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Scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences (30 November 1994)

A scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences was held on 30 November 1994 at the P L Kapitza Institute of Physical Problems. The following papers were presented at this session:

(1) **V S Troitskii** (Institute of Applied Physics, Nizhny Novgorod) "Experimental evidence against the Big Bang cosmology";

(2) **A A Slutskin** (Physicotechnical Institute of Low Temperatures, Kharkov) "'Frozen' electronic phase and high-temperature superconductivity".

Summaries of these papers are given below.

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Experimental evidence against the Big Bang cosmology

V S Troitskii

1. Introduction

A cosmological theory is tested by comparing the observed and theoretical dependences of the apparent luminosity (m) and the angular size (θ) of the galaxies on the red shift (z). In standard cosmology, the expressions for these quantities in terms of linear and stellar magnitudes are as follows:

$$\mathcal{E}(z) = \frac{L(z)}{R^2(z, q_0)\alpha_m^2},$$
$$m(z) = -2.5 \lg \mathcal{E}(z) = 5 \lg R\alpha_m + M(z) - 5, \qquad (1)$$
$$\theta(z) = \frac{l(z)}{R(z, q_0)\alpha_\theta}.$$

The first expression gives the inverse-square dependence of the surface illuminance ($\mathcal{E}$) at the observer as a function of his/her distance R to the observed galaxy whose absolute luminosity $L(z)$ is in watts per steradian. The third expression is a purely geometric relationship governing the apparent angular size of a galaxy whose linear size is $l(z)$. The influence of a specific cosmological theory appears in these expressions only in the nature of the dependence of the distance $R(z, q_0)$ on the red shift of a galaxy and, to a slight extent, on the functions $\alpha_m = z + 1$ and $\alpha_\theta = (z + 1)^{-1}$. These functions are found theoretically and they depend on the assumed red shift mechanism. In particular, the expanding Universe theory predicts for a closed model ($q_0 = 1$) that the metric distance is $R(z) = R_{\mathrm{H}} z/(z + 1)$, where $0 \leqslant z \leqslant \infty$, $R_{\mathrm{H}} = cH_0^{-1}$ and H_0 is the Hubble constant. The dependence $R(z)\alpha_m = R_{\mathrm{H}} z$ has been confirmed by measurements for distances less than 1% of the limiting value (which is $R_{\mathrm{H}} = 6000$ Mpc if $H_0 = 50$ km s^{-1} Mpc^{-1}) and it represents the well-known Hubble law. A theory can be checked and an experimentally tested cosmology can be developed if at least the function $R(z)$ or the product of this function and α are found from measurements carried out over the whole accessible range of the red shifts $10^{-3} \leqslant z \leqslant 5$. The cosmological 'standard candle' and 'standard rod' tests have been proposed for this purpose over 50 years ago. These tests are based on measurement of the apparent luminosity $m(z)$ and of the angular size $\theta(z)$ of galaxies which are located at different distances z, but have the same standard luminosity $L(z) = L_0 =$ const and the same standard linear size $l(z) = l_0 =$ const. It is evident from expressions (1) that the experimental dependences $m(z)$ and $\theta(z)$ determine directly the unknown functions $R(z)\alpha_m$ and $R(z)\alpha_\theta$. The correctness of this approach, in principle, is not in any doubt, but its practical application gives ambiguous results. This is due to the fact that the galactic parameters $L(z)$ and $l(z)$ are random quantities with a very large scatter. The distribution law of the apparent quantities $P(m/z)$ and $P(\lg \theta/z)$ is defined directly for galaxies which are at the same distance $z \pm \Delta z$. Hence, it follows from expressions (1) that the primary parameters $P(M/z)$ and $P[\lg l(z)/z]$ have the same distribution. Sandage et al. [1] found that in the case of the E, SO, and S galaxies in the Virgo cluster, located at approximately the same distance z, the distribution law $P(M/z)$ is normal and the variance is $\sigma = 1.5$ magnitudes. My own and my colleagues' investigations [2, 3] carried out on ensembles of up to 30 000 galaxies and 4000 quasars have demonstrated that the conditional normal distribution law is obeyed rigorously in the range $10^{-3} \leqslant z \leqslant 4$ with a variance $\sigma = 1.2 \pm 0.1$ magnitudes both for the galaxies and the quasars. The Schechter law $P(L) = l^{-x} x^{-1}$, derived earlier for galaxies with $z \leqslant 0.1$ ($x = L/\overline{L}$), is not purely experimental, because it is deduced from the calculated values of L in accordance with expressions (1), so that it cannot be used in our case. The Hubble diagram for the apparent luminosity $m(z) = 5 \lg \mathcal{E}(z)(z + 1) + M(z) - 5$ is thus obtained in the form of a field of random points which lie within a noise band of width $\pm 3\sigma \simeq \pm 3$ magnitudes. This corresponds to variation of the luminosity $L(z)$ at each z by a factor exceeding 10 on either side of the average

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value. The scatter is somewhat less for the random values of $\lg\theta(z)$ and $\lg l(z)$, which also obey the conditional normal distribution. In this case the dynamic relationships $R(z)\alpha_m$ or $R(z)\alpha_\theta$ are masked by the noise in the experimental dependence $m(z)$ and $\lg\theta(z)$, which is due to the random scatter of the absolute luminosity $M(z)$ and $\lg l(z)$. Familiar averaging methods have to be used to reveal the dynamic relationships. A field of random galactic quantities should be characterised by specific statistical relationships such as the distribution law, and by parameters such as the average value, deviation, correlation, etc. Only these relationships and parameters can reveal the dynamics latent in the random quantities. A theory must therefore be checked by employing averaged functions $E(m/z) = \overline{m(z)}$ and $E(\lg\theta/z) = \overline{\lg\theta(z)}$, which are found by the familiar method of regression analysis. A comparison of the regression functions $\overline{m(z)}$ and $\overline{\lg\theta(z)}$ with the theoretical relationships (1) makes it possible to find the averaged functions $\overline{L(z)}$, $[\overline{M(z)}]$, and $\overline{l(z)}$ and also, most important, to determine the dynamic function $R(z)$, which is our main task here. It is quite clear from the above that it is impossible to apply the 'standard candle' and 'standard rod' methods without ways for independent measurement of M and l of galaxies for any value of z. Several decades ago Sandage attempted to solve this problem by proposing that the brightest galaxies in the galactic clusters at various distances should be regarded as the 'standard candle'. This would seem to be based on a fairly reasonable assumption that there should be an upper limit to the absolute luminosity. From the point of view of statistics, this implies the hypothesis of the existence of a sharp discontinuity in the distribution curve of the absolute luminosity $P(M/z)$ at some value $M_{\max}$, which is the same for all z. However, such discontinuities are not observed in this distribution. Galaxies selected in accordance with the Sandage criterion are in fact in the wing of a Gaussian distribution and, therefore, cannot be selected unambiguously. The objects selected in accordance with this idea are rare and exotic galaxies. In view of their high brightness, they are most probably subject to rapid evolution of the luminosity and size, but the selection described above attempts to exclude the influence of such evolution. After many years of application of this method of selection of galaxies 'suitable' for comparison with theory, it has been found that the agreement between the theoretical and observed values of $m(z)$ requires $0.5 \leqslant q_0 \leqslant 5$, whereas in the case of observations of $\lg\theta(z)$ there is no quantitative or qualitative agreement with theory. Therefore, these methods have failed to solve the problem of checking the theory and, as demonstrated in the thorough reviews of Burbidge [4] and Baryshev [5], we are in a blind alley.

2. Initial observational data

Use of the regression analysis to check a theory must be based on the global number of galaxies, so as to ensure that the data ensemble is statistically representative. This should be done taking account of the various effects of selection. The global regression dependence $m(z)$ has been found from an ensemble of 9000 galaxies of all types and all 4000 known quasars in the red shift range $10^{-2.5} \leqslant z \leqslant 4$ within the V band (visible range). Use has been made of 30 recent reviews and catalogues. A study has been made of the influence of the Malmquist effect [2, 3]. Moreover, a K correction, calculated from the average spectra of

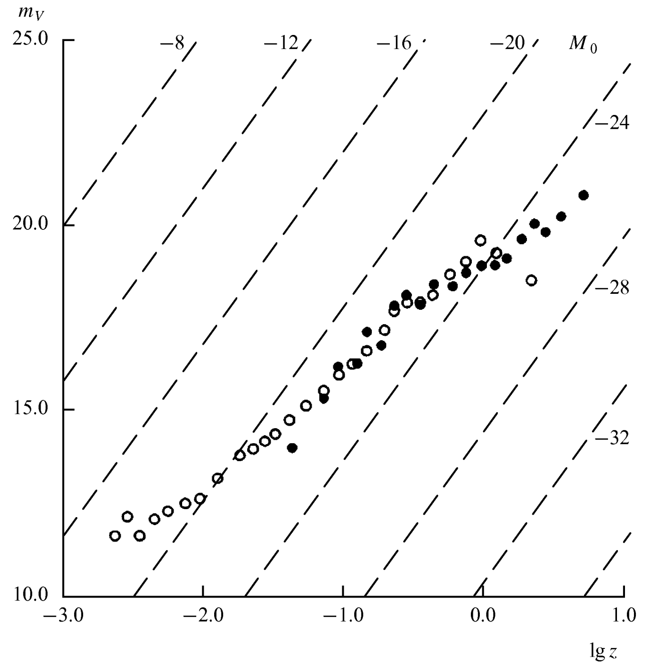

Figure 1. Global Hubble regression diagram: (o) galaxies, (•) quasars; the dashed lines form a family of theoretical dependences $m(z) = 5\lg z + M_0 + 43$ calculated for $q_0 = 1$ and $H_0 = 75$ km cm^{-1} Mpc^{-1}.

galaxies and quasars, has been introduced. In each interval $\Delta\lg z = 0.2$ the normal distribution law $P(m/z)$ has been checked. This has shown that in the case of galaxies and quasars the mean square deviation in each interval of z is $\sigma = 1.2 \pm 0.1$ magnitudes. The influence of an inhomogeneous distribution of the number of data over z is avoided by determination of the regression function on the basis of the values of the average m for intervals Δz. The global regression dependence $\overline{m(z)}$ obtained by the present author and his colleagues [3] is reproduced in Fig. 1 together with a family of theoretical dependences. The analytic dependence in terms of the stellar and physical quantities is

$$m(z) = (2.7 \pm 0.1)\lg z + 18.6\,, \quad \mathcal{E}(z) = z^{-1.10}10^{-7.44}, \quad (2)$$
$$10^{-2.5} \leqslant z \leqslant 4\,.$$

This dependence naturally differs fundamentally from the curves usually plotted on the basis of a few tens of selected 'standard' galaxies.

The global regression dependence $\overline{\lg\theta(z)}$ has been obtained by us for 10 250 normal galaxies in the red shift range $10^{-2.5} \leqslant z \leqslant 0.5$ within the V band. An analysis has been made of all possible systematic distortions of the function $\theta(z)$ when isophote measurements are made. These distortions may be due to the likely evolution of the surface brightness of the galaxies. The results demonstrate the absence of any significant distortions [6]. The logarithmic and linear forms of the regression function are:

$$\overline{\lg\theta(z)} = -(0.55 \pm 0.05)\lg z + 0.93\,,$$
$$\overline{\theta(z)} = \frac{8.5''}{z^{0.55}}\,, \quad 10^{-2.5} \leqslant z \leqslant 0.5\,. \quad (3)$$

The regression function is plotted in Fig. 2 and compared there with the theoretical dependence for the $q_0 = 1$ model.

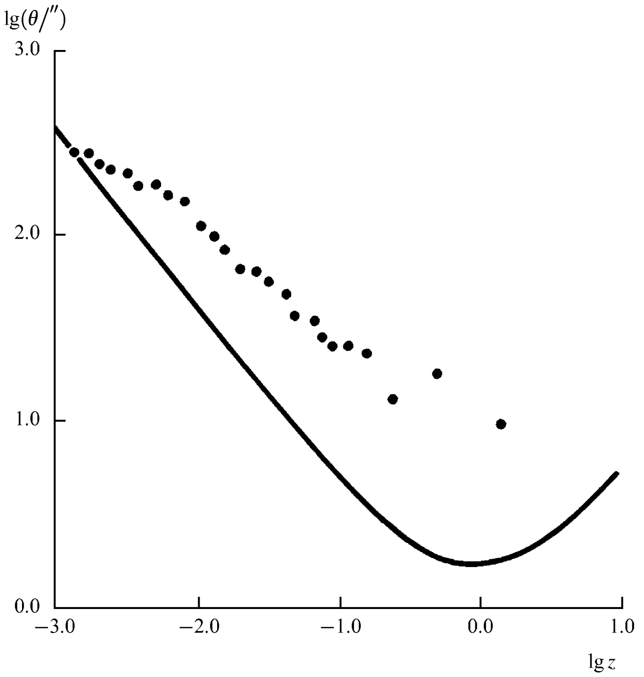

Figure 2. Global regression dependence of the angular size of galaxies (black dots). The continuous curve is the theoretical dependence for $q_0 = 1$ and $l(z) = \mathrm{const}$.

The global regression dependence also differs radically from the dependence obtained for a small number of selected exotic objects, as was done by Sandage [7], Kapahi [8], and Kellermann [9], who obtained $\theta \propto z^{-1}$. The experimental relationships (2) and (3) make it possible to detemine the average surface brightness over a galactic disk, averaged over all the galaxies:

$$\overline{\mu(z)} = \overline{m(z)} + 5\,\overline{\lg\theta''(z)} = (-0.05 \pm 0.1)\lg z + 23,$$
$$10^{-2.5} \leqslant z \leqslant 0.5\,. \tag{4}$$

which is expressed in stellar magnitudes per square of an arcsecond. We can see that this surface brightness is practically independent of z and equal to $\overline{\mu(z)} = 23$ magnitudes. Direct measurements of the surface brightness carried out recently with the aid of charge-coupled devices give $22.0 \leqslant \mu(z) \leqslant 24.0$ magnitudes irrespective of the red shift of the sources in the interval $10^{-2.5} \leqslant z \leqslant 0.5$ (Graham [10], Hoessel et al. [11], Dressler et al. [12], and Peletier et al. [13]). This is a good confirmation of the precision of the independent series of measurements of $\overline{m(z)}$ and $\overline{\lg\theta(z)}$ and, consequently, of relationships (2) and (3). The theoretical expressions (1) can be written in a form more convenient for further comparisons:

$$\mathcal{E}_t(z) = \frac{L(z)}{R_m^2(z)}, \quad \theta_t''(z) = 2\times10^5\,\frac{l(z)}{R_\theta(z)},$$
$$\mu_t(z) = -2.5\,\lg\left(\frac{L(z)R_\theta^2}{l^2(z)R_m^2}\right) + 26.6\,. \tag{5}$$

A comparison of relationships (2) and (3) with the corresponding first two relationships given above yields

$$R_m(z) = z^{0.56}10^{3.72}\left[\overline{L(z)}\right]^{1/2},$$
$$R_\theta(z) = z^{0.55}10^{4.37}\,\overline{l(z)}\,. \tag{6}$$

Hence, by elimination of z, we find that $R_\theta[\overline{L(z)}]^{1/2}/R_m\,\overline{l(z)} = 4.5$. Here, the function $R_\theta(z)/R_m(z)$ and the expression $[\overline{L(z)}]^{1/2}/\overline{l(z)}$ are of different physical origin, so that they are independent and, consequently, each of them is equal to a constant. Obviously, $R_\theta(z)/R_m(z) = \alpha_\theta/\alpha_m = 1$, so that

$$\frac{\left[\overline{L(z)}\right]^{1/2}}{\overline{l(z)}} = \frac{\sqrt{L_0}}{l_0} = 4.5, \quad R_m(z) = R_\theta(z) = R(z)\,. \tag{7}$$

The statistical relationship $[\overline{L(z)}]^{1/2}/\overline{l(z)} = \mathrm{const}$ found in this way is supported by a number of investigations, which are summarised in my recent paper [14]. They are also supported by studies of the correlation between the luminosity and the size of galaxies listed in the UGC catalogue [6]. The required functions are thus found to be

$$R(z) = R_0 z^{0.55}\,\frac{\left[\overline{L(z)}\right]^{1/2}}{L_0}, \quad R_0 = 10^{3.72}\sqrt{L_0}\,,$$
$$\frac{\left[\overline{L(z)}\right]^{1/2}}{\overline{l(z)}} = 4.5. \tag{8}$$

Here, $\overline{L(z)}/L_0$ is an arbitrary function, since for the three required functions $\overline{L(z)}$, $\overline{l(z)}$, and $R(z)$ there are only two relationships: (2) and (3). Therefore, the experimentally determined functions $R(z)$ and α_m/α_θ differ fundamentally from the theoretical functions used in standard cosmology. It would seem that by selecting $\overline{L(z)}$ one can make the expression for the distance (8) agree with the function of the distance in standard cosmology. However, this is impossible even in principle since in standard cosmology there are two expressions for the distance. For example, if $q_0 = 1$, $R_m = R_{\mathrm{H}}z$ and $R_\theta = R_{\mathrm{H}}z(z+1)^{-2}$. Selection of $\overline{L(z)}/L_0 = z^{0.9}$ ensures agreement with the distance $R_m(z)$ and selection of $\overline{L(z)}/L_0 = z^{0.9}(z+1)^{-2}$ provides agreement with $R_\theta(z)$.

3. Statistical cosmological tests

Since the observed astrophysical parameters of galaxies and quasars obey specific statistical laws, a number of new tests can be proposed. For example, one could investigate the dependences on z of the variances of the luminosity and angular size, and also of the average values and variances of the spectral indices of the radiation emitted by galaxies and quasars, and so on. Our investigations [2, 3] show that neither the distribution law of m and $\lg\theta$ nor their variance depend significantly on the red shift and it is found that $\sigma(m) = \mathrm{const}$ applies to galaxies and to quasars throughout the investigated red shift range $10^{-3} \leqslant z \leqslant 4$. The variance is also constant, $\sigma(\lg\theta) = 0.25$, in the range $10^{-3} \leqslant z \leqslant 0.5$ accessible to investigation. The variance and the average value of the spectral index and the continuous spectra of the optical radiation emitted by quasars and galaxies are found to be independent of the red shift [2, 15]. The same result is reported by Hutchings et al. [16] for the microwave emission spectra of quasars. The fact that these statistical characteristics are independent of the positions of the objects in space are evidence of the equilibrium state of the Metagalaxy system. All these tests demonstrate unambiguously the absence of any detectable evolution of the average luminosity $\overline{L(z)}$ and size $\overline{l(z)}$ of galaxies in the investigated range of their existence, which is 7 to 10 billion years. Thus, on the assumption that $\overline{L(z)} = L_0$, we can now find the numerical values of the parameters L_0, $\overline{l(z)} = l_0$, and R_0.

4. Average parameters of galaxies and space

We shall determine R_0, L_0, and l_0 from the average luminosity of groups of galaxies within the radius $z \leqslant 0.02$, which is $M_0 = -21 \pm 0.5$ magnitudes. It is known that $-2.5 \lg L_0 = M_0 - 5$, and hence $L_0 = 10^{10.4} L_\odot = 10^{43}$ erg s^{-1} sr. Finally, it follows from expressions (8) that

$$\begin{aligned} &R(z) = R_0 z^{0.55}\,, \\ &R_0 = (830 \pm 200)\ \text{Mpc}, \\ &l_0 = (35 \pm 8)\ \text{kpc}\,, \\ &\overline{M} = -21 \pm 0.5\ \text{magnitudes}\,. \end{aligned} \tag{9}$$

Here, R_0 is the distance from the galaxies characterised by $z = 1$. The fact that the average values of the luminosity and size, the forms of the spectra of galaxies and quasars, and the variances of these quantities are independent of the positions of the objects in the space of the Metagalaxy fits well the known fundamental cosmological principle of homogeneity and isotropy of the Universe in space and time, established earlier for the average volume density of matter. Naturally, the constancy of these average values does not exclude the possibility of evolution of the luminosity and size of specific galaxies. There is an appropriate analogy here with the average strength of people on the whole of our planet, which remains constant in time, although each man undergoes evolution of his strength. The statistical homogeneity of the characteristics of galaxies in the space of the Universe is evidence of the great age of the Universe, which at least should be an order of magnitude greater than the age of the galaxies, estimated at 15 to 20 billion years.

5. Nature of the red shift

The experimentally determined dependence $z = R^2/R_0^2$ limits greatly the range of hypotheses which can account for the red shift. This range is limited even further if it is postulated that a new explanation of the origin of the red shift should agree with the known and thoroughly investigated physical processes. These conditions are satisfied by the familiar gravitational shift. In fact, following classical physics, a spherical light wave propagating in an infinite medium with a homogeneous density ρ performs work against the gravitational force of matter interacting with the spherical wave. This reduces the wave energy by an amount $-\mathrm{d}\varepsilon = \varepsilon c^{-2}\,\mathrm{d}\varphi$, where $\varepsilon = h\nu$, $\varphi = 4\pi G\rho R^2/3$, and, consequently, $\mathrm{d}\nu/\nu = 8\pi G\rho R\,\mathrm{d}R/3c^2$. Integration in the range from the frequency ν_1 at the moment of emission to ν_0 at the moment of reception of a wave propagating from $R = 0$ to R gives $\nu_1/\nu_0 = (z+1) = \exp(R^2/2r_\mathrm{g}^2)$, where $r_\mathrm{g} = \sqrt{3c^2/8\pi G\rho}$ is the gravitational radius. In the relativistic treatment, confirmed for weak fields, we have $(z+1) = (1 - R^2/r_\mathrm{g}^2)^{-0.5}$. If $R \ll r_\mathrm{g}$, then in both cases we find that $z = R^2/2r_\mathrm{g}^2$. According to expressions (9), we should have $z = R^2/R_0^2$ and then on the assumption that $R_0^2 = 2r_\mathrm{g}^2$, we find the required density of matter is $\rho = 10^{-28}$ g cm^{-3}, which is 50–100 times higher than the published estimates. This conclusion also follows from standard cosmology. It is therefore assumed that 98%–99% of the mass is in a hidden invisible state. It is possible also that other but quite hypothetical explanations of the red shift (such as those proposed by Kropotkin [17]) will prove to be in better agreement with experiments.

6. Microwave background

According to the reported results, the Universe is a practically unbounded system of galaxies. This makes it possible to explain the observed microwave background by thermal emission of both microwave and optical radiation by stars. The stellar microwave radiation flux at the observation wavelength λ_0, collected within a solid angle Ω of the aperture of an antenna at a distance R in an element of volume $\Omega R^2\,\mathrm{d}R$, is $\mathrm{d}p = r^3 nm\Omega F(\lambda, T)\,\mathrm{d}R\,\mathrm{d}\lambda_0$, where $F(\lambda,T) = (2\pi c^2 h/\lambda^5)[\exp(hc/\lambda kT) - 1]$ is the Planck emissivity function of stars whose temperature is T; $\lambda = \lambda_0(z+1)$ is the wavelength of the radiation emitted by a star; r is the average radius of the stars; n is the average density of galaxies; m is the average number of stars in the galaxies. This radiation is screened (absorbed) by galaxies on its way to the observer. The attenuation or screening function is approximately $\gamma = (1 - 0.33nl^2R)$, where l is the average size of the central regions of the galaxies. If $\mathrm{d}p$ is multiplied by γ and integration with respect to R is performed for $R = R_0\sqrt{z}$, the result is the spectral density of the flux at the wavelength λ_0. The equivalent temperature T_b of this microwave background can be found by equating the spectral flux density from a black body in a solid angle Ω at the wavelength λ_0 when the temperature is T_b. This gives

$$\frac{1}{2}\,r^2 nmR_0 \int_0^{z_0} \frac{(z+1)^3\gamma(z)\,\mathrm{d}z}{\sqrt{z}[\exp(hc(z+1)/\lambda_0 kT) - 1]} = \left[\exp\left(\frac{hc}{\lambda_0 kT_\mathrm{b}}\right) - 1\right]^{-1}. \tag{10}$$

Here, z_0 is found from the condition $\gamma(z_0) = 0$. A calculation of T_b, carried out with the use of the well-known parameters of galaxies and stars in the Main Sequence, yields the observed background temperature, which is determined primarily by the stellar radiation obeying the Rayleigh–Jeans law at distances of up to 60 000 Mpc in the range $0 \leqslant z \leqslant 5000$ and is independent of the observation wavelength in the range $0.1 \leqslant \lambda_0 \leqslant 100$ cm. This result makes it possible to estimate small-scale fluctuations, $\Delta T_\mathrm{b}/T_\mathrm{b} = 3 \times 10^{-5}$, in agreement with observations [18]. We can use relationship (10) to account for the mysterious agreement between the energy of the optical radiation integrated over all the frequencies for our Galaxy and the background radiation energy (see Ref. [4]).

In conclusion, it should be pointed out that the conflict between the dependence $R = R_0\sqrt{z}$ and the generally accepted Hubble law $R \propto z$ is not a convincing argument against the results obtained, since the Hubble law has been established for small values of z ($\leqslant 0.02$) when any smooth function, including $R = R_0\sqrt{z}$, is distinguishable from a straight line. There have been new determinations of the dependence $R(z)$ based on the Tully–Fisher law by Arp and van Flandern [19], and measurements of Giraud [20], in good agreement with our empirical dependence $R(z)$, which moreover is supported by the more realistic hypotheses about the red shift. A reduction in the estimates of the distances to galaxies and quasars by a factor of 4 for $z \sim 1$, compared with the estimates obtained from standard theory, eliminates the problem of superluminal velocities of the expansion of matter in these objects, which now become less than the velocity of light. Finally, application of the statistical approach to the problem of checking the Big Bang theory, started almost simultaneously by us [21] and by Segal and Nicoll [22], gives similar results for the dependence $R(z)$.

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'Frozen' electronic phase and high-temperature superconductivity

A A Slutskin

In the thirties, Eugene Wigner demonstrated [1] that a homogeneous Fermi liquid of free electons, which repel one another in accordance with the Coulomb law, can undergo a first-order phase transition to a localised state with a periodic spatial structure (Wigner crystal). According to Wigner, this happens when the electron density is so low that the characteristic Coulomb energy per electron

$$u = \frac{e^2}{\bar{r}} \tag{1}$$

(e is the electron charge and $\bar{r}$ is the average distance between electrons) exceeds the characteristic kinetic energy

$$\Delta = \frac{\hbar^2}{m\bar{r}^2}, \tag{2}$$

which is acquired by an electron of mass m when it is localised in a region $\sim \bar{r}$ because of the quantum uncertainty principle.

Wigner's concept of electron crystallisation was first applied to the conduction electrons in solids by Verwey [2]. Verwey measured the electrical conductivity of metal oxide Fe_3O_4 as a function of temperature T and observed an abrupt change in the conductivity at $T \sim 10^2$ K, which he treated as melting of a Wigner crystal formed by charge carriers in the oxide. Such Verwey transitions have since been discovered in a whole range of metal oxides which are semiconductors. In explaining the Verwey transitions by Wigner's concept it is necessary to assume that the characteristic dimensions of the electon localisation region exceed a typical period α_0 of the conductor lattice. One can then expect the appearance of boson excitations, which are phonons with relatively high velocities and are typical of a Wigner crystal. However, to the best of my knowledge, these excitations have not been detected experimentally. This has made it necessary to postulate the existence of non-Wigner collective mechanisms of electron localisation.

My purpose will be to show that long-range Coloumb forces in homogeneous narrow-band conductors may lead to electron self-localisation of a new type [3, 4], which is of purely dynamic (quantum) nature and thus fundamentally different from the 'thermodynamic' Wigner crystallisation. I shall also show that the macroscopic electronic state formed as a result of such localisation differs qualitatively from a Wigner crystal in respect of its thermodynamic and conducting properties.

The mechanism of the postulated 'dynamic' Coulomb self-localisation can be understood by considering first the possibility of formation of a Wigner crystal in a narrow-band conductor on the basis of the familiar Lindemann criterion. According to this general criterion, a crystal exists if the amplitude of its zero-point vibrations δr satisfies

$$\frac{\delta r^2}{\bar{r}} \leqslant 0.3 , \tag{3}$$

or in the case of a Wigner crystal

$$\frac{\delta r^2}{\bar{r}} = n^{1/6}\left(\frac{m_0}{m^*}\right)^{1/2} \leqslant 0.3 . \tag{4}$$

where $n = N/N_0 \propto \alpha_0^3/\bar{r}^3$ is the number of electrons (per atom) participating in the conduction process; N is the total number of electrons; N_0 is the number of sites in the crystal lattice; m^* is the effective mass of a conduction electron, which is related to the width t of an energy band by

$$m^* \propto \left(\frac{\hbar}{\alpha^0}\right)^2 t^{-1} \tag{5}$$

where m_0 is the mass of a free electron. Since in the case of a narrow-band conductor, we have $m^* \gg m_0$, the Lindemann criterion (4) and the essentially equivalent thermodynamic Wigner criterion

$$\Delta < u , \tag{6}$$

can be satisfied even at 'metallic' values of the electron density ($n \sim 1$). Hence, it follows that the long-range Coulomb interaction leads unavoidably, at a given density n, to the localisation of the whole ensemble of narrow-band electrons at sufficiently low temperatures. However, it is definitely not true that such a localised structure must be a Wigner crystal!

The reason for this is that the dynamics of a narrow-band electron is essentially discrete: it can be described by the tight-binding approximation, according to which an electron undergoes quantum jumps to adjacent sites in the crystal lattice and tunnels between equivalent atomic orbitals. In this situation a very important factor is the ratio of the width t of an energy band to a characteristic variation

$$\delta u \propto u\left(\frac{\alpha_0}{\bar{r}}\right)^2 \tag{7}$$

of the Coulomb energy of an electron in the course of its hopping between adjacent lattice sites. The role of the parameter $t/\delta u$ can be understood if we compare the amplitude δr of the zero-point vibrations in a Wigner crystal with the intersite distance α_0. It follows from expressions (4), (5), and (7) that

$$\frac{\delta r^2}{\alpha_0^2} \propto \left(\frac{t}{\delta u}\right) \propto n^{-1/2}\left(\frac{t}{\varepsilon_0}\right)^{1/2} \tag{8}$$

where $\varepsilon_0 = e^2/\alpha_0$ is a characteristic atomic energy. In the case of a narrow-band conductor we have $t \ll \varepsilon_0$, so that the parameter $t/\delta u$ and, therefore, the ratio $\delta r/\alpha_0$ are both ~ 1 even at a low electron density: $n \sim t/\varepsilon_0$. However, in accordance with the tight-binding approximation, the amplitude δr may be only of the order of or greater than the intersite distance α_0. This suggests that at electron densities

$$n \sim \frac{t}{\varepsilon_0} \ll \left(\frac{t}{\delta u} \sim 1\right) \tag{9}$$

a Wigner crystal should transform (most likely by an infinite series of second-order phase transitions) to a localised phase with a qualitatively different structure.

The nature of the new macroscopic state becomes clear if we consider the limit $t/\delta u \ll 1$. In this case an electron located at a given site of the crystal lattice cannot in general tunnel to any one of the adjacent sites, because the Coulomb fields created by the remaining electrons push apart the energy levels of the nearest orbitals by an amount $\sim \delta u$, which is considerably greater than the width of the energy band t. This means that if $t/\delta u \ll 1$, then the long-range forces of the mutual repulsion between electrons destroy completely the Bloch (current-carrying) states in a narrow band and that all electrons are localised at certain crystal lattice sites. More precisely, if $(1-n) \sim 1$, the stationary states $|\psi\rangle$ of the whole ensemble N of the narrow-band electrons represent, in the leading approximation in terms of the parameter $t/\delta u$, all possible products

$$|\psi\rangle = |\boldsymbol{r}_1\rangle|\boldsymbol{r}_2\rangle \ldots |\boldsymbol{r}_N\rangle \tag{10}$$

of N arbitrarily selected orbital electron states at sites $|\boldsymbol{r}_1\rangle, |\boldsymbol{r}_2\rangle, \ldots, |\boldsymbol{r}_N\rangle$ ($\boldsymbol{r}$ is the vector number of a site). Such a localised macroscopic state, which differs radically from a Wigner crystal, may be called a 'frozen electronic phase' (FEP). The characteristic features of an FEP can be seen most clearly in the limit $t/\delta u \ll 1$. The most important features are as follows.

(A) If the electron density n is fixed, then in the ground state of an FEP, we have

$$|\psi_0\rangle = |\boldsymbol{r}_1^0\rangle|\boldsymbol{r}_2^0\rangle \ldots |\boldsymbol{r}_N^0\rangle \tag{11}$$

and the sites $\boldsymbol{r}_i^0$ $(i = 1, 2, \ldots, N)$ occupied by electrons form generally a very disordered structure of the quasicrystal type, known as an 'electronic glass' [3, 4]. This disorder is a direct consequence of the fact that a periodic (translation-symmetric) electron configuration, which corresponds to the absolute minimum of the energy of the mutual Colomb repulsion of electrons, is incommensurate with the crystal lattice of a conductor in the general case of irrational vaues of n (in the thermodynamic limit when N, $N_0 \to \infty$).

(B) In view of the absence of continuous spatial degrees of freedom in an FEP, its elementary excitations are not phonons, as in a Wigner crystal, but transitions in two-level electron systems localised in regions of $\sim \alpha_0$ size and distributed at random over the whole configuration of an electronic glass. These two-level systems (like the familiar two-level systems of atomic glasses) form because of accidental degeneracy, which appears as follows. In view of the disorder of an electronic glass, its sites must include $\tilde{N} \sim (t/\delta u) N \ll N$ 'lability' sites $\boldsymbol{R}_\alpha$ $(\alpha = 1, 2, \ldots, \tilde{N})$, which are distinguished by the fact that a jump of an electron from site $\boldsymbol{R}_\alpha$ to one of the adjacent sites $\tilde{\boldsymbol{R}}_\alpha$ alters the Coulomb energy of the system by a small (compared with δu) amount δu_α, comparable with the band width t. The tunnelling lifts this degeneracy at the 'lability' sites and this gives rise to the formation of an electronic glass of $\tilde{N}$ two-level systems with the excitation energies

$$\omega_\alpha = [4t^2 + (\delta u_\alpha)^2]^{1/2} \quad (\alpha = 1, 2, \ldots, \tilde{N}). \tag{12}$$

Then, both the ground $|\psi_\alpha^-\rangle$ and excited $|\psi_\alpha^+\rangle$ states of two-level systems represent the following superpositions of orbitals at sites:

$$|\psi_\alpha^\pm\rangle = a_\alpha^\pm|\boldsymbol{B}_\alpha + b_\alpha^\pm|\tilde{\boldsymbol{R}}_\alpha\rangle, \tag{13}$$

where the absolute amplitudes $a_\alpha^\pm$ and $b_\alpha^\pm$ are comparable with unity.

(C) The thermodynamics of an FEP is very complex and its detailed description is outside the scope of this paper. It is important to stress that, in contrast to a Wigner crystal, an FEP is of purely dynamic origin and heating of this phase does not convert it into a Fermi liquid of free electrons: an FEP remains a localised structure without a long-range order.

The electronic glass modification of an FEP is infinitely degenerate, like a spin glass. In other words, there is an infinitely large number of stationary states described by expression (10) and the Coulomb energies of these states are exponentially close (at least in terms of the parameter $N^{1/2}$ to the Coulomb energy of the ground state, but the electron configurations $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N$ differ considerably from one another. A characteristic transition time of the electron system between these states is exponentially large: $\sim (\delta u/t)^N$. This means that an electronic glass should have a number of properties typical of a spin glass: an infinite spectrum of the relaxation times, nonergodic behaviour, and slow relaxation to a thermodynamic equilibrium state when an external perturbation is removed.

These properties of an FEP appear in full measure if the external field created by ions of doping elements is sufficiently homogeneous, namely when the characteristic change in the potential energy of an electron is such a field (experienced as a result of intersite hopping) is less than δu. This condition may be satisfied, for example, in layer (quasi-two-dimensional) conductors if donors or acceptors are located sufficiently far from conducting layers. It is very important to stress that in layer conductors where the distance between the layers L is much greater than the crystal lattice period α_0 it is relatively easy to satisfy along the layers one of the main conditions for the formation of an FEP: the electron – electron long-range interaction. This is because in a layer electron system the screening radius of the electron – electron interaction is always greater than or of the order of L for any (including 'metallic') values of the two-dimensional electron density. Consequently, the mutual Coulomb repulsion of two-dimensional electrons is of the long-range type if the average distance $\bar{r}$ between them is less than or of the order of L. This condition is easily satisfied.

A typical example of layer conductors with $L \gg \alpha_0$ are cuprates known to exhibit high-temperature superconductivity. Therefore, it is interesting to consider the possible role of an FEP as an important factor in the appearance of high-temperature superconductivity.

Let us now consider an *sd* system which consists of electron layers of two types: (a) s layers of light s electrons (with their mass of the order of the free-electron mass) which are in the Fermi-liquid state; (b) d layers of narrow-band d electrons. In the proposed superconductivity scenario of a layer *sd* system there are only Coulomb electron – electron interaction forces. These forces play a dual role: the mutual long-range repulsion between the d electrons is reponsible for the appearance of an FEP (electronic glass) and the Coulomb *sd* interaction ensures that the s electrons exchange (by a virtual mechanism) elementary excitations of an electronic glass (electronic glass excitations) which — as stated above — are transitions in electron two-level systems. Since such electronic glass excitations are Bose-like, this exchange unavoidably leads to an effective *ss* attraction with all the consequences that follow from it: the Cooper instability of the Fermi-liquid subsystem of the s electrons and its transition to a superconducting state at some critical temperature T_c.

A theoretical investigation of the superconductivity of a layer *sd* system [4] reduces to the following. If the $s-d$ hybridisation is ignored (the role of such hybridisation reduces simply to equalisation of the chemical potentials of the s and d layers), the Hamiltonian of such a system is

$$\hat{H} = \hat{H}^s + \sum_l (\hat{H}_l^{\text{eg}} + \hat{H}_l^{sd}), \tag{14}$$

where $\hat{H}^s$ is the Hamiltonian of the Fermi liquid of the s electrons; the index l labels the d layers; $\hat{H}_l^{\text{eg}}$ and $\hat{H}_l^{sd}$ are, respectively, the Hamiltonian of the electronic glass state of the lth layer and the Hamiltonian of the Coulomb interaction of the electronic glass with the s layers which are closest to the lth d layer. Both Hamiltonians $\hat{H}_l^{\text{eg}}$ and $\hat{H}_l^{sd}$ are expressed in terms of the creation (B_α^+) and annihilation (B_α) operators of excitations in the two-level systems in a given d layer, which satisfy the following mixed commutation rules

$$\{B_\alpha^+, B_\alpha\} = 1, \quad B_\alpha^2 = B_\alpha^{+^2} = 0,$$
$$[B_\alpha B_{\alpha'}] = [B_\alpha B_{\alpha'}^+] = [B_\alpha^+ B_{\alpha'}^+] = 0,$$

where, as above, the index α labels two-level systems; $\{\ldots\}$ represents an anticommutator; $[\ldots]$ is a commutator. Then $\hat{H}_l^{\text{eg}}$ is similar in its structure to the Hamiltonian of free phonons and $\hat{H}_l^{sd}$ resembles the Hamiltonian of the electron – phonon interaction:

$$\hat{H}^{\text{eg}} = \sum_\alpha \omega_\alpha B_\alpha^+ B_\alpha + \text{const}; \tag{15}$$

$$\hat{H}^{sd} = \frac{t}{2} \sum_{\alpha, \boldsymbol{r}_s} \omega_\alpha^{-1} (\tilde{\boldsymbol{R}}_\alpha - \boldsymbol{R}_\alpha) \nabla V_{sd}(\boldsymbol{R}_\alpha - \boldsymbol{r}_s) \hat{n}(\boldsymbol{r}_s) (\boldsymbol{B}_\alpha^+ + \boldsymbol{B}_\alpha). \tag{16}$$

Here, the index l is omitted for the sake of simplicity; $\hat{n}(\boldsymbol{r}_s)$ is the operator of the s-electron density at a site $\boldsymbol{r}_s$ in an s layer; $V_{sd}(\boldsymbol{r})$ is the screened Coulomb potential of the *sd* pair interaction; the summation over α extents to all the two-level systems in a given d layer; the summation over $\boldsymbol{r}_s$ applies to all the sites in the s layers adjoining a d layer.

The effective Hamiltonian of the *ss* interaction $\hat{H}_{\text{eff}}^{ss}$ can be found from perturbation theory in terms of the parameters $t/\delta u$ and α_0/L. The procedure is analogous to the familiar technique used in the study of the electron – phonon interaction in what is known as the weak coupling approximation. In this case the matrix element of a transition between pairs of states with oppositely directed momenta $\boldsymbol{p}$ is described by the following expression:

$$\langle \boldsymbol{p}, -\boldsymbol{p} | \hat{H}_{\text{eff}}^{ss} | \boldsymbol{p} + \boldsymbol{p}, -\boldsymbol{p} - \boldsymbol{q} \rangle$$
$$= 8t^2 q^2 \tilde{V}_{sd}^2(q) \int_0^{\varepsilon_{\max}} \frac{g(\varepsilon)\omega^{-1}(\varepsilon)}{\left[E(\boldsymbol{p}+\boldsymbol{q}) - E(\boldsymbol{p})\right]^2 - \omega^2(\varepsilon)} \, d\varepsilon, \tag{17}$$

where $\boldsymbol{q}$ is the transferred momentum: $\tilde{V}_{sd}(\boldsymbol{q})$ is the Fourier transform of the *sd*-interaction potential; $E(\boldsymbol{p})$ is the dispersion law of the s electrons; $\omega(\varepsilon)$ is the excitation energy of two-level systems given by formula (12) and corresponding to the Coulomb splitting $\delta u_\alpha = \varepsilon$; $g(\varepsilon)$ is the number density of two-level systems with the given value of ε, which vanishes when ε is equal to or greater than a certain maximum energy $\varepsilon_{\max} \sim \delta u$. Hence it is clear that the *ss* attraction occurs in that part of the momentum space where $[E(\boldsymbol{p}+\boldsymbol{q}) - E(\boldsymbol{p})]^2$ is less than or of the order of t^2.

The next step is the application of the canonical BCS scheme to the effective Hamiltonian $\hat{H}_{\text{eff}}^{ss}$. The resultant integral equation for the gap width (self-consistency condition) has a number of special features which are due to, on the one hand, the square-root singularity of the number density of two-level states with the limiting energy $\omega = 2t$ and, on the other, the smallness of the ratio α_0/L. This makes it necessary to modify considerably the traditional calculation technique. The final result, which gives the gap width $\Delta(\boldsymbol{p})$ on the Fermi surface at absolute zero $T = 0$ is

$$\Delta(\boldsymbol{p}) = \gamma t \exp\left[-\frac{\sqrt{\pi^2+1}-1}{\pi^2 \lambda(\boldsymbol{p})}\right], \quad \gamma \approx 7.5. \tag{18}$$

The dependence on the momentum p occurs only in the quantity

$$\lambda(p) = \frac{g(0) n_{\text{d}}}{4\pi\hbar v_{\text{F}}^2 v(\boldsymbol{p}) L^3} \sim \left(\frac{\alpha_0}{L}\right)^3 n_d^{-1/2} (\alpha_0 p_{\text{F}})^{-1}, \tag{19}$$

where $n_d(\alpha_0/\bar{r})^2$ is the number of electrons per unit cell in a d layer; v_{F} is the number density of the s electrons in an s layer on the Fermi surface; $v(\boldsymbol{p})$ is the velosity of an s electron at a given point on the Fermi surface; p_{F} is a characteristic Fermi momentum. Therefore, the gap reproduces directly the anisotropy of the distribution of the Fermi velocities and it increases on reduction in $v_{\boldsymbol{p}}$.

The critical temperature T_c is related to the maximum value of the gap, $\max \Delta(\boldsymbol{p})$, by the same simple relationship as the temperature and gap width in the isotropic BCS theory:

$$k_B T_c = C \max \Delta(\boldsymbol{p}), \qquad C = 0.57\,. \tag{20}$$

The distinguishing feature of the expression for the gap (17) is the independent of the parameter λ (and, therefore, of the argument of the exponential function) on the d-band width t, i.e. on the characteristic width of the attraction region. According to the estimate given by expression (19), the value of λ and, consequently, the gap width and T_c, all increase on reduction in the distance between the layers L and on reduction in n_d. However, the value of n_d is limited from below by the long-range condition: $\bar{r} < L$. The optimal situation obviously corresponds to $L \sim \alpha_0$ and $n_d \sim 1$. In this case the argument of the exponential function in expression (17) is comparable with unity and fairly high values of T_c are obtained even for relatively narrow bands. For example, the temperature $T_c \sim 10^2$ K corresponds to the d-band width $\sim 10^{-2}$ eV.

We thus reach the conclusion that the Coulomb electron – electron repulsion in a layer sd system induces a transition between the subsystem of the narrow-band d electrons to an FEP state (i.e. it suppresses the Fermi-liquid behaviour of the subsystem) and can by itself lead to superconductivity with a relatively high value of T_c in a wide range of the electron densities. It follows from the above discussion that the layer nature of the electron structure is needed only to ensure, at all electron densities, the long-range interaction (weak screening) of the Coulomb forces which results in quantum self-localisation ('freezing') of the d electrons.

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