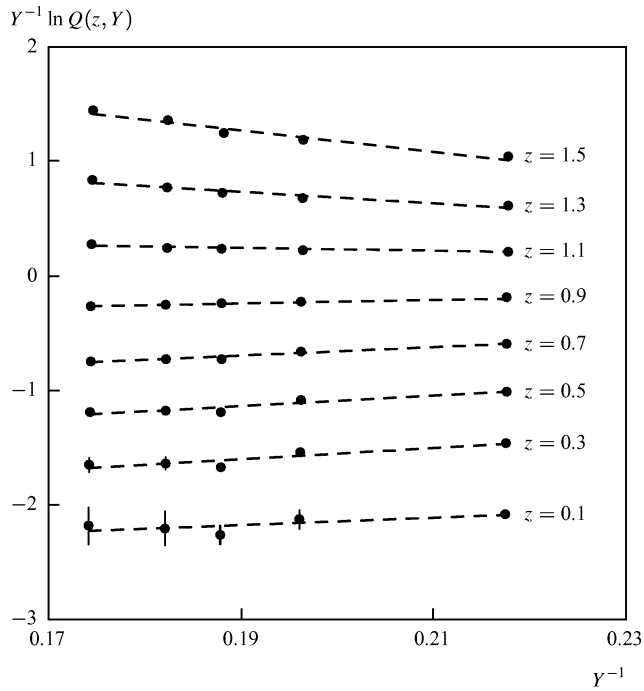

Figure 1. The best approximation of the functions $\ln G(z,Y)/Y$ depending on the energy Y at various activities z presented in Ref. [34]. Let us mention that the data from the experiments in the Z^0-boson region are sufficiently well reproduced by the extrapolation of the straight lines drawn in the figure (see Ref. [41]).

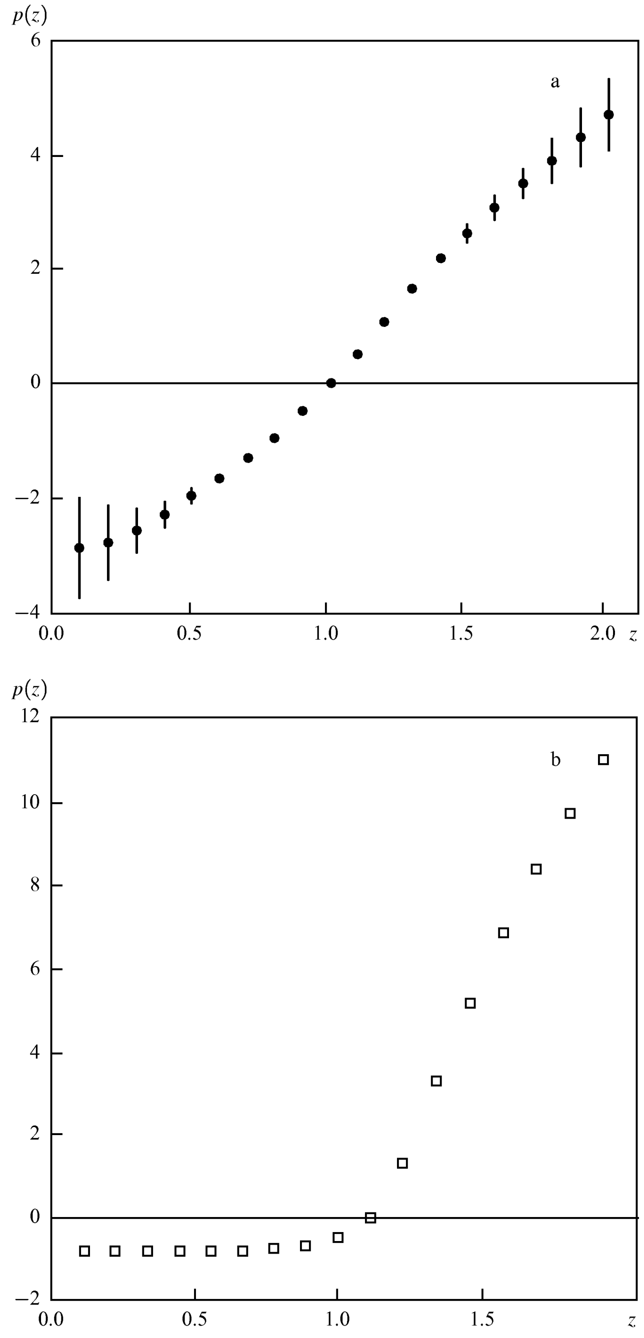

Figure 2. 'Experimental' values extrapolated according to the procedure of Fig. 1 to the point $Y^{-1} = 0$ lead to the presented functional dependencies of pressure [calculated from formula (18)] on the activity z for (a) electron – positron annihilation, (b) nondiffractive hadron – hadron collisions. The figure is taken from Ref. [34].

different energies Y. A linear extrapolation to $Y \to \infty$ (see Fig. 1) allows one to compute the slopes for the different values of z (i.e. for different chemical potentials). The resulting dependence of pressure on the chemical potential (see Figs 2a and 2b) is especially interesting in relation to the problem of the phase transition in such a system. The existence of a phase transition should show up in the discontinuities of the pressure derivatives over z. From Figs 2a and 2b it follows that in e^+e^- processes such discontinuities are absent, whereas in the hadron–hadron interaction the constancy of $p(z)$ at $z \leqslant 1$ turns into a rapid growth at large z. Such a difference in the behaviour of the pressure is interesting in itself, although a conclusion on the existence of some phase transition would be clearly premature.

This problem could be approached from a different perspective [36, 37] by studying the behaviour of the zeros of a grand canonical partition function in a complex z plane as a function of the number of particles in the system under consideration. The point is that at finite energy one always has a certain maximally possible multiplicity N . Therefore, the sum in the formula (16) terminates at this value of N, i.e. the generating functional becomes a polynomial form z of degree N and thus has N zeros in the complex plane. Lee and Yang have used these properties of a partition function [21, 36, 37] and formulated a method of locating the phase transition point by finding a point on the positive real z axis to which the partition function zeros converge at large N. The analysis of the model e^+e^- events at the energy 1000 GeV performed in Ref. [38] has shown that the zeros of the partition function lie on the circle† in the complex z plane and really converge to a real z axis at growing N. The real experimental data on e^+e^- and $p\bar{p}$ interactions at high energies also lead to the zeros of a generating function placed on the circle [40]. In the limiting point to which the locations of the zeros converge one should have a singularity of a total generating function given by the formula (16) (i.e. at $N \to \infty$). Methods of investigating the character of this singular point were proposed in Ref. [41]. The question of which cases the appearance of such a singularity corresponds to the presence of a phase transition of some type in the particle production processes still remains open.

There were many theoretical attempts to describe this transition at the phenomenological level (see e.g. Refs [29–33, 35, 42–47] and sections 4.2.4 and 4.2.5 in Ref. [24]). As in all the phenomenological theories of critical phenomena the main problem is a choice of a corresponding order parameter and such an expression for the partition function that at small values of this parameter the analytically solvable Ginzburg–Landau Hamiltonian is recovered‡. For the order parameter one usually takes either a certain mean field (with the possible Gaussian fluctuations) [29, 33] or a fluctuating field of inclusive distributions [35]. In the latter case the order parameter is local in the momentum space, which actually corresponds to an intrinsically non-local approach in the usual space. In particular, even the 'free' Ginzburg–Landau Hamiltonian leads here to strong correlations [35]. Within such a picture one can find the correlation characteristics described in section 3 and also details of the behaviour of pressure, density, etc. Although the indications of the presence of singularities in these quantities and also in the generating functionals were obtained, the specific values strongly depend on the particular assumptions.

†More complicated configurations of zeros were also discussed [39].

‡Although, the same form of the Hamiltonian can be assumed at large values of the order parameter too.

Thus we shall not present a detailed account of this series of papers, and shall only briefly illustrate the idea with an example of the coherent states [30], which by definition realise the eigenstates of the annihilation operator

$$a(\boldsymbol{p})|\Pi\rangle = \Pi(\boldsymbol{p})|\Pi\rangle , \tag{20}$$

where $\Pi(\boldsymbol{p})$ is some (generally speaking, complex) function of $\boldsymbol{p}$. As has been already mentioned, the coherent states may provide a more convenient representation for the operators, characterising the multiparticle production process, than the particle number representation (in analogy with laser physics). Of course, this does not mean that the created system is always in the coherent state.

In this case the inclusive density ρ_q [Eqn (5)] takes the form

$$\rho_q(1,\ldots,q) = \int \delta\Pi |\Pi(1)|^2 \ldots |\Pi(q)|^2 \exp\left[-F(\Pi)\right] , \tag{21}$$

where $F(\Pi)$ is an arbitrary functional analogous to the free energy in statistical physics. Here the generating function, which follows from [Eqn (10)] at $z=$ const, is written as

$$G(z) = 1 + \sum_{q=1}^{\infty} \frac{z^q}{q!} \int_{\Omega} \rho_q \prod_{j=1}^{q} \mathrm{d}(j)$$
$$= \frac{1}{N} \int \delta\Pi \exp\left[-F(\Pi)\right] \exp\left[z \int \mathrm{d}\boldsymbol{p} |\Pi(\boldsymbol{p})|^2\right] , \tag{22}$$

where the normalisation N is fixed by the relation:

$$N = \int \delta\Pi \exp\left[F(\Pi)\right] . \tag{23}$$

In the absence of the complete dynamical theory, which would allow one to calculate $F(\Pi)$, one is tempted to use a phenomenological ansatz. An analogy with the phenomenological Ginzburg–Landau theory of superconductivity leads to an expression for $F(\Pi)$ of the form

$$F(\Pi) = \int \mathrm{d}\boldsymbol{p} \left[a|\Pi(\boldsymbol{p})|^2 + b|\Pi(\boldsymbol{p})|^4 + c\left|\frac{\partial \Pi}{\partial \boldsymbol{p}}\right|^2 \right] . \tag{24}$$

The calculation of the thermodynamical quantities is then performed in accordance with the formulas (18), (19). Different models correspond to a different choice of the parameters a, b, c and are considered in Refs [29, 30, 33, 35]. As usual, the phase transition point is the one in which the parameter a goes through zero.

In this case it is possible to describe a large variety of the states of a system, from coherent to chaotic ones. Therefore, the whole approach can be considered either as a convenient parametrisation of the data or as an attempt to find out some dynamical features of processes. It is interesting to note that the location of the singularity of the generating function is sensitive to the choice of parameter. It moves from infinity in the case of a coherent state (Poisson distribution) approaching the point $z = 0$ [where the inclusive distributions are calculated, see Eqns (13), (14)] with increasing chaoticity. At the same time the model of coherent states can itself be modified with the squeezed and correlated states taken into account (see, e.g., Ref. [48]).

5. Quark – gluon plasma and multipion states

The problem of the phase transition became especially acute when there appeared an idea about the formation of the quark-gluon plasma in those collisions where a high energy density is reached. The natural density scale in hadronic matter is either the average nuclear or nucleon density or that based on purely dimensional arguments, namely, on the value of the QCD cut-off parameter. All these estimates give values of the same order lying in the range from 0.15 to 0.5 GeV fm^{-3}. At much higher densities (exceeding 1 – 2 GeV fm^{-3}) one can expect the appearance of a plasma of quarks and gluons. The estimates show that such energy densities can hopefully appear in nucleus – nucleus collisions at high energies, or in some fluctuations in hadron interactions. As the QCD coupling constant decreases at high temperatures and densities because of the asymptotic freedom property, one can hope that in this domain the weakly interacting system of quarks and gluons (plasma) can be described theoretically.

The evolution of quark – gluon matter with its subsequent transformation into hadrons is one of the most cardinal problems of QCD and demands the application of the methods of many-body theory. The high-temperature plasma behaviour in a certain frequency domain is described by perturbative QCD, but at lower temperatures the description of the system is already given by the nonperturbative calculations on the lattice in QCD (in the vicinity of the phase transition) and the effective theories of meson and baryon fields in the hadronic phase. It is necessary to stress that even in the high-temperature domain the perturbative approach is applicable only in the lowest orders in perturbation theory at high frequencies and allows one to calculate the energy density, pressure, plasmon damping, etc. [49, 50]. However, in the long-wavelength and lower-temperature domain the perturbation theory is not applicable. In the phenomenological analysis, the influence of the long-wavelength modes is often described with the help of the notion of classical sources leading to the coherent states. At the same time the short-wavelength excitations are considered to be responsible for the quantum correlations, which in a thermalised system are averaged with the Planck distribution (see e.g. Ref. [51]). All the higher correlations are often reduced to the products of the two-particle ones (analogously to the coupled pair approximation [52]). The relative contribution of these domains determines the relation between the chaotic and regular components of correlations, which is also often used in the analysis of the experimental data (see e.g., Ref. [52]).

The lattice QCD calculations have revealed the presence of a phase transition between the equilibrium quark – gluon and hadron phases at a temperature which, according to the different estimates, is of the order of 100 – 160 MeV [53 – 56]. Let us note that such a temperature was mentioned as maximally possible for the hadronic phase in the statistical bootstrap models that were developed in the sixties and seventies (see e.g. Ref. [57]). The question concerning the order of the transition is not completely settled. The conclusions often depend even on the size of the lattice. However, the latest investigations [58 – 60] give some grounds to think that in the theory with two massless dynamical quarks the transition is of the second order. At larger numbers of massless quarks it seems to be of the first order. However when the mass of the third (strange) quark grows, the transition again becomes a second-order one for the masses that are even smaller than that of a strange quark. Thus, the existence of such a transition can be taken as a starting point in the construction of the phenomenological models.

The simplest situation from the point of view of its theoretical description would be the formation of an equilibrium quark – gluon plasma, then its expansion and accompanying cooling, the phase transition to the equilibrium hadronic matter, and its subsequent expansion. This particular scenario lies in the foundation of the majority of papers analysing the possible detection of the quark – gluon plasma in the experiments on heavy ion collisions. The central question here is doubtlessly whether the quark – gluon system, formed as a result of a heavy ion collision, has enough time to thermalise. This determines whether we have a hydrodynamical plasma expansion starting from some moment, or if the process of conversion into hadrons is essentially a nonequilibrium one.

Unfortunately a unified approach describing all the stages of the process still does not exist, and the description is quite fragmentary. Most often one deals with particular models of this or that stage. Naturally the status of a problem will affect the subsequent presentation in this review. We shall begin with the description of the hadronic stage of a process. The dominant fraction of the created particles are the pions, which in the first approximation can be treated as practically massless and having relatively low energies in the centre-of-mass system of colliding high-energy particles.

One of the simplest effects could be the influence of the laws of conservation of quantum numbers on the properties of the pion system. Let us consider for definitiveness the process of the collision of two high energy nucleons, where in the final state, apart from the two nucleons, a certain number of pions has been created. The total isotopic spin of

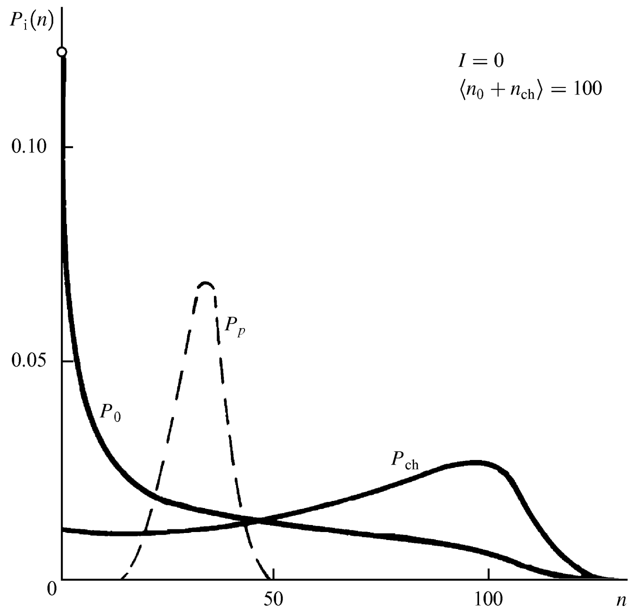

Figure 3. The distributions of the total number of pions, and separately charged and neutral pions in the case of a pion system with zero isotopic spin. The figure is borrowed from Ref. [61].

the pion system is limited, according to the conservation laws, to the values $I = 0, 1, 2$, whereas in the general case it could take values up to $n_{\rm tot}$, where $n_{\rm tot}$ is the total number of pions. This fact significantly affects the pion charge distribution. Even if the distribution in the total pion number $n_{\rm tot}$ is Poissonian (as happens when pions are produced by classical currents), it turns out that the separate distributions of the charged (n_+, n_-) or neutral (n_0) pions are much wider than the Poisson one (see Fig. 3, which is taken from Ref. [61]). The experimental confirmation of this fact could be the 'Centauro' events, where the charged particles are noticeably dominating, or 'anti-Centauro' ones with a large number of neutral pions. Let us illustrate the idea by considering, following the quasiclassical approach of Ref. [62], the production of many pions in a system with zero isotopic spin. The characteristic initial assumption is the possibility of describing the pion system that radiates the final state pions as a classical field [61–69], i.e. that the number of pions per phase space cell is assumed to be big. According to the standard reduction formula, the amplitude of generation of N pions by the source J equals

$$A^{a_1,\ldots,a_n}(k_1,\ldots,k_n) = \lim_{k_n^2 \to m_\pi^2} \int \mathrm{D}\pi^a \int \mathrm{D}\, J^a W[J] \times \exp\left(\mathrm{i}S[\pi] + \mathrm{i}\int \mathrm{d}^4 x \pi^a J^a\right) \prod_{n=1}^{N} \int \mathrm{d}^4 x_n \times \exp(\mathrm{i}k_n x_n)(-\partial_{x_n}^2 - m_\pi^2)\, \pi^{a_n}(x_n) , \tag{25}$$

where the functional integration over J corresponds to the averaging over the characteristics of the pion source. Let us note that the radiation of a classical current exactly reproduces the language of coherent states. The quasiclassical estimate of the amplitude, performed on the assumption of the axial symmetry of the initial interaction and of the isotopic symmetry of the pion system (i.e. of the zero total isospin) [62], leads to the two characteristic conclusions that also appear in other publications on this topic (in particular, such conclusions were reached in the above-mentioned paper, Ref. [61]).

First, only the distribution over the total number of pions is Poissonian. The distributions over the number of neutral and charged pions are much wider (see Fig. 3). For the state with zero isospin the probability of finding $2n$ neutral pions in the system of $2N$ pions has the form [67, 68]:

$$P(n,N) = \frac{(N!)^2 2^{2n}(2n)!}{(n!)^2 2^{2N}(2N+1)!} . \tag{26}$$

At large n, N we have the characteristic distribution [61, 66]:

$$P(n,\, N) \sim \left(\frac{n}{N}\right)^{-1/2} . \tag{27}$$

Second, the conservation of the total isospin leads, for example, to the specific angular correlations between the particles with different charges. Let us give the characteristic formula for the correlation over the azimuthal angle φ, obtained in Ref. [62] for the pions having zero rapidity:

$$\frac{\sigma_{\rm tot}\left[\mathrm{d}\sigma^{\pi^+\pi^-}/(\mathrm{d}k_1\,\mathrm{d}k_2)\right] - 9/10(\mathrm{d}\sigma^{\pi^+}/\mathrm{d}k_1)(\mathrm{d}\sigma^{\pi^-}/\mathrm{d}k_2)}{(\mathrm{d}\sigma^{\pi^+}/\mathrm{d}k_1)(\mathrm{d}\sigma^{\pi^-}/\mathrm{d}k_2)} = \frac{3}{10}\cos^2(\varphi_1 - \varphi_2) . \tag{28}$$

The experimental verification of such predictions is in our opinion very interesting.

We have already seen that only taking into account the isospin conservation within the framework of the quasiclassical approach can lead to dramatic changes of the naive ideas concerning the character of particle multiplicity distributions. From the theoretical point of view the most interesting calculations are the ones attempting to use the effective pion Lagrangians. In principle this gives a chance of getting model-independent predictions.

Before describing the corresponding results, we feel it necessary to make the following comment. The general feature of all the papers using the low-energy model Lagrangians for the description of the distributions of particles and their correlations in the multiparticle production processes is an application of these Lagrangians in the case when the initial energy of the collision is quite high. The question concerning the possibility of neglecting the heavy modes and the interaction between the modes is quite nontrivial. Owing to the practically insurmountable difficulties arising on the way to the complete solution of the problem, it seems reasonable to analyse the consequences of the most radical assumptions concerning the possible dynamical reasons causing the sharp charge asymmetry of the events where a large number of pions is generated.

In the recent literature a 'disoriented chiral condensate' was widely discussed as a possible asymmetry source in the production of charged and neutral pions in some fraction of the events (in particular, in the above-mentioned 'Centauros') [70–77]. Let us recall that the transformation properties of mesons with respect to the chiral transformations are determined according to the corresponding properties of the order parameter, which characterises the spontaneous breaking of chiral symmetry and given, for example, by the average of the bilinear combination of quark fields

$$\varPhi \sim \langle \bar{\mathrm{q}}_{\rm L} \mathrm{q}_{\rm R} \rangle , \tag{29}$$

where $\mathrm{q}_{\rm R(L)}$ are the right (left) states of the massless quarks. For investigating the character of the singularity of thermodynamic functions in the vicinity of the phase transitions, it is desirable to find a solvable model with the same symmetry. Then, according to the universality principle based on the scale invariance near the critical point, the solutions of this model will have the same set of singularities. In such an approach the order parameter (29) can be rewritten in terms of a set of hadron fields having the same symmetry. Therefore, the multipion states become related to the massless quark fields and quark–gluon plasma, i.e. there appears the hope of describing a phase transition between these widely differing phases within such a model. In the realistic case of two massless quark flavours the chiral field can be written in the form

$$\varPhi \sim \sigma \cdot \bar{1} + \mathrm{i}\bar{\tau} \cdot \bar{\pi} , \tag{30}$$

where $\sigma, \bar{\pi}$ are the real fields, $\bar{\tau}$ are the standard Pauli matrices, the π-meson fields $\bar{\pi}$ form an isotriplet and σ is an isosinglet.

In the case of SU(3) algebra the number of such fields is already 18. They form the scalar and pseudoscalar nonets.

The dynamics of these degrees of freedom can be described by the Lagrangian of the linear σ-model

$$L = \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\bar{\pi})^2 - V(\sigma,\, \bar{\pi}) , \tag{31}$$

where V is a potential depending on the combination $\sigma^2 + \bar{\pi}^2$. In the standard version [78] the spontaneous symmetry breaking occurs via the formation of the nonzero vacuum average of the field σ. The isotriplet fields remain massless, i.e. the pions are the goldstones of the chiral group.

Let us now assume [71] that in some region of space the vacuum orientation is different from the standard one, and, for example,

$$\langle\sigma\rangle = f_\pi \cos\theta\,, \quad \langle\bar{\pi}\rangle = f_\pi \bar{n} \sin\theta\,, \tag{32}$$

where $f_\pi = 93$ MeV, and $\bar{n}$ is a unit orientation vector of $\bar{\pi}$. Such an assumption presupposes a specific scenario of the process, which has yet to be studied in detail.

If the field Φ is isotropic with respect to a direction on the 3-dimensional sphere in the 4-dimensional space with the angles defined as

$$(\sigma, \pi_3, \pi_2, \pi_1) = (\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi\sin\eta, \sin\theta\sin\phi\cos\eta)\,, \tag{33}$$

then for the probability distribution of a given state $r \equiv \cos^2\phi$ we have:

$$\int_{r_1}^{r_2} \mathrm{d}r P(r) = \frac{1}{\pi^2}\int_0^{2\pi} \mathrm{d}\eta \int_0^{\pi} \mathrm{d}\theta \sin^2\theta \int_{\arccos r_2^{1/2}}^{\arccos r_1^{1/2}} \mathrm{d}\phi \sin\phi, \tag{34}$$

as was obtained for the first time in Ref. [61] [see Ref. [3]):

$$P(r) = \frac{1}{2\sqrt{r}}\,, \tag{35}$$

i.e. we have again returned to formula (27). Thus, the probability of finding an event with a fraction of the neutral pions being smaller than a certain given r_0 is

$$W(r < r_0) = \sqrt{r_0}\,, \tag{36}$$

which constitutes 10% even at $r_0 = 0.01$. The charge fluctuations in such a system are much larger than those that would follow from the Poisson distribution, where the distributions are concentrated around $r = 1/3$.

Note that the configuration (32) is not a solution of the equations of motion and is therefore unstable. In order to give an accurate quantitative estimate it is necessary to take into account the presence of a domain wall separating the metastable configuration (32) from the surrounding normal vacuum. Unfortunately, such calculations have not yet been performed.

To carry out more detailed investigations, it is necessary to specify the source of the fluctuations of the chiral order parameter. In Ref. [70] this mechanism was considered in the context of a chiral phase transition, when at increasing temperature the chiral symmetry is restored at some temperature T_c. Earlier it was established [55] that the QCD chiral field corresponds to an 0(4)-magnetic [which is natural for the choice of the order parameter in the form of Eqn (30)], for which one has a second-order phase transition. Therefore, a most natural mechanism for the generation of large scale fluctuations is the so-called 'critical slowing'. In the vicinity of a critical point the relaxation time of the long-wavelength degrees of freedom increases, and such fluctuations can thus generate an isotopic disbalance, as the system's expansion can 'freeze' the appearing isotopic inhomogeneities. It turns out, however, that this mechanism is not effective. The reason is that the tiny (on the hadronic mass scale) current quark masses nevertheless generate a sufficient pion mass m_π (in fact, $m_\pi \sim T_c$), for the chiral fluctuations to be indistinguishable from the thermal ones.

This has led the authors of Ref. [70] to consider the alternative possibility, when the fluctuations that initially appear in the high-temperature (unbroken) phase, subsequently evolve according to the zero-temperature equations of motion. Here the amplifying mechanism has quite remarkable character [70]. In the spontaneously broken phase of a linear σ model with the condensate v and the bare goldstone mass μ, the zero mass of the goldstone is provided by the relation

$$m_\pi^2 = -\mu^2 + \lambda v^2 = 0\,. \tag{37}$$

Above the transition point one always has long-wavelength fluctuations, for which $\langle\Phi^2\rangle < v^2$, and during a certain time interval the mass of these fluctuations is negative. Correspondingly the long-range fluctuations will grow until the equations of motion restore the relation $\langle\Phi^2\rangle = v^2$. Such a situation could take place for a rapidly expanding system. The resulting numerical solutions of the corresponding evolution equations [70, 72–75] are somewhat ambiguous. We think that it would be premature to conclude whether the fluctuations of the chiral order parameter can be amplified to an observable scale.

In several papers [63, 64, 66, 69] the problem was considered within a nonlinear σ model. Its Lagrangian can be written in the form (the discussion of the nonlinear σ-model can be found, for example, in Ref. [79]):

$$L = \frac{1}{2f_\pi^2}\,(\partial_\mu\Phi_a)(\partial^\mu\Phi_a) - \lambda\left(\frac{1}{f_\pi^2}\,\Phi_a^2 - 1\right)\,, \tag{38}$$

where λ is a Lagrange multiplier. In Refs [63, 69] the classical solutions of the equations of motion of the nonlinear σ model, describing the isotopic fluctuations of the chiral field, were found. Unfortunately this has not led to more detailed predictions, although the situation was clarified to some extent in Refs [66, 69].

An interesting attempt at obtaining the equations describing the quantum fluctuations of the order parameter was recently made in Refs [76, 77]. The basic physical idea is that, as in the heavy nuclei collisions, the longitudinal expansion of hadronic matter in the pionisation domain at a scale of 8–10 fm dominates over the transverse one and the problem becomes essentially two-dimensional. In Ref. [77] the SU($\mathrm{N_f}$) Wess–Zumino–Novikov–Witten Lagrangian for the chiral field $U = \exp(2\mathrm{i}\bar{\tau}\bar{\pi})$ was obtained as an effective 1 + 1-dimensional Lagrangian. In this model the fluctuations of the order parameter can be computed exactly. In particular, for the two-point rapidity correlations at a given proper time τ one gets

$$\big\langle U(\tau, Y)U'(\tau, Y')\big\rangle \sim \frac{1}{(\sqrt{2}\tau)^{4\Delta}}\,[\mathrm{ch}\,(Y - Y') - 1]^{2\Delta}\,, \tag{39}$$

where $\Delta = 3/20$. Such predictions could be tested experimentally. Let us however note once again that a method of constructing the effective Lagrangians, which mixes the ideas originating from both the high- and low-energy regimes, does not allow one to get a consistent reduction. As a result the whole treatment is rather a guess subject to experimental verification.

Where should we look for the experimental signatures of the existence of a chiral condensate and what are they? An idea of how to answer the first part of the question follows from the fact that 'Centauros' were found in cosmic rays but were not found in accelerator experiments, although the Tevatron does already cover this energy domain. If we do not ascribe the whole effect to the specific conditions of the registration of cosmic rays and their composition, we have to assume that the difference in the results originates from the fact that in cosmic rays one studies the fragmentation region of large (pseudo) rapidities, whereas in the accelerator experiments we have been up to now studying the pionisation domain. According to the estimates of Ref. [71] the 'Centauro' type clusters should be looked for at Tevatron in the pseudorapidity range in CMS of the order of 4 and higher. This is one of the tasks of the T864 experiment. If the charge asymmetry is given by formula (27), it should be quite visible.

As we have mentioned, apart from the distinct signal of charge asymmetry (27), (35) we have also drawn some conclusions on the correlation properties of such objects in azimuthal angle (28) and pseudorapidity (39). Moreover, the electromagnetic decay modes of the hadronic resonances can be sensitive [73, 74] to the presence of such a condensate, because its orientation is misaligned with that of the electroweak vacuum as defined by the Higgs fields.

Concluding this section, let us mention the efforts to look at the collective effects in the pion (pion-resonance) gas within the pragmatic approach using the experimentally known scattering phase shifts and constructing the corresponding optical potential [108].

6. Collective effects in QCD jets

Let us turn now to QCD jets. From the point of view of field theory a typical situation, in which one uses the description in terms of the collective degrees of freedom, arises when the system is considered as being composed from a classical subsystem (e.g. crystal, plasma, etc.) and corresponding quasiparticles (phonons, plasmons, etc.). In QCD it is natural to distinguish between the domain of small distances (quarks and gluons as high energy modes) and large distances (hadronic modes). The high-energy phase is described by the QCD perturbation theory, but in order to extract quantitative predictions, we should be able to estimate the contribution of the low-energy modes to the characteristics that are traditionally computed within the perturbation theory. One of the most interesting objects to look at here is the hadron jet, generated by highly energetic quarks and gluons.

Let us begin with considering the possibility of the phenomenological account of the presence of low-energy modes (confinement) on the evolution of the quark–gluon jets that give rise to hadron jets. In the simplest approximation (leading logarithms) the perturbative evolution of quark–gluon jets is described by the Gribov–Lipatov–Altarelli–Parisi equations (see e.g. Refs [1, 6]). The structure of these equations is very similar to the system of equations describing the evolution of electromagnetic showers in matter [80]. The analogy becomes even more attractive because the interaction with a low-energy subsystem is inherently present in electromagnetic showers: these are the inelastic losses on the ionisation of the atoms of the medium by the particles from the shower. We are pleased to note that precisely this problem was solved in the paper by S Z Belenky and I E Tamm [81], whose centenary this issue of the journal is devoted to. We are following the same line of investigations, but in application to quark–gluon jets.

Let us assume that the influence of confinement (long-range modes related to strings) on the evolution of hard quarks and gluons results in the gradual loss of their energy going into the formation of pions. This process resembles the loss of the electrons from the shower into matter leading to the shower damping. As in the previous paragraph, we get the amplification of the low-energy modes and the attenuation of the hard component. In Ref. [82] a modification of the GLAP equations was proposed, which treats the interaction with the low-energy modes in analogy with the ionisation losses. If we consider for simplicity only a gluon subsystem, the modified equations take the form:

$$\frac{\partial D(E,\tilde{Y})}{\partial \tilde{Y}} = 2\int_E^{\infty} \frac{\mathrm{d}E'}{E'}\, P^{\mathrm{GG}}(E,E')D^{\mathrm{G}}(E,\tilde{Y}) - \int_0^E \frac{\mathrm{d}E'}{E}\, P^{\mathrm{GG}}(E',E)\, D^{\mathrm{G}}(E,\tilde{Y}) + \beta_{\mathrm{G}}\, \frac{\partial D(E,\tilde{Y})}{\partial E}\,, \quad (40)$$

where $D^{\mathrm{G}}(E,\tilde{Y})$ is a distribution function of gluons over the energy E and 'depth'

$$\tilde{Y} = \frac{1}{2\pi b} \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(k^2/\Lambda^2)}\,; \qquad b = \frac{33}{12\pi}\,, \quad (41)$$

Q^2, k^2 are the initial and 'current' invariant mass in a jet, Λ is a cut-off parameter and P^{GG} is a transition (gluon bremsstrahlung) probability,

$$P^{\mathrm{GG}} = c_V x(1-x)\left[1 + \frac{1}{x^2} + \frac{1}{(1-x)^2}\right]; \qquad x = \frac{E'}{E}\,, \quad (42)$$

β_{G} is a parameter determining the magnitude of the nonperturbative inelastic losses. The solution of Eqn (40) reveals the maximum of the parton shower and rapid damping of the shower at low gluon energies owing to the conversion of gluons into hadrons whose influence is phenomenologically described by the last term. No hadronisation model was attempted in Refs [82, 83]. However, the solution of Eqn (40) allows [83] one to find the energy flow from a 'hard' component to a soft one. For the gluon jet with the initial energy, E_0 the average energy of a hard component at the depth $\tilde{Y}$ is equal to

$$\langle E(\tilde{Y})\rangle = E_0\left[1 - \frac{\beta_G}{E_0}\sqrt{\frac{\tilde{Y}}{2c_V \ln(E_0/\beta_G)}}\, I_1 \times\left(2\sqrt{2c_V\tilde{Y}\ln(E_0/\beta_G)}\right)\exp(-\bar{a}\tilde{Y})\right]\,, \quad (43)$$

where $\bar{a} = 101/18$, and I_1 is a modified Bessel function.

We see that the picture is qualitatively attractive, but for providing more accurate predictions one should take into account the modification of the initial equations due to kinematical and interference restrictions on the perturbative evolution. It seems, however, quite probable that the above refinements will significantly alter the distribution function and correlations, but the formula for the energy losses (43) will most probably remain essentially the same. The grounds for such a belief come from the derivation of Eqn (43) in Ref. [83], where we essentially used the energy conservation in the perturbative evolution. This phenomenological approach to the parton's hadronisation is not complete since one should also describe how many hadrons appear, and with which characteristics as a result of parton convolution. This is possible only with Monte Carlo models with the use of even more phenomenological parameters.

An attempt at a field-theoretic realisation of the above scheme was recently proposed in a series of papers [84, 85]. The evolution of parton showers both in space–time and in energy–momentum phase space is analysed. Some arguments in favour of rapid transition from the partonic to the hadronic stage are given. Therefore, the idea of a unified treatment of a high-energy subsystem and low-energy 'thermostat' is realised by a literal addition of the corresponding Lagrangians of the subsystems and of the interaction:

$$L = L_{\rm QCD} + L_{\rm eff} + L_{\rm eff}^{\rm int} , \qquad (44)$$

where $L_{\rm eff}$ is a low-energy Lagrangian accounting for the breaking of chiral and dilatational invariance [86, 87] while the interaction Lagrangian describing the processes at the intermediate scales interpolates between the two subsystems by phenomenologically introducing some effective cut-offs imitating the rapid parton–hadron conversion within a very narrow space–time region. In particular, if the gluon degrees of freedom are considered only, the Lagrangian is chosen in the form:

$$L = -\frac{1}{4}(G^a_{\mu\nu})^2 + \frac{1}{4}\,\xi(\chi)(G^a_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu\chi)^2 - b\left[\frac{1}{4}\chi_0^4 + \chi^4 \ln\frac{\chi}{e^{1/4}\chi_0}\right] , \qquad (45)$$

where

$$\xi(\chi) = \theta(\chi)\left(\frac{\chi}{\chi_0}\right)^3\left(4 - \frac{3\chi}{\chi_0}\right) , \qquad (46)$$

χ is a scalar glueball field, $G^a_{\mu\nu}$ is the gluon field strength, and the interaction of the quantum gluon fields $G^a_{\mu\nu}$ with the 'classical' χ has the form $\xi(\chi)G^2$. The nonvanishing gluon condensate at large scales is denoted by χ_0 and the parameter b is related to the conventional bag constant B by $B = b\chi_0^4/4$. In the chiral limit, the potential has a minimum when $\langle\chi\rangle = \chi_0$ and equals the vacuum pressure B at $\langle\chi\rangle = 0$. From the point of view of the development of a gluon cascade the interaction Lagrangian $L_{\rm eff}^{\rm int}$ leads to the appearance of the new vertices, i.e. to the possibility of a cascade glueball formation. The natural requirement to the modified Lagrangian is the presence of an energy flow (from the quantum component to the classical one) growing with increasing $\tilde{Y}$ which is reminiscent of the results of Ref. [83]. The numerical solutions of the corresponding kinetic evolution equations are indeed demonstrating this property. The elaborated Monte Carlo program provides characteristics of final hadrons in various reactions. The direction of investigations in Refs [84, 85] is rather appealing even though there appear several phenomenological parameters to be approved by later development. One would hope that in the near future the situation here will be clarified, and the problem of the nonperturbative effects in the evolution of the quark–gluon cascades will reach the quantitative level.

Let us mention an interesting possible analogy with a collective effect in photon radiation. It is known that the real part of the elastic scattering amplitude of hadrons becomes positive at large energies. In the terminology of classical physics this means that the refractive index of the hadronic medium exceeds one. In this case the phase velocity of a coloured charge in the hadron medium can be higher than the speed of light. This can lead to the 'colour Cherenkov radiation' [88], which is analogous to the usual Cherenkov radiation, the theory for which was developed by Tamm and Frank [89]. A characteristic feature of such radiation will be its angular distribution with its typical 'ring-like' structure (see the review [90] and the papers [91, 92]). However, its intensity can be damped owing to the small size of the target hadron and the absorption in the hadronic medium (an imaginary part of the scattering amplitude). Besides, a change of the current in the course of emission of colour objects could play its role (let us recall that the photon is electrically neutral and, unlike the gluon, does not change the emitting current at small recoils).

Nevertheless some events having a 'ring structure' were observed in cosmic rays [93], and for larger statistics the processing of the experimental data of the NA22 experiment indicated [94] a statistically significant contribution of such events. They show up as peaks in the pion distributions at the rapidities $|y| \sim 0.3$ in the centre-of-mass frame. Several bands of energies are claimed [91,92] to satisfy the requirements of 'colour Cherenkov radiation'. A continuation of the searches for the possible manifestations of this effect would be desirable.

Let us now consider another aspect of the physics of hadron jets that is also directly related to the considered collective effects. We are now discussing the passage of a fast particle, which subsequently forms a hadron jet, through the hadronic matter. A clear example of the effects that could be expected in this connection is a sharp decrease of the collisional energy losses in quark–gluon plasma near the phase transition point (jet quenching [95]). In fact, from the Bjorken estimate for the energy losses of the parton in plasma at the unit length

$$\frac{{\rm d}E}{{\rm d}x} \sim 6\alpha_s^2 T^2 \ln\frac{4ET}{M^2}\exp\left(-\frac{M}{T}\right)\left(1+\frac{M}{T}\right) \qquad (47)$$

it follows that for the quite realistic values of $\alpha_s = 0.2$, $T = 250$ MeV, and $M = 500$ MeV the energy losses of a parton at unit length in plasma are 10 times smaller than the value of 1 GeV fm^{-2} expected for the usual hadronic matter, where the scale is determined by the string tension. The accuracy of the answer directly depends on how well-studied are the proper collective modes of a system. From classical plasma physics it is well known that the interaction of an external probe with the system has a resonance character at frequencies equal to the eigenfrequencies of the system. In spite of the work on the study of the collisional losses (see e.g. Ref. [96]), the question concerning the quantitative estimate of the effect is in our opinion open. In particular, one does not know sufficiently well the spectrum of collective excitations of the quark–gluon plasma (recently, new modes were found in [97]).

Let us also discuss one more interesting effect illustrating the deep differences between the effective non-Abelian medium in different phases. From consideration of the propagation of a colour charge in the external random chromoelectric field [98] the following formula for the change of the energy of a fast test particle was obtained:

$$\left\langle\frac{{\rm d}E}{{\rm d}t}\right\rangle = \frac{2\pi^{3/2}\alpha_s\langle\mathcal{E}^2\rangle\tau_{\rm c}}{m}\,\frac{C_A}{N_{\rm c}^2-1} , \qquad (48)$$

where $\langle \mathcal{E}^2 \rangle$ is the mean square of the external field strength, τ_c is its characteristic correlation time, m is the mass of the particle, C_A is a corresponding Cazimir operator, N_c is the number of colours (a Gaussian chromoelectric field correlator is assumed). The most interesting thing here is not the quantitative estimate of the possible losses, but the fact that in the QCD vacuum one gets $\langle \mathcal{E}^2 \rangle < 0$ (which is well known from the consideration of heavy quarkonia [99]), whereas the fluctuations of the colour field in the QCD plasma naturally lead to $\langle \mathcal{E}^2 \rangle > 0$. Therefore we have a 'stochastic cooling' (deceleration) in the vacuum in the hadronic phase and the 'stochastic heating' in, for example, QCD plasma, where in some way the stochastic colour fields were generated.

It is necessary to mention the recent growth of interest in the different aspects of the physics of quark – gluon jets. We saw how tightly the analysis of these problems is related to the collective properties of the non-Abelian medium in which the jets propagate.

One of the most interesting things here is the problem of coherent effects in the induced gluon bremsstrahlung during the passage of a fast particle through the non-Abelian medium. We are speaking about the non-Abelian analogue of the Landau – Pomeranchuk effect which is well known in electrodynamics [100 – 102]. The physical essence of this effect is a damping of the soft photon radiation of a fast particle by a high-energy particle owing to the coherent influence of many scatterers. It is interesting that the theory of the electromagnetic effect was verified experimentally only in 1993 [103]! For the physics of quark – gluon jets the question of the intensity of the induced bremsstrahlung radiation is a central one. In particular, the above-mentioned estimates of the parton energy loss [96] are made under the assumption of the negligibly small energy losses at gluon radiation. It may seem that coherent radiation in the QCD medium is simply impossible, because any radiated gluon changes the colour of a test parton, which should exclude any possibility of 'using' the multiple scattering for achieving the coherent action of a number of scatterers on the bremsstrahlung gluon radiation. This point of view was laid into the basis of calculations in Ref. [104], where for the energy losses at unit length the authors obtained

$$-\frac{\mathrm{d}E}{\mathrm{d}x} \sim \mathrm{const}\, \alpha_s \mu^2 \,, \tag{49}$$

where μ is a screening length in the medium, which is fundamentally different from the electrodynamic result [100 – 102].

This statement was recently objected to in Ref. [105], where the authors showed that a coherent action of a set of non-Abelian scatterers can be achieved by taking into account the regeneration of the initial colour by the gluon, emitted by the parent test parton. Ideologically this situation resembles the colour coherence effect in the quark – gluon jets, where some gluons 'see' only the total charge of a jet [6], which is analogous to the Chudakov effect [106], and is well known in the physics of electromagnetic showers. An account of the above mechanism in the formula for the radiative energy losses in the non-Abelian medium takes the form [105]

$$-\frac{\mathrm{d}E}{\mathrm{d}x} \sim \alpha_s \sqrt{\frac{E\mu^2}{\lambda_g}} \ln\left(\frac{E}{\lambda_g \mu^2}\right) \,, \tag{50}$$

where λ_g is a mean free path of gluons. Formula (50) differs from the classical result of Migdal [101] by a logarithmic factor appearing because of the correct treatment of collisions with a large momentum transfer. An analogous factor also appears in the corresponding electrodynamic formula.

The problem of the parton energy losses in the non-Abelian medium is of decisive importance for the description of the spectrum of jets in collisions with nuclei. The experimental study of hadron jets in such collisions has already a relatively long history. In particular, there exists a number of experimental results, which distinctly show the role of multiple scattering in the formation of two-jet configurations [107]. The continuation of the study of this problem will undoubtedly shed light on the properties of the extremely complicated medium, in which the jets are born and where they propagate.

Here it is tempting to point out an analogy with cosmic rays, from which we obtain information on the structure of fields in the intergalactic space. It is possible that the rich experience accumulated in the physics of cosmic rays will also help in the study of the 'cosmic rays' of hadron physics, i.e. of the quark – gluon jets.

Of course, our discussion did not cover all the possible manifestations of the collective effects in the quark – gluon plasma. One of the most widely discussed effects is the possible absence of J/Ψ suppression in the quark – gluon plasma [109] due to a Debye screening, which seems to be the best established and understood collective effect in QGP. However, one should mention that there exist alternative explanations of the effect within the quark – gluon string model approach. The analogous effects with open charm production are also widely discussed nowadays.

7. Discussion

We have tried to give a general description of the theoretical attempts to find the collective effects in the multiparticle systems formed in the collisions of high-energy particles. In some way they are all of a phenomenological character and are inspired by the hope of revealing certain features of confinement and QCD vacuum structure and their role in the multiparticle production process. The diversity of ideas, methods, and approaches underlines the absence of a unifying picture, but undoubtedly is a necessary element on the way to its creation. We are sure that eventually it will appear. If we look back only 60 years, we shall see (see the article of E L Feinberg on page 773 of this issue) that the very notion of the pion was still not unanimously accepted, and now we are trying to understand the properties of multipion states and their relation to the quark and gluon degrees of freedom of an excited hadronic medium. The absence of general conclusions means only that they are in the stage of formation and dynamic development.

Acknowledgements. One of the authors (IMD) is grateful to R C Hwa for the hospitality at the University of Oregon where the final draft of the paper was done. This work was supported by the Russian Fund for Fundamental Research under grant 93-02-3815 and, in part, by the NATO grant CRG930025.

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